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Numerical and Arithmetical Cognition: Performance of Low- and Average-IQ Children

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Neuropsychological and developmental models of number, counting, and arithmetical skills, as well as the supporting working memory and speed of articulation systems, were used as the theoretical framework for comparing groups of low- and average-IQ children. The low-IQ children, in relation to their average-IQ peers, showed an array of deficits, including difficulties in retaining information in working memory while counting, more problem solving errors, shorter memory spans, and slower articulation speeds. At the same time, the low-IQ children's conceptual understanding of counting did not differ from that of their higher-IQ peers. Implications for the relation between IQ and mathematics achievement are discussed.

Intelligence (IQ) is the best single predictor of academic achievement, years of schooling completed, and many other important outcomes (Jensen, 1998). Basically, intelligence represents the ease with which individuals can acquire novel and complex material, whether the content of the material is social, academic, or employment related (Gottfredson, 1997a, 1997b).[1] One of the more

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[1]A related theoretical issue is the relation between IQ and evolved, or primary, and non-evolved, or secondary, abilities (Geary, 1995). Much of the research on the relation between IQ and learning involves the acquisition of secondary abilities. Geary (1998) argued that, functionally, IQ indexes the ability to adapt evolved cognitive systems, such as those that subserve language, for non-evolved uses, such as reading. The mechanisms involved in adapting these primary systems are not yet understood, but they probably involve working memory (see Geary, 1998, for further discussion).

complex domains that must be mastered in school is mathematics, and, given this, individual differences in mathematics achievement will be influenced to some degree by individual differences in intelligence (Dark & Benbow, 1991). However, the mechanisms that govern the relation between intelligence and academic achievement in general and mathematics achievement in particular are not well understood (see Jensen, 1998, for a review). Previous studies suggest that individual differences in intelligence are related to individual differences in working-memory capacity and the speed with which information can be processed (Fry & Hale, 1996; Keating & Bobbitt, 1978; Vernon, 1983), and these same factors might mediate, in part, the relation between IQ and mathematics achievement (Baroody, 1987, 1988; Dark & Benbow, 1991; Geary & Brown, 1991).

In a series of studies of the relations among IQ and verbal and mathematical precocity (i.e. 12- and 13-year-olds who scored above the average of high-school girls on the Verbal and Mathematics sections of the Scholastic Achievement Test), Dark and Benbow (1990, 1991) demonstrated that high-IQ, mathematically talented adolescents had an enhanced ability to manipulate numerical and spatial information in working memory, relative to their average-IQ and verbally talented peers. In a related study, Geary and Brown (1991) found that in relation to average-IQ children, high-IQ children showed faster overall reaction times (RT) for solving simple addition problems, such as 5 + 3. The high-IQ children's speed-of-processing advantage was evident whether they solved the problems by counting on their fingers, counting verbally, or retrieving the answer from long-term memory. However, a componential analysis of the RTs indicated that the high-IQ children did not execute all of the elementary operations underlying arithmetical problem solving more quickly than did their average-IQ peers but rather appeared to reach faster asymptotic levels of processing speed with fewer practice trials. In other words, the relation between speed of processing and IQ might be more evident after extensive exposure to a domain than with initial exposure to the domain (Siegler & Kotovsky, 1986).

Either way, the results from Dark and Benbow's (1991) and Geary and Brown's (1991) studies indicate that contrasts of groups of children who differ in IQ on mathematical cognition tasks provide valuable information on the factors that mediate the relation between IQ and mathematics achievement, as well as information on the cognitive systems (e.g. working memory) that contribute to individual differences in mathematics achievement (see Geary & Burlingham-Dubree, 1989; Geary & Widaman, 1992; Siegler, 1988). The present study followed the approach used in this previous research but extended the comparisons to include low-IQ children and extended the domains of study to number and counting, as well as simple addition, working memory, and speed of processing. The first section below provides an overview of neuropsychological and developmental models of number, counting, and arithmetical competencies, and the second provides an introduction to the relation between working memory, speed of articulating words, and arithmetical development. A brief summary of the current study is provided in the final section.

Numerical and Arithmetic Cognition

Overviews of models of number processing, counting knowledge, and arithmetical development are presented in the respective sections below.

Number Processing

The ability to recognize, comprehend, manipulate, and produce numbers and number words is essential to the development and maintenance of even the most basic mathematical skills (Fuson, 1988). To the best of our knowledge, the present study provides the first assessment of the relation between IQ and number production and comprehension skills in children.

On the basis of patterns of deficit following brain injury, McCloskey, Caramazza, and Basili (1985) proposed a model of the cognitive mechanisms underlying basic numerical competence (see also McCloskey, 1992). The model consists of a number-processing system that contains modules for comprehending and producing numbers, and a calculation system that interacts with the number-processing system and provides the procedural and factual information required to carry out arithmetical calculations. In this model, movement of the number between the comprehension, production, and calculation systems is accomplished via an area of semantic representation. This transition of numbers from one form to another form is termed transcoding. The main issue here is the ability to represent numbers in different modalities, verbal and written, and to transform those representations from one form to the other. For instance, several processing steps are involved in transcoding the spoken numeral /fīv/ to its arabic counterpart, 5 (see Campbell & Clark, 1988, and Dehaene, 1992, for alternative views).

Errors in transcoding are often used to make inferences about the integrity of specific systems that support numerical processing, as well as the development of these systems. Seron and Fayol (1994), for instance, evaluated transcoding errors made by children in order to gauge the development of the number comprehension and production system. In tasks of writing arabic numbers from dictation, Seron and Fayol found that children tended to make transcoding errors at the number production level and that these errors tended to be syntactical in nature. For instance, the verbal number eighty-two might be transcoded incorrectly as 802. The ability to transcode a numerical representation from verbal to arabic form and vice versa then appears to be one important competency supporting children's emerging numerical abilities and is thus a potential source of individual differences in mathematics achievement.

In addition to transcoding, an essential feature of number processing is an understanding of the underlying magnitude or quantity represented by the number (Dehaene, 1992; Gallistel & Gelman, 1992; McCloskey, 1992). One method that has been used to assess whether the individual understands that specific numbers represent specific values is to have the person indicate which of two presented numbers, such as 7 and 9, is smaller or larger. These tasks are often based on the verbal and arabic presentation of number pairs in order to determine if the semantic representation of numbers is amodal, as suggested by McCloskey, or whether there are separate semantic systems associated with the processing of verbal and arabic representations of numbers (e.g. Campbell & Clark, 1988).

