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COMMENTARY

Sex Differences in Mathematical Abilities: Commentary on the Math-Fact Retrieval Hypothesis

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The hypothesis that a male advantage in speed of math-fact retrieval underlies the sex difference, favoring males, in mathematical abilities is unique and provocative. However, the hypothesis does not provide an explanation for the male advantage in mathematical domains, such as geometry, that do not require arithmetic, nor does it accommodate the sex differences in social and occupational interests that contribute to the sex difference in mathematical achievement. An alternative hypothesis focusing on the sex difference in three-dimensional spatial cognition is favored over the math-fact retrieval hypothesis. © 1999 Academic Press

Royer and his colleagues (this issue) are addressing an issue that is of scientific and social importance, the nature and the source of the sex difference in mathematical abilities. Their proposal that the sex difference in mathematical abilities is largely due to a sex difference, favoring boys and men, in the speed of arithmetic-fact retrieval is unique and provocative. My commentary on this proposal and their empirical work addresses two general issues. The first is addressed below and focuses on the relation between automaticity of arithmetic-fact retrieval and performance on mathematical ability and achievement tests. The second section focuses on the potential sources of the sex difference in mathematical abilities.

FACT RETRIEVAL AND MATHEMATICAL ACHIEVEMENT

One of the more important empirical contributions of Royer and colleagues' series of studies is the demonstration of a moderate to strong correlation between the speed with which simple and more complex arithmetic problems are solved and performance on standard tests of mathematical

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achievement and ability. The speed with which individuals solve simple and complex arithmetic problems is not, as assumed by Royer et al., simply related to speed of fact retrieval, however. The speed of solving arithmetic problems is also related to the speed of executing arithmetical procedures (e.g., carrying and counting) and the frequency with which nonretrieval processes are used in problem solving (LeFevre, Sadesky, & Bisanz, 1996; Siegler, 1987; Widaman, Geary, Cormier, & Little, 1989). Nonetheless, the overall findings are consistent with previous studies and are of theoretical and practical importance (Geary & Burlingham-Dubree, 1989; Geary & Widaman, 1992; Siegler, 1988). Practically, these results indicate that as one feature of their mathematics education, children should learn basic computational and procedural skills in arithmetic and other areas of mathematics (e.g., algebra) to the point of automaticity.

Theoretically, the results are consistent with Royer and colleagues' position that the automatic execution of the processes that support the solving of arithmetic problems, such as fact retrieval and carrying, reduces the working memory demands associated with solving such problems. These working memory resources can then be devoted to other aspects of more complex problem solving, such as problem representation in the solving of word problems (Mayer, 1985). In theory, however, a relation between the automatic execution of component processes in arithmetic (e.g., fact retrieval) and skill at solving more complex problems, such as arithmetical word problems, should be found only for more complex problems that actually require simple arithmetic. Geary and Widaman (1992), for instance, found that individual differences in skill at solving multistep arithmetical word problems were related to individual differences in working memory capacity and to the speed of arithmetic-fact retrieval and carrying (as in 65 + 49). The larger the individual's working memory capacity and the faster the individual could execute the retrieval and carrying processes, the better her performance on the arithmetical word problems.

Royer et al. found correlations between speed of solving arithmetic problems and performance on an array of more complex mathematical tests. Without information on the content of these mathematics tests and specifically the degree to which the associated items require arithmetic, it is difficult to make causal arguments based on these correlations. While I agree, as noted above, with the argument that the achievement of automaticity of arithmetical processes should free working memory resources and thereby improve performance on more complex arithmetic problems, it is not clear how the automaticity of arithmetical processes would improve performance on mathematics items (e.g., geometry) that do not require arithmetic. At this point, the basis for the correlation between speed of arithmetical processing and performance on mathematics items that do not require arithmetic are not known and might simply represent a third variable—overall exposure to mathematics—that affects both speed of arithmetical processing and performance in other areas of mathematics.

