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Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability

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Abstract

Groups of first-grade (mean age = 82 months), third-grade (mean age = 107 months), and fifth-grade (mean age = 131 months) children with a learning disability in mathematics (MD, $n = 58$) and their normally achieving peers ($n = 91$) were administered tasks that assessed their knowledge of counting principles, working memory, and the strategies used to solve simple (4 + 3) and complex (16 + 8) addition problems. In all grades, the children with MD showed a working memory deficit, and in first grade, the children with MD used less sophisticated strategies and committed more errors while solving simple and complex addition problems. The group differences in strategy usage and accuracy were related, in part, to the group difference in working memory and to group and individual differences in counting knowledge. Across grade-level and group, the switch from simple to complex addition problems resulted in a shift in the mix of problem-solving strategies. Individual differences in the strategy mix and in the strategy shift were related, in part, to individual differences in working memory capacity and counting knowledge.

Keywords: Learning disabilities; Mathematical disabilities; Working memory; Counting knowledge; Arithmetic development; Strategy choices

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Introduction

Between 5 and 8% of children suffer from some form of learning disability (LD) in mathematics (MD; Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974; Ostad, 1997; Shalev et al., 2001). Yet, with the exception of some aspects of competencies in the domains of number and arithmetic, little is known about the phenotypic expression of this form of LD or the underlying brain and cognitive deficits (Geary, 1993; Geary & Hoard, 2001, in press). Even in the comparatively well studied domain of arithmetic, most of the research has focused on how young children with MD solve simple problems (e.g., 4 + 5; Garnett & Fleischner, 1983; Geary, 1990; Jordan & Montani, 1997), and less commonly on the cognitive mechanisms that may underlie these performance characteristics (Bull & Johnston, 1997; Geary, Bow-Thomas, & Yao, 1992). The goals of this study were to fill some of these knowledge gaps. Specifically, we extended the assessment of arithmetic competencies to children with MD who are older than those typically studied and assessed performance for solving simple (e.g., 3 + 5) as well as more complex (e.g., 16 + 7) problems. We also assessed working memory capacity and knowledge of counting principles and tested predictions regarding how group differences in these domains might contribute to group differences in arithmetic performance. In the following sections, we review normal development and the characteristics of children with MD for arithmetic development, working memory, and counting knowledge, respectively. Specific predictions of this study are presented in the final section.

Arithmetic development

Normal development. The most thoroughly studied developmental and schooling-based improvement in arithmetical competency is change in the distribution of strategies children use during problem solving (Ashcraft, 1982; Carpenter & Moser, 1984; Geary, 1994; Siegler, 1996; Siegler & Shrager, 1984). When first learning to solve simple addition problems (e.g., 5 + 3), children initially count both addends. These counting procedures are sometimes executed with the aid of fingers, the finger counting strategy, and sometimes without them, the verbal counting strategy (Siegler & Shrager, 1984). The two most commonly used counting procedures, whether children use their fingers or not, are called min (or counting on) and sum (or counting all; Fuson, 1982; Groen & Parkman, 1972). The min procedure involves stating the larger-valued addend and then counting a number of times equal to the value of the smaller addend, such as counting 5, 6, 7, 8 to solve 5 + 3. Sometimes children will start with the smaller addend and count the larger addend, which is the max procedure. The sum procedure involves counting both addends starting from 1. The development of procedural competencies is related, in part, to improvements in children's conceptual understanding of counting and is reflected in a gradual shift from frequent use of the sum procedure to the min procedure (Geary et al., 1992; Siegler, 1987).

The use of counting also results in the development of memory representations of basic facts (Siegler & Shrager, 1984). Once formed, these long-term memory representations support the use of memory-based problem-solving processes. The most common of these are direct retrieval of arithmetic facts and decomposition. With direct retrieval, children state an answer that is associated in long-term memory with the presented problem, such as stating "eight" when asked to solve 5 + 3; direct retrieval is typically used for problems for which both addends are less than 10. Decomposition involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem 6 + 7 might be solved by retrieving the answer to 6 + 6 (i.e., 12) and then adding 1 to this partial sum. The use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer. Children with a rigorous criterion only state answers that they are certain are correct, whereas children with a lenient criterion state any retrieved answer, correct or not (Siegler, 1988). The transition to memory-based processes results in the quick solution of individual problems and reductions in the working memory demands that appear to accompany the use of counting procedures (Delaney, Reder, Staszewski, & Ritter, 1998; Geary, Bow-Thomas, Liu, & Siegler, 1996; Lemaire & Siegler, 1995).