Counting

Children's counting knowledge and behaviour appear to emerge from both inherent and experiential factors (Briars & Siegler, 1984; Geary, 1995; Gelman & Gallistel, 1978; Wynn, 1996). Early inherent constraints are best represented by Gelman and Gallistel's five implicit principles: (a) One-to-one correspondence—one, and only one, word tag (e.g. "one," "two") is assigned to each counted object, (b) The stable order principle—the order of the word tags must be invariant across counted sets, (c) The cardinality principle—the value of the final word tag represents the quantity of items in the counted set, (d) The abstraction principle—objects of any kind can be collected together and counted, (e) The order-irrelevance principle—items within a given set can be tagged in any sequence. The principles of one–one correspondence, stable order, and cardinality define the "how to count" rules, which, in turn, provide constraints on the nature of preschool children's counting behaviour and provide the skeletal structure for children's emerging knowledge of counting (Gelman & Meck, 1983).

Children also appear to make inductions about the basic characteristics of counting through observation of standard counting behaviour and the associated outcomes (Briars & Siegler, 1984; Fuson, 1988). This induced knowledge reflects both essential features of counting, such as those identified by Gelman and Gallistel (1978), and unessential features of counting (Briars & Siegler, 1984). These unessential features include start at an end (i.e. counting starts at one of the end points of an array of objects), adjacency (i.e. a consecutive count of contiguous objects), pointing (i.e. counted objects are typically pointed at but only once), and standard direction (i.e. counting proceeds from left to right). By the age of 5 years, many children know the essential features of counting but also believe that adjacency and start at an end are essential features of counting. The latter beliefs indicate that young children's conceptual understanding of counting is rather rigid and immature and is influenced by the observation of standard counting procedures.

The actual counting behaviour of children reflects a combination of this conceptual knowledge and their procedural skills, such as pointing to objects as they are counted. In order to assess conceptual knowledge in a way that is not confounded by procedural skills, an error-detection task must be used (Gelman & Meck, 1983). The error-detection task involves using a puppet that does the procedural part of counting, but sometimes counts in accordance with the counting principles and sometimes violates them. The child is asked to determine whether the puppet has counted correctly or not. It is assumed that if the child detects the violation of a principle—that the count was wrong—then the child implicitly understands the principle whether or not he or she can explicitly articulate it.

The relation between IQ and children's counting knowledge, as defined by Gelman and Gallistel (1978) and Briars and Siegler (1984), has not been assessed. Geary and his colleagues have, however, conducted one potentially relevant study of the counting knowledge of academically normal and mathematically disabled (MD) children (Geary, Bow-Thomas, & Yao, 1992). Following earlier work, the assessment of counting knowledge was based on the error-detection task. The results indicated that MD children understood one-to-one, stable order, and cardinality, but did not understand the order-irrelevance principle or, from Briars and Siegler's perspective, believed that adjacency was an essential feature of counting. In other words, MD children understood counting as a rote, mechanical activity.

Many MD children also failed to detect double-counting errors. On this task, the puppet double-counted (i.e. tagged one of the counted items with two number words, such as counting "one, two" for the first item) either the first or the last item counted in an array of items. Both the MD and academically normal children indicated that double counting the last item was wrong, indicating an implicit understanding of one-to-one correspondence, but many of the MD children stated that the count was correct when the puppet double-counted the first item. The pattern across the first and last items suggests that many MD children have difficulties keeping information in working memory while monitoring the counting process. If the failure to detect errors when the first item is double-counted is in fact related to working-memory resources, then performance on these items should vary across IQ status, given that IQ is associated with working memory capacity (Fry & Hale, 1996; Jensen, 1998).

Arithmetic

Children use a variety of strategies to solve simple arithmetic problems, such as 5 + 3 (Siegler & Shrager, 1984). For American first graders, the most common of these strategies involves counting, sometimes with the aid of their fingers and sometimes without them (the counting-fingers and verbal-counting strategies, respectively). Whether involving fingers or not, children use a variety of procedures during the counting process. The sum, or counting-all, procedure involves the child counting both numbers starting from 1, as in counting "1, 2, 3, 4, 5, 6, 7, 8" to solve 5 + 3 (Groen & Parkman, 1972). With the max procedure, the child states the value of the smaller number and then counts a number of times equal to the value of the larger number. Min, or counting-on, is the most efficient counting procedure and involves stating the value of the larger number and then counting a number of times equal to the value of the smaller number.

Arithmetic problems can also be solved through memory-based processes, as in retrieving the answer directly from long-term memory (Ashcraft, 1995). The development of memories for arithmetic facts appears to result from the formation of associations between problems (e.g. 5 + 3) and answers (e.g. 8), associations that appear to form as the child uses the above-described counting procedures (Siegler & Shrager, 1984). Two other retrieval-based procedures are fingers and decomposition (Siegler, 1987). With the fingers strategy, the child represents the addends by raising a corresponding number of fingers and then states the answer without counting the fingers; representing the addends on fingers appears to prompt retrieval. Decomposition involves deriving the answer from easily retrievable facts. For example, to solve 5 + 3, the child might first retrieve the answer to 3 + 3 (6) and finally add, through counting or retrieval, 6 + 2. The use of memory-based processes is moderated, however, by a confidence criterion. The confidence criterion represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer. Children with a rigorous confidence criterion only state answers that they are certain are correct, whereas children with a lenient criterion state any retrieved answer, correct or not (Siegler, 1988).

In one relevant study, Geary and Brown (1991) found that in comparison to average-IQ children, high-IQ children used finger counting and verbal counting less frequently and direct retrieval more frequently with fewer retrieval errors. However, not all of these differences were statistically significant (see also Geary & Burlingham-Dubree, 1989). Although it is likely that low-IQ children will use the same types of strategies as their higher-IQ peers to solve simple addition problems (Siegler, 1993), it is not currently known whether the distribution of strategy choices, error patterns, and so on will differ comparing low-IQ with higher-IQ children. Baroody (1988, 1996) found that children with IQs between 31 and 66—IQs below those of our low-IQ group—almost never retrieved correct answers when asked to solve addition problems (e.g. 2 + 5) as quickly as possible, although they sometimes used counting-on.

Supporting Cognitive Competencies

In addition to assessing basic competencies in number, counting, and simple addition, it is useful to assess the cognitive systems that support these competencies. Two such systems are working memory and processing speed (Geary, 1990, 1993; Jensen, 1998; Swanson, 1993). For the latter, speed of articulating familiar words and non-words is of theoretical interest, because differences in the speed of articulating these two classes of words provide a means of assessing the ease of retrieving information, such as arithmetic facts, from long-term memory (Gathercole & Adams, 1994).

Working Memory

Previous research indicates that working-memory resources are engaged during the solving of simple and complex arithmetic problems (Dark & Benbow, 1990, 1991; Geary & Widaman, 1992; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994). Siegel and Ryan (1989), for instance, showed that MD children exhibit deficits in span tasks involving counting but not those involving language. Hitch and McAuley (1991) confirmed these findings and went on to show that poor mathematics achievement was associated with slow counting speeds and difficulties in retaining numbers in working memory during the act of counting.

