

Uncommitted Couples: Some Efficiency and Policy Implications of Marital Bargaining

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Abstract

This paper studies married couples' dynamic investment and consumption choices under the assumption that the couple cannot commit across time to not to renegotiate their decisions. The inability to commit is shown to lead to dynamic inefficiency and to affect the level of savings. As an application of the model, the efficiency properties of different divorce asset division regimes are compared and a Common Law Regime is shown to permit the couple to make dynamically efficient choices even without explicit commitment technology.

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JEL Codes: *D1, D91, J12*

1 Introduction

This paper studies the implications of inability commit across time for economic efficiency and decision-making. While there exists a vast literature on the effects of the inability to commit to a given solution in many applications (including investment decisions and monetary policy), the implications of this imperfectness have not been studied much in the context of a household. This paper asks how the fact that a (married) couple cannot necessarily credibly

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commit to a future division of consumption and labor affects the decision made today.

The setup of this paper is simple. A couple needs to decide today on how to divide current consumption between spouses and how much to save. The couple cannot credibly agree on how to divide consumption in the future, since they cannot commit not to renegotiate the current agreement in the future. Furthermore, the “balance of power” in the family may be different in the future, so that one spouse may have comparative advantage in the family now that will decay in the future. This potential variation of “balance of power” is shown to lead to an economic inefficiency within the household since the spouses cannot complete Pareto-improving trades between themselves across periods due to lack of commitment.¹

This paper remains mostly agnostic about the sources of the difference of “balance of power” across time periods by not explicitly modeling the phenomenon in most cases. It is only assumed that for some reason, the relative “welfare weight” in the household’s objective function varies across time periods and is function of past choices. The reason why the current objective function is not necessarily aligned with the future objective can be justified, e.g., by assuming that the spouses engage in period-by-period bargaining. If, loosely speaking, the relative outside options of the spouses differ from period to period, then welfare weights will differ too. Reasons for non-constant relative outside options abound: dynamic effects of labor force attachment of spouses (through human capital acquisition), spouse-specific educational investments, the details of divorce laws, remarriage prospects, health shocks and time-varying differing relative attachment to the community outside the family.

An exception to the agnostic attitude toward the “balance of power” phenomenon is taken in the section dealing with divorce. This paper makes two contributions to theory of divorce. First it shows how divorce rules can have an effect on savings for couples that stay married through their effect on the marital bargaining. In a tractable repeated Nash-bargaining model (a special case of the general model), this effect can be positive or negative depending on the elasticity of intertemporal substitution of the utility functions. This effect is completely separate from the traditional “insurance against bad outcome” effect of divorce on savings considered in Cubeddu and

¹It is also possible that actions taken today (like labor force participation) will affect future “balance of power”. This possibility is also considered in the model.

Ríos-Rull (1997).

The second contribution to the theory of divorce is efficiency comparison between different divorce regimes. It is shown in the section 4 that a stylized common-law property regime attains full efficiency under special conditions while the community property regime is unlikely to lead to full efficiency. Thus the choice between common-law and community property regimes involves potentially an equity-efficiency trade-off.

This paper is a part of the growing literature of family decision making that models the family not as a single aligned entity, that can be modeled as if it were an individual agent, but as group of agents whose preferences are not completely aligned. While this line of inquiry goes back to Becker (1973), the papers that initiated the more recent interest in this research topic are papers on Nash bargaining models of family (Manser and Brown, 1980; McElroy and Horney, 1981) and papers on the efficient contracting models of family (starting from Chiappori, 1988). Many empirical papers in this literature implicitly assume lack of full commitment across time periods (and legislative regimes) in household decision-making. Examples of this are papers using the effects of law changes to test the single-utility-function view of the household, like Aura (2005), Duffo (2000) and Lundberg et al. (1996).

This paper can be seen as an extension of the framework set out in the series of papers by Chiappori and his coauthors into dynamic setting. Mazzocco (2000) extends similar ideas as this paper, although its focus is different. Mazzocco's paper differs from this paper by emphasizing the empirical implementation and not the analysis of the normative and positive implications of the intertemporal inefficiency.

There are two separate branches of literature that are closely related to this paper. First, the literature on savings decisions and family bargaining. The paper perhaps closest to this one in this literature is Browning (2000), which presents a two-period game-theoretic model in which the non-cooperatively made savings and investment decisions yield full Pareto efficiency.

The other related literature is the growing literature on informal insurance arrangements. Two examples of this are Attanasio and Ríos-Rull (2000) and Ligon et al. (2000). This literature emphasizes the same issue as this paper: the inability to commit across time (and states of the world). However, since most of this literature is on agricultural communities in developing countries, they do not see savings as the most interesting aspect (since the crop cannot be stored indefinitely) and emphasize risk-sharing almost solely. Ligon et al.

do consider savings, but while coming close to the results of the first part of this paper, they do not emphasize the role of savings in their model.

The paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes the effect of lack of commitment to the level of savings in a specific version of the general model. Section 4 analyzes the efficiency properties of different divorce regimes. Section 5 concludes.

2 The basic model and results

Can a married couple commit to a given consumption path and sharing rule across time? One reason to think that the answer to this question may be negative is that the outside options (outcomes, in the event of divorce) of the spouses may evolve in time. This could make the agreement based on yesterday's balance of power unsustainable today. If this is the case, then in today's decision making the couple has to take into account the effect of today's choices on future decisions. This dynamic linkage, through the fact that future behavior is constrained by the outcome of the future renegotiation process and the fact that today's choices affect this process, is the central focus in this paper. In order to proceed, this paper makes three further assumptions on the decision making process.

