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Individual Differences in Cognitive Arithmetic

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SUMMARY

Unities in the processes involved in solving arithmetic problems of varying operations have been suggested by studies that have used both factor-analytic and information-processing methods. We designed the present study to investigate the convergence of mental processes assessed by paper-and-pencil measures defining the Numerical Facility factor and component processes for cognitive arithmetic identified by using chronometric techniques. A sample of 100 undergraduate students responded to 320 arithmetic problems in a true–false reaction-time (RT) verification paradigm and were administered a battery of ability measures spanning Numerical Facility, Perceptual Speed, and Spatial Relations factors. The 320 cognitive arithmetic problems comprised 80 problems of each of four types: simple addition, complex addition, simple multiplication, and complex multiplication. The information-processing results indicated that regression models that included a structural variable consistent with memory network retrieval of arithmetic facts were the best predictors of RT to each of the four types of arithmetic problems. The results also verified the effects of other elementary processes that are involved in the mental solving of arithmetic problems, including encoding of single digits and carrying to the next column for complex problems. The relation between process components and ability measures was examined by means of structural equation modeling. The final structural model revealed a strong direct relation between a factor subsuming efficiency of retrieval of arithmetic facts and of executing the carry operation and the traditional Numerical Facility factor. Furthermore, a moderate direct relation between a factor subsuming speed of encoding digits and decision and response times and the traditional Perceptual Speed factor was also found. No relation between structural variables representing cognitive arithmetic component processes and ability measures spanning the Spatial Relations factor was found. Results of the structural modeling support the conclusion that information retrieval from a network of arithmetic facts and execution of the carry operation are elementary component processes involved uniquely in the mental solving of arithmetic problems. Furthermore, individual differences in the speed of executing these two elementary component processes appear to underlie individual differences on ability measures that traditionally span the Numerical Facility factor. More generally, the present study provides evidence for continuity of intellectual abilities identified with the use of factor-analytic methods and elementary component processes isolated with the use of reaction-time techniques.

Concepts of human intelligence include models for the structure of mental abilities as well as models identifying the cognitive processes underlying mental abilities. The use of factor-analytic methods has resulted in a well-defined, replicable taxonomy of human intellectual abilities. More recently, the information-processing approach to the study of human intelligence has provided a method for identifying the processes underlying specific mental abilities. Numerical facility has been recognized as an important and stable aspect of human intelligence throughout this century, and both the factor-analytic and information-processing methods have been used extensively in the study of numerical abilities. The present study, which uses both methods, was designed to test concurrently the cognitive components model for mental addition recently proposed by Widaman, Geary, and Cormier (1986) across arithmetic operations and to determine the component processes underlying individual differences on measures traditionally spanning the Numerical Facility factor.

The Factor-Analytic Approach to Intelligence

Over the past 70 years, taxonomies of human mental abilities were developed with the factor-analytic methodology (Cattell, 1963; Guilford, 1972; Horn, 1968; Horn & Cattell, 1966; Spearman, 1927; Thomson, 1951; Thurstone, 1938; Thurstone & Thurstone, 1941; Vernon, 1965). The factor-analytic method enabled the isolation of latent variables underlying individual differences on tests of mental abilities. Factorial descriptions of the ability domain proposed by individual scientists (e.g., Spearman, 1927; Thurstone, 1938) varied because of differences in statistical methodology (e.g., the criterion for deciding the number of factors needed to represent the correlations among a set of mental tests) as well as theoretical differences regarding the relative importance of specific latent variables for explaining individual differences on mental tests. However, in spite of this variability, tests of numerical facility were typically included in batteries spanning the ability domain, and a Numerical Facility factor was included in different taxonomies either as a separate ability dimension (e.g., Thurstone, 1938) or as an indicator of a more general mental ability (e.g., crystallized intelligence; Horn & Cattell, 1966).

More specifically, researchers who have used the factor-analytic method have repeatedly shown that unities exist in the processes involved in solving arithmetic problems of varying operations (i.e., addition, subtraction, multiplication, and division). Spearman (1927) indicated that "these four arithmetical abilities have much in common over and above such *g* as enters into them respectively" (p. 251). Thurstone (1938; Thurstone & Thurstone, 1941) demonstrated that the Numerical Facility factor was defined by tests in which an arithmetic operation was required for solution of problems presented and not tests that simply contained numbers as stimuli. Within the theory of intelligence proposed by Thurstone, the Numerical Facility factor referenced an important and replicable primary mental ability. More recently, Numerical Facility has been included as a lower strata ability in the three-level model of intellectual structure proposed by Gustafsson (1984). Factors at lower strata reflect rather specific abilities, that is, abilities involved uniquely in restricted domains of cognition (in contrast to *g*, which spans all cognitive abilities). In all, the use of factor-analytic techniques has repeatedly demonstrated that Numerical Facility is a highly replicable and important dimension of human intellectual ability.

Critiques of Factor-Referenced Theories

Despite the utility of factor-analytic methods for the development of taxonomies of human intelligence, theories of ability structure that were developed through the use of such methods have been criticized. For example, Sternberg (1977, 1980, 1985) argued that factor analysis is of limited use in theory testing because factor solutions are indeterminate. That is, factor solutions are not unique because an infinite number of alternative orientations of axes are possible, each of which provides an equally acceptable mathematical representation of the data. On the surface, this appears to be an appropriate criticism, particularly for any single set of data (Gorsuch, 1983). However, the problem of indeterminacy may be largely mitigated with use of appropriate psychometric criteria (e.g., rotation to simple structure) that are used to select a final factor solution (Carroll, 1980), and indeterminacy is not an issue when confirmatory methods are used (Jöreskog, 1969).

A second criticism is that factor-referenced abilities are static; that is, factor analysis is not appropriate for the study of the processes that constitute human intelligence (Sternberg, 1977, 1985). Indeed, factor analysis is probably not an appropriate method for identifying real-time processes. However, careful consideration of the utility of factor-analytic methodology for making inferences regarding latent variables that underlie observed measures of cognitive processes will lead to a conclusion different from that reached by Sternberg (1977, 1985). Specifically, it is not the factor-analytic methodology that is inappropriate for the study of processes comprising human intelligence. Rather, it is the nature of the data that have been used in factor-analytic studies that does not allow strong inferences to be made regarding the nature of cognitive processes; that is, factor-analytic methods have traditionally been used to represent the structure underlying the products of cognitive activity.

As we will demonstrate later, the factor-analytic method is quite appropriate for the differentiation of latent variables representing psychologically distinct component processes and for defining unities in observed variables representing psychologically similar processes that may span an array of cognitive tasks. If two distinct components are highly correlated across subjects, they may appear as a single factor if exploratory methods are used. However, with the use of confirmatory factor-analytic methods (Jöreskog & Sörbom, 1984), which enable the a priori specification of a factor pattern, the fit of the resulting solution would provide a powerful test of the extensive validity (Sternberg, 1977) of the distinct component processes. A factor-analytic study of the dimensional structure of observed variables representing cognitive processes must be preceded by experimental studies that identify the unique processes involved in the solving of cognitive tasks, a procedure suggested by several theorists who relied on factor-analytic methods (e.g., Thurstone, 1947; Vernon, 1965).

Information-Processing Models for Cognitive Arithmetic

As noted earlier, factor-analytic studies of ability measures have consistently identified a Numerical Facility factor as representing an important ability dimension (Coombs, 1941; French, 1951; Gustafsson, 1984; Thurstone & Thurstone, 1941). Experimental studies based on the measurement of response latencies have also been extensively used for the study of

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numerical abilities and have enabled the modeling of the process components involved in solving arithmetic problems (Ashcraft, 1982; Groen & Parkman, 1972; Moyer & Landauer, 1967; Restle, 1970; Widaman et al., 1986). Process models for the solving of arithmetic problems have included analog, counting, and memory retrieval models, each of which will be summarized briefly.

Processing Models for Mental Addition

Analog models. Early research found reaction time (RT) for addition (Restle, 1970) and number-comparison (Moyer & Landauer, 1967) tasks to be a function of the difference between the two presented numbers. Restle (1970) argued that addition problems were solved by transforming the numbers to be added into analog magnitudes, which were represented as distances along an internal number line. Addition involved the concatenation of the shorter line segment onto the longer line segment, and the sum was represented by the endpoint of the concatenated line segments.

Counting (digital) models. To compute sums, alternative processes to the analog model include digital, or counting-based, models. Groen and Parkman (1972) outlined five counting-based models. Each of the counting models involved the manipulation of an internal incrementing device. The counting models differed as a function of the value to which the incrementing device, or internal counter, was initially set. Parkman and Groen (1971) found that adult RT to simple addition problems was best predicted by the smaller, or minimum, addend. The hypothesized process consistent with this result involved setting the internal counter to the larger addend and then incrementing the counter a number of times equal to the value of the smaller addend until a sum is obtained (Groen & Parkman, 1972).

Direct memory access/counting model. Although the minimum addend (Min) was the strongest predictor of RT to simple addition problems, using the sum of the two addends (Sum) as a predictor explained nearly as much adult RT variance as did the Min. Furthermore, the Min slope (20 ms) for adults differed from the Min slope (400 ms) for children by a factor of 20. A counting rate of 20 ms per incrementation seemed unreasonably fast, as the implicit counting rate for adults required at least 150 ms per digit. Thus, Groen and Parkman (1972) interpreted these results, coupled with the finding of uniform RT to tie problems, as reflecting direct memory access for most addition facts, with occasional memory retrieval failure for some problems. Groen and Parkman reasoned that proficient adults would retrieve addition sums directly from memory 95% of the time and that adults revert to the more reliable Min counting strategy with memory retrieval failure.

