Second Best Taxation and Regulation with Differentiated Land Quality

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Abstract

We derive rules for the optimal taxation of commodities and regulation of development when land is differentiated by access to commuting destinations, but not all revenue can be raised in a lump sum fashion. The welfare effects of policies that reduce land consumption, such as growth boundaries and gas taxes, are ambiguous even without traffic or environmental externalities. Labor taxes justify such policies, but when land profit taxes are not available, property taxes justify policies that distort land consumption upward. The optimal property tax rate increases with population and commuting costs.

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1 Introduction

This paper derives rules for the optimal taxation of commodities and regulation of development when land is differentiated by access to commuting destinations, but not all revenue can be raised in a lump sum fashion. We obtain three major results. First, time costs of travel and distortive labor taxes imply that land consumption generates a fiscal externality. The externality reflects a reduction in time costs of travel and associated cheapening of land brought on by the reduction of wages through income taxes that lead to increased travel and reduced labor supply. Second, when land rents must be taxed simultaneously with elastically supplied components of housing, the optimal composite tax rate typically increases in population and travel costs. Whether pro- or anti-"sprawl" policy is desirable, and more generally whether complements to land should be taxed or subsidized, thus hinges critically on the extent of labor taxation and on the time costs of travel. Our third result is that distortive property taxes create the possibility that other policy tools should be used to subsidize or mandate increased individual land consumption.

We establish these results by combining the Ramsey commodity tax framework familiar to public economists with the workhorse monocentric city model of urban economics. Our application of the monocentric city is notable for a transformation of the measure of distance from linear units to consumer location rank.

Given the large revenue shares of labor and property taxes, considerable subsidies to housing, and the large consumer expenditure shares on housing and transportation throughout the world, it seems natural that important interactions would arise between housing and tax policies. To our knowledge, however, land economics and optimal tax theory have an almost empty intersection. Whereas applications of commodity tax theory have focused on labor market distortions, the monocentric city policy analyses have focused primarily on housing, land use, and transportation.

The complementarity between labor taxes and land use constraints reflects a novel consideration: when commuting takes time, *ex ante* identical consumers have different time endowments, and hence different expenditure functions, depending on where they live. When individuals consume more land, they lengthen other consumers' commutes and hence increase others' marginal utility of leisure. Absent labor taxes, this external effect is pecuniary, and priced in the land market without violating the welfare theorems. With labor taxes, the externality becomes fiscal and can justify land market interventions.

This result amplifies an insight due to Corlett and Hague (1953): governments relying on distortive labor taxes should typically tax commodities complementary with leisure at higher rates than substitutes for leisure. Our result differs from the standard complementarity view of commodity taxation, however. Even if land and structures are no more complementary than other goods for leisure at the consumer level, the general equilibrium land market effect still arises. Commodities' complementarity with land demand thus generally affects tax policy. For example, gasoline taxes punish commuting and may be desirable for previously unexplored long run reasons.[1]

Our result that property taxes rise with the ratio of land rents to development and construction costs is not surprising. It has been known since as long ago as Ricardo (1821) that taxes on pure land rents are efficient, but that taxes on land development generate distortions. Mieskowski (1972) shows that a property tax is borne chiefly by land owners when the "share of land rents in total costs" is high. The result also reflects the importance of complementarities with sources of profits when profits are not directly taxed, identified by e.g. Dasgupta and Stiglitz (1971), Munk (1978, 1980), and Myles (1989). The use of taxes that distort land consumption as a back door to profit taxation generate a desire to subsidize the development of raw land at the urban fringe.

It might well be asked why distortive taxes should be considered in a discussion of optimal commodity taxation with a single consumer type and in the presence of land rents. Indeed, one of our results is that a combination of distortive land taxes and development subsidies achieve the first best.[2] With a single, homogeneous consumer, lump sum taxation is feasible. While our consumers are heterogeneous due to location, we prefer to justify the absence of lump sum taxes on the grounds that our model of a single consumer type is meant to highlight issues likely also to arise with multiple types. The latter analysis would be more complicated, and we briefly discuss natural extensions in that direction in the conclusion.

Absent direct individual lump sum taxes, we explicitly consider the possibility that profit taxes might be levied at a rate of 100%, but fail to raise needed revenue. We do not have strong views on whether a feasibly small lump sum tax on top of a full profit tax would typically satisfy government requirements. Some of our results do not specify the level of profit taxation and we

[1] West and Williams (2005) emphasize direct complementarity between gas and leisure, but do not consider the land market effects.

[2] Arnott and Petrova (2002) summarize and add to the literature on non-distortive land taxation.

simulate economies both with and without profit taxes.

The fact that effective real estate profit tax rates (e.g. gains on sale of farmland) are typically far below 100% suggests that evaluating distortive land taxes is a useful exercise. We thus consider cases where profit taxes are not available, allowing for the possibility that taxes can separate total land value from structure value. That is a weaker constraint on taxes than the restriction to common taxes on all components of property value that almost all jurisdictions appear to obey.[3] Adding structures to land rents and land development costs in its base affects the optimal level of property taxes, but not the comparative static results and demonstrated in simulations.

The paper proceeds as follows: Section 2 details the land, labor, and other commodity markets the government confronts when setting taxes. Section 3 describes optimal tax rules under different constraints and also describes conditions under which constraints on land development are welfare increasing or decreasing. Such "anti-sprawl" constraints are often blamed for high housing prices, but to our knowledge, their welfare properties have not been considered in a world with other taxes. Section 4 provides some parsimonious calibrations that illustrate some unproven conjectures by example, and also illustrate the importance of land markets to optimal tax policy. A final section summarizes, discusses the allocation of property tax burden and growth controls across the US in the context of our findings, and discusses how relaxations of some of our simplifying assumptions might affect our results.

2 Formal Model

In this section we present a formal model of a linear monocentric city and the problem of a government seeking to minimize the utility loss to consumers from raising a fixed revenue requirement. We begin with the market setup, next describe the government's problem, and then detail a useful transformation of the model. Notation is detailed in Table 1.

2.1 Setup of the model

We use the monocentric city model to capture three crucial and readily observed features of real world land markets: First, there is variation in land quality and hence land price. Second, this

[3] Pittsburgh, PA, has employed a land tax, as have some Commonwealth countries (Andelson (2000) surveys historical use of land taxes).

variation provides profits to owners of better than marginal quality land. Third, much of the variation in land quality comes from proximity to commuters' destinations.

The starting point of our model is a continuum of *ex-ante* identical consumers of measure N. After the government sets tax rates, each consumer maximizes his or her utility by choosing a consumption bundle of goods, labor supply, a location r where they live, and land consumption at r. We index locations by distance r from the city center and assume that everyone has to commute to the (spaceless) city center each day. Our model would not require meaningful modification if there were multiple commuting destinations, as long as these locations were fixed. We assume round trip travel from location r costs $\rho_m r$ in terms of the numeraire and $\rho_t r$ in units of time lost to commuting.[4]

We assume that every location has one unit of land. This means that the geometry of the "city" is a band of width 1 and endogenous length b. While a linear city is only an approximation, we will show that it greatly simplifies the analysis, and we see no reason why the nature of our results would change with different geometry.