First, the within-period decision-making is assumed to Pareto-efficient with respect to the constraints of the problem. Second, we assume full information and of lack of risk or uncertainty. While many of the results presented here could be extended to one type of uncertainty (exogenous risks) the issues relating to asymmetric information (hidden knowledge or hidden actions) within marriage are not considered. Third, the risk of divorce is not considered in this section of the paper.²

To illustrate the decision-making problem that the couple faces when it cannot commit to a time-consistent solution, consider the following problem. Let the world last for T periods.³ Assume that there is no uncertainty and

²The simplistic view on divorce taken in this paper is the following. Each period two states of the world are possible: under the normal circumstances, the couple continues together if their utility from staying together under the cooperative regime is higher than in the case of divorce. However, in each period, there is a small chance of a random shock that irrespective of economic variables makes the couple incompatible with each other (say one partner is caught cheating). The latter (an unmodelled phenomenon) is what is called the risk of divorce (or breakup) in this paper.

³The assumption of fixed number of time periods is only used to intuitively justify

that the only decision the couple faces at period t is how much to save and how to divide the current consumption between spouses. Assume that the life-time utility functions of respective spouses are defined as time-separable utilities over their own consumption only:

$$U^m = \sum_{t=1}^T u_t^m(c_t^m)$$

and $U^f = \sum_{t=1}^T u_t^f(c_t^f),$

where superscripts m and f refer to husband and wife respectively.⁴

The inability to commit means that future behavior is taken into account as a constraint on the constrained Pareto-efficient decision program at every period. This means that at period T the problem that couple solves is:⁵

$$V_T^m(A_T) = \max_{c_T^m, c_T^f} u_T^m(c_T^m)$$

subject to

$$A_T = c_T^m + c_T^f$$

$$u_T^f(c_T^f) = V_T^f(A_T),$$

for some function $V_T^f(A_T)$ that represent wife's period specific utility level at time T in the optimum.⁶ The $V_T^m(A_T)$ and $V_T^f(A_T)$ functions are reduced form representations of the household decision-making process, representing the point in the utility possibility frontier that the household will choose. A leading example of structural form representation that could be characterized

the use of backward induction and value functions in the dynamic optimization. All the results of the paper would hold in an infinite-horizon setting.

⁴Note that the period specific sub-utility functions are indexed by time, thus allowing for discounting and other time variations in the marginal utility of consumption.

⁵As long as we assume that there is only one private consumption good the period T maximization is trivial.

⁶This is nothing more than the usual characterization of Pareto-efficient choice, except that for generality it is assumed that the wife's utility level is a function of the wealth holdings of the household at period T . The dependence of V_T^f on the wealth holding makes it possible to characterize general efficient forms of household decision-making (like Nash-bargaining or social welfare function maximization) where an increase of wealth available in the period T will in general have effect on the utilities and consumptions of both spouses.

by these functions is Nash-bargaining between spouses (being a member of the class of Pareto efficient decision-making processes).

Working backwards, at period $T - 1$ the household now has three choice variables, current consumption of respective spouses and the assets level at period T . The inability to commit across time is captured by the fact that they cannot contract on the respective consumption levels of spouses at period T . Instead, they have to take the decision process in the period T as a constraint while making decisions at time $T - 1$. Completing this reasoning using backward induction this leads to following characterization of the problem:

$$V_t^m(A_t) = \max_{c_t^m, c_t^f, A_{t+1}} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1}) \quad (1)$$

subject to

$$A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1 + r_t)}$$

$$V_t^f(A_t) = u_t^f(c_t^f) + V_{t+1}^f(A_{t+1}).$$

The first constraint is the usual budget constraint, where A_t is the remaining life-time wealth of the couple at period t .⁷ The second constraint is the usual Pareto-efficiency requirement altered to take into account that the couple is constrained by the process characterizing their decision-making in the future and cannot commit to (potentially better) future consumption allocations that are incompatible with that process. Naturally, a completely equivalent characterization of the process is the maximization of the wife's life-time utility subject to constraint on the husbands utility level. This interchangeability will become useful in providing short proofs for the theorems.⁸

The characterization above is meant to describe the optimum in a same way as Pareto-frontier characterizes possible optima in a usual exchange economy setting. While we can study properties of the optimum using this characterization, we cannot derive comparative statics with respect to changes of

⁷All the analysis in this paper would go through when A_t is construed as the net wealth of the couple at time t , and in each period each partners gets an additional amount (possible negative) of income I_t^m and I_t^f respectively. While completely equivalent characterizations of the dynamic budget constraint in the current environment, in the extensions this allows for the respective I_t -processes to be contingent on the divorce. Thus the results do extend to a more general specification of the household's state-contingent budget constraint.

⁸Note that the non-negativity constraints on consumption are assumed to be non-binding throughout the paper.

the economic parameters without specifying the structural process (including the effects through intrafamily redistribution) by which the couple arrives into the solution.⁹ Thus, if we find a policy intervention that can lead to a first-best solution, we can only make the claim that it will make at least one member of the couple better off.

An example of a structural model characterized by the framework presented is a model where in each period spouses determine the period-specific savings and consumption by Nash-bargaining while fully understanding that in the next period the process is repeated and that the outside options in the future are affected by the current savings decisions. In what follows this will be called repeated Nash bargaining without commitment.

To characterize the efficiency properties and to describe the nature of inefficiencies that can arise because of the lack of commitment across time periods, the following assumption will be made.