Memory network models. Ashcraft and Battaglia (1978) empirically tested the aforementioned counting and direct access/counting models as well as a memory network retrieval model. Adult RT to simple addition problems was best predicted by the square of the correct sum (Sum^2). The latter finding was inconsistent with both the counting and direct access/counting models. Ashcraft and Battaglia concluded that their results were consistent with a model in which the correct sum for a simple addition problem is obtained through retrieval of the sum from a memory network of addition facts. The memory network was conceptualized as a square matrix with column and row entry nodes, with values 0 through 9, representing the two addends. The correct sum for a given simple addition problem was assumed to be stored at the intersection of the entry nodal values corresponding to the two addends. Because RT increased exponentially with the size of the correct sum, Ashcraft and Battaglia argued that the matrix was "stretched" in the region of larger sums, which resulted in longer vector distances and, therefore, longer RTs for these sums.

Recently, Miller, Perlmutter, and Keating (1984) reported that RT to simple addition and simple multiplication problems was best predicted by the correct product of the problem digits (Prod). This result and analyses of errors suggested that both addition and multiplication facts were retrieved from a similar memory network. Furthermore, Widaman et al. (1986) reported that the correct product was the best predictor of RT to simple addition problems and that the column-wise sums of complex addition problems were best predicted by the column-wise product. Widaman et al. argued that the product of addends was also consistent with retrieval of addition facts from a memory network.

The product structural variable allows for a conceptual model of the memory network that is simpler than models previously proposed. Widaman et al. (1986) suggested that the product was consistent with a tabular matrix representing a memory network similar to that proposed by Ashcraft and Battaglia (1978). The memory network is conceptualized as a square symmetric matrix with two orthogonal axes representing nodes for the integers to be added. However, the distance between the nodal values is assumed to be constant, not "stretched" in the region of larger sums as in the Ashcraft and Battaglia model. The memory network is entered at the origin, and the rate of activation of the memory network is assumed to be a constant function of the area of the network activated. The product structural variable represents the total area of the matrix activated, and the product is therefore linearly related to search time required to arrive at the correct answer.

Regardless of the specific mathematical representation (e.g., Sum^2, Prod) of the memory network for arithmetic facts, research by Ashcraft & Battaglia (1978), Miller et al. (1984), and Widaman et al. (1986) strongly suggests that response latencies to simple arithmetic problems are better accounted for by some form of memory retrieval process (e.g., Prod) than by any alternative digital (e.g., Min) or analog algorithm.

A General Model for Mental Addition

The use of chronometric procedures in the cognitive arithmetic area has led to the identification of a memory retrieval process involved in solving arithmetic problems and at least one additional elementary component processe: the carry operation for complex problems (Ashcraft & Stazyk, 1981). However, process models for adults have been limited in scope and have encompassed only those basic processes involved in the solving of simple problems and/or problems of a single operation. To overcome these limitations, Widaman et al. (1986) outlined a general model identifying the elementary processing components required for the mental solving of both simple and complex addition problems, a model that may be easily modified to accommodate other arithmetic operations.

The Widaman et al. (1986) model is an elaboration of the model described by Ashcraft (1982) and includes the same basic processing stages: encoding, search/compute, decision, and response. The first stage involves encoding of problem operation (e.g., addition) and the initial two integers (e.g., in the units column in complex problems) to be summed. Once in working memory, a sum for these integers is obtained through either a counting or a memory-search process. For simple addition involving two single-digit addends, the obtained sum is then compared with the stated sum (for verification tasks), and a decision (true or false) is made and executed. For more complex problems, recycling loops corresponding to the summing of more than one column or more than two rows of integers are included in the model. For multicolumn problems, the encoding and search/compute processes for simple problems are recycled until sums are obtained for each column. This recycling loop is potentially modified in two respects: First, a carry operation is required if the preceding column sum is greater than nine; and, second, complex multicolumn problems may be self-terminated if an error in the stated sum is encountered before the entire problem is processed. The processing of multicolumn problems is terminated and the response "false" is executed when the first column-wise error is encountered. Self-termination of complex problems implicates the existence of a metacognitive process involved in the mental solving of addition problems. This metacognitive process monitors the course of problem solving, which results in the selection and execution of the most efficient component process at each step.

Widaman et al. (1986) assessed the internal validity (Sternberg, 1977), both intensive and extensive forms, of their cognitive components model with the a priori specification of hierarchical regression equations embodying the component processes outlined in the model. Regression equations representing RT to simple and complex forms of addition problems included independent structural variables for each component process proposed in the model (Widaman et al., 1986). Intensive validity of the hypothesized components is assessed by the goodness of fit of a full regression model representing RT to problems of a given type. Extensive validity of the component processes is established by the demonstration that identical structural variables are necessary to model RT across tasks theoretically involving the same component processes. Intensive validity of the model was demonstrated with the finding that RT for each of the four types of addition problems tested was predicted rather well by the full regression equations (R^2s = .71–.84). Furthermore, each structural variable representing a unique process component in the model explained statistically significant amounts of RT variance for each problem type.

Extensive validity of the Widaman et al. (1986) model was clearly supported by the finding that identical structural variables, where appropriate, predicted RT for each of the four types of addition problems. Furthermore, in analyses of a data set containing all four problem types, RT was predicted quite well with a single regression equation specifying structural variables for each component process proposed in the model (R^2 = .89). Within the single equation, regression estimates were constrained to be equal across problem type when such a constraint was justified by the data (e.g., for the carry operation). Enforcing these equality constraints allowed Widaman et al. to argue that not only is the same component process executed in the solving of addition problems of varying complexity, but also the temporal duration of parallel component processes is identical across problem type. Structural variables for which equality constraints were not justified were represented with linearly constrained regression estimates for parallel processes across problem type. For example, time to encode digits was constrained to increase linearly as the number of digits in the problem increased, a finding similar to that found for number comparison tasks (Poltrock & Schwartz, 1984). The combined analysis demonstrated that identical structural variables with highly similar regression estimates predicted RT for addition problems of varying complexity, thereby providing further support for the extensive validity of the model for mental addition. Finally, support for the metacognitive process was demonstrated with the finding that full model R^2s increased an average of .25 with models representing self-terminating processing relative to comparable models reflecting exhaustive processing.

Process Models Across Arithmetic Operations

Although Widaman et al. (1986) tested their model with addition problems only, the same elementary process components (e.g., carry) and perhaps the same search/compute component (e.g., product) may represent process strategies for other arithmetic operations. Indeed, unities in the processes involved in the mental solving of various types of arithmetic problems have been suggested by other researchers (Miller et al., 1984; Parkman, 1972; Siegler & Shrager, 1984; Svenson & Hedenborg, 1979). For example, several lines of evidence suggest that addition and multiplication facts are represented in an interrelated memory network (Miller et al., 1984; Parkman, 1972; Stazyk, Ashcraft, & Hamann, 1982; Winkelman & Schmidt, 1974).

Parkman (1972) found that RT to simple multiplication problems was best predicted by the same structural variables predicting RT to simple addition problems (i.e., Min, Sum). An incrementing strategy for processing these problems seemed unlikely, as this would involve the incrementation of numbers of varying size at the same speed, and Parkman argued that multiplication problems were solved through retrieval of the product from a memory network. Furthermore, the memory network for multiplication facts was hypothesized to be hierarchically related to the memory network for addition facts (Parkman, 1972). Results from experiments presenting confusion problems, where the stated answer is correct for one operator (e.g., addition) but incorrect for the given operator (e.g., multiplication), supported the conclusion by Parkman. Reaction time and error rates for confusion problems have been found to be higher than those for nonconfusion problems (Stazyk et al., 1982, Experiment III; Winkelman & Schmidt, 1974).

Further evidence suggesting unities in component processes across arithmetic operations was reported by Miller et al. (1984). As noted earlier, Miller et al. (1984) reported that the product of two single-digit numbers provided the best fit to RT data for both addition and multiplication problems but not for number comparison tasks. Results from the Miller et al. (1984) experiment as well as confusion experiments (Stazyk et al., 1982; Winkelman & Schmidt, 1974) are consistent with the hypothesis that there is an interrelated memory network for addition and multiplication facts. However, unities in additional elementary component processes (e.g., carrying to the next column) across arithmetic operations have not been systematically demonstrated. Factor-analytic studies of ability measures defining the Numerical Facility factor have repeatedly suggested identities in the processes underlying the solution of problems of different arithmetic operations. Despite this, there has been no research, to date, fitting a cognitive components model for arithmetic across operations and concurrently assessing the relation between the component processes for cognitive arithmetic and measures spanning the Numerical Facility factor.

The Present Study

The present study was designed to assess concurrently the validity of the cognitive components model for addition proposed by Widaman et al. (1986) across arithmetic operations and to assess the relation between component processes specified in the model and traditional measures of numerical facility. The former involved the attempt to establish the internal validity of the model for both simple and complex forms of addition and multiplication problems. Intensive validity of the model would be established with the demonstration that regression equations specified to reflect independent component processes in the model accurately predicted RT to both simple and complex forms of addition and multiplication problems. Extensive validity of the Widaman et al. model would be supported with the demonstration that identical structural variables, where appropriate and with comparable regression weight estimates, represented RT to arithmetic problems of varying complexity and operations.