POTENTIAL CAUSES OF THE SEX DIFFERENCE IN MATHEMATICAL ABILITIES

As aptly reviewed by Royer et al., the magnitude of the sex difference in mathematical abilities varies with the mathematical content of the tests and with the age and overall ability of the individuals assessed (see also Geary, 1996). The male advantage is most evident in high-ability samples and for the solving of word problems and items that require complex spatial competencies (e.g., the SAT-M; Benbow, 1988; Casey, Nuttall, & Pezaris, 1997; Lummis & Stevenson, 1990). The three sections below briefly address factors that potentially contribute to the advantage of boys and men in these domains. The first section focuses on the social sex differences that might contribute to the sex difference in mathematics achievement. The second and third sections focus on cognitive sex differences as related to mathematics ability, specifically math-fact retrieval and spatial cognition, respectively.

Biology, Experience, and Social Sex Differences

The relations among biological factors (e.g., sex hormones), experience, and sex differences in brain, cognition, and social behavior are very complex (Andersson, 1994; Geary, 1998). For instance, in those species in which it has been studied, it has been found that the brains of females and males sometimes respond (e.g., in terms of growth of dendrites) differently to the *same* experiences, apparently through the action of sex hormones (Kolb, Forgie, Gibb, Gorny, & Rowntree, 1998). Sex hormones can also contribute to a sex difference in the types of environmental niches and the types of experiences within those niches that females and males seek (Collaer & Hines, 1995). In other words, sex hormones, and perhaps more direct genetic influences (Arnold, 1996), can influence the ways in which the brain responds to environmental input and the types of environmental input the individual seeks.

Given this, Royer et al.'s conclusion that the sex difference in mathematical abilities is likely to be experience based and that any such sex difference in math-related experiences does not have a biological basis is premature. In other words, it cannot be assumed that the different experiences of boys and girls and men and women are driven only by cultural factors (e.g., gender roles) and even with the same experiences it cannot be assumed that the cognitive and brain development of boys and girls will be the same. In fact, biologically driven sex differences in experiences and in responsiveness (in terms of cognitive and brain development) to these experiences is likely (Geary, 1998).

As an example, sex differences in vocational interests, including interest in math-intensive careers, might be *indirectly* related to evolved and biologically based sex differences. For societies in which women and men are free to choose their own careers, the occupational interests and choices of men and women have been found to consistently differ from each other. On vocational interest tests, ''young women [score] higher than young men on domestic, artistic, writing, social service, and office service vocational interests and young men [score] higher than young women on business, law, politics, mathematics, science, agriculture, athletics, and mechanical interests'' (Willingham & Cole, 1997, p. 178). The sex difference in vocational interests is especially striking among mathematically gifted youths. When they are in their 20s, for every mathematically gifted woman who is working toward or who aspires to earn an advanced degree in mathematics, engineering, or the physical sciences, there are eight equally talented men (Lubinski & Benbow, 1994).

For these gifted individuals, the sex difference in the pursuit of an advanced education in math-intensive areas cannot simply be attributed to cognitive factors, given that nearly all of these women have the mathematical competencies necessary to succeed in these careers, nor can the difference be attributed solely to a bias against women. For example, gifted women, as a group, do not view mathematics as a ''male'' occupation and are not discouraged from pursuing math-intensive careers (Lubinski & Humphreys, 1990; Raymond & Benbow, 1986). Rather, the sex difference in the pursuit of math-intensive careers appears to largely stem from the occupational and social interests of these gifted men and women.

People—men and women—who enter math-intensive fields tend to have a ''low need for people contact'' (Lubinski, Benbow, & Sanders, 1993, p. 701) and tend to prefer occupations that involve a high degree of theoretical and investigative activities (i.e., abstract disciplines that involve some form of discovery). Mathematically gifted men who enter these fields do indeed show this pattern of occupational and social interests. As a group, mathematically gifted women, in contrast, ''are more socially and esthetically oriented and have interests that are more *evenly divided* among investigative, social, and artistic pursuits'' (Lubinski et al., 1993, p. 702). In short, many mathematically gifted women choose not to enter math-intensive fields because they have broader social and occupational interests than their equally gifted male peers and therefore more frequently pursue occupations outside of these math-intensive areas.