Land is owned and developed by competitive firms whose shares are equally divided among the N consumers, independent of consumers' location choice. Developing a unit of land anywhere costs a developer c_l units of numeraire. Non-residential land is useless.

We assume zero profits in all industries other than land development. Furthermore we assume that production technology that is linear in labor and then, without loss of generality, normalize the gross wage to one.

The government has a budget requirement that needs to be satisfied with taxation of a varying subset of land profits, land consumption, goods consumption or labor income.

An equilibrium in our model is defined as a situation where:

1. Goods and labor markets clear
2. Land markets clear at every location
3. Each individual attains the same utility level
4. Land is developed up to the distance b from the center at which the producer price $p_l(b) = c_l$.

[4]If time spent travelling is at all enjoyable, ρ_t is less than the time spent commuting.

5. Government meets its budget requirement

The land market clearing and equal utility level of each individual define the particular features of the urban model. The key variables for land market clearing are the land price at each location and the distance of the development boundary b from the center. Prices ensure equal utility across locations and the boundary distance satisfies zero profits on the margin (absent government regulation).

Table 1: Notation

Symbol	Interpretation
$p_l(r)$ or $p_l(n)$	Land price at location r or for the consumer living nth most distant from the border b
p_i	Producer price of good i, $p_i = 1$ for goods other than land, structures
c_l	Constant cost per unit of land developed
$c_s = p_s$	Fixed cost per unit of structures
τ_π	Tax on profits, exogenously fixed
$\tau_i,\ i \neq \pi$	Tax on commodity i
P	Indicator for whether the tax rate on land also applies to structures.
$q_i = (1+\tau_i)p_i$	Consumer price of good i
z (or x_z)	Labor supply
l	Land consumption
s	Structure consumption
r	Distance from "downtown" location 0
b	Maximal distance where land is developed
$f(r)$	Population frequency at location r
λ_G	Shadow value of an additional dollar of government revenue
λ_π	Shadow value of an additional dollar of land development profits
$x_i(r)$	consumption of good i at location r.
η_{ij}	Compensated demand elasticity for good i wrt. consumer price q_j.
$\rho_t(\rho_m)$	Cost in money (time) of travelling one unit of distance.
$\hat{\rho}$	$\frac{\rho_m+\rho_t(1-\tau_z)}{1+\tau_l}$ Slope of producer land price in consumer space.

In the beginning of this section we index demands (and labor supply) by location (e.g. $l(r)$) and later by cumulative population up to that location, if at all. These are notational shortcuts for the fact that these realized values of demand are products of consumers' maximization and hence functions of all prices and the reference utility level.

2.2 Government's Optimal Tax Problem

The government's optimal tax problem can be stated as:

$$\min_{\{\tau,\Pi,f(r),b\}} e(1-\tau_z,(1+\tau_l)p_l(0),c_s(1+P\tau_l),(1+\tau_1),\ldots,(1+\tau_K);u)-\Pi/N \quad (1)$$

subject to

$$\forall r\le b:\ \frac{dp_l}{dr}=-\frac{\rho_m+\rho_t(1+\tau_z)}{l(r)(1+\tau_l)} \quad (2)$$

$$G(N)=\int_0^b\left((\tau_l+\tau_\pi)(p(r)-c_l)+\sum_{i\neq l}\tau_i x_i p_i f(r)+\tau_l c_l\right)dr \quad (3)$$

$$\Pi=(1-\tau_\Pi)\int_0^b(p_l(r)(1-\tau_\pi)-c_l)dr \quad (4)$$

$$p_l(b)=c_l \quad (5)$$

$$\forall r\le b:\ f(r)l(r)=1 \quad (6)$$

$$N=\int_0^b f(r)dr \quad (7)$$

$$\tau_\pi\le T_\pi \quad (8)$$

The objective (1) is to minimize the expenditures net of profits for someone living in the center of the city (no travel cost) to attain a utility level u.[5] Constraint (2) determines the evolution of the producer price of locations so that each location provides equal utility to consumers (for derivation, see e.g. Fujita (1989)). Equation (3) is the government's budget constraint, equation (4) is the accounting equation for land profits, equation (5) is the zero profit condition on the marginal land, equation (6) is the point-wise land market clearing condition and equation (7) is the overall population constraint. Equation (8) constrains profit taxes from exceeding 100% and allows institutional constraints. We specify government revenues to be linear in N in simulations presented below.

[5] This is the dual problem to maximizing the utility of the consumer at $r=0$ subject to their budget constraint. This is equivalent to maximizing utility everywhere by the equal utility constraint. All the analysis in the paper is done using utility compensated demands. This leads to a much more economical presentation of the government's problem, see Diamond and McFadden (1974).

2.3 Transformation of the Optimal Tax Problem

The problem as stated in equations (1) through (7) is difficult to analyze, so we transform this problem by a change of variable in integration. We choose as the variable of integration $n(r)$, the total population from the border of the city up to point r instead of the raw location variable r. Hence $n(b) = 0$ and $n(0) = N$.

A key to the power of this transformation is proved in the Appendix: that with only money costs of travel and conditional on consumer prices, the expenditure function does not change with location. The introduction of time cost of travel will change this conclusion with respect to labor supply, but in a very simple way: the "gross labor supply" which sums time working and time spent commuting is independent of location, so that labor supply z (net of travel costs) varies linearly with distance, conditional on prices. Other consumer choices are fixed across locations, conditional on prices. Because rents vary by location, consumers at different locations make different choices. Concretely, denoting the expenditure function for the nth most distant consumer from the border by e^n, we have:

$$e^n(1-\tau_z, q_l(n), \dots; u) = e^N(1-\tau_z, q_l(n), \dots; u) + (1-\tau_z)\rho_t r(n). \tag{9}$$

Note that in the equilibrium the location of individual in position n is

$$r(n) = \int_0^{r(n)} 1 d\nu = \int_n^N l(v) dv. \tag{10}$$

When there are time costs of travel, holding rents constant, expenditures relative to a consumer with no travel requirement are raised by exactly the opportunity cost of time multiplied by a consumers' distance from the center. The change in rents exactly compensates for this lump sum time cost plus the monetary costs of travel.

Equation (9) implies that labor supply for any consumer n is given by

$$z(n) = z(1-\tau_z, q_l(n), \dots; u) - \rho_t \int_n^N l(\dots, q_l(n), \dots) dv, \tag{11}$$

where $z(\cdot)$ is a function common to all consumers. We will generally drop the arguments of z

and other consumer demands. We can write the total derivative (including general equilibrium labor market effects) of consumer i's labor supply with respect to the price of good i as: $\frac{\partial z}{\partial q_i} + \rho_t \int_n^N \frac{\partial l}{\partial q_i} dv$. By Slutsky symmetry of the expenditure function e^N and the fact that $z(1-\tau_z)$ enters the expenditure function with a negative sign, this derivative can be rewritten: $-\frac{\partial x_i}{\partial(1-\tau_z)} + \rho_t \int_n^N \frac{\partial l}{\partial q_i} dv$. For notational convenience, we write the sum of individual consumers' elasticities of good i with respect to the price of all other non-labor goods plus the negative elasticity with respect to wages as: $\sum_j \hat{\eta}_{ij}$, with $\hat{\eta_{iz}} \equiv -\frac{\partial x_i}{\partial(1-\tau_z)} \frac{1-\tau_z}{x_i}$.