Finally, external validation (Sternberg, 1977) of the Widaman et al. (1986) model would require the demonstration of a strong relation between component scores (when RT is the dependent measure, raw regression weights are termed *component scores*) for processes specified in the model and measures spanning the Numerical Facility factor, and no relation between the same component scores and measures of other primary mental abilities. The former would support the convergent validity of the component processes, whereas the latter would address the issue of the discriminant validity (Campbell & Fiske, 1959) of the same component processes. In the present study, the relation among component scores estimated from RT to simple and complex forms of addition and multiplication problems and traditional measures defining the Numerical Facility, Perceptual Speed, and Spatial Relations (Pellegrino & Kail, 1982) factors were assessed. Numerical facility measures were chosen to assess the convergent validity of the cognitive arithmetic component processes. Perceptual Speed measures were chosen for two reasons: (a) to assess the discriminant validity of substantive processes potentially underlying the Numerical Facility factor and (b) to assess the relation between more basic processes (e.g., decision and response time) associated with the RT method that may underlie individual differences on the Perceptual Speed factor (Lansman, 1981). Finally, measures of spatial relations were chosen specifically to assess the discriminant validity of the cognitive arithmetic component processes specified in the Widaman et al. (1986) model.

Method

Subjects

Subjects were 45 male and 55 female undergraduates enrolled in psychology courses at the University of California at Riverside. Each subject received $3 or course credit for participating in this experiment.

Reaction-Time Problem Sets

A total of 320 arithmetic problems served as stimuli. The global set consisted of 80 problems of each of four types of arithmetic: simple addition, complex addition, simple multiplication, and complex multiplication. The four sets of problems were presented independently.

Simple addition. Each of the 80 simple addition problems consisted of two vertically placed single-digit integers with a stated sum. Forty of the problems were selected from the 90 possible nontie pair-wise combinations of the integers 0 through 9 as the first addend and the same integers as the second addend; the 40 problems were presented with the correct sum. The frequency and placement of all integers were counterbalanced. That is, each integer (0 through 9) appeared eight times across the 40 problems, and each integer appeared equally often as the first addend and as the second addend. The remaining simple addition problems were the same 40 pairs of addends, but these were presented with a stated sum incorrect by ± 1 or ± 2. The magnitude of the error was counterbalanced across the 40 false stimuli. No repetition of either integer or of the stated sum was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Complex addition. Each of the 80 complex addition problems consisted of two vertically placed double-digit integers with a stated sum. The 40 correct problems were constructed from 80 of the 90 possible integers 10 through 99. The larger integer was the first addend for one half of the problems, and the frequency of individual digits 0 through 9 was counterbalanced for position. Within a given problem, all four digits were unique. Finally, for one half of the problems, the stated sum for the units column was greater than 9, which therefore required a carry operation. The remaining 40 problems consisted of the same 40 pairs of integers, but these were presented with a stated sum incorrect by ± 1, ± 2, ± 10, or ± 20. The placement of the error was counterbalanced; that is, each of the eight possible values of difference between true and false stated sums (e.g., $+1$ or -2) occurred five times. Location of the error in the stated sum, in the units or tens column, was crossed with presence versus absence of a carry operation. No repetition of either addend or the stated sum was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Simple multiplication. Each of the 80 simple multiplication problems consisted of two vertically placed single-digit integers presented with a stated product. Problems with identical integers (ties) and problems including the integer 0 were excluded because of inconsistent performance with these problems in previous research (e.g., see Stazyk et al., 1982). Accordingly, simple multiplication problems consisted of the remaining 36 unique nontie and nonzero pairs of digits 1 through 9 and four randomly selected repeated pairs. The pairs of digits were then used as multiplier and multiplicand in simple multiplication problems, which ensured that the larger value integer was placed in the top position for half of the problems. Thus, each unique integer 1 through 9 appeared four or five times in the top position and four or five times in the bottom position across the 40 problems. The 40 incorrect problems consisted of the same 40 pairs of integers but with a stated product deviating from the correct product by ± 1, ± 2, or ± 10. Across the 40 problems, the stated product deviated from the correct product by ± 1 or ± 2 for 24 problems, and ± 10 for 16 problems. No repetition of either integer or of the stated product was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Complex multiplication. Each of the 80 complex multiplication problems consisted of a double-digit multiplicand placed vertically over a single-digit multiplier and presented along with a stated product. Multiplicands consisted of a sample of 40 of the 90 integers from 10 through 99. The integers 1 through 99 served as multipliers. Across the 40 problems the integers 1 through 9 served as the multiplier four or five times each; the units place for the multiplicand contained each integer 0 through 9 four times each; and the tens place for the multiplicand contained the integers 1 through 9 four or five times each. Within each problem, all digits were unique. The incorrect problems consisted of the same 40 pairs of multiplicands/multipliers but were presented with a stated product deviating from the correct product by ± 1, ± 2, ± 10, ± 20, or ± 100. The placement of these errors was counterbalanced across the stated product columns. That is, there were 14 errors in the units column, 14 errors in the tens column, and 12 errors in the hundreds column. No repetition of the stated product or of multiplicands or multipliers was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Apparatus. The arithmetic problems were presented at the center of a 30-cm × 30-cm video screen controlled by an Apple II Plus microcomputer. A Cognitive Testing Station clocking mechanism ensured the collection of RTs with ± 1 ms accuracy. Subjects were seated approximately 70 cm from the video screen and responded "true" by depressing a response button on the side of their dominant hand and "false" by depressing a response button using their nondominant hand.

For each problem, a READY prompt appeared at the center of the video screen for a 500-ms duration, followed by a 1000-ms period during which the screen was blank. Then, an arithmetic problem appeared on the screen and remained until the subject responded, at which time the problem was removed. If the subject responded correctly, the screen was blank for a 1000-ms duration, and the READY prompt for the next problem appeared. If the subject responded incorrectly, a WRONG prompt with a 1000-ms duration followed the removal of the stimulus and preceded the 1000-ms interproblem blank period.

Procedure. Subjects were tested individually in a quiet room. We told subjects that they were going to be presented with four individual sets of arithmetic problems in a set order: simple addition, complex addition, simple multiplication, and complex multiplication. They were told that their task was to respond "true" or "false"to each presented problem by pressing the appropriate button. Equal emphasis was placed on speed and accuracy. Subjects were told the type of problem to be presented before each set, and a practice set of eight problems was presented at the beginning of each set. Finally, a short rest period followed each of the sets. The entire testing session lasted approximately 45 min.

Ability Test Battery

Three sets of ability tests were used in the study: tests spanning the Spatial Relations, Numerical Facility, and Perceptual Speed factors. Three measures of each of these mental abilities were administered, and alternate forms of each individual measure were administered.

Spatial Relations. The three measures of Spatial Relations (Pellegrino & Kail, 1982) were the Mental Rotation Test (MRT; Vandenberg & Kuse, 1978), the Card Rotation Test (S-1; Ekstrom, French, & Harman, 1976), and the Cube Comparison Test (S-2; Ekstrom et al., 1976). Both forms of all three measures were administered. The score for each form, for all three measures, was the number of items correct minus the number of items incorrect. The total score for each measure was the sum of scores on the two forms.

Numerical Facility. The three measures of Numerical Facility were taken from the Educational Testing Service (ETS) test battery (Ekstrom et al., 1976). The three measures were the Addition Test (N-1), the Division Test (N-2), and the Subtraction and Multiplication Test (N-3). Both forms of all three measures were administered. The score for each form was the total number of items answered correctly. The total score for each measure was the sum of both forms.

Perceptual Speed. The three measures of Perceptual Speed were taken from the ETS test battery (Ekstrom et al., 1976). The three measures were Finding *A*s (P-1), Number Comparison (P-2), and Identical Pictures (P-3). Both forms of all three measures were administered. The score for each form of the Finding *A*s test was the total number of words marked correctly. The score for each form of the remaining two measures was the number of items correct minus the number of items incorrect. The total score for each measure was the sum of both forms.

Procedure. The nine ability tests were administered in a large classroom to subject groups ranging in size from 10 to 20 subjects. Each group completed the nine tests within a single testing session that lasted approximately 60 min. The nine tests were timed according to instructions in the manuals (Ekstrom et al., 1976; Vandenberg & Kuse, 1978) and were administered in the following order: MRT, S-1, S-2, N-1, N-2, N-3, P-1, P-2, and P-3. Approximately one half of the subjects participated in the group session before the reaction-time measures were administered, and the remaining subjects received the reversed order of conditions.

Analytic Procedures

To facilitate description of the analyses, the results section will be presented in three sections: information processing (IP), test battery, and combined (IP and test battery). Information-processing models were fit to RT data by using hierarchical regression techniques (Cohen & Cohen, 1983). Regression models were fit a priori, on the basis of the general processing model for mental addition proposed by Widaman et al. (1986).

Analyses of the ability measures and the combined data used structural equation modeling that followed the LISREL VI program (Jöreskog & Sörbom, 1984). Indexes of fit of structural models included both statistical and practical criteria. The statistical criterion was the likelihood ratio test statistic available with maximum likelihood estimation of parameters. The likelihood ratio statistic is distributed as a chi-square variable and reflects the difference in fit between a proposed restricted model and a completely saturated model (Bentler & Bonett, 1980). An acceptable restricted model would yield a statistically nonsignificant p value for the associated χ^2. However, the χ^2 measure is directly related to the sample size, and may yield an acceptable (nonsignificant) p value for a model that does not represent the data well if sample size is rather small. Alternatively, if sample size is rather large, even small residual covariances associated with a well-fitting model may lead to a significant χ^2 value, which would suggest rejection of the model.