Assumption 1 (More wealth is better for both in every period). $V_t^{m'} \geq 0$ and $V_t^{f'} \geq 0$ for all time periods in the optimum solution.

The justification for assumption 1 is that it bounds the potential bargaining effect of wealth to not dominate the direct budget set expansion effect of wealth. A problem with assumption 1 is that it is in terms of both $V_t^{m'}$ and $V_t^{f'}$. While intuitively appealing, this is not completely satisfactory, since only one of the V_t -functions should be taken as fundamental of the problem (i.e. reduced form presentation of the bargaining process), the other being a quantity derived from the optimization solution. Several alternative conditions, and some that are derived from more primitive assumptions, guaranteeing that assumption 1 holds are given as Lemma 1 in the appendix.

Definition 1 (Undersaving and oversaving). *Define the situation where $u_t^{m'} < (1 + r_t)u_{t+1}^{m'}$ and $u_t^{f'} < (1 + r_t)u_{t+1}^{f'}$ as undersaving and situation where both inequalities are reversed as oversaving.*

The justification for the definition of undersaving is that it defines a situation where a both spouses would prefer to transfer current consumption into

⁹A reduced-form way of doing this would be to parametrize the V_t^f -function family.

An analogy of this into the exchange economy setting is that while we can study the properties of the Pareto-frontier, in order to say how the equilibrium changes with respect to the outside parameters, we need to make additional assumptions (like price-taking and utility maximization) to have predictions about the effects of parameter changes.

next period consumption. It is noteworthy that the definition here applies to consecutive time periods only.

Theorem 1. *Let assumption 1 be satisfied. In the optimum there cannot be undersaving nor oversaving.*

Proof. Let μ_t and λ_t be the Lagrange-multipliers for the budget constraint and the wife's utility level constraint on the period t -suboptimization (1). Using the envelope theorem and the first order condition with respect the husband's consumption the intertemporal first order condition can manipulated as:

$$\begin{aligned} \mu_{t+1} - \lambda_{t+1}V_{t+1}^{f'} - \frac{\mu_t}{1+r_t} + \lambda_t V_{t+1}^{f'} &= 0 \Leftrightarrow \\ u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'}. \end{aligned}$$

By considering the same problem from the wife's perspective (and using the relationship between the Lagrange coefficients in the two problems), the intertemporal first order condition for the wife can be written as:

$$u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} = \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) V_{t+1}^{m'}.$$

By applying the envelope theorem, the dynamic first order conditions can be now written as:

$$\begin{aligned} u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'} \\ u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= u_{t+1}^{m'} \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) \left(1 - \frac{V_{t+1}^{f'}}{u_{t+1}^{f'}} \right), \end{aligned}$$

or equivalently as

$$\begin{aligned} u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= u_{t+1}^{f'} (\lambda_{t+1} - \lambda_t) \left(1 - \frac{V_{t+1}^{f'}}{u_{t+1}^{f'}} \right) \\ u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) V_{t+1}^{m'}. \end{aligned}$$

The result is immediate from the above. □

It is straightforward to generalize Theorem 1 to include multiple private goods and public goods within the household. Theorem 1 would hold for all private goods, but unfortunately, the Euler equation for the household public goods does not yield any interesting economic intuition.

An interesting extension of Theorem 1 is to include labor supply decisions (or leisure consumption) as one of the choice variables. In the simplest case, where both spouses have an exogenously given period-specific wages, labor supply is like any other additional consumption good. However, the labor supply decisions in this dynamic model becomes extremely interesting when one allows for the the following generalizations:

1. The current-period wage is affected by the past labor supply decisions.
2. The current-period wages of the spouses affect their relative positions in the family decision-making.

Formally these two effects can be taken into account by specifying that $w_t^f = g_t^f(w_{t-1}^f, L_{t-1}^f)$ and $w_t^m = g_t^m(w_{t-1}^m, L_{t-1}^m)$, where w stands for wage and L for labor supply, and that $V_t^f = V_t^f(A_t, w_t^m, w_t^f)$. Three interesting results arise from this extension. First is that Theorem 1 continues to hold for the consumption goods (but not for labor supply). The second is that even under full commitment the labor supply decisions must take the dynamic (human capital) effects of the labor supply decisions into account (so marginal disutility of labor is equated with the sum of the wage and the marginal dynamic effect of labor supply to the future wages). The third is that without commitment, labor supply becomes a “strategic” variable: because, through future wages, it affects future bargaining, in setting the labor supply this effect has to be taken into account. Thus the model is compatible with the effect put forward in Wells and Maher (1998), where they argue that for strategic reasons the lower-earning capacity spouses may not specialize in home-production because this would diversely affect her future bargaining position.

3 Effect of lack of commitment on the level of savings

Beyond the question of efficiency (i.e., would more or less savings be Pareto-improving), it is interesting to also ask whether lack of commitment causes

the couple to save more or less than in a world where they could commit from day one to a consumption plan. As explained earlier, this is a question that cannot be asked in the most general framework, since this change in environment will almost always include redistribution effects too. Hence, one needs more specific model to answer this question. The model presented in this section will give one answer to this question: in this model, the effect of inability to commit is similar to the effect of a decrease of the assets return.

This section also makes general point about the feedback of divorce rules to savings behavior. In the previous literature (like in Cubeddu and Ríos-Rull, 1997) the effect of divorce on savings behavior is an insurance effect: savings are positively linked to higher probability of divorce because the marginal utilities of consumption are assumed to be higher in divorce state. This effect could be incorporated to the model of this section, but to save on notation and to concentrate on the effect introduced here it is omitted. The feedback effect of divorce rules on savings that is presented in this section operates through a different channel: the future divorce property division rules affect bargaining power in the future and this can have (positive or negative) effects on savings in a model where decisions are renegotiated each period.