The size of the city is:

$$b = \int_0^b 1 dr = \int_0^N l(n) dn. \tag{12}$$

The Jacobian term for our transformation from physical distance r to consumer distance rank n is $\frac{-1}{f(r)}$ which is equal to $-l(r)$ by the point-wise land-market clearing. Here the linearity of the city plays a crucial role, since we don't have to keep track of the (constant) amount of land available in each location. Combining this with equations (2) and (5), and introducing the notation $\hat{\rho}$ we obtain the result that:

$$p_l(n) = c_l + \frac{\rho_m + \rho_t(1-\tau_z)}{1+\tau_l} n \equiv c_l + \hat{\rho} n \tag{13}$$

Equation (13) demonstrates that the producer price paid by a given consumer depends only on the rank of the consumers' location from the border, relative to other consumers' locations. It follows directly that the consumer price for land is:

$$q_l(n) = (1+\tau_l)c_l + (1+\tau_l)\hat{\rho} n. \tag{14}$$

Dropping the arguments of consumer demands, the government's problem can now be written:

$$\begin{aligned} \min_{\{\tau,\Pi\}} V = {} & e(1-\tau_z, (1+\tau_l)\left(c_l + \hat{\rho} N\right), (1+\tau_1), \ldots, (1+\tau_K); u) - \Pi/N \\ & + \lambda_g \int_0^N \left[l\left((\tau_l + \tau_\pi)\hat{\rho} n + \tau_l c_l\right) + \sum_{i \neq l} \tau_i x_i - \tau_z \rho_t \int_n^N l dv \right] dn - G(N) \\ & + \lambda_\pi \left(\int_0^N \hat{\rho}(1-\tau_\pi) n l dn - \Pi \right) + \mu \left(\tau_\pi - T_\pi\right). \end{aligned} \tag{15}$$

Some results assume that V is convex in all policy parameters, and thus has a unique optimum.

3 Optimal Taxation and Regulation

3.1 Optimality of Profit Taxes

In the absence of a government revenue requirement, it can be shown (e.g. Fujita (1989)), that the first and second welfare theorems hold. In that case, taxes that support a competitive equilibrium are preferred to distortive taxes. Thus, the first best can be obtained by a profit tax as long as the revenue requirement G is less than profits.

That profit taxes avoid distortions can be seen mathematically by taking the derivatives of the objective function (15) with respect to the tax rate τ_π and with respect to profits Π when τ_π is unconstrained so $\mu = 0$. We find that $\lambda_g = -\lambda_\pi, = -\frac{1}{N}$. Thus a dollar of profits to be distributed lump sum among the N residents has the same value as a dollar given to the government to pay down the revenue obligation.

Before proceeding with the analysis of optimal taxation when τ_π is bounded above, two auxiliary results are useful. The first result is that if τ_π is bounded above, so that $\mu > 0$, then $\lambda_g < \lambda_\pi$. In these cases, a dollar reduction of the government budget constraint is more helpful than a dollar distributed lump sum to consumers. The first order condition for profits continues to imply $\lambda_\pi = -\frac{1}{N}$.

A second result is that even if land taxes fail to separate costs of land development from locational rents, government quantity setting policy can undo the distortion.

Result 1. *If the government must set $\tau_\pi = 0$, but can impose a flat tax or subsidy per acre on development or require developers to build up to an arbitrary distance from the center, then the optimal combination of a land tax and development subsidy or requirement generates no deadweight loss.*

Proof. With a land tax in place and only land taxes, the consumer price of land $q_l(n)$ is $c_l(1+\tau_l)+n(\rho_t+\rho_m)$. Consumers pay the portion of the tax assessed on development costs and firms pay the tax on profits. Absent a government quantity control, land is developed up to the point where the development cost c_l equals consumer willingness to pay.

Suppose now that the government mandates that land be developed to a point where consumers would be willing instead to pay $(c_l + k)(1 + \tau_l)$, so that the producer price paid to the firm by any consumer n becomes $c_l + k + n\frac{\rho_t+\rho_m}{1+\tau_l}$. If the government sets $k = -\frac{c_l\tau_l}{1+\tau_l}$, then consumer land prices are restored to their level in a non-taxed (or profit tax) equilibrium, $c_l + n(\rho_t + \rho_m)$. Hence consumer behavior is unaffected by the land tax.

The tax revenue raised under this regime is:

$$\Delta G = \tau_l \int_0^N p(n)l(n)dn = \int_0^N \frac{\tau_l\,(c+\rho n)}{1+\tau_l} l(n)dn.$$

The change in profit redistributed to consumers is:

$$\Delta\Pi = \int_0^N \left(\frac{c+\rho n}{1+\tau_l} - (c+\rho n)\right) l(n)dn = -\int_0^N \frac{\tau_l\,(c+\rho n)}{1+\tau_l} l(n)dn \tag{16}$$

Thus $\Delta G = \Delta\Pi$. With no change in consumer prices and government revenues equal to lost profit income, we see that a lump sum profit tax has been achieved. □

Because the government requires development beyond the location at which profit maximizing developers would build, we have the result that with distortive land taxes, land use controls that limit the extent of "urban sprawl" would induce further deadweight loss. This will be verified mathematically below, in section 3.3. A further logical step suggests that if the net burden on property is negative (e.g. through housing subsidies), then land use controls may be desirable, even without environmental concerns, because they undo a distortion that causes excess land consumption. In the remainder of the paper, we show that the validity of these intuitions depends primarily on the way in which land demand interacts with the rest of the tax base.

A universal unit subsidy given to developers for each acre developed in the amount k also supports a first best, with the land tax set so that revenues are $G + k\int_0^N l(n)dn$. Fees per home, as opposed to per acre, are common practice, but have more complicated effects on density.

3.2 Welfare Effects of Land and Property Taxes

We now consider a government that cannot raise $G(N)$ through taxes on land profits or lump sum taxes. This could be because $\Pi < G(N)$ or because the government is unable both fully to control land development (e.g. state and federal governments in the US) and to distinguish land development costs from land rents, so that $\tau_\pi = 0$. We wish first to inquire whether a positive tax rate on residential land is desirable. We will consider the possibility that taxes on residential land must also be assessed at the same rate on housing structures, as they almost always are. To identify that case, we will define the variable P as follows:

$$P = \begin{cases} 1 \text{ if } \tau_l \text{ is assessed on land and structures} \\ 0 \text{ if only assessed on land} \end{cases}$$

The derivative of the objective (15) with respect to a common tax at rate τ_l on both land and structures is:

$$\begin{aligned}\frac{\partial V}{\partial \tau_l} &= l(N)c_l + Ps(N)c_s \\ &+ \lambda_g \int_0^N \left(l(n)\left(\hat{\rho} n + c_l\right) + Pc_s s(n) + \tau_l c_l \left(Pc_s \frac{\partial l}{\partial q_s} + c_l \frac{\partial l}{\partial q_l} \right) + P\tau_l c_s \left(c_l \frac{\partial s}{\partial q_l} + c_s \frac{\partial s}{\partial q_s} \right) \right) dn \\ &- \lambda_g \int_0^N \tau_z \rho_t \int_n^N \left(Pc_s \frac{\partial l(v)}{\partial q_s} + c_l \frac{\partial l(v)}{\partial q_l} \right) dv dn \\ &+ \left(\lambda_\pi(1-\tau_\pi) + (\tau_l + \tau_\pi)\lambda_g\right) \int_0^N n\hat{\rho} \left(Pc_s \frac{\partial l}{\partial q_s} + c_l \frac{\partial l}{\partial q_l} - \frac{1}{1+\tau_l} \right) dn \\ &- \int_0^N \lambda_g \sum_i \tau_i p_i \left(P \frac{\partial x_i}{\partial q_s} c_s + \frac{\partial x_i}{\partial q_l} c_l \right) dn. \end{aligned} \quad (17)$$