Therefore, for the present study two measures of practical fit were chosen to aid in evaluating the goodness of fit of various structural models. Practical measures of fit are relatively unrelated to sample size. The two measures chosen were ρ (Bentler & Bonett, 1980) and the chi-square/degrees of freedom ratio used in recent studies (e.g., see Marsh & Hocevar, 1985). The measure ρ is a relative measure of covariation among variables explained by the model and is calculated in the following manner:

$$\rho = \frac{(\chi_n^2/df_n) - (\chi_s^2/df_s)}{(\chi_n^2/df_n) - 1}, \tag{1}$$

where χ_n^2 is the chi-square associated with the null model (estimating only unique variances), df_n is the degrees of freedom for the null model, χ_s^2 is the chi-square associated with a substantive model, and df_s is the degrees of freedom associated with the substantive model.

The χ^2/df ratio is a simple ratio of the chi-square associated with the substantive model divided by the degrees of freedom for the model. The χ^2/df ratio has an expected value of unity.

Substantive models yielding ρ values greater than .90 (Bentler & Bonett, 1980) or χ^2/df ratios less than 2.0 (e.g., see Marsh & Hocevar, 1985) are typically considered acceptable. These two criteria are inversely related and adopting both criteria provided somewhat redundant information. However, because there is no single generally accepted index for representing the practical goodness of fit of structural models, both measures of practical fit were presented to aid in accepting or rejecting alternative substantive models.

Results and Discussion

Information-Processing Tasks

Overall error rate in the matrix of 32,000 RTs was 4.9% (range = 3.2%–8.5% across sets), and less than 1.0% of the RTs were deleted as outliers (using Dixon's test; Wike, 1971). All analyses were performed with the error and outlier RTs excluded. Models for mental arithmetic were fit to average RT data using hierarchical regression techniques. Structural variables for the search/compute process included the five counting-based models proposed by Groen and Parkman (1972), the square of the correct sum (Ashcraft, 1982), and the correct product (Miller et al., 1984; Widaman et al., 1986). In addition, structural variables for important elementary processes (e.g., carrying to the next column) specified in the Widaman et al. model for mental addition were included in the regression equations.

Specifically, additional component processes were represented by structural variables estimating (a) intercept differences between correct and incorrect problems for verification tasks (truth: coded 0 for correct and 1 for incorrect problems), (b) speed of encoding digits (NI: coded the total number of digits in the problem including the stated sum, e.g., 6 or 7 for complex addition problems, but coded 3 for complex problems that were self-terminated following an error in the units column), and (c) speed of carrying to the next column for complex problems (carry: coded 0 for the absence and 1 for the presence of a carry). Furthermore, structural variables were coded so as to represent self-terminating processing of complex problems. Self-termination of a complex problem should occur if a units column error were encountered. At this step, the processing of the problem would stop, and the response "false" would be executed. No independent structural variable represented the self-terminating process; rather, structural variables for any process following a units column error (e.g., carry) were coded 0. The a priori assumption of self-terminating processing was based on results reported by Widaman and his colleagues (Geary, Little, Widaman, & Cormier, 1985; Widaman et al., 1986), which clearly demonstrated the superiority of models representing self-terminating processing relative to comparable models reflecting exhaustive processing of complex addition problems.

Independent regression models for simple addition and simple multiplication problems were fit using each of the seven search/compute structural variables and the truth parameter. Models for simple problems were initially fit with an independent structural variable specified so as to estimate the speed of encoding digits (i.e., NI). Estimates (about 50 ms) for the NI parameter were almost identical for simple addition and simple multiplication, were close to encoding speed estimates reported by Widaman et al. (1986), and were similar to speed estimates for retrieving name codes for letters (Hunt, Lunneborg, & Lewis, 1975; Posner, Boies, Eichelman, & Taylor, 1969). However, the partial F ratios for the NI parameter were not significant for either simple addition or simple multiplication. The nonsignificance of the NI variable likely resulted from a lack of variance in the number of integers in simple problems (i.e., 3 or 4). The NI parameter was therefore excluded from the final regression models for simple problems. As a result, speed of encoding digits was incorporated into the intercept term.

Models for complex problems were fit according to the processing stages outlined by Widaman et al. (1986). The regression equations were specified as assuming (a) column-wise processing of problems, in which sums or products for complex problems were obtained one column at a time and (b) self-termination of processing following a units column error. Final regression models for complex problems included structural variables for encoding speed (NI), search for/computing of the units column sum (or product), carrying to the next column (carry), search for/computing of the tens column sum (or product), and true/false intercept differences (truth). It is unclear what processes are represented by the truth parameter, although these processes may include, in part, a re-encoding of integers in false stated sums (Widaman et al., 1986). Regardless of the processes represented by this parameter, true/false intercept differences are often obtained in verification task performance (Farell, 1985) and are orthogonal to component processes involved in the solving of addition problems (Widaman et al., 1986). For complex multiplication problems, an additional parameter representing the value of the carry following the units column multiplication (carry remainder) was included in the regression equations. To illustrate, consider the problem 36 × 8. Following the units column multiplication (6 × 8), the remainder of this operation (4) must be held in short-term memory while concurrently performing the tens column multiplication; the carry remainder (4) must then be added to the provisional tens column product (24) to complete the problem. Table 1 presents a summary and further explanation for the coding values for each of the foregoing structural variables.

The *Number of items encoded* column in Table 1 indicates the number of digits in the problem, except when the stated units column answer is incorrect, in which case the digits in the tens column are not encoded, as we assumed the problem would be terminated at this point. The value coded for NI multiplied by its regression weight estimates the time required to encode all of the digits in the problem required for problem

Table 1
Summary of Coding of Values on Structural Variables

	Structural variables					
Problem	No. of items encoded	Units column product	Carry/self-terminate	Carry remainder	Tens column product	Truth
3 + 4 = 7	3	12	—	—	—	0
7 × 3 = 20	4	21	—	—	—	1
23 + 79 = 102	7	27	1	—	21	0
23 + 79 = 104[a]	3 (7)	27 (27)	0 (1)	— (2)	0 (21)	1
75 × 6 = 450	6	30	1	3	42	0
75 × 6 = 550	6	30	1	3	42	1

[a] In this problem, the stated units column answer is incorrect, and the problem is self-terminated. The processing of the carry operation and the tens column information does not ensue, and the coding changes for number of items encoded, carry, and the tens column product are changed accordingly. Thus, the coded values are appropriate for representing the self-terminating processing, whereas the values in parentheses reflect exhaustive processing of problems.

solution. The next column, *Units column product*, is the correct product of the digits presented in the units column, or simply the correct product for simple addition and simple multiplication. The value coded for this variable multiplied by its regression weight estimates the time required to retrieve the answer from long-term memory. The *Carry/self-terminate* variable is coded 1 for the presence of a carry operation, and 0 for the absence of a carry operation. If a problem is self-terminated because of an error in the stated units column, then carry is coded 0. The regression weight for the carry variable estimates the time required to execute this operation. The fifth structural variable column, *Tens column product*, is the correct product of the digits presented in the tens column, except when the problem is self-terminated, in which case this variable is coded 0. If a carry operation was required, a one was added to the first addend. Therefore, the value coded for the tens column was the sum of the first addend and the carry value multiplied by the second addend. The interpretation of this variable is identical to the interpretation of the units column product. As noted earlier, complex multiplication problems require a *carry remainder*. This variable, which is coded the value of the remainder following the units column multiplication, multiplied by its regression weight, estimates the amount of time required to increment the remainder onto the provisional tens column product. The final column in Table 1 presents the coding values for true–false intercept differences (*truth*). Truth is coded 0 for correct problems and 1 for incorrect problems. The response constant plus the regression weight for truth represents the intercept value for false problems.

Summary results for full model regression equations that included the three best predictive search/compute parameters for both addition and multiplication problems are presented in Table 2. Inspection of Table 2 reveals that for each problem type, independent regression models that included the same three search/compute structural variables provided the best description of RT. Reaction times to simple and complex forms of addition and multiplication were best fit by two variables representing retrieval of arithmetic facts from a memory network, the Prod and Sum^2 structural variables, and one variable reflecting a counting process, the Min structural variable. Each of these search or compute parameters was fit within an independent regression equation, and, as noted earlier, each regression equation included additional elementary component processes required for solution of the problem (e.g., carry). Furthermore, the existence of a self-terminating strategy was supported for both addition and multiplication problems: full model R^2s increased an average of .31 when equations were specified to reflect self-terminating processing, relative to comparable equations specified to reflect exhaustive processing.

Across the four problem types, equations including the product and column-wise product for simple and complex problems, respectively, provided better fit to RT than did models using the Sum^2 or the Min structural variables. Differences in the level of fit, however, for regression equations including Prod, Sum^2, and Min do not appear to be large. The three structural variables are highly correlated and statistically differentiating the variables appears difficult. However, for each type of problem we tested statistically the difference in level of fit between the regression equation including the Prod structural variable and the equation showing the next best level of fit. First, the structural variable (Sum^2 or Min) showing the second best level of fit was included in the regression equation that included the Prod variable. Next, the Sum^2 or Min variable was dropped

Table 2
Full Model R^2s for the Three Best Fitting Search/Compute Parameters for Addition and Multiplication

Search/Compute parameter	Addition		Multiplication	
	Simple	Complex	Simple	Complex
Min	.681	.859	.707	.865
Sum^2	.689	.866	.684	.866
Prod	.737	.867	.721	.878

Note. Min = column-wise minimum addend; Sum^2 = square of the correct column-wise sum; Prod = column-wise product. All models are significant, $p < .0001$.