As noted earlier, working memory, along with processing speed, has also been implicated as an important component underlying individual differences in intelligence (Fry & Hale, 1996; Jensen, 1998). Fry and Hale explored the causal relations among working memory, processing speed, and intelligence in a cross-sectional study of children ranging in age from 7 to 19 years. The results showed that age-related improvements on the Raven IQ test (i.e. raw scores) were related to age-related improvements in working memory, which in turn were related to improvements in processing speed (see also Kail, 1991). In other words, individual and developmental differences in intelligence are explained in part by individual differences in working memory and processing speed (e.g. Keating & Bobbitt, 1978), with more intelligent individuals showing larger working-memory capacities and faster speed of processing information (e.g. binary decision-making) than their less intelligent peers. Given the relations among working memory, basic counting and arithmetic skills, and intelligence, it is likely that groups of children that differ in IQ will also differ in working memory and the counting and arithmetic skills supported by working memory.

Articulation Speed

The ability to accurately store and retrieve information, especially numbers (names and meaning) and basic arithmetic facts, from long-term memory is crucial to the development of mathematical abilities. Lack of automatized retrieval of basic facts inhibits the development of arithmetical competencies and has been implicated as one factor contributing to poor mathematics achievement (Ashcraft, Donley, Halas, & Vakali, 1992; Bull & Johnston, 1997; Geary, 1993). As noted above, the formation of representations of arithmetic facts in long-term memory appears to be dependent on the execution of the counting procedures described earlier (Siegler & Shrager, 1984). The problem/answer associations, in turn, provide the basis for automatised retrieval. The relation between counting and the formation of problem/answer associations in long-term memory indicates that the articulatory systems that support counting (e.g. phonemic memory) are likely to be involved, in part, in the development of long-term memory representations of arithmetic facts (Baddeley & Hitch, 1974; Geary, Brown, & Samaranayake, 1991).

Moreover, the ease with which lexical information—such as number names—can be accessed from long-term memory during the act of counting and the speed with which the number words can be articulated are potentially related to the ease with which problem/answer associations can be formed (Cowan et al., 1994; Geary, Bow-Thomas, Fan, & Siegler, 1993). Slow articulation speeds and difficulties in accessing number words might inhibit the formation of these associations.

Speed of word articulation is thus theoretically related to normal arithmetical development and potentially to MD. Speed of word articulation is also potentially related to IQ, given the broad relation between speed of processing in many domains and intelligence (Jensen, 1998). On the basis of these relations, word articulation speeds should differ across IQ groups, which, in turn, might eventually lead to IQ-related differences in the ease with which basic arithmetic facts are learned. In addition to ease of learning, IQ-related differences are found in the speed of accessing information, such as words, from long-term memory, with lower IQ scores being associated with slower rates of information access (Hunt, 1978). IQ-related differences in articulation speed might then be related to a more fundamental deficit in representing (e.g. activating the long-term-memory representations) the to-be-articulated words in working memory—which is needed to articulate these words—rather than speed of talking per se.

If so, then IQ should be related not only to speed of articulation, it should also be related to differences in the speed of articulating familiar and unfamiliar words. By definition, familiar words are represented in long-term memory, and access to these representations appears to facilitate their encoding into working memory and might facilitate the speed with which these words can be articulated (Bow-Thomas, 1994; Dark & Benbow, 1991; Wagner & Torgesen, 1987). Non-words, in contrast, are not represented in long-term memory, and thus there are no direct long-term memory advantages for encoding or articulating these words (see Gathercole & Adams, 1994, for further discussion). Thus, differences in the speed of articulating familiar words and unfamiliar non-words might provide a useful means of assessing the ease with which lexical information can be retrieved from long-term memory; a large difference indicates fast access to familiar information.

Given this, IQ should vary with the speed of articulating both words and non-words, and the relative articulation-speed advantage for familiar words should also vary with IQ. The familiarity advantage should be smaller for low-IQ than for average-IQ children, and thus the *difference* in the speed of articulating familiar and unfamiliar words should be larger for average-IQ than for low-IQ children.

THE PRESENT STUDY

The present study examined the relations among IQ and specific competencies in number, counting, arithmetic, and theoretically related supporting cognitive systems (i.e. working speed and articulation speed). Although the study is largely exploratory in nature, as was just described for articulation speeds there are reasons to believe that IQ will be systematically related to performance in one or more of these domains (e.g. Baroody, 1996, 1999). If such relations are found, then this will provide useful insights into the cognitive mechanisms that mediate the relation between IQ and mathematics achievement (Dark & Benbow, 1991), as well as the cognitive mechanisms that underlie individual differences in mathematics achievement.

Method

Participants

The participants were selected from a larger group of 114 first graders (50 boys, 64 girls; mean age = 82 months) from five elementary schools in Columbia, Missouri. These children comprised two cohorts of children who were participating in a longitudinal study of children at risk for MD. While the longitudinal research is focused on the relation between numerical and arithmetic cognition in children with low normal or better IQ scores and low concurrent mathematics-achievement scores, the current study focuses on the relation between numerical and arithmetic cognition and IQ, as described above. The relation between these forms of cognition and IQ was assessed through comparisons of average-IQ children with average or better academic-achievement scores and low-IQ children, regardless of achievement scores. As would be expected based on the relation between IQ and academic achievement (Jensen, 1998), nearly 80% of the low-IQ children's achievement scores were low—that is, nearly a standard deviation or more below the mean of the average-IQ group for performance on standardized reading and mathematics achievement tests (described in the Results section).

All of the children were administered the Vocabulary and Block Design subtests of either the Wechsler Intelligence Scale for Children–III (Wechsler, 1991) or their counterparts from the Stanford-Binet Intelligence Scale (Thorndike, Hagen, & Sattler, 1986), and an estimated IQ was derived based on performance on these two scales (Sattler, 1982; Siegel & Ryan, 1989). The participants' data were then grouped into three IQ levels: low (IQ $<$ 85), average (IQ = 85–115, inclusive), and high (IQ $>$ 115). Initial analyses revealed few significant differences when comparing the average- and high-IQ groups, due largely to ceiling effects. Thus, these groups were collapsed, and all analyses reported here are based on two groups—low (IQ $<$ 85) and average (IQ $\geq$ 85). The low-IQ group included 19 children (9 boys, 10 girls; mean age = 84.0 months, *SD* = 3.7), and the average-IQ group included 43 children (15 boys, 28 girls; mean age = 81 months, *SD* = 3.95). The difference in mean age between groups was significant, $F(1, 60) = 7.89$, $p < .01$; the low-IQ group was slightly (3.1 months) older than the average-IQ group ($p < .05$).