Grouping terms and defining η_{ij} as the elasticity of demand for good i with respect to good j, we have:

$$\begin{aligned}\frac{\partial V}{\partial \tau_l} &= \left(l(N) + \lambda_g \int_0^N l(n)dn \right) c_l + P \left(s(N) + \lambda_g \int_0^N s(n)dn \right) c_s \\ &+ \int_0^N \lambda_g \left(\tau_l c_l l \left(P \frac{\eta_{ls}}{1+\tau_l} + \frac{c_l}{q_l}\eta_{ll} \right) + P\tau_l c_s s \left(\frac{\eta_{ss}}{1+\tau_l} + \eta_{sl}\frac{c_l}{q_l} \right) \right) dn \\ &+ \int_0^N \lambda_g \sum_{i \neq l, Ps} \tau_i p_i \left(Psc_s \hat{\eta}_{si} + c_l l \hat{\eta}_{li} \right) dn \\ &+ \int_0^N l(n) n\hat{\rho} \left(\lambda_g + \left(\lambda_g(\tau_l + \tau_\pi) + \lambda_\pi(1-\tau_\pi)\right) \left(P\frac{\eta_{ls}}{1+\tau_l} + \eta_{ll}\frac{c_l}{q_l} - \frac{1}{1+\tau_l} \right) \right) dn \\ &- \lambda_g \int_0^N \int_n^N \tau_z \rho_t l(v) \left(P\frac{\eta_{ls}}{1+\tau_l} + \eta_{ll}\frac{c_l}{q_l} \right) dv dn. \end{aligned} \quad (18)$$

The first three lines of equation (18) reflect standard commodity tax tradeoffs. Revenue is raised at the expense of consumers and tax revenue from land and complementary goods is reduced. There are also changes in government revenues from other commodity taxes induced by demand changes resulting from the changes in consumer prices for land and structures.

The last two lines of equation (18) reflect the existence of land rents and the fact that an individual's land consumption exerts a negative influence on labor supply for consumers living farther from the center. These considerations give rise to the following:

Result 2. *If $P\eta_{ls}$ and η_{ll} are sufficiently small in magnitude, and if all other elasticities are bounded, then a positive property tax (land tax if $P = 0$) is better than no property tax for sufficiently large $N\left(\rho_m + \rho_t(1-\tau_z)\right)$ when the government must rely on distortive taxes.*

Proof. The assumptions guarantee that $\frac{\int_0^N l(n)\hat{\rho}n dn}{\int_0^N (c_l l + c_s s) dn}$ becomes arbitrarily large and that the term $P\eta_{ls} + \eta_{ll}\frac{c_l}{q_l}$ becomes arbitrarily small so it is sufficient to show that the term multiplying $l(n)n\hat{\rho}$ in equation (18) is strictly negative. Under the assumptions, this term is equal to $\lambda_g + (1+\epsilon)(\lambda_g\tau_\pi - \lambda_\pi(1-\tau_\pi)$, where ϵ is arbitrarily close to zero. Rearranging terms, this expression is equal to

$$(\lambda_g - \lambda_\pi)(1+\tau_\pi) + \epsilon\left(\lambda_g\tau_\pi - \lambda_\pi(1-\tau_\pi)\right).$$

Reliance on distortive taxes implies $\lambda_g < \lambda_\pi$. Since the inequality is strict and ϵ is arbitrarily close to zero, the result is proved. □

Result 3. *For some a and b sufficiently negative, if for all n, $P\eta_{ls} < a$ and $\eta_{ll} < b$, then from a starting point of only labor taxes, for ρ_t sufficiently large, a positive property tax (or land tax if $P = 0$) is better than no property tax.*

Proof. With sufficiently negative $P\eta_{ls}$ and η_{ll}, as $\rho_t \to \infty$, l, $l\hat{\rho}n$ and Ps approach zero. Hence, only the last line of equation (18) must be considered. By the assumptions and negativity of λ_g, the last line is negative. □

The discussion above and inspection of the first order condition (18) suggest the following pair of conjectures:

Conjecture 1. *1. If the profit tax rate is sufficiently small and the sum of the elasticities of demand for land with respect to its own price and the price of structures is less than one in absolute value, then the optimal property tax rate rises with the fraction* $\frac{N(\rho_m+\rho_t)}{c_l+c_s}$.

2. Property taxes are more desirable when the travel cost of time is larger and the cross partials of demand for labor with respect to land and structure prices more positive.

The conjectures would be *bona fide* results if λ_g could be held constant while changing parameters.[6] We verify the conjectures in several numerical examples.

Summarizing, inelastic land demand makes property an attractive tax target, as do a high ratio of profits to costs in land development. However, even if land demand is elastic, if time costs of travel are high, a property tax can also be attractive independent of the profit effect. The mechanism is that reducing land consumption reduces travel times and hence the average marginal utility of leisure. This increases labor supply, undoing the distortion induced by a wage tax. Symmetrically, the labor tax distorts time costs of travel downward, thereby inducing excessive land demand by flattening the land price gradient. This calls for corrective land taxation.

3.3 Growth Controls and Other "Anti-Sprawl" Measures

A positive tax rate on housing induces a distortion to demand for land. This effect is negative even if structures are taxed at the same rate as land, assuming that structures and land are not strong Hicksian complements. This is a weak assumption, given that estimates of Marshallian demand suggest roughly offsetting income and substitution effects.[7]

The fact that a pure land tax can be rendered lump sum by encouraging growth past the equilibrium boundary suggests that worsening the property tax distortion by constraining growth is a bad idea. If taxes net subsidize housing, the same consideration suggests that growth controls would be advisable, ignoring the rest of the tax base.

Consideration of the rest of the tax base suggests an additional force that should guide land

[6] This conjecture and other conjectures could be stated as a theorems with large amounts of restrictive assumptions in the following fashion: "Assume that we are comparing two economies that are otherwise similar, have the same shadow cost of government's revenue and all (other) demand elasticities that appear in the relevant first order conditions. Then the change in equilibrium utility of relaxing a zero property tax constraint in the economy with (e.g.) higher travel cost of time is larger." We refer to these results loosely as conjectures since none of the assumptions needed to prove them in a theorem form (e.g. zero property taxes) is a necessary assumption.

[7] See, e.g. Thorsnes (1997).

use decisions: distorting land demand downward increases labor supply when there are positive time costs of travel. Alternatively, an income tax distorts time costs of travel downward, thereby adding to aggregate travel and disorting land rents downward, so that increasing land costs may be a helpful correction. We consider correction of travel costs below.