Children use similar strategies when solving more complex problems, such as 17 + 6 (Fuson, Stigler, & Bartsch, 1988; Geary, 1994). With initial learning, the most common of these involves use of the min procedure, as in counting 17, 18 ... 23, or decomposition. As an example, decomposition would involve breaking the 6 into two 3s, and then adding these in succession, 17 + 3 = 20 + 3 = 23. With formal schooling and especially with complex problems, such as 27 + 38, children will use the commonly taught columnar strategy (i.e., summing the ones-column integers and then summing the tens-column integers).

Children with MD. During the solving of simple arithmetic problems (e.g., 4 + 3), children with MD use the same types of strategies (e.g., verbal counting) as their normally achieving peers, but differ in the strategy mix, strategy accuracy, and in the pattern of developmental change (Geary, 1990; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). These differences have been found in the United States (Geary & Brown, 1991; Jordan & Montani, 1997; Jordan, Hanich, & Kaplan, 2003a), Europe (Barrouillet, Fayol, & Lathulière, 1997; Ostad, 1997, 1999, 2000; Svenson & Broquist, 1975), and Israel (Gross-Tsur et al., 1996). As an example, Geary and colleagues (Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999) found that in first and second grade, children with MD committed more counting errors and used the developmentally immature sum procedure more frequently than did their normally achieving peers or children with reading disability (RD). In keeping with models of normal arithmetical development, in first and second grade, the children with RD and the normally achieving children shifted from heavy reliance on finger counting to verbal counting and retrieval, and committed fewer errors. The children with MD, in contrast, did not show this shift, but instead relied heavily on finger counting in both grades, and continued to commit finger and verbal counting errors. The most consistent finding in this literature is that children with MD have difficulty retrieving basic arithmetic facts from long-term memory (Barrouillet et al., 1997; Garnett & Fleischner, 1983; Geary, 1990, 1993; Jordan & Montani, 1997; Ostad, 1997, 2000). Unlike the use of counting strategies, the ability to retrieve basic facts apparently does not substantively improve across the elementary-school years for most children with MD, suggesting the retrieval difficulties result from a persistent cognitive deficit, as contrasted with delayed development (Geary, 1993).

Russell and Ginsburg (1984) compared fourth-grade children with MD to normally achieving children in third and fourth grade on a variety of number and arithmetic tasks. Included among these were tasks that assessed the ability to solve simple (e.g., 3 + 9) arithmetic problems mentally and to solve more complex (e.g., 17 + 34) arithmetic problems mentally and with paper and pencil. For the mental solving of simple problems, the children with MD correctly retrieved fewer answers than did the normally achieving children in third or fourth grade, in keeping with the research already described. For the solving of more complex problems, children in all of the groups tended to count, or use decomposition or regrouping (e.g., adding the units column values and then the tens column values). For the complex problems, the children with MD did not differ from the normally achieving third graders, but they did not perform as well as the normally achieving fourth graders. The children with MD used the same types of counting and decomposition strategies as their normally achieving fourth-grade peers, but committed more execution and working memory errors, in keeping with the results described in the next section.

Working memory and counting span

Working memory is the ability to maintain explicitly a mental representation of some amount of information, while being engaged simultaneously in other mental processes. According to Baddeley (1986, 2000), working memory is dependent on a central executive that is expressed as attention-driven control of information represented in three slave systems, a language-related phonetic system, a visuospatial sketch pad, and an episodic buffer. Debates regarding the nature of these components of working memory are discussed elsewhere (Miyake & Shah, 1999). The issues here concern developmental change in the overall capacity of working memory in normally achieving children, and the working memory of same-age children with MD.