Experimental Tasks

Standardized Achievement. Academic achievement was assessed using the Mathematics Reasoning subtest of the Wechsler Individual Achievement Test (Wechsler 1992) and the Word Attack and the Letter-Word Identification subtests of the Woodcock Johnson Psycho-Educational Battery—Revised (Woodcock & Johnson, 1989/1990). The Mathematics Reasoning subtest assesses basic arithmetic skills such as counting and subtraction, as well as some more advanced skills such as graph reading and time telling. The Word Attack subtest assesses the ability to apply the rules involved in the pronunciation of written words (e.g. sounding out non-words), while the Letter-Word Identification subtest assesses the ability to understand that written symbols or groups of symbols (icons, letters, words) represent objects, letters, or words.

Number Production and Comprehension. The items were developed based on an adaptation of the Johns Hopkins Dyscalculia Battery for children (McCloskey, Alminosa, & Macaruso, 1991; Shalev, Manor, & Gross-Tsur, 1993) and assess the ability to name and reproduce visually and auditorily presented numbers and to compare the magnitude of visually and auditorily presented numbers. The first number-recognition/production stimulus consisted of a series of four integers (3, 8, 5, 12) arranged vertically on an otherwise blank sheet of paper in a large font. The second set of stimuli was a series of four integers (2, 5, 7, 13) presented by dictation. The first set of magnitude-comparison stimuli consisted of four pairs of single-digit integers (1 9, 3 2, 5 7, 9 8) arranged vertically on an otherwise blank sheet of paper in a large font, and the second set of stimuli was comprised of four pairs of single-digit integers (6 3, 2 5, 5 6, 8 1) presented by dictation. Both sets of magnitude-comparison pairs were constructed so as to include numbers that represented relatively small (e.g. 2) and large (e.g. 9) magnitudes, and relatively small (e.g. 5 7) and large (e.g. 1 9) distances between the two magnitudes.

These measures provided four items for each of five basic number-processing competencies: number naming (arabic to verbal transcoding), number writing with visual presentation, number writing with auditory presentation (verbal to arabic transcoding), magnitude comparison with visual presentation, and magnitude comparison with auditory presentation. A reliability estimate could not be calculated for the number writing with visual presentation task (i.e. number copying) because of ceiling or near-ceiling performance on most of the items. For number naming, number writing with auditory presentation, and magnitude comparison with auditory and visual presentation, Cronbach's alpha was used as an estimate of task reliability. The resulting estimates were .68, .58, .98, and .94, respectively.

Counting Knowledge. The stimuli were horizontally presented arrays of 5, 7, or 9 poker chips of alternating red and blue colour. Following previous studies of children's counting knowledge (Briars & Siegler, 1984; Geary et al., 1992; Gelman & Meck, 1983), four types of counting trials were administered: correct, right–left, pseudo, and error. For correct trials, the chips were counted sequentially and correctly, from the child's left to the child's right. Right–left involved counting the chips sequentially and correctly, but from the child's right to the child's left. For pseudo trials, the chips were counted correctly from left to right, but first one colour was counted, and then, returning to the left-hand side of the row, the count continued with the other colour. For error trials, the chips were counted sequentially from left to right, but the first chip was counted twice. Each counting-trial type occurred once for each array size (i.e. 5, 7, 9), with one additional pseudo trial (for seven chips). The additional pseudo trial was added so that two trials started with red chips and two with blue chips. In all, there were 13 experimental counting trials. A reliability estimate could not be calculated for the correct counting items because of near-ceiling performance. The reliability estimates, again using Cronbach's alpha, were 0.80, 0.96, and 0.92 for the right–left, pseudo, and error counting measures, respectively.

Addition Strategy Assessment. Stimuli for this task were 14 single-digit addition problems presented horizontally (4 + 5 =) on a computer screen. The stimuli consisted of the integers 2 through 9, with the constraint that the same two integers were never used in the same problem (i.e. no doubles problems such as 2 + 2, 6 + 6, 9 + 9). Across stimuli, each digit was presented two to four times, and half of the problems summed to 10 or less, inclusive. For half of the problems, the larger-valued integer was presented in the first position, and for the remaining problems the smaller-valued integer was presented in the first position. The order of problem presentation was determined randomly, with the constraint that no integer was presented in the same position across consecutive trials.

The stimuli were presented on the 13″ monochrome monitor of an IBM PS/2 Model 30 microcomputer. For each problem, a "READY" prompt appeared at the center of the screen for 1,000 msec, followed by a 1,000-msec period during which the screen was blank. Then, an addition problem appeared on the screen and remained until the subject responded. Answers were spoken into a microphone that activated a Gerbrands G1341T voice-operated relay that was interfaced with the microcomputer. RTs were generated using a Cognitive Testing Station hard card timing mechanism with ±1.0 msec accuracy and were simultaneously written to floppy diskette. The experimenter initiated each problem-presentation sequence via a control key.

Digit Span. This task included the forward and backward sections of the Digit Span subtest of the WISC–III (Wechsler, 1991).

Articulation Speed. The stimuli for this task were triads of one-digit number names (2, 9, 5), common one-syllable words (school, tree, cake), and one-syllable non-words (lote, dake, pog).[2] Non-words and words were a subset of those used by Bow-Thomas (1994) in a study of MD children which, in turn, were based on Cowan's (1986) and Edwards' (1978) list of monosyllabic English words and non-words. Reliability estimates, using Cronbach's alpha, were uniformly high; .82, .80, and .82, for number, word, and non-word articulation speeds, respectively.

Experimental Procedures

All children were tested twice during the academic year, once in the autumn and once in the spring. Autumn testing included the just-described experimental tasks, while the spring testing included the IQ and achievement tests. For autumn testing, the experimental tasks were administered in a randomly determined order for each child. For spring testing, the tests were administered in the following order: Letter-Word Identification, Word Attack, and Mathematics Reasoning, followed by the Vocabulary and Block Design subtests of the WISC–III or Stanford-Binet. For both testing periods, the children were assessed individually and in a quiet room at their school. Research assistants who were trained and well practiced on the administration of each task did the testing. Testing time was about 40 min and 30 min for the autumn and spring assessments, respectively. Mean time between testing periods was 126 days (SD = 31), and time elapsed did not differ significantly across groups, $F(1, 59) = 0.84$, $p > .10$.

Number Production and Comprehension. For the first series of items, the experimenter showed the child the four numbers in sequence and asked the child to name each number shown and then write the number next to the printed copy. Next, the experimenter spoke a series of four numbers one at a time and asked the child to write each number on the same paper. For the magnitude-comparison items, the child was first shown a series of four pairs of numbers and was asked to decide, one pair at a time, "Which is bigger, which is more?" The same task was repeated with four new number pairs that were presented, one pair at a time, by dictation.