Assumption 2 (CRRA utilities). *Let both spouses have lifetime utility function that can be written in the time separable CRRA-form:*

$$U^m = \frac{1}{1 - \theta^m} \sum_{k=1}^T (\beta^m)^{k-1} (c_k^m)^{1-\theta^m}$$

$$U^f = \frac{1}{1 - \theta^f} \sum_{k=1}^T (\beta^f)^{k-1} (c_k^f)^{1-\theta^f}$$

Assumption 3 (Nash-Bargaining with divorce outside options). *Let the household decision making process be repeated symmetric Nash-bargaining without commitment across time periods, so the period- t decision making-*

process can be characterized as:

$$\begin{aligned} & \max_{c_t^m, c_t^f, A_{t+1}} \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + V_{t+1}^m(A_{t+1}) \right) - \tilde{V}_t^m(A_t) \right) \\ & \quad * \left(\frac{1}{(1-\theta^f)} \left((c_t^f)^{1-\theta^f} + V_{t+1}^f(A_{t+1}) \right) - \tilde{V}_t^f(A_t) \right) \\ & \text{subject to } A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)}, \end{aligned}$$

where $V_{t+1}^m(A_{t+1})$ and $V_{t+1}^f(A_{t+1})$ are the value functions characterizing the utility value of future periods for a given asset level in period $t+1$ and where $\tilde{V}_t^m(A_t)$ and $\tilde{V}_t^f(A_t)$ characterize the outside options of spouses should the negotiation break down and divorce occur.

Thus, the outside option functions incorporate all the relevant information on the institutions (e.g. divorce assets division rules) and environment (e.g. remarriage prospects) relevant to spouses should they divorce. Furthermore, let us assume that once divorce happens, it is final.

Assumption 4 (Proportional cost of divorce). *Let the outside option functions be of the form:*

$$\begin{aligned} \tilde{V}_t^q(A_t) &= \max_{\{\tilde{c}_i^q\}_{i=t}^T} \frac{1}{1-\theta^q} \sum_{k=t}^T (\tilde{\beta}^q)^{k-t} (\tau_k^q \tilde{c}_k^q)^{1-\theta^q} \\ & \text{subject to } \psi_t^q(A_t) = \sum_{k=t}^T \frac{\tilde{c}_k^q}{\prod_{j=t}^k (1+r_j)^{j-t}}, \end{aligned}$$

where $q \in \{m, f\}$.

This means that the direct individual utility function is from the same family of utility functions regardless of the marital state. The parameters τ_k^q (typically < 1) represent how much utility loss is there in each period from being divorced.

Assumption 5 (Linear sharing of property). *Let the sharing rule of property in case of divorce be*

$$\psi_t^q(A_t) = \begin{cases} \alpha_t A_t, & \text{if } q = m \\ (1-\alpha_t) A_t, & \text{if } q = f \end{cases}.$$

Assumption 6 (Identical discount factor and CRRA parameter). *Let $\theta^m = \theta^f \equiv \theta$ and $\beta^m = \beta^f \equiv \beta$.*

Theorem 2. *Let assumptions 2-6 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.*

The proof of Theorem 2 is provided in the appendix.

Theorem 2 provides a special case where inability to commit across time-periods acts analogously as decrease of the return on the asset. Thus, when elasticity of intertemporal substitution is higher than 1 (i.e. $\theta < 1$) the inability to commit decreases savings. When elasticity of intertemporal substitution is less than 1 (i.e. $\theta > 1$) the inability to commit increases savings.

4 Divorce

This section continues to consider the case where divorce outcomes define the outside options of the spouses. The question asked in this section is whether the family of models considered here can give policy recommendations, i.e. whether changes in divorce laws could be used to restore first-best solutions. The answer to this is shown to depend crucially on how one views divorce. If divorce is just an off-equilibrium path event, then the first-best solution can be easily restored. However, adding additional risk of getting into the divorce state will modify this conclusion.

It is interesting to compare the results of this section to the real world existing divorce property division regimes. The two-assets model of this section can be viewed as stylized common-law property regime, where both spouses can own assets while married without automatic joint ownership. A special case of the single assets model is a community property regime where the property accumulated during marriage is split evenly in case of divorce. Generally, community property regime is viewed as more progressive and more “pro-women” or “pro-weaker spouse” (Weitzman, 1992). Dnes (1999) argues that a community-property regime is likely to be more efficient, since it provides lesser incentives for costly litigation in the case of divorce than community property and since, at least in England, the discretion that judges

have in common-law property regime creates excess uncertainty.¹⁰

The results of this section are not supportive of Dnes' conclusion (although they do not consider the effects that Dnes emphasizes). Consider a community property regime through the following description of marriage. In the beginning of marriage, the spouses each own some assets. These assets can be covered by prenuptial contract to assign permanent property rights on them. The assets accumulated during marriage are divided equally in case of divorce. Now if the assets are the only thing that matters for relative outside options (as in the examples in the previous sections) and the pre-marital wealth was evenly divided in the prenuptial contract then there is no reason to expect that the community property regime does not yield the first-best solution in the simple model that has no exogenous divorce risk. However, other factors also can affect the outside options and these factors can lead to a non-constant time path of relative outside options and to dynamic inefficiency. These factors can include remarriage prospects, attachment to community outside the couple and human capital accumulation that are unlikely to be included (at least perfectly) in the community property valuation. In contrast, Theorem 3 states that without exogenous divorce risk under a stylized common-law setting the couple can correct these disparities themselves by trading future property rights to current consumption and attain first-best efficiency.