We first consider the effect of a government policy that forces developers to act as if the cost of development is equal to $c_l + k$, rather than c_l. We start with the simplest case of an "unfunded mandate" placed on developers. This will change the producer price at n to $c_l + k + n\frac{\hat{\rho}}{1+\tau_l}$, with the consumer price multiplied by $1 + \tau_l$. An alternative policy would be to actually charge k per unit of land developed. This would have a welfare effect that we ignore: transferring $k \int_0^N l dn$ from profits to the government.

Adding these changes to consumer and producer prices (and hence profits and revenues), from equation (15), we obtain the first order condition for k:

$$\frac{\partial V}{\partial k} = (1+\tau_l)l(N) + \int_0^N (\lambda_g(\tau_l + \tau_\pi) + \lambda_\pi(1-\tau_\pi)) \left(l(n) + (1+\tau_l)\left(k + n\hat{\rho}\right) \frac{\partial l}{\partial q_l} \right) dn$$
$$+ \lambda_g \int_0^N (1+\tau_l) \left(\tau_l c_l \frac{\partial l}{\partial q_l} + \sum_{i \neq l} \tau_i p_i \frac{\partial x_i}{\partial q_l} - \tau_z \rho_t \int_n^N \frac{\partial l}{\partial q_l} dv \right) dn. \quad (19)$$

The following result of integrating by parts will prove useful:

$$\int_0^N l(n)dn = Nl(N) - \int_0^N nl(n)\frac{\partial l}{\partial n}dn = Nl(N) - \int_0^N \frac{\partial l}{\partial q_l}\frac{\partial q_l}{\partial n} ndn. \quad (20)$$

Combining equations (19) and (20) with the fact that $\frac{\partial q_l}{\partial n} = (1+\tau_l)\hat{\rho}$, we obtain:

$$\frac{\partial V}{\partial k} = l(N)\left(1 + \tau_l + N(\lambda_g(\tau_l + \tau_\pi) + \lambda_\pi(1-\tau_\pi))\right)$$
$$+ (1+\tau_l) \int_0^N \left(\left((\lambda_\pi + \lambda_g(\tau_l + \tau_\pi))\, k + \lambda_g \tau_l c_l \right) \frac{\partial l}{\partial q_l} \right) dn$$
$$+ \lambda_g (1+\tau_l) \int_0^N \left(\sum_{i \neq l} \tau_i p_i \frac{\partial x_i}{\partial q_l} - \tau_z \rho_t \int_n^N \frac{\partial l}{\partial q_l} dv \right) dn. \quad (21)$$

Using Slutsky symmetry and elasticities, and observing that $\lambda_\pi = \frac{-1}{N}$, from a starting point of

$k = 0$, (21) can be written:

$$\frac{\partial V}{\partial k} l(N)^{-1} = (N\lambda_g + 1)(\tau_l + \tau_\pi) + \lambda_g(1+\tau_l)\tau_l c_l \int_0^N \eta_{ll} \frac{l(n)}{l(N)} dn$$
$$+ \lambda_g(1+\tau_l) \int_0^N \left(\sum_{i \neq l} \frac{\tau_i}{p_i} \hat{\eta}_{li} \frac{l(n)}{l(N)} - \int_n^N \tau_z \rho_t \eta_{ll} \frac{l(v)}{l(N)} dv \right) dn \quad (22)$$

The first line of equation (22) reflects the direct land market welfare and revenue effects of increasing prices at each location. The sign of these summed effects depends on the own price elasticity of demand for land. If η_{ll} is small in magnitude, land consumption everywhere will be roughly identical and the last integral will be small. Since $\lambda_g < \frac{-1}{N}$, this would imply that the first line is negative. If η_{ll} is large in magnitude, the ratios $\frac{l(n)}{l(N)}$ are also large, and the last integral will dominate, so that the first line may be positive.

The second line of equation (22) reflects the change in revenues from non-land taxes induced by increasing land prices. Assuming land is a substitute for other taxed goods, these effects become more positive as the own price elasticity of land (and hence the ratios $\frac{l(n)}{l(N)}$) increases in magnitude.

This analysis yields the following results and conjectures:

Result 4. *From a starting point of only labor taxes and with $\rho_t > 0$, if the elasticity of land demand with respect to the wage rate $\frac{\partial l}{\partial(1-\tau_z)} \frac{1-\tau_z}{l}$ is bounded above by a sufficiently small number , then $\frac{\partial V}{\partial k} < 0$ from a starting point of $k = 0$. Hence a small growth boundary is better than no fee or boundary.*

Proof. When $k = 0$ and there are only labor taxes, the expression (22) reduces to its second line. Because the cross partial η_{lz} (which enters negatively) is assumed close to zero and because η_{ll} must be strictly negative for any well defined expenditure function, the second line is strictly negative. □

Result 5. *If all revenue is raised by a land tax, then the optimal value of k is $-\frac{\tau_l c_l}{1+\tau_l}$.*

Proof. Because the cost of raising a dollar of revenue is equivalent to the shadow value of profits (Result 1), the first line in equation (21) vanishes. The second line also vanishes with the assumed level of k. The last line with no non-land taxes. □

If all parameters could be held constant, the following would also be results:

Conjecture 2. *All else equal, quantity constraints on land development have more favorable welfare effects when ρ_t rises.*

3.4 Transportation Taxes

If the only travel is to and from work, then a tax on miles travelled (approximately a gas tax), is equivalent to an increase in ρ_t that also earns revenue on each unit of distance travelled by each consumer. Label this increase τ_t, and differentiate expression (15) to find:

$$\frac{\partial V}{\partial \tau_t} = Nl(N) + \int_0^N n\lambda_g \left(\sum_{i \neq l} \tau_i q_i \frac{\partial x_i}{\partial q_l} + \tau_l c_l \frac{\partial l}{\partial q_l} + \int_n^N \left(l + (\tau_t - \tau_z \rho_t) v \frac{\partial l}{\partial q_l} \right) dv \right) dn$$
$$+ \int_0^N n \left(\lambda_g(\tau_l + \tau_\pi) + \lambda_\pi(1 - \tau_\pi) \right) \left(\frac{l}{1 + \tau_l} + \frac{\partial l}{\partial q_l} \frac{\hat{\rho} + \tau_t}{1 + \tau_l} \right) dn. \quad (23)$$

The analysis is very close to that for growth controls, with much of the derivative of the objective pointwise multiplied by $\frac{n}{1+\tau_l}$. Hence the transportation tax has greatest effect near the center and least effect near the border. This should have a greater effect on labor supply than an acreage tax per dollar of revenue, but have less effect on reducing the size of the region. By the same logic as above, we have:

Result 6. *From a starting point of only labor taxes, a small transportation tax is welfare increasing if the elasticity of land demand with respect to wages is bounded above by a number that is sufficiently small and the own price elasticity of demand for land is always less than one in absolute value.*

Proof. Under the elasticity condition, the second line of equation (23) is strictly negative. With only labor taxes, the first line of equation (23) reduces to

$$l(N) \left(N + \int_0^N n\lambda_g \left(-\frac{l(n)}{l(N)} \eta_{lz} + \int_n^N \frac{l(v)}{l(N)} \left(1 - \tau_z \frac{\rho_t v}{q_l} \eta_{ll} \right) dv \right) dn \right).$$