Table 3
Statistical Summaries of Regression Analyses: Product Structural Variable

Equation	R^2	F	df	MS_e
	Addition			
Simple				
RT = 889 + 9.87 (Prod) + 156 (Truth) Partial *F*s = 180.54, 35.0	.737	107.77	2,77	118.4
RT = 1,151				
Complex				
RT = 727 + 179 (NI) + 8.04 (Unitprod) + 367 (Carryst) + 8.04 (Tenprod) + 232 (Truth) Partial *F*s = 91.40, 54.46, 91.40, 54.46, 18.82	.867	122.38	4,75	197.8
RT = 2,272				
	Multiplication			
Simple				
RT = 904 + 10.06 (Prod) + 155 (Truth) Partial *F*s = 170.92, 23.09	.721	99.50	2,77	131.4
RT = 1,232				
Complex				
RT = 1,143 + 105 (NI) + 13.53 (Unitprod) + 482 (carryst) + 13.53 (Tenprod) + 160 (Carrem) + 191 (Truth) Partial *F*s = 5.17, 31.38, 12.88, 31.38, 14.84, 4.57	.878	104.78	5,74	362.0
RT = 2,840				

Note. All models are significant, $p < .0001$; all partial F ratios are significant, $p < .01$. Prod = product; Truth = intercept differences comparing true with false problems; NI = number of items encoded; Unitprod = units column product; Carryst = self-terminating carry operation; Carrem = the value of the remainder following the units column multiplication; Tenprod = tens column product.

from the full model equation, and the decrease in the R^2 associated with dropping the variable was tested using an incremental F test (Cohen & Cohen, 1983). The Prod variable was then dropped from the full model equation (Sum^2 or Min was added back into the equation), and the decrease in the R^2 associated with dropping the Prod variable was tested. The significance of the F test indicated the importance of the dropped search/compute variable "above and beyond" the importance of the alternative search/compute variable in explaining RT variance.

Results from the foregoing procedures indicated that dropping the Sum^2 or Min variable from the full model equation never significantly decreased the regression model R^2 (all $ps > .25$). However, dropping the Prod variable from the full model equation resulted in statistically significant decreases in the regression model R^2 for simple addition, $F(1, 76) = 12.87$, $p < .01$, simple multiplication, $F(1, 76) = 5.75$, $p < .05$, and complex multiplication, $F(1, 73) = 5.68$, $p < .05$, but not for complex addition, $F(1, 74) = 1.54$, $p > .10$.

The better fit of regression models incorporating the Prod structural variable for each of the problem types, when compared with all other alternative search/compute parameters, is consistent with recent research in the mental arithmetic area (Miller et al., 1984; Widaman et al., 1986) and suggests a somewhat different memory network for arithmetic facts than the network proposed by Ashcraft and Battaglia (1978). As we noted earlier, the memory network reflected by the product variable is conceptualized as a square, symmetric matrix, with orthogonal axes representing nodes for the two integers to be added or multiplied. Following Ashcraft and his colleagues (Ashcraft, 1982; Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981), we assume that correct answers are stored at the intersection of nodal values corresponding to the single-digit numbers to be added or multiplied. The network is entered at the origin, and the rate of activation of information in the network is assumed to be a constant function of the area of the network activated. The product structural variable represents the area of the network activated, and the product is then linearly related to search time required to arrive at the correct answer (Widaman et al., 1986). The precise nature of the memory network for arithmetic facts is still a topic of debate (e.g., see Hamann & Ashcraft, 1985; Siegler & Shrager, 1984). However, the product structural variable provided a better representation of the long-term memory network for addition and multiplication facts for each type of problem than did any alternative structural variable.

The Prod structural variable, however, is not an essential aspect of the Widaman et al. (1986) model, and the present analysis may not be definitive because of both quite high correlations among alternative structural variables included in our study and the high correlation of these variables with indexes reflecting subjective difficulty of problems or frequency of problem presentation (e.g., Wheeler, 1939). The Prod structural variable is used to represent the memory network retrieval process, which is only one of several processes presumably involved in solving arithmetic problems. It might well be that an index reflecting the associative strength of each problem with potential answers to the problem (e.g., Siegler & Shrager, 1984) may later provide a better statistical fit than does the Prod variable. In this case, the associative strength index would be preferable to the Prod variable for representing the memory retrieval process within the framework of the overall model specified by Widaman and his associates (Widaman et al., 1986). Next, the full model equations, including Prod, for each of the four types of arithmetic problems will be discussed.

Simple addition. The best fitting full model regression equation for each problem type is presented in Table 3. Inspection of the first equation in Table 3 reveals, as noted earlier, that RT to simple addition problems was best predicted by the Prod structural variable, along with the truth variable, $R^2 = .737$. The regression estimate for the truth variable was identical across the equations including the Prod, Sum^2, and Min, which suggests that the process for arriving at the correct sum was independent of the process reflected by the truth parameter (Sternberg, 1969). The structural independence of the two variables included in the simple addition equation was supported by the finding that the Prod by truth interaction was very nonsignificant, $F(1, 76) = 0.00$, $p > .90$.

Complex addition. The second equation presented in Table 3 is the best fitting full model equation predicting RT to complex addition problems. Inspection of this equation reveals that, in addition to the column-wise product parameter, all of the structural variables representing independent elementary component processes specified in the Widaman et al. (1986) model had highly significant partial F ratios (all $ps < .01$). For this equation, the Prod structural variable was initially estimated separately for each column. Inspection of the initial results revealed the column-wise slope estimates to be highly similar. Accordingly, the product slope estimates for the units and the tens columns were constrained to be equal. The significance of this equality constraint was evaluated using an incremental F test (Cohen & Cohen, 1983) of the significance of the decrease in R^2 associated with enforcing the equality constraint. Constraining column-wise slope estimates to be equal resulted in a rather nonsignificant decrease in the full model R^2, $F(1, 75) = 0.17$, $p > .50$. Identical slope estimates for the units and the tens columns are therefore presented in the second equation.

Finally, product by truth interactions were estimated separately for each column, and the significance of these interactions was tested with an incremental F test. The addition of these interactions to the second equation resulted in a nonsignificant increase in the full model R^2, $F(2, 72) = 2.00$, $p > .10$. Accordingly, the memory retrieval process appears to be orthogonal to the process modeled by the truth variable, as was found for simple addition.

Simple multiplication. The best fitting equation predicting RT to simple multiplication problems is presented as the third equation in Table 3. Inspection of the third equation in Table 3 reveals that RT to simple multiplication problems was best predicted by structural variables identical to those best predicting RT to simple addition problems, that is, the Prod and truth variables, $R^2 = .721$. Furthermore, comparison of the first and third equations in Table 3 reveals highly similar intercept values and regression weights for Prod and truth for simple addition and multiplication. As with the preceding types of problems, the independence of the memory retrieval and truth processes was supported by the finding that the Prod by truth interaction was not significant, $F(1, 76) = 1.27$, $p > .25$.

Complex multiplication. The final equation in Table 3 presents the best fitting full model regression equation predicting RT to complex multiplication problems. Inspection of the final equation in Table 3 reveals that, in addition to the column-wise product, all of the structural variables representing elementary component processes specified in the Widaman et al. (1986) model had highly significant partial F ratios (all $ps < .01$). With the exception of the carry remainder parameter, which was unique to complex multiplication, structural variables best predicting RT to complex multiplication problems were identical to the structural variables best predicting RT to complex addition problems; regression weigh estimates for parallel structural variables did, however, vary somewhat across operations. For this final equation, the Prod structural variable was once again initially estimated separately for each column. Inspection of these results revealed the column-wise slope estimates to be highly similar. Accordingly, the parameter estimates for the Prod variable for the units and the tens columns were constrained to be equal. Constraining column-wise slope estimates to be equal resulted in a rather nonsignificant decrease in the full model R^2, $F(1, 74) = 0.02$, $p > .90$. Identical slope estimates for the units and the tens columns are therefore presented in the final equation in Table 3.

Finally, product by truth interactions were estimated separately for each column slope, and the significance of these interactions was tested with an incremental F test. The addition of these interactions to the final equation in Table 3 again resulted in a nonsignificant increase in the full model R^2, $F(2, 71) = 2.29$, $p > .05$.

Summary of IP models for addition and multiplication. Regression equations were specified independently for the prediction of RT to simple and complex addition and multiplication problems to represent the component processes outlined in the general model for mental addition proposed by Widaman et al. (1986). The intensive validity of the Widaman et al. model was demonstrated independently for each of the four problem types, as full regression models provided a good representation of RT for each problem type ($R^2 = .72$–$.88$). Furthermore, important elementary component processes, such as carrying to the next column, were statistically reliable and of interpretable magnitude for each of the four types of arithmetic tested.

Extensive validity of the Widaman et al. model was also demonstrated with the IP results. Specifically, identical structural variables, where appropriate, were found to accurately represent the processing components necessary to mentally solve each of the four types of arithmetic. Extensive validity of the model was supported further with the finding that regression estimates for parallel structural variables were generally of a comparable and an interpretable magnitude across the four

Table 4
Descriptive Statistics for Measures in the Ability Test Battery

Test	*M*	*SD*	Spearman-Brown reliability estimates
Spatial Relations			
Mental rotation	12.13	8.66	.729
Card rotation	101.05	37.51	.908
Cube comparison	17.30	9.51	.759
Numerical Facility			
Addition	42.62	11.91	.894
Division	37.40	14.08	.905
Subtraction/multiplication	62.74	19.35	.939
Perceptual Speed			
Finding *A*s	65.75	17.04	.871
Number comparison	25.88	5.79	.790
Identical pictures	76.02	14.10	.850

types of cognitive arithmetic. For example, the increase of 74 ms in the NI parameter for complex addition relative to the regression estimate for the NI parameter for complex multiplication is consistent with the linear increase in encoding time associated with each added digit in addition problems found by Widaman et al. (1986).