The point of Theorem 3 is not to say that a common-law regime is necessarily more desirable than a community property regime. Instead Theorem 3 is meant to highlight the possible efficiency-equity tradeoff between these two regimes. Under the common-law regime the couple attains a point in the unrestricted lifetime utility possibilities frontier. This point does not necessarily Pareto-dominate the solution under community property. Thus, by forcing the community property rules on the couple the government can possibly attain distributional goals, but since this typically involves a departure from the first-best intertemporal and interpersonal allocation of consumption

¹⁰In the US community property states are: Arizona, California, Idaho, Louisiana, Nevada, New Mexico, Texas, Washington and Wisconsin. In addition, Puerto Rico is a community property jurisdiction. However, since the 1970s, all of the common-law states (with the exception of Mississippi) have enacted so called "equitable division" clauses, that make their legislation be somewhere between the two extremes considered here. In the UK, Scotland has a community property regime, while Wales and England have common-law regimes (Dnes, 1999). Many continental European countries have community property regime (Dnes, 1999).

within the couple, this possible equity gain comes with an efficiency cost.

4.1 No-equilibrium path divorce

Consider the environment of Theorem 1 with the following modifications to allow for difference in property rights for different assets:

Assume that there is more than one asset (without loss of generality, we can assume that there are only two assets), with possibly different rates of rates of return.¹¹ Assume that what drives the power in decision-making is the outcome for the spouses should they divorce. A structural model having this implication is repeated Nash-bargaining with divorce as outside option.

The decision problem can now be written as:

$$V_t^m(A_t^m, A_t^f) = \max_{c_t^m, c_t^f, A_{t+1}^m, A_{t+1}^f} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1}^m, A_{t+1}^f) \quad (2)$$

subject to

$$A_t^m + A_t^f = c_t^m + c_t^f + \frac{A_{t+1}^m}{(1+r_t^m)} + \frac{A_{t+1}^f}{(1+r_t^f)}$$

$$u_t^f(c_t^f) = V_t^f(A_t^m, A_t^f) - V_{t+1}^f(A_{t+1}^m, A_{t+1}^f),$$

where A_t^m (A_t^f respectively) represents the asset that has stronger effect on the husband's (wife's) utility in the future since he has larger marginal claim on this asset in the case of divorce. As always in this paper, assume that there are no liquidity constraints.

Now, make the following assumptions.

Assumption 7 (Different marginal effects of assets on outcomes). *There does not exist a pair of A_t^m, A_t^f for any t such that $\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} = \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f}$.*

Assumption 8 (Equal rates of return). *Let $r_t^m = r_t^f \equiv r_t$ for all t .*

Assumption 7 can be justified in a case of divorce as outside option Nash-bargaining, where the marginal rights to assets in case of divorce are different, since while an increase in either asset has the same effect on the budget set, they will have differing effects on the future outside options.

¹¹For simplicity, it is assumed that the rates of return are non-stochastic.

Theorem 3. *Let Assumptions 7 and 8 hold and let an interior optimum exist. The resulting outcome without commitment is fully efficient.*

Proof. The first order conditions of problem (2) can be rearranged to yield:

$$\left(\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} - \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f} \right) (\lambda_t - \lambda_{t+1}) = 0.$$

Since this is true for all time periods, this means that $\lambda_t = \lambda_{t'} \quad \forall t, t'$. This combined with the dynamic first-order conditions

$$\begin{aligned} u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) \frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} \\ u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) \frac{\partial V_{t+1}^m}{\partial A_{t+1}^m}, \end{aligned}$$

will yield the fully efficient solution:

$$\begin{aligned} u_t^{m'} &= (1+r_t)u_{t+1}^{m'} \\ u_t^{f'} &= (1+r_t)u_{t+1}^{f'}. \end{aligned} \quad \square$$

The intuition for Theorem 3 comes from incomplete markets analogy. With just one asset, the couple is constrained in the way it can transform current consumption to future consumption. With two assets (“his” and “her” accounts) and Assumptions 7 and 8, they have two assets that span the whole space of required transactions.

An interesting point to note about this model (now for generality, assume that there can be more than two assets) is that the model is consistent with an interior optimum without any constraints in cases where one asset’s return is dominated by some other assets return. This is obvious from the first-order conditions, if the property rights (i.e. the effects on the next period’s value functions) are different and the resulting optimum is dynamically inefficient, an interior optimum with positive holdings of dominated assets is a possibility for strategic reasons.

Another interesting point is the interaction of the divorce law-regime and the labor supply decisions. Under pure common law there is no reason to think that the labor supply will be distorted due to dynamic strategic reasons (that work through human capital accumulation and/or labor force

attachment as explained earlier), since the assets can be used to accomplish the right bargaining positions for future periods. This is not the case under community property legislation and therefore the dynamic strategic effects of labor supply become relevant. An interesting test of the relevance of the model would be to use data from the enactment of “equitable division” clauses in the US and see what kind of labor supply effects these law changes had.

4.2 Independent risk of divorce

The results of Theorem 3 changes if one adds little bit of empirical relevance to the model. In real life, divorces do occur. This section takes another simplistic view (ignoring the link between past actions and divorces) by assuming that divorce is a random event that strikes the couple with an exogenous probability. This could be called the “suddenly the love died” view of divorce: in the beginning of each period a random event (no divorce or divorce) is realized. After that the couple renegotiates their allocation (e.g. by Nash bargaining) still taking into account the divorce outcomes as threat points in their decision making, since they always have the option to divorce if the negotiations breaks down.