For small η_{lz}, the result holds for any maximal $\eta_{ll} > -1$ since the last inequality guarantees that the ratios $\frac{l(v)}{l(N)}$ all exceed one. The term multiplying these ratios in the last integrand is greater than one by negativity of η_{ll}. Since $\lambda_g < \frac{-1}{N}$, $\int_0^N n \int_n^N \lambda_g dv dn < -N$. This proves the result. □

Conjecture 3. *Optimal transportation taxes rise with ρ_t.*

3.5 Taxes on Other Goods

The presence of land markets introduces considerations previously missed in the optimal tax literature, namely general equilibrium effects on profits, land tax revenues, and on labor supply through commuting, by way of land consumption. The first two considerations argue that complements to land consumption should be subsidized, to undo the distortions to land consumption from land or property taxes. The second consideration suggests that complements to land consumption should be taxed, to encourage labor supply. This leads to the following:

Conjecture 4. *Commodities that are substitutes for land should be relatively heavily taxed when travel time costs are low relative to money costs and when land is lightly taxed or subsidized. Complements should be relatively heavily taxed under the opposite conditions.*

4 Numerical Examples

In this section we describe results from simulations with several different parameterizations of our model. These simulations illustrate the theorems and verify the conjectures. While calibration is challenging, we aim to present a plausible range of optimal policy parameters, observe comparative statics of these parameters, and to understand the relative magnitudes of welfare losses associated with different failures to follow optimal policy.

Across four urban scenarios, we vary a welfare maximizing government's access to different policy tools: profit taxes, pure land taxes, property taxes, transportation taxes, and land development quantity controls. We then calculate certain equilibrium statistics and optimal policy choices, described below. We also provide estimates of the deadweight loss from taxation under each regime. Deadweight loss is measured as the compensation in numeraire required to provide equal utility to consumers in a given policy environment to consumers in a world with no taxation, less the government's revenue need. The average cost "AC" of taxation is computed as the total compensation divided by the revenue need.[8]

[8] To attain the same utility level in a world with revenue raised by distortive taxes as in a world with no revenue needs requires compensation equal to the revenue need plus deadweight loss. This compensation gives the consumer lump sum income, and this creates an incentive for the government to tax commodities other than labor. To eliminate this artificial consideration from the analysis, we endow consumers with minus the revenue requirement in the original simulation. This leaves a lump sum endowment equal to deadweight loss in each simulation, but the distortion on optimal taxes from a world with no lump sum income other than profits is small, because the deadweight loss turns out to be small.

We simulate four scenarios by varying both the population size and the composition of travel cost. Result 2 and the conjectures show the importance of these considerations for optimal policy. In population size we consider a small city (population $N = 1$) and a big city ($N = 1,000$). The ratio of land producer prices in the center of the city to the producer cost of development (set to one unit) suggest comparison to cities with population 10,000 and 1,000,000.[9] Our range of urban ratios thus encompasses the range from rural to moderately large metropolitan areas. For each of the two city sizes, we consider two transportation cost scenarios. In the first scenario, monetary costs of travel are five times larger than time costs in the absence of taxes. The second scenario inverts this ratio.[10]

Consumers' preferences are characterized by a five good translog cost function. Our baseline case is a Cobb-Douglas utility function (a special case of the translog), with budget (net of commute costs) shares (α's) of 40%, 20%, 10%, 10% and 20% for leisure, numeraire, land, structures and other good consumption respectively.[11] Each consumer is endowed with one unit of leisure (so the maximum labor supply is 1). The cost of developing land and building structures is normalized to one. The government's revenue requirement is set at .2 units of numeraire per capita. This turns out to be close to 35% of the total labor production in almost all our simulations. In line with US averages reported in the *Consumer Expenditure Survey*, aggregate spending on commuting never exceeds 10% of all spending.

We employ a Cobb-Douglas baseline specification of expenditures because of its common use and because its absence of involved substitution patterns. The simple structure satisfies the Corlett and Hague (1953) conditions for non-labor taxes to be optimally set to zero in our setup, absent land rents. We are unaware of any estimated cost function breaking out land and structure demand for urban residents. However, we allow some complementarities guided by empirical results in other specifications, acknowledging that these complementarities are unlikely to be correctly offset in the rest of the expenditure function. For example, Thorsnes (1997) justifies the Cobb-Douglas

[9] Land price ratios vary from approximately two to approximately 1,000 across our scenarios. US agricultural land price per acre (inclusive of agricultural structures) was \$1,213 in the 2002 Agricultural Census, with considerable variation. In the Austin, TX, metropolitan area, with a population of 1.2 million, Morton (2006) reports residential land prices downtown of approximately 2.5 million per acre.

[10] Barnes and Langworthy (2003) present estimates of monetary commuting costs of approximately \$0.15 per mile. Given plausible wage and speed ranges, either ratio strikes us as plausible.

[11] The traditional consumer shares for both numeraire and other good consumption is 1/3 and for is 1/6 each for land and structures.

Table 2: Land and property taxes: Cobb-Douglas utility, no profit taxes

	(1)	(2)	(3)	(4)
Population	1	1	1,000	1,000
Time cost of travel	0.2	1	0.2	1
Money cost of travel	1	0.2	1	0.2
Ratio of producer prices at center to border (labor tax only)	2.132	1.847	1131.8	806.5
Labor supply at center / supply at border (labor tax only)	.963	1.041	.770	1.528
Optimal land tax rate with labor and land taxes only	17%	23.70%	927.80%	1371.80%
Optimal property tax rate with labor and property taxes only	8.30%	11.10%	21.80%	29.50%
AC of taxes with labor tax and 20% property subsidy	1.108	1.116	1.123	1.373
AC of taxes with labor tax only	1.085	1.088	1.086	1.118
AC of taxes with labor and land taxes	1.084	1.085	1.031	1.037
AC of taxes with labor and property taxes	1.082	1.082	1.077	1.102

Marshallian cross price elasticities of one for land and structures, but the own price elasticity of one for the composite of land and structures is too high based on micro estimates. Below, we discuss a fuller specification for transportation demand that allows for leisure travel as well as commuting.

Table 2 presents simulation results relating to land and property tax rates. Here, we assume that the government does not have access to pure profit taxes, and that the composite other good cannot be taxed, so that lump sum individual taxation is also ruled out. We consider optimal tax rates on land (when land profits are indistinguishable from development costs) and on property (combining land and structures). The only other tax allowed for the moment is on labor income.

We find confirmation of Conjectures 1.1 and 1.2. When population increases, land taxes rise dramatically, from approximately 20% in a city with population measure 1 to approximately 1,000% in a city with population measure 1,000. When travel costs are dominated by time rather than money, the property and land tax rates rise by roughly 35% in both small and large cities. The justification for these differences can be found in the considerable variation in the locational gradient of labor supply. When time costs dominate, labor supply is much greater in the large city's center than at the border. The opposite is true when time costs dominate.[12]

[12]The ratio of labor supply at city center to the labor supply of median individual varies between 0.96 and 1.05. This means that while the ratio of labor supply between city center and border residents can be very large, most of the population is very close in behavior to the residents of city center.

In larger cities, we find considerable costs of property subsidies, with the average cost of raising a dollar of tax revenue equal to 1.373 with a property subsidy when commuting costs are mostly time as opposed to 1.118 with a labor tax only in the same setting.