Normal development. The capacity of working memory increases from preschool through the elementary school years. As an example, preschool children can hold three to four items of some forms of information, such as numbers, in working memory, whereas a typical fourth grader can hold five to six items (Kail, 1990). The mechanisms underlying these developmental changes appear to include an improved ability to use strategies, such as rehearsal, to keep the information active in working memory (e.g., Kreutzer, Leonard, & Flavell, 1975), and changes in more fundamental components that support age-related improvements in working memory capacity. The latter include one or some combination of an improved ability to control the focus of attention, increased speed of processing information represented in the slave systems, or slower decay of information represented in the slave systems (e.g., Cowan, Saults, & Elliott, 2002; Kail, 1991). In a recent review, Cowan et al. determined that all of these fundamental components improve as part of normal development in childhood, and thus each contributes to the observed increase in working memory capacity.

Children with MD. Children with MD do not perform as well as their same-age peers on a variety of working memory tasks (Geary, Brown, & Samaranayake, 1991; Geary et al., 1999; Hitch & McAuley, 1991; McLean & Hitch, 1999; Siegel & Ryan, 1989; Swanson, 1993). One often-used task, counting span, is highly relevant in terms of the working memory processes involved in the use of counting procedures (Hitch & McAuley, 1991; Siegel & Ryan, 1989); thus, we used this task in this study. Here, children must maintain one or a series of integers in working memory while engaged in the act of counting; counting span is the number of integers that can be accurately held in working memory. However, the mechanisms underlying group differences in counting span and other working memory tasks are unclear. Some studies suggest that the differences reside in fundamental differences in speed of processing, as in speed of articulating number words (Bull & Johnston, 1997; Hitch & McAuley, 1991), whereas other studies suggest that the differences reside in poor information representation in the slave systems (Geary, 1993), or in executive/attentional control (Bull, Johnston, & Roy, 1999; McLean & Hitch, 1999).

Whatever the causes, the relation between the working memory deficit of children with MD and the group differences in the developmental maturity and mix of the strategies used to solve arithmetic problems has not been explored systematically. Geary (1990, 1993) hypothesized that children with MD rely heavily on finger counting and commit more counting errors as a result of poor working memory resources. If so, then measures of working memory, such as counting span, should be related to individual differences in use of finger counting and frequency of finger and verbal counting errors, and should contribute to differences in these strategy variables comparing children with MD to their normally achieving peers. The importance of working memory may also increase with increases in task novelty and complexity, and thus may be more important during the solving of complex compared to simple arithmetic problems, and during the initial stages of learning (Ackerman, 1988).

Counting knowledge

Normal development. Children's early counting knowledge and counting behavior can be represented by Gelman and Gallistel's (1978) five implicit and perhaps inherent principles. These principles include one–one correspondence (one and only one word tag, such as "one," "two," is assigned to each counted object); stable order (the order of the word tags must be invariant across counted sets); cardinality (the value of the final word tag represents the quantity of items in the set); abstraction (objects of any kind can be collected together and counted); and, order-irrelevance (items within a given set can be tagged in any sequence). The principles of one–one correspondence, stable order, and cardinality define the "how to count" rules, which, in turn, appear to constrain the nature of preschool children's counting behavior and to provide the skeletal structure for children's emerging knowledge of counting (Gelman & Meck, 1983).

In addition, children make inductions about the basic characteristics of counting by observing standard counting behavior and the associated outcomes (Briars & Siegler, 1984; Fuson, 1988). These inductions may elaborate Gelman and Gallistel's counting rules (1978) and result in a belief that certain unessential features of counting are essential (Briars & Siegler, 1984). In particular, young children often induce that the unessential features of adjacency (items must be counted contiguously) and start at an end (counting must start from the leftmost item) are in fact essential. The latter beliefs indicate that young children's conceptual understanding of counting is rigid and influenced by the observation of standard counting procedures.

Children with MD. Geary et al. (1992) contrasted the performance of first-grade children with MD and their normally achieving peers for tasks that assessed all of Gelman and Gallistel's (1978) basic principles and most of Briars and Siegler's (1984) unessential features of counting. The procedure involves asking children to help a puppet learn how to count. The child watches the puppet count a series of objects. The puppet sometimes counts correctly and sometimes violates one of Gelman and Gallistel's counting principles or Briars and Siegler's unessential features of counting. The child's task is to determine if the puppet's count was "OK" or "not OK and wrong." In this way, the puppet performs the procedural aspect of counting (i.e., pointing at and tagging items with a number word), leaving the child's responses to be based on her conceptual understanding of counting.