Now, for the sake of argument, assume that there are three assets available to the couple: an asset that pays in the case of the couple not divorcing and divorce insurance accounts for both spouses. A divorce insurance is an insurance product, that delivers income to each spouse in the case of divorce (regardless of whether divorce happens because of an exogenous shock or because of joint welfare maximization).¹²

With divorce risk the decision problem can be written as:

$$V_t^m(A_t, I_t^m, I_t^f) = \max_{\left\{ \begin{array}{l} c_t^m, c_t^f, A_{t+1}, \\ I_{t+1}^m, I_{t+1}^f \end{array} \right\}} u_t^m(c_t^m) + (1-p)V_{t+1}^m(A_{t+1}, I_{t+1}^m, I_{t+1}^f) + p\tilde{V}_{t+1}^m(I_{t+1}^m)$$

subject to

$$A_t = c_t^m + c_t^f + \frac{(1-p)A_{t+1}}{(1+r_t)} + \frac{p(I_{t+1}^m + I_{t+1}^f)}{(1+r_t)}$$

$$u_t^f(c_t^f) = V_t^f(A_t, I_t^m, I_t^f) - (1-p)V_{t+1}^f(A_{t+1}, I_{t+1}^m, I_{t+1}^f) - p\tilde{V}_{t+1}^f(I_{t+1}^f)$$

¹²The huge moral-hazard and asymmetric-information problems related to divorce insurance accounts are ignored in this section. Divorce insurance is considered in this section only to illustrate that even perfect insurance markets would not restore full efficiency.

where \tilde{V}_{t+1}^m and \tilde{V}_{t+1}^f represent the indirect utilities of spouses in case of divorce and p is the exogenous probability of divorce.

Theorem 4 (Inefficiency theorem). *With independent divorce risk, generically the decision will not be efficient.*

Proof. Full efficiency requires that

$$\begin{aligned} u_t^{m'} &= (1 + r_t)u_{t+1}^{m'} = (1 + r_t)\tilde{V}_{t+1}^{m'} \\ u_t^{f'} &= (1 + r_t)u_{t+1}^{f'} = (1 + r_t)\tilde{V}_{t+1}^{f'}. \end{aligned}$$

This means that full efficiency is a condition on 6 objectives (marginal utilities of consumption to be equated). The couple only has five choice variables to achieve this, so generically it cannot do this while satisfying the budget constraint. \square

The sense in which word “generically” is used in Theorem 4 is that starting from a model where the solution of the problem without commitment is fully efficient; by slightly perturbing the problem (e.g. by changing how the divorce threat points affects current bargaining) we will always find solutions that are not Pareto-efficient. Conversely, if we start with a solution that is not fully efficient, a slight perturbation of the problem will not lead to a solution that is fully efficient.

The basic problem in the model with independent divorce risk is that divorce insurance is used to for two purposes: to equate individual marginal utilities between current consumption and future consumption in the divorce state; and to equate the ratios of marginal utilities of the two spouses between current consumption and future consumption in the marriage state.

With divorce insurance, the couple has set of assets that span the future state-space. Since even with the complete spanning they cannot always reach Pareto-efficient allocation, they generally cannot do that with less complete assets selection. The potential implication of the Theorems 3 and 4 is that the married couples with stable marriages (where the exogenous divorce risk is low enough to be ignored, but divorce may still be relevant as an outside option) will make more efficient dynamic choices (including division between market and non-market work) under a common-law regime than under a community property regime, but that this relationship does not hold necessarily for unstable marriages (with large exogenous divorce risk).

5 Conclusion

The main conclusion of this paper is that in theoretical model of family decision-making, the inability to commit across time matters for economic efficiency. The use of seemingly dominated assets and inefficient labor supply choices can be explained as an attempt to overcome the problems related to incomplete commitment. Theorem 2 provides an added reason why divorce as a phenomenon can lead couples to save more than they would otherwise do. This justification has nothing to do with the traditional “saving for a rainy day” argument for increased savings (self-insuring against divorce). Instead, it is shown that the fact that divorce threat-points affect the balance of power within the family while still married, can lead to higher saving through an effect that is analogous to decrease in the return on assets. In Theorem 3, it is shown that in a simple model a stylized common-law divorce property division regime is likely to lead to an efficient solution. This result can be viewed as an extension of Coase’s Theorem: in absence of transaction costs, assigning property rights leads to a Pareto-efficient outcome. However, this result is shown to depend on the assumption of no exogenous divorce risks. Taking into account this caveat and also taking into account the possible differing distributional impact of different divorce regimes means that the advantage of common-law over community property regime in the theoretical model should be taken only as a tentative result. Further research on optimal divorce property division regimes incorporating distributional and risk-sharing aspects of the alternative regimes is clearly needed.