4.1 "Anti-Sprawl" Measures: transportation taxes and growth controls

Table 3 presents simulation results related to transportation taxes and growth control policies. These simulations provide support for Conjectures 2 and 3.

We find, pursuant to Conjecture 2, that growth controls are undesirable except when income taxes have a significant effect on time costs of travel. In fact, we find that the government chooses to encourage sprawl by mandating borders that are roughly twenty percent farther from downtown than the market would choose in the case of property taxation in the small population city with mostly time costs. In this setting, because of the substitution between leisure and land, in the more distant locations, more work is done. This, combined with a positive property tax, justifies a growth mandate. In the large population city with relatively high time costs of travel, the optimal boundary is over 10% closer to the center than the free market border in the case of a property tax. With a 20% property subsidy, the increased leisure with distance due to travel time combines with the subsidy's upward distortion of land demand to render a growth control of over 20% of free market distance optimal.

Transportation taxes are positive across every scenario, reflecting the link between transportation demand and land demand, with taxation of the latter desirable. We find confirmation of Conjecture 3 in that freedom to set growth boundaries yields a greater welfare gain than the ability to tax transportation gas taxes in all scenarios except the one with large population and relatively large time costs of travel. In the first three scenarios, growth mandates undo the distortions of land taxes. Directly increasing the border is a more efficient means of increasing land consumption than reducing the gas tax below an otherwise desirable level. When the optimal growth control is to reduce land consumption due to fiscal travel time externalities, gas taxes are a more efficient way of remedying the problem.

Informed by West and Williams (2005), we consider additional consumption of gasoline for non-commuting purposes that are complementary with leisure. We change the Cobb Douglas specification to a truly translog cost function by changing the log cost function to include the term

Table 3: Transportation taxes and growth regulations: Cobb Douglas utility

	(1)	(2)	(3)	(4)
Population	1	1	1,000	1,000
Time cost of travel	0.2	1	0.2	1
Money cost of travel	1	0.2	1	0.2
Ratio of producer price of land at city center to producer price at border with only labor tax	2.132	1.847	1,131.8	806.5
Labor supply at center / supply at border (labor tax only)	.963	1.041	.770	1.528
Optimal transportation tax rate with labor and property taxes	3.4%	29.2%	11.2%	33.9%
Free market border distance to center with labor and property taxes	0.044	0.046	0.337	0.409
Optimal border distance with labor and property taxes	0.054	0.054	0.364	0.353
Free market border distance to center with labor tax and 20% property subsidy	0.049	0.053	0.319	0.431
Optimal border distance with labor tax and 20% property subsidy	0.057	0.057	0.353	0.345
AC of taxes with labor, property, and gas taxes	1.084	1.083	1.083	1.075
AC of taxes with labor and property taxes and optimal boundary	1.071	1.078	1.078	1.101

$\frac{1}{2}\gamma(1-\tau_z)q_{gas}$. Per their analysis, we set γ=-.005, noting that γ's appropriate value depends on land costs, that West and Williams (2005) have a cost function with different goods than ours, and that they allow income effects (and thus have an AIDS cost function, of which ours are special cases). We consider three gas tax configurations: taxes on this non-commuting travel commodity only, taxes on commuting only when this good exists with $\gamma = -.005$, and taxes that must be simultaneously levied on both commuting and the other gas commodity at the same rate.[13]

Results are presented in Table 4. Here we consider gas taxes from a starting point of taxes on labor income and property only. In one set of simulations we assume that there are no profit taxes, in the other we assume that profits are taxed at 100%. We find that in the absence of profit taxes, the commodity that is relatively complementary to leisure is taxed most heavily in the city with large population and high time costs of travel (column (4)). This distribution of commodity taxes makes sense because that city has the greatest pre-existing labor distortions. That result changes, however, when profits are fully taxed. In that case, the cost of raising public funds is lower in the larger cities than in the smaller cities, because non-distortive profit taxes are a larger per capita source of revenue. It remains the case that this complement with leisure is more heavily taxed in the communities with large time costs of travel relative to monetary costs. It is a general result that when this good is taxed at the same rate as commuting, the tax rate is higher than when the good is taxed in isolation.

The equilibrium links among gas, labor supply, and land consumption have important effects on the optimal tax treatment of gas. Within institutional constraint environments, differences in city structure have very large effects on the optimal tax rate on the joint commodity, with tax rates ranging from 3.41% to 33.89% with no profit taxes and from 5.13% to 38.78% with profit taxes.

We know that leisure complementarity and land complementarity both affect the optimal tax rate on commodities. We now consider the relative roles of labor versus land complementarities for commodity taxation in different urban settings. Table 5 reports the tax rate on the non-numeraire commodity with a 6.6% expenditure share (but for complementarities), that has a complementarity ($\gamma = -.005$) with either land or leisure.[14] This good is now stripped of any direct linkage to

[13] We also changed other "γ" parameters in our translog specification to satisfy the required adding up and symmetry conditions. We did this by adjusting the interaction terms of gasoline and leisure demand with numeraire consumption and the own price elasticity of numeraire. We also gave gasoline a 3.3% budget share absent complementarity considerations.

[14] We followed similar procedure to adjust other gammas as earlier in order to keep our expenditure function proper.

Table 4: Gas taxes with direct complementarities between gas and leisure consumption

	(1) Small City Mostly money	(2) Small City Mostly Time	(3) Large City Mostly Money	(4) Large City Mostly Time
	Commodity Tax Only, No Profit Tax			
Commodity tax rate	19.39%	19.28%	18.04%	22.00%
Average Cost of taxes	1.082	1.084	1.076	1.100
	Commuting Tax Only, No Profit Tax			
Commuting tax rate	3.41%	29.17%	11.21%	33.89%
Average Cost of taxes	1.084	1.083	1.075	1.075
	Joint Tax, No Profit Tax			
Joint tax rate	14.10%	23.86%	13.22%	32.22%
Average Cost of taxes	1.083	1.082	1.074	1.075
	Commodity Tax Only, 100% Profit Tax			
Commodity tax rate	12.91%	14.63%	7.36%	8.19%
Average Cost of taxes	1.058	1.063	1.024	1.032
	Commuting Tax Only, 100% Profit Tax			
Commuting tax rate	43.27%	65.55%	4.23%	20.18%
Average Cost of taxes	1.055	1.055	1.024	1.024
	Joint Tax, 100% Profit Tax			
Joint tax rate	25.83%	38.78%	5.13%	18.61%
Average Cost of taxes	1.055	1.057	1.024	1.024

commuting. We find that complementarity with leisure has a much greater effect on the optimal commodity tax rate in small cities. In the larger cities, however, the complementarity with land is more important than complementarity with leisure. The sign of the tax depends on whether the land taxes distortion land consumption (money costs of travel dominate) or equilibrium labor market effects of land (time costs dominate) are more important.