The mental solving of simple and complex forms of both addition and multiplication problems requires identical component processes, where appropriate, with similar execution times for parallel processes. Specifically, addition and multiplication problems are processed in a column-wise fashion. Column-wise sums or products are retrieved from an interrelated memory network of arithmetic facts, and complex problems are self-terminated when an error in the units column of the stated sum or product is encountered. Important additional elementary components for the mental solving of addition and multiplication problems are encoding of single integers and carrying to the next column for complex problems.

Structural Models for the Ability Test Battery

Table 4 presents descriptive statistics and reliability estimates for the three sets of ability tests. Total score (form 1 + form 2) reliability estimates, obtained with the Spearman–Brown prophecy formula, ranged from .73 to .94 and were comparable to reliability figures reported in the ETS manual (Ekstrom et al., 1976).

Pearson product-moment correlations among the ability tests were computed. The factor structure of the ability tests was assessed by fitting a confirmatory factor analytic model to the data by using the LISREL VI program (Jöreskog, 1969; Jöreskog & Sörbom, 1984). First, a null model hypothesizing no common factors was estimated for the correlation matrix. The null model was clearly rejectable, $\chi^2(36) = 368.39$, $p < .000$. Next, on the basis of previous research (Coombs, 1941; Ekstrom et al., 1976; French, 1951; Pellegrino & Kail, 1982; Thurstone & Thurstone, 1941) a three-common-factor model was formulated. The three hypothesized factors were Spatial Relations, Numerical Facility, and Perceptual Speed; the indicators for these factors were as denoted in Table 4. The loadings of each ability test on its respective common factor were estimated, as were interfactor correlations, in the first nested model. This first model showed acceptable statistical, $\chi^2(24) = 34.28$, $p < .080$, as well as practical indexes of fit ($\rho = .954$; $\chi^2/df = 1.43$). However, modification indexes provided by the LISREL VI program (Jöreskog & Sörbom, 1984) indicated that this model could be substantially improved by allowing the Identical Pictures Test to load on the Spatial Relations factor. The estimation of this loading, which led to the second model, resulted in a statistically significant improvement in model fit, $\chi^2(1) = 7.06$, $p < .01$, better overall statistical, $\chi^2(23) = 27.22$, $p = .247$, and practical indexes of fit ($\rho = .980$; $\chi^2/df = 1.18$). Parameter estimates for the second model are presented in Table 5.

Inspection of Table 5 reveals that all of the factor loadings were of reasonable magnitude and were statistically significant, as each factor loading was at least twice as large as its standard error ($p < .05$). Thus, the expected factor structure of the ability tests was confirmed, with the slight modification that the Identical Pictures Test loaded on two factors. Furthermore, the presented solution provides empirical support for the highly similar factor solutions and substantive interpretations based on exploratory factor analyses of the same or similar ability measures (Coombs, 1941; Pellegrino & Kail, 1982; Thurstone & Thurstone, 1941). The only unexpected result of the analysis was the very high correlation between the Numerical Facility and Perceptual Speed factors, which was probably due to the highly speeded and overlearned nature of the two types of tasks (Coombs, 1941).

Structural Models for the Combined Data

Regression weights, or component scores, for each subject for all structural variables in the information-processing (IP) analyses were obtained. Inspection of individual-level regression weights revealed that five subjects had rather large negative weights for several IP parameters. Data for these five subjects were therefore eliminated. Furthermore, the carry remainder parameter for complex multiplication had no counterpart from the other types of problems and was also eliminated from the analyses. Finally, initial structural equation models were fit with the NI (encoding speed) parameter estimates from the complex addition and multiplication tasks defining the same latent variable as the four intercept values and then, in a separate analysis, defining an independent factor. The loadings for the NI variables were rather unstable, probably because of the high negative correlations with their respective intercept values (−.76 for complex addition and −.84 for complex multiplication); therefore, regression equations for the complex problem types were re-estimated with no independent structural variable for encoding speed. As a result, encoding speed was incorporated into the intercept value in all equations.

All remaining component scores derived from the four IP problem types were used in the combined analyses. These component scores included the four intercept values, four estimates of intercept differences between true and false problems (truth), four memory-search rate estimates for the Prod structural variable, and estimates for performing the carry operation in the

Table 5
Results of Confirmatory Factor Analysis of Measures in the Ability Test Battery

Variable	Factor: Spatial Relations	Perceptual Speed	Numerical Facility	Unique variance
		Factor pattern		
Mental rotation	.679* (.100)			.539* (.100)
Card rotation	.805* (.098)			.352* (.099)
Cube comparison	.714* (.099)			.490* (.098)
Identical pictures	.312* (.110)	.387* (.109)		.666* (.105)
Number comparison		.784* (.101)		.385* (.104)
Finding *A*s		.598* (.108)		.642* (.107)
Addition			.862* (.084)	.257* (.060)
Division			.747* (.090)	.442* (.075)
Subtraction/multiplication			.877* (.084)	.232* (.059)
		Factor intercorrelations		
Factor				
Spatial Relations	—			
Perceptual Speed	.362* (.126)	—		
Numerical Facility	.203 (.116)	.796* (.075)	—	

Note. The latent variable variances were fixed in order to identify the model. Tabled values are loading estimates; associated standard errors are in parentheses. Empty cells signify parameters fixed at zero.
* $p < .05$.

two complex problem types. In all, component scores for 14 structural variables across the four arithmetic problem types were used from the IP analyses.

Pearson product-moment correlations among the component scores for the 14 IP variables and the nine ability tests were computed. The resulting correlation matrix was analyzed with the LISREL VI program (Jöreskog & Sörbom, 1984). First, a null model was specified, a model embodying the hypothesis of zero correlation among all variables. Table 6 presents overall goodness-of-fit indexes for all of the structural equation models. Inspection of Table 6 reveals that the null model had a rather unacceptable level of statistical fit, $\chi^2(253) = 1{,}318.4$, $p < .000$. Thus, the hypothesis of lack of correlation among the IP components and the ability tests was clearly rejectable.

Next, the initial measurement model, termed Model 1, was estimated. Model 1 included the three common factors for the measures in the test battery, factors described earlier, and four trait factors for the IP variables. The IP trait factors consisted of (a) a Memory Search latent variable for which the component scores for the Prod structural variable for each problem type served as indexes, (b) a Carry latent variable for which the component scores for the carry operation from the two complex

Table 6
Goodness-of-Fit Indexes for Structural Equation Models Relating Information-Processing Parameters to Ability Test Measures

Model	*df*	χ^2	*p*	χ^2/df	*p*
		Overall fit of alternate models			
Null	253	1,318.4	.001	5.21	—
1: Seven trait factors and four method factors	215	559.6	.001	2.60	.619
2: Model 1 plus two correlated second-order trait factors	212	470.8	.001	2.22	.710
3: Model 2 plus direct path from Arithmetic Processing second-order factor to Numerical Facility test factor	211	423.1	.001	2.01	.761
4: Model 3 plus direct path from Speed second-order factor to Perceptual Speed test factor	210	393.3	.001	1.87	.793
5: Model 4 plus four correlated uniqueness terms[a]	206	292.2	.001	1.42	.901

[a] The four correlated uniqueness terms were (a) the column-wise product variable for complex addition with the carry variable for complex addition (−.289), (b) the column-wise product variable for complex multiplication with the product variable from simple multiplication (.166), (c) the simple addition intercept with the product variable from simple multiplication (.166), and (d) the complex addition intercept with the truth variable from complex addition (−.144).

Table 7
Indexes of Difference Between Nested Structural Equation Models Relating Information-Processing Parameters to Ability Test Measures

Comparison	Differences χ^2	df	p	Differences χ^2/df	ρ
Null vs. Model 1	758.8	38	.001	2.61	—
Model 1 vs. Model 2	88.8	3	.001	0.38	.091
Model 2 vs. Model 3	47.5	1	.001	0.21	.051
Model 3 vs. Model 4	29.8	1	.001	0.14	.032
Model 4 vs. Model 5	101.1	4	.001	0.45	.108

problem types served as indicators, (c) a Truth factor with loadings from each of the four component scores for the truth variables, and (d) a combined Encoding-Decision-Response latent variable with loadings estimated for each of the four intercept scores. Model 1 also included one method factor for each of the four IP problem types. That is, all parameter estimates from simple addition loaded on one method factor, all parameter estimates for complex addition loaded on a second method factor, and so forth for the parameter estimates for simple and complex multiplication (several of these relations were estimated as correlated uniqueness terms in the final structural model, see Table 6). The four method factors were included because regression parameters from the same equations may have idiosyncratic patterns of intercorrelation. Specification of the method factors would therefore isolate any idiosyncratic parameter intercorrelations from the substantive portion of the structural models. Three-method-factor correlations were allowed, and the three correlations among the ability test latent variables were allowed. All other latent variable intercorrelations were fixed at zero, and all nondefining factor loadings were fixed at zero.

Table 7 presents indexes of difference in fit between nested structural equation models. Inspection of Table 7 reveals that estimation of Model 1, which included the seven trait and four method factors, resulted in a rather large improvement in statistical fit, $\chi^2(38) = 758.8$, $p < .001$, as well as a large decrease in the χ^2/df ratio (Δ 2.61). However, as was shown in Table 6, Model 1 had unacceptable levels of both statistical fit, $\chi^2(215) = 559.6$, $p < .0001$, and practical fit, χ^2/df ratio = 2.603 and $\rho = .619$. Thus, Model 1 did not provide an adequate representation of these data.

