Level Set Evolution without Re-initialization
Outline

- Parametric active contour (snake) models.
- Concepts of Level set method and geometric active contours.
- A level set formulation without reinitialization.
- Mumford-Shah functional.
- Piecewise constant and piecewise smooth models.
- Local binary fitting model.
Image Segmentation and Applications

- Image segmentation: extract objects of interest in images.
- Image segmentation is a fundamental step in computer vision and image analysis.
- Applications of image segmentation:
  1. Shape recovery, analysis, recognition...
  2. Measurement
  3. Visualization
  4. Medical applications: tissue measurement, diagnosis, study of anatomical structures, computer-integrated surgery ...
Classical Methods

An image of blood vessel

Thresholding

Edge detection
An Advanced Method: Active Contour Model
Parametric Active Contours (Kass et al 1987)

For a contour \( C(s) = [x(s), y(s)] \), define energy:

\[
E = \int_0^1 \frac{1}{2} (\alpha |C'(s)|^2 + \beta |C''(s)|^2) ds + \int_0^1 E_{\text{ext}}(C(s)) ds
\]

\[
E_{\text{ext}}(x, y) = -|\nabla (G_\sigma \ast I(x, y))|^2.
\]
Evolution of Active Contours

Gradient descent flow:

\[
\begin{cases}
\frac{\partial}{\partial t} C(s, t) = \alpha C'''(s, t) - \beta C''''(s, t) - \nabla E_{\text{ext}}, \\
C(s, 0) = C_0(s)
\end{cases}
\]

Advantages:
- Smooth and closed contour
- Sub-pixel accuracy.

Disadvantages:
- Cannot change topology.
- Initial contour must be close to the object boundary.
Geodesic Active Contours (Caselles et al, 1997)

- Minimize a weighted length of $C$

\[ L_g(C) = \int_0^1 g(C(p)) |C'(p)| dp \]

where

\[ g = \frac{1}{1 + |\nabla G_\sigma \ast I|^2} \]

- Gradient descent flow:

\[ \frac{\partial C}{\partial t} = (\kappa g - \langle \nabla g, \mathcal{N} \rangle) \mathcal{N} \]

- Add balloon force:

\[ \frac{\partial C}{\partial t} = (\alpha + \kappa g - \langle \nabla g, \mathcal{N} \rangle) \mathcal{N} \]
Level Set Representation of Curves

$z = \phi(x, y)$
Level Set Method (Osher and Sethian, 1988)

- Curve evolution
  \[ \frac{\partial C}{\partial t} = FN \]
  where $F$ is the speed function, $N$ is normal vector to the curve $C$

- Level set formulation
  \[ \frac{\partial \phi}{\partial t} = F|\nabla \phi| \]
Geodesic Active Contour: Level Set Formulation

• Curve evolution of geodesic active contour:

\[
\frac{\partial C}{\partial t} = (\alpha + \kappa g - \langle \nabla g, \mathbf{N} \rangle) \mathbf{N}
\]

• Level set formulation of geodesic active contours:

\[
\frac{\partial \phi}{\partial t} = g|\nabla \phi| \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|} + \nabla g \cdot \nabla \phi + \alpha g|\nabla \phi|
\]
Drawbacks of Geodesic Active Contour

- Unstable evolution, requires periodic reinitialization to signed distance function.
- Balloon or pressure force cause boundary leakage.
- Slow evolution due to small time step.
Variational Level Set Method without Reinitialization (Li et al, 2005)

Define an energy functional on level set function:

$$\mathcal{E}(\phi) = \mu \mathcal{P}(\phi) + \mathcal{E}_{\text{ext}}(\phi)$$

where

$$\mathcal{P}(\phi) \triangleq \int_{\Omega} \frac{1}{2} (|\nabla \phi| - 1)^2 dxdy$$  \hspace{1cm} \text{Level set regularization}$$

Internal energy:
• Penalize the deviation from a signed distance function

External energy:
• Drive the motion of the zero level set
External Energy for Image Segmentation

Edge indicator function for image $I$

$$g \triangleq \frac{1}{1 + |\nabla G_\sigma * I|^2}.$$ 

Define external Energy:

$$\mathcal{E}_{g,\lambda,\nu}(\phi) = \lambda \mathcal{L}_g(\phi) + \nu \mathcal{A}_g(\phi)$$

Weighted length:

$$\mathcal{L}_g(\phi) = \int_\Omega g\delta(\phi)|\nabla \phi| \, dx \, dy$$

Weighted area:

$$\mathcal{A}_g(\phi) = \int_\Omega gH(-\phi) \, dx \, dy.$$
Energy Functional and Gradient Flow

Define energy functional:

\[ \mathcal{E}(\phi) \triangleq \mu \mathcal{P}(\phi) + \mathcal{E}_{g,\lambda,\nu}(\phi) \]