Appendix

Lemma 1. *Any one of the following assumptions is sufficient for Assumption 1 to be satisfied:*

- a) *Value functions are independent of wealth for one of the spouses: $V_t^{f'} = 0 \forall t, A_t$ or $V_t^m = 0 \forall t, A_t$.*
- b) *Two period world and wealth good for both in second period: let $T = 2$ and let the second period solution be characterized by a sharing rule for wealth: $c_2^m = \psi_2(A_2)$ and $c_2^f = A_2 - \psi_2(A_2)$. Furthermore let $0 \leq \psi_2 \leq 1$.*
- c) *T -period world, with last period described as in b) and letting the series of functions $V_t^f(A_t)$ satisfy following conditions: $V_t^{f'} \geq 0, V_t^{f''} \leq 0 \forall t, A_t$ and*

$$(1 + r_t)V_{t+1}^{f'}(A) \geq V_t^{f'}(A) \quad \forall t, A.$$

- d) Like c), but applying the restrictions on $V_t^m(A)$ -functions.
- e) CRRA-utilities and outside options in repeated Nash-bargaining: Let Assumptions 2-6 hold of the section 3 hold.
- f) $V_t^{m'} \geq 0$ and $V_t^{m'} \leq u_t^{m'} \quad \forall t$ in the solution; or equivalently $V_t^{f'} \geq 0$ and $V_t^{f'} \leq u_t^{f'} \quad \forall t$ in the solution.

Proof. Case by case:

- a) Direct consequence of envelope theorem and the first order conditions yielding $V_t^{m'} = u_t^{m'} - \frac{u_{t+1}^{m'}}{u_{t+1}^{f'}} V_t^{f'}$.
- b) Under the assumptions, $V_2^m(A_2) = u_2^m(\psi_2(A_2))$ and $V_2^f(A_2) = u_2^f(A_2 - \psi_2(A_2))$. Therefore $V_2^{m'} \geq 0$ and $V_2^{f'} \geq 0$ iff $0 \leq \psi_2 \leq 1$.
- c) By induction. Since last period is like period 2 in b) the claim holds for last period. Now, the first order condition of the problem yields $u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} = (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'}$. Using the fact that by induction assumption $V_{t+1}^{f'} \leq u_{t+1}^{m'}$ this can be manipulated to yield $\frac{u_t^{f'}}{1+r_t} \geq V_{t+1}^{f'}$. Now using the assumption on the inequalities that V^f -functions will satisfy, concavity of V^f and the fact $A_{t+1} \leq A_t$ yields $u_t^{f'} \geq (1 + r_t)V_{t+1}^{f'}(A_{t+1}) \geq V_t^{f'}(A_{t+1}) \geq V_t^{f'}(A_t)$. By the calculation done in a) this means that claim holds.
- d) Same as c), except with roles of m and f reversed.
- e) Follows from the positivity of γ_j and δ_j constants in Lemma 3.
- f) Follows from the same calculation as a). □

The point of this is to illustrate that Assumption 1 covers a large class of interesting problems. Some of these alternative conditions need further commenting: first f) is nothing more than restatements of Assumption 1 using envelope theorem to derive more interpretable conditions. It suffers from the same weakness as Assumption 1, it refers to quantities (marginal utilities of consumption) that are defined in the optimum.

Condition a) provides an interesting special case, where one of the spouses, say the husband, has constant outside options. This means, that in the optimum, the wife will attain full efficiency in her consumption plan even though the husband's consumption plan might be distorted.

Condition e) provides an example of a structural model that satisfies the Assumption 1.

Conditions b), c) and d) are the most fundamental. Condition b) uses two-stage budgeting in the last period as the starting point (the sharing rule of assets). This can be viewed as fundamental description of the bargaining and not as an outcome.¹³ Conditions c) and d) extend this with relatively strong assumptions into T-period setting. However, the wide class of problems that will satisfy Assumption 1 will become evident from the proof of condition c). From that proof we can see that condition c) is just a very strong sufficient condition that can be violated while Assumption 1 still holds.¹⁴

Proof of Theorem 2

Theorem 2 is proved in by first stating and proving two lemmas.

Lemma 2. *Let assumptions 2-5 be satisfied. The Nash-Bargaining problem in period t can be written as:*

$$\begin{aligned} \max_{c_t^m, c_t^f, A_{t+1}} & \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + \sum_{k=t+1}^T (\beta^m)^{k-t} (\gamma_k A_{t+1})^{1-\theta^m} \right) - A_t^{1-\theta^m} \tilde{v}_t^m \right) \\ & * \left(\frac{1}{(1-\theta^f)} \left((c_t^f)^{1-\theta^f} + \sum_{k=t+1}^T (\beta^f)^{k-t} (\delta_k A_{t+1})^{1-\theta^f} \right) - A_t^{1-\theta^f} \tilde{v}_t^f \right) \\ \text{subject to } & A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)}, \end{aligned}$$

for some constants \tilde{v}_t^m and \tilde{v}_t^f and series of constants γ_k and δ_k .

¹³Unfortunately this two-stage budgeting does not extend to any other than last period. In all other periods there are three goods, one of which is a public good (assets in the next period).

¹⁴Condition c) is too strong in two senses. First, it requires that the inequality holds for all values of possible values of A . However, this is only done to avoid a (circular) reference to quantities relating to the optimum. Another sufficient condition is that $(1+r_t)V_{t+1}^{f'}(A_{t+1}) \geq V_t^{f'}(A_t)$, where A_{t+1} and A_t are the quantities chosen in the optimum (this form is implied by the for all values of A condition, concavity of $V_t^{f'}$ and by the fact that $A_t \geq A_{t+1}$). Even this is not necessary, since this inequality condition is also just a sufficient condition, that may not be satisfied while Assumption 1 still holds.