Theoretical considerations as well as modification indexes were used to improve the level of fit for these data; the first modification, adding the specification of two second-order factors for the four IP trait factors, resulted in Model 2. Two first-order IP factors, Memory Search and Carry, represented basic arithmetic processes and loaded on the first second-order factor, labeled Arithmetic Processes. The remaining two first-order IP factors represented basic speed processes, such as response time, and loaded on the other second-order factor, labeled Speed. To identify the model, the two factor loadings for each of the second-order factors were constrained to be equal. The loadings of two first-order factors on a given second-order factor are based on the single correlation between the two first-order factors. If an equality constraint were not imposed, the LISREL program would estimate separate loadings for each of the first-order factors. However, in such cases, the separate factor loadings are often highly unstable even if they are mathematically identifiable. The equality constraints were therefore invoked to improve the empirical identification of parameter values (Jöreskog & Sörbom, 1984). Inspection of Table 7 reveals that estimation of the two correlated second-order factors resulted in a significant improvement in statistical fit, $\chi^2(3) = 88.8$, $p < .001$, as well as improvements in both the χ^2/df ratio (Δ 0.38) and in the ρ value (Δ 0.091). However, Model 2 did not provide an acceptable representation of these data, as neither the χ^2/df ratio (2.22) nor the ρ value (.710) attained acceptable levels, as was shown in Table 6.

Further respecification of the structural equation model was based both on theoretical considerations and modification indexes. Theoretically, component processes derived using the additive factors paradigm (Sternberg, 1969) should represent the processes underlying measured ability on traditional mental tests (Sternberg, 1980; Hunt et al., 1975). On the basis of this premise, directed paths were estimated between IP factors and ability test factors; the directed paths with the largest modification indexes were estimated. For Model 3, a direct path from the second-order Arithmetic Processes factor to the Numerical Facility common factor was estimated. As was shown in Table 7, allowing this direct path resulted in a significant improvement in the model chi-square, $\chi^2(1) = 47.5$, $p < .001$, as well as improvements in both the χ^2/df ratio (Δ 0.21) and in the ρ value (Δ .051). However, inspection of Table 6 reveals an unacceptable χ^2/df ratio (2.01) and ρ value (.761) for the overall model.

Next, a direct path from the second-order Speed factor to the Perceptual Speed common factor was estimated, which resulted in Model 4. Model 4 fit the data significantly better than did Model 3, both statistically, $\chi^2(1) = 29.8$, $p < .001$, and practically ($\Delta\ \chi^2/df = .14$, and $\Delta\ \rho = .032$; see Table 7). As shown in Table 6, the χ^2/df ratio (1.87) was acceptable for Model 4, but the ρ value (.793) was still unacceptable. Therefore, to improve the level of fit of the structural equation model, correlated uniqueness terms were added sequentially, based on modification indexes and substantive considerations. The addition of four correlated uniqueness terms to Model 4 resulted in a rather low χ^2/df ratio (1.42) and an acceptable ρ value (.901), as well as a highly significant improvement in the model chi-square, $\chi^2(4) = 101.1$ $p < .001$. Thus, Model 5 was accepted as providing an adequate representation of these data. In terms of statistical fit, all models were rejected, including Model 5, which suggested that further improvement in model fit might have been possible (Jöreskog & Sörbom, 1984). However, examination of the modification indices from Model 5 indicated that any respecification of the model would not have led to substantial improvements in model fit.

Trait-, method-, and unique-factor loadings for Model 5 are presented in Table 8. Inspection of Table 8 reveals that all of the trait factor loadings were of a reasonable magnitude and were statistically significant. Furthermore, the trait factor loadings were generally higher than were the method factor loadings.

Table 8
Estimates From Structural Equation Model 5

	Trait factor		Method factor		Unique factor	
Observed measures	Loading	*SE*	Loading	*SE*	Variance	*SE*
			Information-processing parameters			
Memory search parameter						
Simple addition	.862	.135	.372[a]	.081	.107	.066
Complex addition	.745	.131	.426[b]	.081	.267	.081
Simple multiplication	.519	.100	.532	.077	.310	.072
Complex multiplication	.524	.111	.659	.083	.350	.078
Carry parameter						
Complex addition	.718	.146	—	—	.524	.112
Complex multiplication	.673	.134	.153	.091	.517	.100
Truth parameter						
Simple addition	.564	.180	.372[a]	.081	.555	.109
Complex addition	.310	.139	.426[b]	.081	.718	.124
Simple multiplication	.436	.148	.592	.098	.380	.094
Complex multiplication	.568	.181	.399	.094	.514	.100
Intercept						
Simple addition	.686	.175	—	—	.430	.071
Complex addition	.888	.218	—	—	.227	.054
Simple multiplication	.532	.160	−.669	.102	.406	.095
Complex multiplication	.748	.188	.628	.062	.050[c]	.000
			Ability tests			
Numerical facility						
Addition	.877	.176	—	—	.241	.052
Division	.774	.162	—	—	.410	.070
Subtraction/multiplication	.866	.174	—	—	.260	.054
Perceptual speed						
Identical pictures	.419	.114	—	—	.622	.101
Finding *A*s	.586	.125	—	—	.640	.107
Number comparison	.759	.142	—	—	.398	.096
Spatial relations						
Mental rotation	.675	.103	—	—	.545	.103
Card rotation	.809	.101	—	—	.345	.101
Cube	.717	.102	—	—	.485	.101
Identical pictures	.312	.106	—	—	.622	.101

Note. All reported loadings are significant, $p < .05$, except for the method factor loading for the multiplication carry operation, which dropped to nonsignificance with the addition of the correlated residuals. All remaining, nonreported loadings were fixed at zero.
[a] Parameter estimates constrained to be equal.
[b] Parameter estimates constrained to be equal.
[c] Parameter fixed at this value.

To achieve stability of factor loadings and to improve the empirical identification of parameters in the model, equality constraints were imposed for the two simple addition method factor loadings and for the two complex addition method factor loadings.

In Figure 1, the final structural relations among the seven first-order trait factors and the two second-order factors from Model 5 are presented. The important estimates of structural relations between IP and ability test measures, embodied in the path coefficients for the directed paths from the second-order IP factors to the ability test common factors, were rather large. The estimate for the directed path from the second-order Arithmetic Processes factor to the Numerical Facility common factor was extremely large, −.879, and provided results analogous to IP-trait factor relations reported for other domains in previous research (Lansman, 1981; Palmer, MacLeod, Hunt, & Davidson, 1985; Sternberg & Gardner, 1983). The path coefficient shows that the second-order Arithmetic Processes factor explained 77% of performance variability on the latent variable defined by the traditional numerical facility tests. On the basis of this relation, the substantive component processes underlying numerical facility appear to be speed of executing the elementary operations of retrieval of information from a network of stored arithmetic facts and execution of the carry operation.

The remaining directed path from the second-order Speed factor to the Perceptual Speed common factor was estimated on the basis of substantive considerations as well as a large modification index for this relation. The second-order Speed factor explained 50% of performance variability on the Perceptual Speed factor. The component processes that were included as indicators of the second-order Speed factor comprised encoding

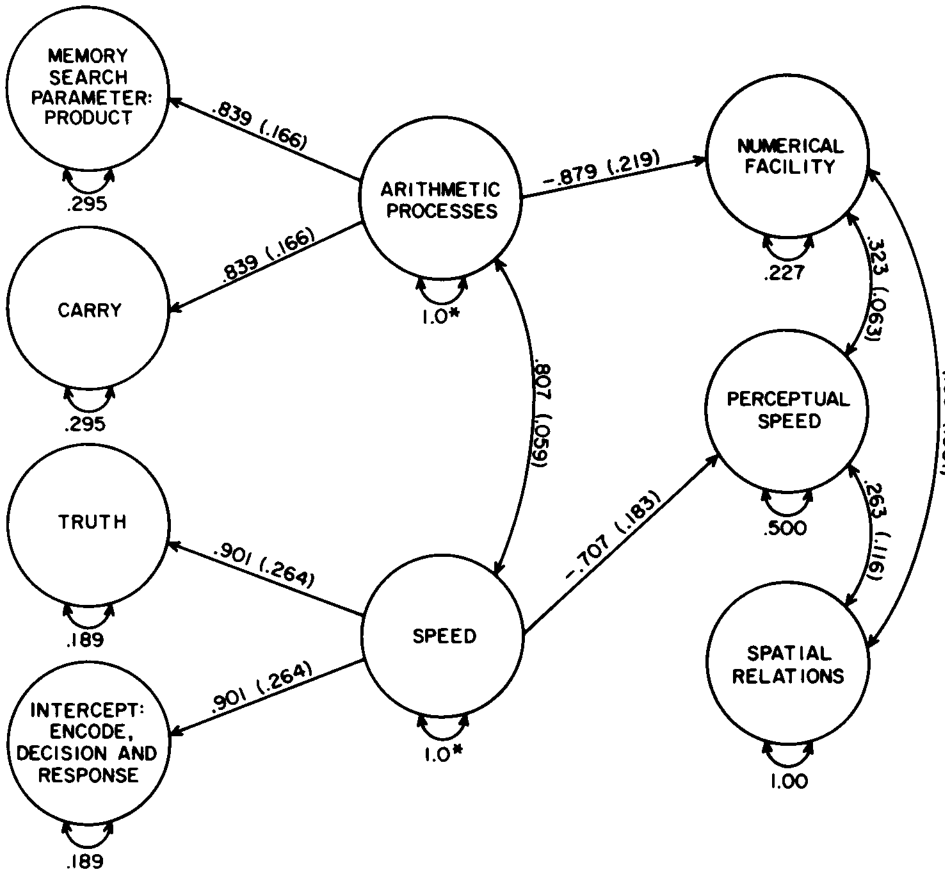

Figure 1. Standardized estimates from Model 5 of structural relations among first- and second-order information-processing trait factors and test battery factors.

of single digits, decision, and response times (all reflected in the intercept estimates) and RT latency differences between true and false problems (associated with the truth parameter estimates). The Speed second-order factor appears to represent simpler component processes related to speed of encoding overlearned information and making noncomplex decisions.