Geary (1996) argued that many of the sex differences in occupational interests are *indirectly* related to sexual selection. For instance, the tendency for girls and women to value the development and maintenance of intimate and reciprocal social relationships more than boys and men is found across cultures, from infancy to old age, and appears to have an evolutionary history (see Geary, 1998); a similar pattern of social sex differences has been found in the chimpanzee (*Pan troglodytes;* de Waal, 1993). In modern society this sex difference appears to contribute to the sex difference in the relative attractiveness of math-intensive careers. More generally, it appears that gifted women are more interested in careers that involve living things (e.g., biology and medicine) as opposed to inorganic things (e.g., physics and engineering), whereas gifted men show the opposite pattern, on average. These sex differences would appear to be a continuation of an object versus people orientation that emerges in infancy and is evident in the play patterns and social interests of boys and girls, as well as in the social motives of men and women (Geary, 1998; Haviland & Malatesta, 1981; McGuinness & Pribram, 1979; Pratto, 1996). Geary (1998) argued—and presented evidence to support this argument—that most of these sex differences have evolved by means of sexual selection and are a reflection of the different reproductive strategies of women and men.

Fact Retrieval

Sexual and natural selection are also potentially related to the cognitive sex differences that contribute to the sex difference in mathematical abilities. Nonetheless, I agree with Royer et al.'s argument that any sex difference in the speed of solving simple and complex arithmetic problems is not likely to be biologically based. Although humans and related species (e.g., chimpanzees) do appear to have a system of evolved numerical and arithmetical abilities, these competencies almost certainly do not include arithmetic-fact retrieval and the procedures (e.g., carrying) used to solve complex arithmetic problems (Boysen & Berntson, 1989; Geary, 1995). In fact, the principle process—sexual selection (Darwin, 1871)—that drives the evolution and therefore the biological basis of sex differences does not appear to have directly affected evolved numerical and arithmetical competencies, as noted in Geary (1996).

Moreover, as noted by Royer et al., boys and men do not appear to have an advantage in the more primary memory systems that support math-fact retrieval, suggesting that any male advantage in speed of solving simple arithmetic problems is not secondary to a more basic (i.e., biologically based) sex difference in speed of memory retrieval; in fact, an advantage of girls and women in speed of accessing language-related information would be expected (Geary, 1998), as found by Royer et al. Either way, it is not clear how a male advantage in speed of math-fact retrieval could contribute to the sex difference, favoring boys and men, on mathematical tasks that do not require arithmetic (e.g., visualizing geometric shapes; Lummis & Stevenson, 1990).

Spatial Cognition

Sexual selection can, however, indirectly influence sex differences in mathematical abilities, to the extent that the cognitive and brain systems that support mathematical cognition have been shaped by evolution (see Geary, 1998). As noted by Royer et al., I argued that certain forms of spatial cognition (there are many different types of visual–spatial abilities; Milner & Goodale, 1995) indirectly contribute to the advantage of boys and men on tests such as the SAT-M (Geary, 1996). In particular, my position was (and still is) that the cognitive and brain systems that have evolved to enable movement in and the representation of three-dimensional space are more highly elaborated in boys and men than in girls and women (see Geary, 1998, for discussion of the associated evolutionary mechanisms). The neurobiological architecture of these systems represents an implicit understanding of Euclidean geometry, given that Euclidean geometry is a mathematical representation of the structure of three-dimensional space (Shepard, 1994). In other words, sexual selection (male–male competition in particular) has resulted in a greater elaboration of the cognitive and brain systems that support navigation in physical space in men than in women. One feature of these systems is an intuitive understanding of Euclidean geometry. One indirect, or secondary, effect of the sex difference in navigational competencies is that boys and men have an advantage in mathematical areas that require an understanding of geometry and involve the use of spatial representations of mathematical information, including the use of spatial representations to solve complex word problems (Johnson, 1984).