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[2]For the second cohort, the stimulus "pog" was replaced with "vog", because it came to our attention that "pog" referred to a children's game.

Counting Knowledge. The child was first introduced to a puppet who was just learning how to count and therefore needed assistance from the child to know if his counting was "OK and right, or not OK and wrong". During each of the 13 trials, a row of 5, 7, or 9 poker chips of alternating colour (e.g. red, blue, red, blue, red) were aligned behind a screen. The screen was then removed, and the puppet counted the chips. The child was then queried on the correctness of the counting. The experimenter noted on a protocol whether the child stated the puppet's count was "OK, or Not OK and wrong." Any miscellaneous statements made by the child about the puppet's count were also recorded on a trial-by-trial basis (very few children made miscellaneous statements).

Addition Strategy Assessment. Following the presentation of three practice problems, the 14 experimental problems were presented one at a time. The child was asked to solve each problem as quickly as possible without making too many mistakes—that is, speed and accuracy were balanced. It was emphasised that the child could use whatever strategy was easiest for his or her to get the answers, and the child was instructed that as soon as he or she had the answer, he or she was to speak it into the microphone.

During problem solving, the experimenter watched for physical signs, such as regular movements (e.g. fingers, mouth), that indicated counting. For these trials, the experimenter initially classified the strategy as finger counting or verbal counting, depending on whether or not the child used his or her fingers to count. If the child upraised a number of fingers to represent the addends and then stated an answer without counting them, then the trial was initially classified as fingers. If the child spoke the answer quickly, without hesitation, and without obvious counting-related movements, then the trial was initially classified as retrieval.

After the child had spoken the answer, the experimenter queried the child on how he or she got the answer. If the child's response (e.g. "just knew it") differed from the experimenter's observation (e.g. saw the child mouthing counting), then a notation indicating disagreement between the child and the experimenter was made. If counting was overt, then the experimenter classified it as a counting strategy. If the trial was ambiguous, then the child's response was recorded as the strategy. Previous studies indicate that this method provides a reliable measure of children's trial-by-trial strategy choices (e.g. Geary et al., 1991; Siegler, 1987). With the current study, agreement between the child's description and the experimenter's observation was found for nearly 94% of trials. Based on the child's description and the experimenter's observation, each trial was classified into one of five strategies: counting fingers, fingers, verbal counting, retrieval, or decomposition (Siegler & Shrager, 1984). Counting trials were further classified based on where counting began—that is, min, sum, or max.

Digit Span. The forward and backward sections of the Digit Span subtest were administered using standard protocol. The score is the value of the longest digit string correctly repeated. In addition, a difference score was calculated by subtracting the backward score from the forward score.

Articulation Speed. Each word triad was presented to the child by dictation. The child was encouraged to repeat the triad until she could remember all three words. Next, the child was asked to say the triad as quickly as possible, two times in a row. A stopwatch was used to time the speed of articulation for each triad. This procedure was performed a total of three times for each word, number, and non-word trial, yielding three articulation-speed estimates per triad.

Results

For ease of presentation, the results are presented in six sections. In the first, group differences on the standardised achievement measures are described. The second section presents results for the number-naming, number-writing, and magnitude-comparison tasks, while the third and fourth sections present results from the counting-knowledge and addition-strategy tasks, respectively. An examination of group differences in digit span (working memory) and articulation speed are presented in the final two sections. In all analyses, a least-squares solution was used to control for unequal cell sizes.

Standardised Achievement Tests

As shown in Table 1, the mean scores of the low-IQ children were significantly below the mean scores of the average-IQ children for Letter-Word Identification, $F(1, 60) = 32.19$, $p < .0001$, Word Attack, $F(1, 60) = 39.00$, $p < .0001$, and Mathematics Reasoning, $F(1, 60) = 91.27$, $p < .0001$.

Number Production and Comprehension

The mean percentage of correct responses for the number-naming and number-writing tasks are shown in Table 2. Inspection of the table reveals perfect to near perfect performance for all three tasks for children in the average-IQ group. Likewise, the performance of the children in the low-IQ group was near perfect for copying numbers (i.e. number writing with visual presentation), but these children showed fewer correct responses, on average, than the average-IQ children for number naming and number writing with auditory presentation. A 2 (group) × 3 (task type) mixed ANOVA, with group as a between-subjects factor and type as a within-subjects factor, confirmed significant group, $F(1, 60) = 13.48$, $p < .001$, type, $F(2, 120) = 13.61$, $p < .0001$, and Type × Group, $F(2, 120) = 7.98$, $p < .001$, effects. Follow-up ANOVAs for each task type revealed that the group differences were significant for both the number-naming task, $F(1, 60) = 19.21$, $p < .0001$, and the number writing with auditory presentation task, $F(1, 60) = 6.61$, $p < .015$, although these results need to be interpreted with caution because of the ceiling performance of the average-IQ children.

TABLE 1
Mean Achievement and Intelligence Scores

	IQ			Achievement Tests: Mathematical Reasoning		Achievement Tests: Word Attack		Achievement Tests: Letter-Word Identification	
Group	*n*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Low	19	78	5.6	20	15.0	21	19.3	24	21.4
Average	43	108	11.3	68	19.1	59	22.6	64	27.8

Note. Achievement test scores are national percentile rankings, based on age.

TABLE 2
Mean Percentage of Correct Responses for Number Processing Tasks

Group	*Number Naming*	*Number Writing: Visual Presentation*	*Number Writing: Auditory Presentation*
Low	84	99	84
Average	100	100	96

For the number-naming task, examination of performance on individual trials indicated that the group difference was largely the result of 9 of the 19 low-IQ children failing to name the number "12", 1 low-IQ child who was unable to name any of the four stimuli. For the number-writing task, 6 of 19 low-IQ children could not write the number "13", although most of these children correctly wrote at least one number greater than or equal to 10. One child was unable to write correctly any of the numbers from dictation.

The mean percentage of correct comparisons for the magnitude-comparison tasks are shown in Table 3. A 2 (group) × 2 (presentation mode) mixed ANOVA, with group as a between-subjects factor and presentation mode as a within-subjects factor, revealed a significant main effect for group, $F(1, 60) = 23.81$, $p < .0001$; the presentation mode, $F(1, 60) < 1$, and the interaction, $F(1, 60) < 1$, were not significant. The average-IQ group was at ceiling, and follow-up ANOVAs confirmed that they significantly outperformed their low-IQ peers for both visual presentation, $F(1, 60) = 30.26$, $p < .0001$, and auditory presentation, $F(1, 60) = 11.69$, $p < .01$. Again, these significant group differences need to be interpreted with caution given the ceiling effects for the average-IQ group.