Separation of the second-order IP factors was justified on both theoretical and statistical grounds. First, the original indexes of the second-order factors were substantively different. The original indexes for the Arithmetic Processes factor represent theoretically important process components for solving arithmetic problems: memory network search (Ashcraft, 1982; Miller et al., 1984) and carrying to the next column (Ashcraft & Stazyk, 1981; Widaman et al., 1986). In contrast, the indexes for the Speed factor reflect elementary processes that might be involved in other types of RT tasks, such as decision and response time in many types of verification tasks. Thus, processes indexed by the second-order Arithmetic Processes factor are unique to the mental solving of arithmetic problems, whereas processes indexed by the Speed factor are processes potentially identifiable across a variety of IP tasks. Finally, forcing the two second-order factors to correlate perfectly resulted in a statistically significant worsening of fit, $\chi^2(1) = 61.0$, $p < .001$, and an unacceptable ρ value (.830).

Examination of modification indexes for both Model 4 and Model 5 revealed that no other directed paths from the IP latent variables to the ability test factors would have substantially improved the fit to these data (i.e., indexes were < 5.00). Thus, for these data, there were no significant direct relations between the IP latent variables and the Spatial Relations factor.

In summary, the combined analysis indicated a rather strong relation between a subset of component processes for mental arithmetic and performance on traditional measures of numerical facility. Speed of information retrieval from long-term memory and facility of executing a carry operation were highly related to performance on traditional numerical facility tests; the greater the facility in executing these two component processes, the higher the score on the Numerical Facility factor. Furthermore, the component processes subsumed by the Arithmetic Processes factor appear to be unique to the mental solving of arithmetic problems, as no direct relation between the Arithmetic Processes factor and either the Perceptual Speed or the Spatial Relations factor was required by the data. The finding of no direct relation between the Perceptual Speed factor and the Arithmetic Processes factor indicates that the processes subsumed by the latter factor represent psychological operations that are distinct from simple speed-of-processing information. However, replication of these results is necessary because of the rather ad hoc nature of portions of the model fitting.

General Discussion

Factor-analytic studies of traditional paper-and-pencil measures have repeatedly suggested unities in the processes underlying ability tests that span the Numerical Facility factor (i.e., addition, multiplication, subtraction, & division; Coombs, 1941; Spearman, 1927; Thurstone & Thurstone, 1941). Furthermore, studies that have used chronometric techniques have identified similar processes involved in the mental solving of arithmetic problems of different operations (Miller et al., 1984; Parkman, 1972; Siegler & Shrager, 1984; Svenson & Hedenborg, 1979). The present study demonstrated a strong convergence in the operations assessed by traditional paper-and-pencil measures of numerical facility and two elementary component processes involved in the mental solving of addition and multiplication problems that were identified by using chronometric techniques. The substantive elementary component processes representing the numerical facility construct were information retrieval from a long-term memory network of arithmetic facts and carrying to the next column for complex problems. These processes were identified as important elementary component processes in the Widaman et al. (1986) model for mental addition. These two component processes have also been identified by others (Ashcraft & Stazyk, 1981; Miller et al., 1984) as important component processes involved in the mental solving of arithmetic problems. However, prior to the Widaman et al. study these processes were not encompassed within a theoretically based cognitive components model (Pellegrino & Glaser, 1979) for mental arithmetic.

The convergence in operations assessed by measures defining the Numerical Facility factor and a subset of component processes specified in the cognitive components model for addition provided strong support for the convergent validity of the substantive processes specified in the model proposed by Widaman and his colleagues (Widaman et al., 1986). The discriminant validity of the component processes specified in the model was supported with the finding of no direct relation between component scores for the substantive processes underlying cognitive arithmetic, information retrieval from a long-term memory network of arithmetic facts and the carry operation, and the Perceptual Speed and Spatial Relations factors. Thus, the present study provided strong support for the external validity, both convergent and discriminant, of the cognitive components model proposed by Widaman et al.

Both the intensive and extensive types of internal validity of the Widaman et al. (1986) model were strongly supported by the results of the study. Intensive validity was clearly supported with the finding that each of the component processes, where appropriate, specified in the model for mental addition proved to be important and psychologically meaningful variables in the representation of RT to each of the four types of arithmetic assessed: simple addition, complex addition, simple multiplication, and complex multiplication. The intensive validity of the mental addition model for representing both simple and complex forms of multiplication problems provided evidence for the general utility of the model for representing the component processes involved in solving arithmetic problems of operations other than addition. The extensive validity of the cognitive arithmetic component processes was supported by the factor structure of the component scores for problems of different arithmetic operations and for arithmetic problems of varying complexity. Specifically, structural variables representing theoretically identical component processes across the four types of arithmetic problem always defined a single factor and were concurrently estimated with loadings of zero on nondefining factors. The factor structure of the component scores also demonstrated the utility of the factor-analytic method for representing the underlying structure of psychologically similar and psychologically distinct component processes spanning an array of cognitive tasks.

The finding of a moderate relation between the second-order Speed factor, which subsumes speed of encoding digits and decision and response times, and the Perceptual Speed factor appears to represent a simple, predictable outcome; persons more efficient at encoding highly overlearned symbols (e.g., digits) and making and executing simple decisions are faster at the simple perceptual discriminations required by perceptual speed tests. This finding is similar to one in a study by Lansman (1981), in which a significant negative correlation was reported between component scores for an encoding process in a sentence verification task and Perceptual Speed factor scores. The preceding findings do, however, stand in contrast to findings from studies of reasoning tasks by Sternberg and his associates. For example, Sternberg (1977) found component scores for encoding in analogical reasoning tasks to be more strongly related to scores on a Reasoning factor than to scores on a Perceptual Speed factor, and Sternberg and Gardner (1983) reported low, nonsignificant correlations between encoding component scores based on inductive reasoning tasks and both Reasoning and Perceptual Speed factor scores. The key to resolving these contrary sets of findings lies most probably with the complexity of the stimuli and discriminations involved. Tests that typically define the Perceptual Speed factor index the speed of making simple perceptual comparisons, consistent with traditional interpretations of the factor (e.g., Thurstone & Thurstone, 1941) and not unlike the encoding of digits required in the arithmetic tasks in the present study. Thus, the finding from the present study of a strong direct relation between the second-order Speed factor and the Perceptual Speed factor was not unexpected.

On the other hand, in the studies by Sternberg (1977) and Sternberg and Gardner (1983), the stimuli consisted of verbal material more complex than simple digits. On such tasks, it is likely that there are individual styles in the way in which stimuli are encoded; such individual styles would affect process duration, but they would also likely be more strongly related to some form of strategy choice involving reasoning about the demands of the given problem than to the efficiency in making simple perceptual discriminations that is at the core of the construct of perceptual speed. Thus, Sternberg (1977, pp. 244–247) concluded that his method of data collection probably led to encoding component scores that were confounded with strategy choice variance. In summary, studies that use such complex stimuli should find lower correlations of encoding component scores with tests of perceptual speed than with tests of reasoning. On the other hand, if rather simple and overlearned stimuli are used, as they are in the present study, then there is a strong theoretical link between the encoding process and individual differences tapped by tests of perceptual speed, a theoretical link that should be accompanied by a direct empirical relation.

In more general terms, the present study provided evidence for continuity of intellectual abilities identified with the use of the factor-analytic method and elementary component processes isolated with the use of chronometric techniques. Continuity in theoretically similar abilities assessed by the two methods was also evident in the research conducted by Hunt and his colleagues (Hunt et al., 1975, Lansman, 1981; Palmer et al., 1985), as well as in research conducted by Sternberg (1977; Sternberg & Gardner, 1983). Furthermore, there appears to be an interesting parallelism in the patterns of correlation among component scores and traditional ability measures for numerical facility, as found in the present study, and for verbal ability (Hunt et al., 1975; although verbal ability is probably not as "factorially pure" as numerical facility). Specifically, for both numerical facility and verbal ability, individuals showing greater measured ability on traditional tests seem to be more efficient, or faster, at retrieving content-relevant information from long-term memory and appear to have better facility at manipulating such information in short-term memory. The particular long-term memory networks accessed in the two types of tasks are likely rather different and not highly related; the short-term memory manipulations may also be rather different. Nevertheless, the results of the present study, in combination with the results of Hunt and his colleagues (Hunt et al., 1975), suggest that individual differences in facility of information retrieval from long-term memory and facility of information retention and manipulation in short-term memory (or working memory; Case, 1985) may be important processes for various ability dimensions and may be the processes underlying individual differences in such ability domains.

Finally, the present study clearly demonstrated that the use of factor-analytic and reaction-time methods in concert provides a powerful technique for the study of basic processes comprising human intelligence and for the study of individual differences in measured cognitive abilities. The two methods do not necessarily lead to different conclusions regarding the nature of human intelligence, but they may enable converging operations for representing individual differences in intellectual abilities. The present study found just such convergence for elementary processes involved in numerical facility using factor-analytic and reaction-time methods.

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