The results revealed that children with MD differed from normally achieving children on two types of counting trials, pseudoerror and error. Pseudoerror trials involved counting, for instance, the first, third, fifth, and seventh items and then returning to the left-hand side of the array and counting the second, fourth, and sixth items. Technically the count is correct, but violates the adjacency rule and assesses the child's understanding of the order-irrelevance principle. Error trials involved double counting either the first or the last item. Children with MD correctly identified these counts as errors when the last item was double counted, suggesting that they understood the one–one correspondence principle. Double counts were often labeled as correct when the first item was counted, suggesting that many children with MD have difficulties holding information in working memory—in this case noting that the first item was double counted—while monitoring the act of counting. A follow-up study that controlled for group differences in intelligence (IQ) confirmed these findings (Geary et al., 1999; Geary et al., 2000). Children with MD, regardless of their reading achievement levels, performed poorly on pseudoerror trials in first and second grade and on error trials (double counting the first item in a series) in first grade. The pattern suggests that even in second grade, many children with MD do not fully understand counting concepts, and in first grade, many children with MD may have difficulty holding information in working memory while monitoring the counting process (Hoard, Geary, & Hamson, 1999).

Geary et al. (1992) found that performance on pseudoerror trials was correlated ($r = .47$) with use of the min procedure when finger counting or verbal counting was used to solve simple addition problems, and explained the difference in use of min counting comparing children with MD to their normally achieving peers. Ohlsson and Rees (1991) predicted that children's counting knowledge and skill at detecting counting errors would enable them to correct these miscounts and thus eventually result in fewer counting errors. In support of this prediction, Geary et al. found that a combination of pseudo and error-trial scores from the counting knowledge task was significantly related to the frequency of finger and verbal counting errors while solving simple addition problems ($r = -.44$), and explained the group difference in the frequency of these errors.

The current study

This study was the first to simultaneously assess working memory capacity (i.e., counting span) and counting knowledge as they contribute to individual and group differences in the pattern of strategy usage and strategy accuracy during arithmetical problem solving. This study also adds to the literature by extending the assessment across the elementary school years and with the inclusion of both simple and more complex addition problems. First, performance on the counting knowledge task was predicted to correlate with individual and group differences in counting errors and use of the min procedure, based on the findings just described (Geary et al., 1992, 2000; Ohlsson & Rees, 1991). Second, we predicted that working memory/counting span would correlate with the use of finger counting as a problem-solving strategy, would correlate with the frequency of counting errors, and would contribute to group differences on both of these dimensions. These predictions follow from earlier suggestions that the use of fingers during counting is a working-memory aid (Geary, 1990), and that the poor working memory skills of children with MD (Hitch & McAuley, 1991) may contribute to their tendency to rely heavily on finger counting and to commit finger and verbal counting errors frequently (Geary et al., 2000; Jordan & Montani, 1997). The predictions do not preclude working memory contributions to the use of other procedures, such as verbal counting, but rather were specifically derived as an attempt to understand the mechanisms that contribute to group differences in the use of finger counting and counting errors.

In contrast, we predicted working memory capacity would be a less important contributor to individual or group differences in the execution of more automatized memory-based processes, particularly direct retrieval. This prediction was based on Ackerman's (1988; Ackerman & Cianciolo, 2000) findings that working memory is most important during the initial phases of skill acquisition and becomes less important with learning, and Siegler's (1996; Siegler & Shrager, 1984) model of strategy choice and patterns of normal arithmetical development. As noted earlier, use of counting during problem solving appears to result in the formation of associations between the problem and the answer generated by means of counting. Eventually, the associations result in the automatic retrieval of the answer, and at this point, working memory is predicted to be less important in terms of understanding group and individual differences. Working memory capacity may still be correlated with retrieval frequency, because children with poor working memory resources may execute counting procedures more slowly and less accurately than other children and thus not easily form the associations needed to support direct retrieval (Geary et al., 1996). In short, poor working memory may result in slow acquisition of basic facts, but should not be as important for the dynamics of retrieving those facts that do become committed to long-term memory.