As noted by Royer et al., the research of Casey and her colleagues supports this hypothesis (Casey, Nuttall, Pezaris, & Benbow, 1995; Casey et al., 1997), as does the finding that gifted boys outperform gifted girls on the SAT-M before being exposed to the mathematics on the SAT-M in school (Benbow, 1988). In the model I have presented, the latter would be expected, if boys have a more elaborated implicit understanding of Euclidean geometry (which is assessed on the SAT-M) and are more facile than same-age girls in spatially representing mathematical information. Even so, I agree with Royer et al. that the issue of the nature and the source of the male advantage in certain mathematical domains is not yet resolved, but I predict that the male advantage in spatial cognition will ultimately prove to be a more important contributor to these sex differences than will any male advantage in speed of solving simple and complex arithmetic problems.

SUMMARY AND CONCLUSION

Royer and colleagues have provided a unique and interesting approach to the issue of the sex difference, favoring boys and men, in certain mathematical domains. Their demonstration of a moderate to strong correlation between speed of solving simple and complex arithmetic problems and performance on tests of mathematical ability and achievement is important, as is the finding that by fourth grade boys have an advantage over girls in speed of solving arithmetic problems, at least in the United States. The source of this sex difference is not known, although Rover et al.'s argument that it is experienced based is feasible and merits further work.

However, the position that an advantage of boys and men in speed of solving basic arithmetic problems is the primary source of the sex difference in certain mathematics domains is not entirely convincing. Their hypothesis does not explain why boys and men would have an advantage on mathematical items that do not require arithmetic (e.g., visualizing geometric shapes; Lummis & Stevenson, 1990) and does not appear to enjoy as much theoretical or empirical support as the hypothesis that sex differences in social and occupational interests and in spatial cognition are largely responsible for the sex difference in certain mathematical domains (Casey et al., 1995, 1997; Geary, 1996).

REFERENCES

Andersson, M. (1994). *Sexual selection.* Princeton, NJ: Princeton Univ. Press.

Arnold, A. P. (1996). Genetically triggered sexual differentiation of brain and behavior. *Hormones and Behavior,* **30,** 495–505.

Benbow, C. P. (1988). Sex differences in mathematical reasoning ability in intellectually talented preadolescents: Their nature, effects, and possible causes. *Behavioral and Brain Sciences,* **11,** 169–232.

Boysen, S. T., & Berntson, G. G. (1989). Numerical competence in a chimpanzee (*Pan troglodytes*). *Journal of Comparative Psychology,* **103,** 23–31.

Casey, M. B., Nuttall, R., Pezaris, E., & Benbow, C. P. (1995). The influence of spatial ability on gender differences in mathematics college entrance test scores across diverse samples. *Developmental Psychology,* **31,** 697–705.

Casey, M. B., Nuttall, R. L., & Pezaris, E. (1997). Mediators of gender differences in mathematics college entrance test scores: A comparison of spatial skills with internalized beliefs and anxieties. *Developmental Psychology,* **33,** 669–680.

Collaer, M. L., & Hines, M. (1995). Human behavioral sex differences: A role for gonadal hormones during early development? *Psychological Bulletin,* **118,** 55–107.

Darwin, C. (1871). *The descent of man, and selection in relation to sex.* London: Murray.

de Waal, F. B. M. (1993). Sex differences in chimpanzee (and human) behavior: A matter of social values? In M. Hechter, L. Nadel, & R. E. Michod (Eds.), *The origin of values* (pp. 285–303). New York: de Gruyter.