Proof. Consider first the outside for the husband:

$$\begin{aligned}\tilde{V}_t^m(A_t) &= \max_{\{\tilde{c}_i^m\}_{i=t}^T} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} (\tau_k^m \tilde{c}_k^m)^{1-\theta_m} \\ &\text{subject to } \alpha_t A_t = \sum_{k=t}^T \frac{\tilde{c}_k^m}{\prod_{j=t}^k (1+r_j)^{j-t}}.\end{aligned}$$

By rewriting the objective as

$$\begin{aligned}\tilde{V}_t^m(A_t) &= \max_{\{\tilde{c}_i^m\}_{i=t}^T} A_t^{1-\theta_m} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} (\tau_k^m \frac{\tilde{c}_k^m}{A_t})^{1-\theta_m} \\ &\text{subject to } \alpha_t = \sum_{k=t}^T \frac{\tilde{c}_k^m/A_t}{\prod_{j=t}^k (1+r_j)^{j-t}}\end{aligned}$$

it is easy to see that $\tilde{V}_t^m(A_t) = A_t^{1-\theta_m} \tilde{V}_t^m(1)$. The outside option value function for the wife is handled similarly. The rest of the proof is a simple induction argument. Consider the Nash-bargaining problem in period T :

$$\begin{aligned}\max_{c_t^m, c_t^f} &\left(\left(\frac{c_T^m}{(1-\theta^m)} \right)^{1-\theta^m} - A_T^{1-\theta^m} \tilde{v}_T^m \right) \left(\left(\frac{c_T^f}{(1-\theta^f)} \right)^{1-\theta^f} - A_T^{1-\theta^f} \tilde{v}_T^f \right) \\ &\text{subject to } A_T = c_T^m + c_T^f.\end{aligned}$$

By the similar homogeneity argument as in the case of the outside options, the optimum solution for $\{c_T^m, c_T^f\}$ is just a linear scaling of the optimal solution in the case where $A_T = 1$. The induction step, assuming that the claim holds for period $t+1$, and showing that the claim holds for period t follows using exactly the same homogeneity of the objective function argument as in the period T . \square

Lemma 3. *Let the assumptions 2-5 hold. Consider maximization of the linear combination of lifetime utilities of spouses (with weight λ_t on wife's utility, this being one characterization of a Pareto-efficient solution subject to constraints):*

$$W(A_t, \mu_t) = \max_{A_{t+1}, \{c_i^m, c_i^f\}_{i=t}^t} \frac{1}{1-\theta} \left(\sum_{k=t}^T \beta^{k-t} (c_k^m)^{1-\theta} + \lambda_t \sum_{k=t}^T \beta^{k-t} (c_k^f)^{1-\theta} \right)$$

subject to

$$A_t = c_t^m + c_t^f + \frac{1}{1+r_t} A_{t+1}$$

$$c_j^m = \gamma_j A_{t+1}$$

$$c_j^f = \delta_j A_{t+1},$$

where $j = t + 1, \dots, T$ and where the series of constants γ, δ satisfy the budget constraint for future periods $1 = \sum_{k=t+1}^T \left(\frac{\gamma_k + \delta_k}{\prod_{j=t+1}^k (1+r_j)^{j-t-1}} \right)$. For $\theta > 1$ the savings A_{t+1} are higher than in the optimal unconstrained solution. For $\theta = 1$ (log utility) the savings A_{t+1} are always at the first best level. For $\theta < 1$ the savings A_{t+1} are lower than in the optimal unconstrained solution.

Proof. Substitute the constraints for future consumption into the objective and consider the first order condition:

$$\begin{aligned} c_t^m &= \lambda_t^{-1/\theta} c_t^f \\ c_t^m &= (1+r_t)^{-1/\theta} A_{t+1} * \omega_t^{-1/\theta} \\ \text{where } \omega_t &= \sum_{k=t+1}^T \beta^{k-t} \gamma_k^{1-\theta} + \lambda_t \sum_{k=t}^T \beta^{k-t} \delta_k^{1-\theta} \end{aligned}$$

The second part of the claim is immediate from above: in the case of $\theta = 1$ the choice of the coefficients does not affect the savings level.¹⁵ Next consider the case where $\theta > 1$ and consider ω_t as a function of $(\gamma_k, \delta_k)_{k=t+1}^T$. Subject to the constraint $1 = \sum_{k=t+1}^T \left(\frac{\gamma_k + \delta_k}{\prod_{j=t+1}^k (1+r_j)^{j-t-1}} \right)$, the function ω_t has an unique minimum (since it is a convex function) at the choice $(\gamma_k, \delta_k)_{k=t+1}^t$ that correspond to the unconstrained optimal solution of joint utility maximization. This fact, budget constraint and the first order conditions imply that for any feasible choice of $(\gamma_k, \delta_k)_{k=t+1}^t$ the wealth holdings in period $t + 1$ will be higher than in the unconstrained optimum. For $\theta < 1$, similar reasoning will imply that the capital stock in period $t + 1$ will be lower than in the unconstrained optimum (since ω_t now is a concave function). \square

Theorem 2 (from the main text). *Let assumptions 2-6 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.*

Proof. Application of Lemmas 2 and 3. By Lemma 2 the problem in any period can be written as Nash-Bargaining with future consumption allocation being a linear transformation of tomorrows wealth. Since any Nash-bargaining solution is

¹⁵Naturally, a separate treatment of the log-utility would confirm this result.

Pareto-efficient (with respect to constraints) Lemma 3 applies here for some λ_t . To prove the theorem, it now suffices to notice that under the assumption of equal discount rates and θ -parameters for spouses, any fully Pareto-efficient solution will imply same levels of wealth holdings, so the fact that the fully efficient (with commitment) solution corresponds to a (potentially) different welfare λ_t becomes irrelevant. \square

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