Method

Participants

A total of 228 first-, third-, or fifth-grade children from four schools in Columbia, Missouri, were administered standardized achievement tests in mathematics and reading, and a standard intelligence (IQ) measure. Following an earlier study (Geary et al., 2000) and in line with goals of the current study, children with IQ scores less than 80 ($n = 11$) or greater than 120 ($n = 26$), and children with a combination of low reading scores (<30th percentile) and average or better scores (>30th percentile) in mathematics ($n = 42$) were excluded from this study. Of the 149 remaining children, 4 were Asian, 29 were Black, 7 were of mixed race, 105 were white, and 4 were of an unknown ethnic background. The ethnic breakdown across grade and group is shown in Table 1.

None of the children with MD at any grade level received any remedial instruction in mathematics, and the majority of children in both the MD and normal groups came from classrooms and used textbooks that followed a constructivist perspective on mathematics education. That is, the instruction focused on children's conceptual knowledge and tended to deemphasize the memorization of mathematical procedures and arithmetic facts.

Standardized tests. All children were administered the Vocabulary and Pattern Analysis subtests of the Stanford–Binet Intelligence Scale (Thorndike, Hagen, & Sattler, 1986). An IQ score was estimated based on performance on these two subtests, as prescribed by the test manual. The children were also administered the Mathematics Reasoning subtest of the Wechsler Individual Achievement Test (Wechsler, 1992) and the Test of Word Reading Efficiency (Torgesen, Wagner, & Rashotte, 1999). The latter is comprised of two subtests, both of which are speeded reading tests in which the child reads aloud as many words as possible from a printed protocol in 45 s. The Sight-Word Efficiency subtest assesses the child's ability to recognize and

Table 1
Ethnic background and mean age, intelligence, and achievement scores

Group	*N*		Age		Ethnic			IQ		Mathematics reasoning		Reading	
	Boy	Girl	*M*	*SD*	Black	White	Other	*M*	*SD*	*M*	*SD*	*M*	*SD*
Grade 1													
MD	10	11	84	5	8	10	3	95	11	19	8	23	18
Normal	15	26	80	4	2	35	4	104	20	67	18	71	22
Grade 3													
MD	7	9	107	5	5	9	2	90	6	19	7	33	34
Normal	9	10	106	4	2	15	2	108	9	64	14	76	17
Grade 5													
MD	15	6	132	4	9	10	2	93	10	19	7	19	21
Normal	12	19	129	4	3	26	2	106	9	70	21	64	21

Note. Age is in months; achievement scores are national percentile rankings, based on age. Other refers to mixed race, Asian, or unknown ethnic status. With the exception of one first-grade and two third-grade Hispanic children (in the normal groups), White refers to non-Hispanic Caucasian.

quickly read words, whereas the Phonemic Decoding Efficiency subtest assesses the child's ability to use phonetic rules to read nonsense words quickly. The total score is based on performance across the two subtests. The Mathematics Reasoning subtest assesses basic arithmetic skills such as counting and subtraction, as well as some more advanced skills such as graph reading and time telling.

Classification scheme. Children with mathematics reasoning scores less than the 30th percentile were classified as MD, independent of their reading achievement scores. Reading achievement scores were not used in the classification, because for the tasks used in this study, the same pattern of deficits is evident for children with MD, whether or not they show comorbid RD (e.g., Geary et al., 2000; Jordan et al., 2003a). Nonetheless, 14 of the 21 first-grade children with MD had total scores on the Test of Word Reading Efficiency less than the 30th percentile, as did 11 of the 16 third-grade children with MD and 18 of the 21 fifth-grade children with MD. In all, 43 of the 58 children with MD showed evidence of a more general learning disability, but because the remaining 15 children did not have a reading deficit and because the focus is on arithmetic competencies, all of the children are referred to as MD.

It should be noted that the cutoff of the 30th percentile does not fit with the earlier described estimation that between 5 and 8% of children have some form of MD. The discrepancy results from the nature of standardized achievement tests and the often rather specific memory and (or) cognitive deficits of children with MD. Standardized achievement tests sample a broad range of arithmetical and mathematical topics, whereas children with MD often have severe deficits in some of these areas and average or better competencies in others. The result of averaging across these topics is a level of performance that overestimates the competencies of children with MD in some areas and underestimates them in others (Jordan, Hanich, & Kaplan, 2003b).