Examination of performance on individual visual-presentation items revealed that 5 of the 19 low-IQ children missed the small-distance item (i.e. 8 9). There

TABLE 3
Mean Percentage of Correct Comparisons for Magnitude Comparison Tasks

	Presentation Mode	
Group	*Visual*	*Auditory*
Low	84	87
Average	100	100

was no discernible pattern of errors for the auditory-presentation task except that one low-IQ child was unable to discern correctly magnitude differences in any trial. The subset of low-IQ children who had difficulty with the magnitude-comparison task tended to be the same children who had had difficulty with the naming and copying tasks.

Counting Knowledge

The mean percentages of correct identifications across counting tasks are shown in Table 4. A 2 (group) × 3 (size: 5, 7, 9) × 4 (type: correct, right–left, pseudo, error) repeated measures ANOVA revealed a main effect for type, $F(3, 174) = 16.07$, $p < .0001$, and significant two-way interactions for Type × Group, $F(3, 174) = 6.09$, $p < .001$ and Type × Size, $F(6, 348) = 2.37$, $p < .05$; the main effect for group approached significance, $F(1, 58) = 3.22$ $p < .08$, but all other effects were non-significant ($ps > .10$). Follow-up analyses done for each counting type showed that the significant findings were due to the average-IQ group outperforming their low-IQ peers on error counting trials, with the magnitude of this effect increasing with increases in set size, as shown in Fig. 1 ($ps < .05$). For the error counting trials, an ANOVA showed a significant main effect for group, $F(1, 58) = 22.79$, $p < .0001$, and size, $F(2, 116) = 4.59$, $p < .05$, with the Size × Group interaction approaching significance, $F(2, 116) = 2.48$, $p < .09$.

Examination of performance on individual items revealed that 6 of the 19 low-IQ children and 1 average-IQ child indicated that error counting was “OK” on all trial sizes. Nearly half of the low-IQ children indicated that error counting was “OK” when the set size was largest (i.e. 9). Finally, it should be noted that misidentification of pseudo counting as incorrect was common for children in both IQ groups.[3]

[3]An additional question was added to the counting knowledge assessment in the third year of the longitudinal study, asking the child if the pseudo-counting trial had the wrong method and wrong answer, or the wrong method and right answer. Of those still participating in the longitudinal study, and still responding that pseudo counting was incorrect, 66% of the low-IQ responders indicated that both the method and answer were wrong, while all of the average-IQ children responded that the method was wrong, but the answer was right.

TABLE 4
Mean Percentage of Correct Identifications Across Counting Tasks

	Counting Task			
Group	*Correct*	*Right–Left*	*Pseudo*	*Error*
Low	98	98	61	59
Average	96	93	63	97

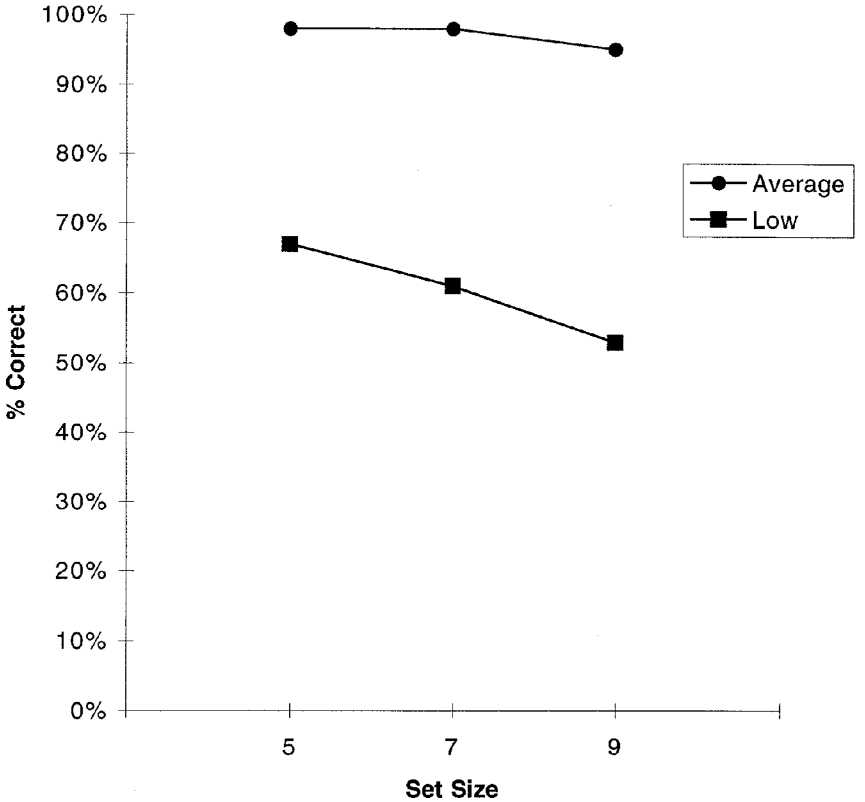

FIG. 1. Mean percentage of double-counting errors identified as errors across IQ group and set size.

Addition Strategy Assessment

Table 5 shows that the low- and average-IQ children used the same types of strategies to solve simple addition problems, but differed in their strategy mixes. Most of the children, regardless of IQ status, relied on some form of counting (either with fingers or verbally) to solve the majority of problems, but significant

TABLE 5
Characteristics of Addition Strategies

	Mean Percentage of Trials[a]		*Mean Percentage of Errors*		*Mean Percentage of Min Trials*		*Mean Reaction Time (sec)*[b]	
Strategies	*Low*	*Average*	*Low*	*Average*	*Low*	*Average*	*Low*	*Average*
Counting fingers	53	61	49	21	22	35	11.4	11.4
Fingers	0	3	—	37	—	—	—	—
Verbal counting	8	23	43	23	23	54	13.3	6.8
Retrieval	32	9	97	22	—	—	—	4.0
Decomposition	2	1	40	0	—	—	—	—

[a]Trials do not total 100% because mixed trials were not included; mixed trials involve the use of two strategies, such as starting the problem solving by means of counting and then retrieving the answer before the count was finished. For the low- and average-IQ groups, 5% and 3% of the trials were classified as mixed.

[b]For the counting-fingers strategies, RTs are based on correct sum trials. For the verbal-counting strategies, RTs are based on correct min trials. RTs are not presented for the fingers and decomposition strategies because of the small number of trials. Similarly, a mean retrieval trial RT is not presented for the low-IQ group because of the small number of correct trials.

group differences were found, favouring the low-IQ children, for use of retrieval, $F(1, 59) = 8.92$, $p < .01$. The low-IQ children also committed significantly more retrieval, finger-counting, and decomposition errors than did their average-IQ peers (ps $< .01$).