Geary, D. C. (1995). Reflections of evolution and culture in children's cognition: Implications for mathematical development and instruction. *American Psychologist,* **50,** 24–37.

Geary, D. C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences,* **19,** 229–284.

Geary, D. C. (1998). *Male, female: The evolution of human sex differences.* Washington, DC: Am. Psychol. Assoc.

Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology,* **47,** 175–192.

Geary, D. C., & Widaman, K. F. (1992). Numerical cognition: On the convergence of componential and psychometric models. *Intelligence,* **16,** 47–80.

Haviland, J. J., & Malatesta, C. Z. (1981). The development of sex differences in nonverbal signals: Fallacies, facts, and fantasies. In C. Mayo & N. M. Henley (Eds.), *Gender and nonverbal behavior* (pp. 183–208). New York: Springer-Verlag.

Johnson, E. S. (1984). Sex differences in problem solving. *Journal of Educational Psychology,* **76,** 1359–1371.

Kolb, B., Forgie, M., Gibb, R., Gorny, G., & Rowntree, S. (1998). Age, experience and the changing brain. *Neuroscience & Biobehavioral Reviews,* **22,** 143–159.

LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition,* **22,** 216–230.

Lubinski, D., & Benbow, C. P. (1994). The study of mathematically precocious youth: The first three decades of a planned 50-year study of intellectual talent. In R. F. Subotnik & K. D. Arnold (Eds.), *Beyond Terman: Contemporary longitudinal studies of giftedness and talent* (pp. 255–281). Norwood, NJ: Ablex.

Lubinski, D., Benbow, C. P., & Sanders, C. E. (1993). Reconceptualizing gender differences in achievement among the gifted. In K. A. Heller, F. J. Monks, & A. H. Passow (Eds.), *International handbook of research and development of giftedness and talent* (pp. 693–707). London, England: Pergamon Press.

Lubinski, D., & Humphreys, L. G. (1990). A broadly based analysis of mathematical giftedness. *Intelligence,* **14,** 327–355.

Lummis, M., & Stevenson, H. W. (1990). Gender differences in beliefs and achievement: A cross-cultural study. *Developmental Psychology,* **26,** 254–263.

Mayer, R. E. (1985). Mathematical ability. In R. J. Sternberg (Ed.), *Human abilities: An information processing approach* (pp. 127–150). San Francisco, CA: Freeman.

McGuinness, D., & Pribram, K. H. (1979). The origins of sensory bias in the development of gender differences in perception and cognition. In M. Bortner (Ed.), *Cognitive growth and development: Essays in memory of Herbert G. Birch* (pp. 3–56). New York: Brunner/Mazel.

Milner, A. D., & Goodale, M. A. (1995). *The visual brain in action.* New York: Oxford Univ. Press.

Pratto, F. (1996). Sexual politics: The gender gap in the bedroom, the cupboard, and the cabinet. In D. M. Buss & N. M. Malamuth (Eds.), *Sex, power, conflict: Evolutionary and feminist perspectives* (pp. 179–230). New York: Oxford Univ. Press.

Raymond, C. L., & Benbow, C. P. (1986). Gender differences in mathematics: A function of parental support and student sex typing? *Developmental Psychology,* **22,** 808–819.

Shepard, R. N. (1994). Perceptual-cognitive universals as reflections of the world. *Psychonomic Bulletin & Review,* **1,** 2–28.

Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General,* **116,** 250–264.

Siegler, R. S. (1988). Individual differences in strategy choices: Good students, not-so-good students, and perfectionists. *Child Development,* **59,** 833–851.

Widaman, K. F., Geary, D. C., Cormier, P., & Little, T. D. (1989). A componential model for mental addition. *Journal of Experimental Psychology: Learning, Memory, and Cognition,* **15,** 898–919.

Willingham, W. W., & Cole, N. S. (1997). *Gender and fair assessment.* Mahwah, NJ: Erlbaum.