Table 5: Importance of land vs. leisure complementarity for commodity tax rates

Travel Cost	Mostly Money			Mostly Time		
Complementarity	None	Leisure	Land	None	Leisure	Land
Population	N=1					
Profit Tax	None					
Other Good Tax	2.96%	11.01%	2.62%	2.65%	10.81%	3.45%
Profit Tax	100.00%					
Other Good Tax	-.27%	6.15%	-.85%	.42%	7.42%	1.12%
Population	N=1000					
Profit Tax	None					
Other Good Tax	3.25%	10.46%	7.98%	4.62%	13.14%	36.45%
Profit Tax	100.0%					
Other Good Tax	-.63%	3.33%	-6.04%	-1.08%	3.51%	8.72%

5 Conclusions

In this paper we have developed a framework for evaluating the optimal level of property taxes under different institutional settings and for understanding the interactions among property taxes, income taxes, other commodity taxes, and land use regulations. We have also identified two important sets of tax rules applying to economies where immutable travel costs are the fundamental source of real estate value.

First, we find that with sufficiently high population and travel costs, property taxes are positive. In a Cobb-Douglas example, with parameters set to roughly match a range of economic conditions within the US, property taxes are always positive as long as land rents are not taxed. There are considerable costs to property subsidies from a starting point of labor taxes only. These costs grow with city size and with the ratio of time costs to monetary costs of travel. We also confirm a conjecture that land and property taxes rise with these parameters. Reducing the own price elasticity of land demand increases the optimal property tax rate, but does not affect the nature of comparative statics.

These theoretical considerations have practical implications for property taxes and deductions in the United States and elsewhere. Standard deductions and progressive income taxes imply that the heaviest burden on housing in the US is in areas where land rents and population are small relative to construction costs. Empirically, Gyourko and Sinai (2003) show that housing subsidies are concentrated in areas with high population density and plausibly relatively greater opportunity costs of time than monetary travel costs. Proposition 13 guarantees that California, where land values are very high, has among the least reliance on property tax revenues. The allocation of property tax burdens across US regions thus appears to be backwards.

The second set of results demonstrate the importance of the general equilibrium relationship between land consumption and labor supply. In the absence of labor market distortions, environmental considerations, or congestion, anti-sprawl policies exacerbate the deadweight loss that arises when property taxes fail to distinguish land (and structure) development costs from pure land rents. With only labor market distortions, such policies may be welfare improving, because reducing aggregate commutes undoes the labor market distortion of income taxes. Equivalently, income taxes distort time costs of travel downward and hence call for corrective increases in commuting costs. When commute costs cannot be increased directly through taxes on commuting, the resultant excess land demand may be addressed. Anti-sprawl policies are thus most appropriately applied where income taxes and time costs of travel are high. We speculate that in the US, such policies are better allocated across markets than are property tax burdens.

In the absence of environmental externalities West and Williams (2005) have pointed out the importance of understanding the effects of complementarities between gasoline and leisure. That analysis, however, misses the long-run equilibrium effects on profits, land consumption, and labor supply through increased land prices and hence reduced land consumption and commutes. Depending on the relative importance of travel time in commuting costs versus tax distortions to land, this implies that the welfare costs of gas taxes have either been over- or under-estimated. Notably, we obtain the possible desirability of a gas tax to change transportation patterns even in the absence of congestion or direct complementarity between labor supply and travel.

In simulations that confirm conjectures, we find that development constraints are desirable in markets with high travel time costs and large populations, but that growth should be encouraged in other types of markets under property taxation. We recommend that the focus of the surging study of land use constraints move toward studying whether the types of land use constraints commonly used can act as limitations on commuting, and whether these policies are justifiable based on tax distortions of land or labor markets.

A large body of previous research has addressed public policy relating to housing markets and we now briefly discusses some considerations missing from our approach. First, our static model, meant to approximate a steady state, does not address capital accumulation directly. Papers that have addressed housing markets and capital e.g. Berkovec and Fullerton (1992) have typically assumed that housing is net subsidized by the federal tax system. Our analysis allows for a housing subsidy, but also for the possibility that property taxes are sufficiently large that the net effect on demand for the elastically supplied components of housing is negative. The fundamental tradeoffs we identify should survive an expansion of the set of commodities to include claims on future capital output and to durability of the housing stock.

The result that property taxes should be higher where land value is greater should be robust to changing the specification of what drives land value and to considerations of congestion or environmental externalities. However, our results concerning the desirability of "anti-sprawl" measures would require modification if we allowed for the development of endogenous employment or leisure subcenters, as in Helsley and Sullivan (1991) and Lucas and Rossi-Hansberg (2002). We confine analysis to an uncongested monocentric city, not because we believe externalities do not exist, but rather because it is a natural starting point for analysis. The key assumption we make in this regard is that employment centers are invariant to residential densities across locations. It is difficult to know how commercial development would react to, say, increased commuting costs. Businesses might relocate to suburban locations to respond to workers' increased commuting costs. Alternatively, businesses might further agglomerate downtown in response to consumers' migration inward. The former reaction would render our results less relevant.

We consider taxation in a "city state," in that only a single public good is financed by income, property, and commodity taxes and in that there is no potential for mobility or sorting across communities. To the extent that local and federal spending needs are fixed, we can interpret the optimal level of the property tax as reflecting the optimal sum of local and federal taxes on property. With multiple labor markets, federal taxation might optimally provide incentives to move to more expensive markets, but Albouy (2006) shows that housing subsidies are an inefficient way to do so.

This study has ignored heterogeneity. Atkinson (1977), following Atkinson and Stiglitz (1971), lays out necessary conditions for housing subsidies to be desirable when redistributive considerations are present in a spaceless, single government economy. With land rents present, stronger conditions are presumably required for a subsidy. Any such modification would be complicated by questions of who owns land in which locations. Heterogeneity could soften the labor market distortions of labor taxation if individuals with low commuting costs tend to live in more distant locations. Heterogeneity would also introduce redistributive considerations, inter-jurisdictional strategic behavior, and local political economy considerations into commodity taxation. Land use controls play an important role in the extent of Tiebout sorting, so we view our results on their desirability as complementary to results found elsewhere.

Appendix: Time cost and behavior of expenditure function

This appendix proves shows how the time cost of travel affects expenditure functions and Hicksian demands.

Someone living in location r with time cost ρ_t and net wage rate w, and choosing quantities x of all goods, including land, faces the following minimization:

$$e(p, U; r) = \min_{x,z} -wz + \sum p_i x_i \tag{24}$$

$$\text{subject to} \tag{25}$$

$$u(z + \rho_t r, x) = U \tag{26}$$

This means that the first unit of labor for some one who lives at location r that they get paid for is $\rho_t r$. Call the value of labor that maximizes the problem above "net labor". It is useful to reparameterize the problem in terms of gross labor $z^* = z + \rho_t r$. Now the problem can be written as:

$$e(p, U; r) = \min_{x,z^*} -wz^* + \sum p_i x_i + w\rho_t r \tag{27}$$

$$\text{subject to} \tag{28}$$

$$u(z^*, x) = U \tag{29}$$

This problem has the following useful properties:

1. It is identical to the problem for a maximizing consumer with no location dimension, except that we add a constant $w\rho_t r$ to the minimum value.

2. Because of the first property, the values of z^* and c that minimize the problem are the same as the values of z and c that minimize the "location-free" problem (i.e. the problem in city center).

3. The second property implies that consumption (demand) and gross labor supply function are independent of location.

4. Net labor supply function on the other hand decreases at rate $\rho_t r$ holding everything constant.

5. The derivatives of net and gross labor supply are the same.

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