Analyses of RTs based on correct min trials for the verbal-counting strategy and correct sum trials for the counting-fingers strategy revealed a significant group difference, favouring the average-IQ children, for the verbal-counting (min procedure) strategy, $F(1,16) = 8.43$, $p < .05$, and a non-significant difference for the counting-fingers (sum procedures) strategy ($p > .10$). However, these analyses need to be interpreted with some caution, given the group differences in error rates and the frequency with which the min procedure was used.

Digit Span

Mean scores for performance on the forwards and backwards sections of the Digit Span subtest, as well as a backward – forward difference score are shown in Table 6. An ANOVA confirmed no significant group differences for forwards digit span, $F(1, 60) = 1.00$, $p > .10$, but the group difference was significant for backwards digit span, $F(1, 60) = 28.46$, $p < .0001$, as well as for the difference score, $F(1, 60) = 12.34$, $p < .001$. The low-IQ children had substantially lower mean backwards digit span and higher difference scores than did the average-IQ children.

Articulation Speed

Table 7 reveals that the mean articulation speeds (i.e. the average of the three articulation speeds recorded for each type of stimulus) of the average-IQ children were faster than those of the low-IQ children. Because mean articulation times varied across stimulus types, the scores were standardised with a mean of 0 and a *SD* of 1. Based on the earlier-noted hypothesis, articulation speed for familiar words (i.e. words + numbers) and unfamiliar words (non-words) and differences between these two speeds are plotted in Fig. 2. The overall pattern is consistent with previous findings—that processing speed varies inversely with IQ status (Jensen, 1998)—and with the hypothesis that the difference score will vary across

TABLE 6
Mean Digit Span Scores

	Forward		*Backward*		*Difference*	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*
Low	4.6	0.8	1.7	1.4	2.9	1.3
Average	4.8	1.0	3.0	0.5	1.8	1.0

TABLE 7
Mean Articulation Speeds

	Numbers		Words		Non-words	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*
Low	1.8	0.4	3.9	1.5	2.8	1.1
Average	1.7	0.4	3.2	1.0	2.6	0.8

Note. Values are in seconds.

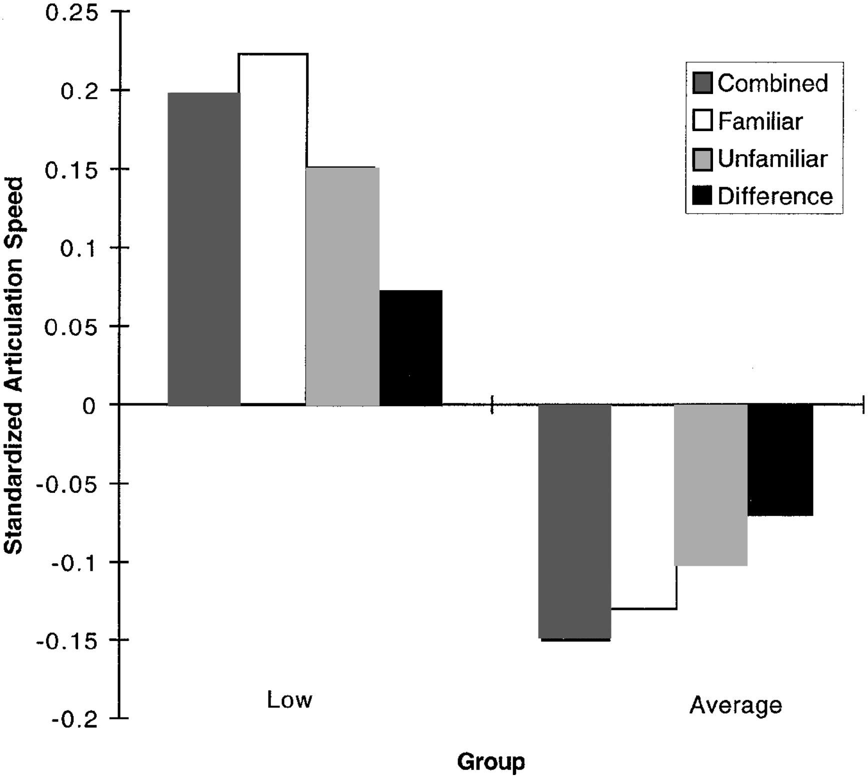

FIG. 2. Mean standardized articulation-speed scores. Scores less than 0 indicate faster-than-average articulation speeds. Combined scores are the sum of the standard scores for word, number, and non-word articulation speeds. Familiar scores are the sum of the standard scores for word and number-articulation speeds, whereas unfamiliar scores are those for non-word-articulation speeds. Difference is the standard score for familiar articulation speeds minus the standard score for unfamiliar articulation speeds.

IQ groups, although, as noted below, not all of these differences were statistically significant.

The standardized articulation-speed scores were submitted to three sets of analyses. First, an ANOVA confirmed significant group differences for combined (i.e. averaged across word type) articulation speeds, $F(1, 58) = 4.62$, $p < .05$. The second and third analyses focused on group differences in speed of articulating familiar and unfamiliar words and differences in these articulation speeds, respectively. A 2 (group) × 2 (familiarity) repeated measures ANOVA revealed a marginally significant main effect for group, $F(1, 58) = 3.64$, $p < .07$, but no other significant effects (ps > .50). Follow-up ANOVAs confirmed a significant group difference, favouring the average-IQ children, for speed of articulating familiar words, $F(1, 58) = 5.17$, $p < .05$, but not for unfamiliar (non-) words, $F(1, 58) = 1.29$, $p > .10$. No significant effects were found for the difference score, $F(1, 58) = 0.42$, $p > .50$), but the presence of a difference score (familiar – unfamiliar = difference score) that is positive in sign for the low-IQ children suggests that they articulated unfamiliar words faster than familiar ones.

DISCUSSION

The present study provided an exploratory analysis of the relation between IQ status and basic competencies in number, counting, and arithmetic, as well as between IQ and some of the cognitive systems that support these competencies—that is, working memory and articulation speed. Because many of the predictions regarding the relation between numerical and arithmetical cognition and IQ stem from the relations among mathematical cognition, IQ, working memory, and processing speed, the first section below focuses on the areas of working memory and articulation speed. In the second section, the relations between IQ status and specific number, counting, and arithmetic competencies are explored. The final section presents a brief summary.

Working Memory and Articulation Speed

On the basis of the relation between IQ and working memory (Jensen, 1998), it was hypothesised that groups of children who differed in IQ would also differ in working-memory resources and in the counting and arithmetic skills supported by working memory. The most complex working memory measure used in this study was backwards digit span, and the low-IQ children scored 1.4 *SD*s lower than did their average-IQ peers. The finding of no significant group differences on forwards digit span, combined with significant group differences on the backwards digit span and difference measures, suggests that the poor performance of the low-IQ children is not the result of difficulties in simply processing numbers in working memory (Butterworth, Cipolotti, & Warrington, 1996). Rather, this pattern suggests that their low performance is the result of difficulties in manipulating numerical information once it is encoded into working memory.