The gradient flow of the functional is the evolution equation:

\[
\begin{cases}
\frac{\partial \phi}{\partial t} = \mu [\Delta \phi - \text{div}(\frac{\nabla \phi}{|\nabla \phi|})] + \lambda \delta(\phi) \text{div}(g \frac{\nabla \phi}{|\nabla \phi|}) + \nu g \delta(\phi), \\
\phi(0, x, y) = \phi_0(x, y)
\end{cases}
\]
To explain the effect of the first term, which is associated to the internal energy $\mu P(\phi)$, we notice that the gradient flow

$$\Delta \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) = \text{div} \left[ (1 - \frac{1}{|\nabla \phi|}) \nabla \phi \right]$$

has the factor $(1 - \frac{1}{|\nabla \phi|})$ as diffusion rate. If $|\nabla \phi| > 1$, the diffusion rate is positive and the effect of this term is the usual diffusion, i.e. making $\phi$ more even and therefore reduce the gradient $|\nabla \phi|$. If $|\nabla \phi| < 1$, the term has effect of reverse diffusion and therefore increase the gradient.
We use the regularized Dirac $\delta_\varepsilon(x)$ with $\varepsilon = 1.5$, for all the experiments in this paper. Because of the diffusion term introduced by our penalizing energy, we no longer need the upwind scheme [4] as in the traditional level set methods. Instead, all the spatial partial derivatives $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are approximated by the central difference, and the temporal partial derivative $\frac{\partial \phi}{\partial t}$ is approximated by the forward difference. The approximation of (10) by the above difference scheme can be simply written as

$$\frac{\phi_{i,j}^{k+1} - \phi_{i,j}^k}{\tau} = L(\phi_{i,j}^k)$$

(12)

where $L(\phi_{i,j})$ is the approximation of the right hand side in (10) by the above spatial difference scheme. The difference equation (12) can be expressed as the following iteration:

$$\phi_{i,j}^{k+1} = \phi_{i,j}^k + \tau L(\phi_{i,j}^k)$$

(13)
Figure 1. Evolution of level set function $\phi$. Row 1: the evolution of the level set function $\phi$. Row 2: the evolution of the zero level curve of the corresponding level set function $\phi$ in Row 1.
Figure 2. Isocontours plot of the level set function $\phi$, using the proposed initial function $\phi_0$ defined by (14) with $\rho = 6$. Seven isocontours are plotted at the levels -3, -2, -1, 0, 1, 2, and 3, with the dark thicker curves being the zero level curves.
Figure 3. Result for a real image of a cup and a bottle, with $\lambda = 5.0$, $\mu = 0.04$, $\nu = 3.0$, and time step $\tau = 5.0$. 
Figure 4. Result for a real microscope cell image, with $\lambda = 5.0$, $\mu = 0.04$, $\nu = 1.5$, and time step $\tau = 5.0$. 
Figure 5. Result for an ultrasound image of carotid artery, with \( \lambda = 6.0, \mu = 0.1, \nu = -3.0 \), and time step \( \tau = 2.0 \).
Results

by Chunming Li

A Movie of Chunming Li
3D Segmentation of Corpus Callosum
Conclusion

The proposed variational level set formulation has three main advantages over the traditional level set formulations:

• First, a significantly larger time step can be used for numerically solving the evolution partial differential equation, and therefore speeds up the curve evolution.

• Second, the level set function can be initialized with general functions that are more efficient to construct and easier to use in practice than the widely used signed distance function.

• Third, the level set evolution in our formulation can be easily implemented by simple finite difference scheme and is computationally more efficient.
Region-based Methods
Mumford-Shah Functional

- Piece wise smooth model
  --- Approximate image by piecewise smooth functions

\[ F_{MS}(u, C) = \int_{\Omega} (u - u_0)^2 dx dy + \mu \int_{\Omega \setminus C} |\nabla u|^2 dx dy + \nu |C| \]
Active Contours without Edges
(Chan & Vese 2001)

- Define a region-based energy functional:

\[
F_1(C) + F_2(C) = \int_{inside(C)} |u_0(x, y) - c_1|^2 \, dx \, dy \\
+ \int_{outside(C)} |u_0(x, y) - c_2|^2 \, dx \, dy
\]
Level Set Formulation of Chan-Vese Model

\[
ECV(\phi, c_1, c_2) = \mu \int \delta(\phi(x, y))|\nabla\phi(x, y)|dxdy
\]
\[
+ \int_\Omega H(\phi(x, y))|I(x, y) - c_1|^2dxdy
\]
\[
+ \int_\Omega (1 - H(\phi(x, y)))|I(x, y) - c_2|^2dxdy
\]

\[
\frac{\partial \phi}{\partial t} = \delta(\phi)[\mu\text{div}(\frac{\nabla\phi}{|\nabla\phi|}) - (I - c_1)^2 + (I - c_2)^2]
\]

\[
c_1 = \frac{\int_\Omega I(x, y)H(\phi(x, y))dxdy}{\int_\Omega H(\phi(x, y))dxdy}
\]

\[
c_2 = \frac{\int_\Omega I(x, y)[1 - H(\phi(x, y))]dxdy}{\int_\Omega [1 - H(\phi(x, y))]dxdy}
\]
Results
Piecewise Smooth Model (Vese and Chan, 2002)