Although it is not certain, the latter finding might be related to group differences in the central-executive component of working memory (Baddeley & Hitch, 1974). In other words, if low-IQ children have a central-executive deficit, and if the central executive controls attention and coordinates the manipulation of information in working memory, then their backward digit span may suffer. This is because the backward digit span is, in effect, a dual task that requires not only storage of information, but manipulation of the stored information before retrieval (Jensen, 1998).

It was also hypothesised that word-articulation speeds and differences between the speed of articulating familiar and unfamiliar words would vary across IQ groups. The former was based on the more general relation between IQ and speed of information processing (Jensen, 1998), as noted above, and the latter on the relation between IQ and the speed with which familiar information (such as letters and words) can be retrieved from long-term memory (Hunt, 1978). Higher IQ should be associated with faster articulation speeds, across word types, and larger differences between the speed of articulating familiar and unfamiliar words; large differences indicate fast access to information stored in long-term memory.

The present results were in accord with these predictions. In relation to their average-IQ peers, low-IQ children were slower at articulating familiar and unfamiliar words and showed a positive difference score—that is, they were relatively faster at articulating unfamiliar than familiar words—although not all of these effects were statistically significant. Nonetheless, the result for speed of articulating familiar words suggests that lower-IQ children are slower at accessing long-term memory representations of familiar information, in keeping with previous studies (Hunt, 1978).

Numerical and Arithmetical Cognition

Number Production and Comprehension. Number processing has been studied extensively in the brain injury and dyscalculia literatures (McCloskey, 1992; McCloskey et al., 1985; Seron & Fayol, 1994; Temple, 1991), but has not been studied specifically in relation to IQ. Group differences on the number production and comprehension tasks produced a guarded—because of ceiling effects—conclusion that the low-IQ children showed difficulties in number naming (transcoding from arabic to verbal) and magnitude comparison (a combination of transcoding and number comprehension), in relation to their average-IQ peers. On the basis of McCloskey et al.'s (1985) model of number processing, this pattern might be interpreted as deficits in the systems that support transcoding numbers between arabic and verbal form and the semantics of number. Another possibility is that many of the low-IQ children simply did not know the number-word names that corresponded with the associated arabic representations (e.g. 12). Indeed, these number words are expected to be difficult to learn, as the teens number words do not show a one-to-one correspondence between the name and the associated quantity (e.g. "twelve", as compared to "ten two" in Asian languages: Geary et al., 1993; Miller, Smith, Zhu, & Zhang, 1995).

Counting Knowledge. Most of the children, regardless of IQ status, recognized the standard left-to-right count as correct (Briars & Siegler, 1984; Geary et al., 1992; Gelman & Meck, 1983). The generally correct identifications of the right-to-left count showed that nearly all of the children also knew that the direction of the count was not an essential feature of counting. Although no significant group differences were found for the pseudo trials (as reported above, for these trials, chips of one colour were counted, and then, returning to the end, the chips of the other colour were counted), it is important to note the overall low scores of both groups and that the average-IQ children had a low percentage of correct identifications. Recall that the pseudo-counting trials reflect Gelman and Gallistel's (1978) order-irrelevance principle and Briars and Siegler's (1984) adjacency principle (an unessential feature of counting). The results for this study indicate that these features of counting are not well understood by many first-grade children and that, at least at this age, IQ does not predict whether or not a child understands this principle; whether IQ is related to differences in the age in which ceiling performance is achieved is not currently known.

It was also predicted that performance on counting tasks—specifically, detecting double-counting errors—which caused a load on the working memory system would vary with IQ status. As shown in Fig. 1, for double-counting trials the performance of the low-IQ children varied systematically with array size. The longer the count, the more frequently the low-IQ children, as a group, indicated that double counting as "OK". The pattern across array size suggests that the poor performance of the low-IQ children on these trials was related to working memory and not to a poor conceptual understanding of one-to-one correspondence. If these children failed to understand the one-to-one principle, then performance on this task should have been uniformly low across set size. In this view, it is likely that these children initially recognized the double count as an error, but failed to retain this information in working memory with longer counts. The longer amount of time the error notation needed to be kept active in working memory, the less likely the error was to be remembered (Hitch, 1978).

The overall pattern suggests that low-IQ children's conceptual understanding of basic counting principles does not differ from that of their higher-IQ peers, at least at this age, but that low-IQ children are disadvantaged on counting tasks that demand working-memory resources.

Addition Strategy Assessment. Consistent with previous studies, the children in this study, regardless of IQ status, used a variety of strategies to solve simple addition problems (Geary & Brown, 1991; Siegler, 1993; Siegler & Shrager, 1984), although IQ-related differences were evident in the strategy mix (Geary & Brown, 1991). Basically, errors in retrieval, finger counting, and decomposition were less frequent for average-IQ children than for low-IQ children. The low-IQ children also used retrieval more frequently than their higher-IQ peers but almost always retrieved an incorrect answer. The very high percentage (97%) of retrieval errors for the low-IQ group may, in part, be a reflection of a low confidence criterion—that their confidence in the answer being correct does not have to be very high for them to state any retrieved answer or any number currently active in working memory (Baroody, 1988; Siegler, 1988).

In summary, efficient use of memory-based arithmetic strategies—that is, correct retrieval and decomposition—varied with IQ. At this point, it is not clear whether low-IQ children will show fact-retrieval deficits, as is the case with higher-IQ MD children (Geary, 1993), or whether their poor retrieval performance (i.e. high error rates) is simply the result of a low confidence criterion. The findings for the articulation speed tasks—specifically their slow articulation speeds for familiar words—suggest that low-IQ children may indeed have later fact-retrieval problems, on top of a low confidence criterion (Baroody, 1988).

SUMMARY

Despite a slight age advantage, the low-IQ first graders in this study displayed specific features of arithmetic ability and the cognitive competencies supporting that ability that differentiated them from their average-IQ peers. These children had lower backwards digit spans and slower articulation speeds for familiar words, both indictors of possible working-memory difficulties. In addition, the low-IQ children showed deficits in number naming and in number writing from dictation as well as for magnitude comparisons, indicating possible transcoding deficits or poor knowledge of number words. Low-IQ children displayed deficits in the ability to detect counting errors as set size increased and, while using similar strategies for solving simple addition problems, displayed significantly higher error rates than their average-IQ peers. Many of these deficits can be understood in terms of working-memory difficulties, in particular difficulties in the ability to retain information in working memory while monitoring other activities and difficulties in manipulating information in working memory.

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