Minimize the energy functional:

\[
F(u^+, u^-, \phi) = \int_\Omega |u^+ - u_0|^2 H(\phi) \, dx \, dy + \int_\Omega |u^- - u_0|^2 (1 - H(\phi)) \, dx \, dy \\
+ \mu \int_\Omega |\nabla u^+|^2 H(\phi) \, dx \, dy + \mu \int_\Omega |\nabla u^-|^2 (1 - H(\phi)) \, dx \, dy \\
+ \nu \int_\Omega |\nabla H(\phi)|
\]
Solve PDEs:

\[ u^+ - u_0 = \mu \Delta u^+ \text{ in } \{(x, y) : \phi(t, x, y) > 0\}, \]
\[ \frac{\partial u^+}{\partial n} = 0 \text{ on } \{(x, y) : \phi(t, x, y) = 0\} \cup \partial \Omega, \]

\[ u^- - u_0 = \mu \Delta u^- \text{ in } \{(x, y) : \phi(t, x, y) < 0\}, \]
\[ \frac{\partial u^-}{\partial n} = 0 \text{ on } \{(x, y) : \phi(t, x, y) = 0\} \cup \partial \Omega, \]

\[ \frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi) \left[ \nu \nabla \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - |u^+ - u_0|^2 - \mu |\nabla u^+|^2 \right. \]
\[ + |u^- - u_0|^2 + \mu |\nabla u^-|^2 \]
\]

\[ , \]
Examples
Local Binary Fitting Active Contours/Surfaces
Local Binary Pattern in General Images

Assumption: image $I$ can be locally approximated by a binary image.
Local Binary Fitting
Level Set Formulation

The LBF energy functional on a contour C

\[ E_{x}^{LBF}(C, f_1(x), f_2(x)) = \lambda_1 \int_{\text{outside}(C)} K(x - y)|I(y) - f_1(x)|^2 dy + \lambda_2 \int_{\text{inside}(C)} K(x - y)|I(y) - f_2(x)|^2 dy \]

is equivalent to the level set formulation:

\[ E_{x}^{LBF}(\phi, f_1, f_2) = \lambda_1 \int K_\sigma(x - y)|I(y) - f_1(x)|^2 H(\phi(y)) dy + \lambda_2 \int K_\sigma(x - y)|I(y) - f_2(x)|^2 [1 - H(\phi(y))] dy \]
Level Set Formulation (Cont’d)

For extracting the entire object boundary, the local binary fitting energy is integrated over all x in the image domain:

\[ E^{LBF}(\phi, f_1, f_2) = \lambda_1 \int \int K_\sigma(x-y)|I(y) - f_1(x)|^2 H(\phi(y)) \, dy \, dx \]
\[ + \lambda_2 \int \int K_\sigma(x-y)|I(y) - f_2(x)|^2 (1 - H(\phi(y))) \, dy \, dx \]

Add two terms for regularization of the contour and the embedding level set function, and define the following energy functional:

\[ F(\phi, f_1, f_2) = E^{LBF}(\phi, f_1, f_2) + \nu L(\phi) + \mu P(\phi) \]

Data fitting term  Length term  Level set regularization
Energy Minimization Using Gradient Flow

The minimization of the energy functional $F$ is achieved by solving the gradient flow:

$$
\frac{\partial \phi}{\partial t} = -\delta_\varepsilon(\phi)(e_1 - e_2) + \lambda \delta_\varepsilon(\phi) \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \mu \left( \nabla^2 \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \right)
$$

where

$$
e_1(x) = \int_{\Omega} K_\sigma(y - x)|I(x) - f_1(y)|^2 dy
$$

$$
e_2(x) = \int_{\Omega} K_\sigma(y - x)|I(x) - f_2(y)|^2 dy
$$

$$
f_1 = \frac{K_\sigma \ast (H_\varepsilon(\phi)I)}{K_\sigma \ast H_\varepsilon(\phi)}
$$

$$
f_2 = \frac{K_\sigma \ast [(1 - H_\varepsilon(\phi))I]}{K_\sigma \ast [1 - H_\varepsilon(\phi)]}
$$
Result

Synthetic noisy image

by Chunming Li
2D Segmentation of Real Color Images

A real image of potatoes
2D Vessel Segmentation
Segmentation of White Matter in MR images
Effect of the Level Set Regularization

Without level set regularization

Final zero level contour

Final level set function
Comparison with Piecewise Smooth Model

Comparison of computational efficiency

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<th>Image 1</th>
<th>Image 2</th>
<th>Image 3</th>
<th>Image 4</th>
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Comparison with Piecewise Smooth Model

Our method

PS model
3D Vessel Segmentation

MRA Vessel Segmentation

by Chunming Li
Summary

- Variational and level set methods for image segmentation.
- My recent works on variational level set methods:
  1. A new level set formulation without the need for reinitialization (CVPR 05).
  2. A region-based model that draws upon local image information. (CVPR 07).
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