In any event, numbers of boys and girls, mean age, IQ, and achievement scores are shown in Table 1. The number of boys and girls did not differ across groups in first, $\chi^2(1) < 1$, or third, $\chi^2(1) < 1$, grade, but there were more boys with MD than girls with MD in fifth grade, $\chi^2(1) = 5.37$, $p < .05$. A two (group) by three (grade) Analysis of Variance (ANOVA) revealed a significant group difference in mean age, $F(1, 143) = 15.80$, $p < .001$, but a nonsignificant grade by group interaction, $F(2, 143) < 1$. The children with MD were, on average, from 1 to 4 months older than the same-grade normally achieving children. There was also a significant group difference in mean IQ scores, $F(1, 143) = 63.19$, $p < .001$, but nonsignificant grade, $F(2, 143) < 1$, and grade by group, $F(2, 143) = 2.64$, $p > .05$, effects. A two (group) by three (grade) Analysis of Covariance, with IQ as the covariate, confirmed significant group differences in mathematics reasoning, $F(1, 142) = 194.47$, $p < .001$, and reading achievement, $F(1, 142) = 71.46$, $p < .001$. These findings indicate that based on IQ, the children with MD have lower than expected academic achievement in mathematics and reading, in keeping with the classification of these children as LD.

Experimental tasks and procedures

Counting span/working memory. Visual counting span under memory load was assessed using a procedure similar to that of Hitch and McAuley (1991). Target (red) and distractor (blue) $\frac{1}{2}$-in.-diameter dots were placed randomly on 5 × 8-in. white index cards. The number of target dots followed the integer pattern in the digit span subtest of the WISC-III (Wechsler, 1991). So, if the digit span item was "3 9 5," then the corresponding counting-span cards included 3, 9, and 5 target dots, respectively, across three cards. Distractor dots were used to prevent subitzing, and their number was randomized such that the total number of dots on each card ranged from 3 to 16. The number of target red dots was then subtracted from this number and the remainder determined the number of distractor blue dots. A random number generator was then used to determine the placement of the target and distractor dots within a grid of 84 positions, with the exception that target dots were not placed in adjacent or diagonal positions.

The goal of the task (to remember the number of red dots on each card) was explained to the child and a practice set of two cards was presented. The task began by placing the two cards face down on the table. The first card was turned face up and the child was asked to count the red dots on the card. After the count was complete, the card was returned to a face down position, and the next card was turned over for the child to count the red dots. That card was then returned to the face-down position and the child was asked to remember how many red dots were on the first card and how many were on the second card. The experimental trials began with two series at the two-card level. The length of the series was increased by one card after two presentations of each length. After the child failed to repeat correctly two consecutive presentations at the same level, the test was terminated. The score was the value of the longest series of cards correctly counted and repeated.

Counting knowledge. The child was first introduced to a puppet (e.g., Big Bird) that was just learning how to count and therefore needed assistance to know if his counting was "OK and right" or "not OK and wrong." During each of the 13 trials, a row of 7, 9, or 11 poker chips of alternating color (e.g., red, blue, red, blue, red) were aligned behind a screen. The screen was then removed, and the puppet counted the chips. The child was then queried on the correctness of the counting. The experimenter recorded whether the child stated the puppet's count was "OK" or "not OK and wrong."

Following previous studies, four types of counting trials were administered (Briars & Siegler, 1984; Geary et al., 1992; Geary et al., 2000; Gelman & Meck, 1983): correct, right-left, pseudoerror, and error. For correct trials, the chips were counted sequentially and correctly, from the child's left to the child's right. Right-left involved counting the chips sequentially and correctly, but from the child's right to the child's left. For pseudoerror trials, the chips were counted correctly from left to right, but first one color was counted, and then, returning to the left-hand side of the row, the count continued with the other color. For error trials, the chips were counted sequentially from left to right, but the first chip was counted twice. Each counting trial type occurred once for each array size (i.e., 7, 9, 11), with one additional pseudoerror count (for 7 chips) as the last trial. If the child stated that this final count was "not OK," the experimenter asked:

> For that last problem, and the others like it where Big Bird counted all of one color first and then all of the other color, you said that he counted the wrong way. Do you think that he

counted the wrong way and got the *wrong* answer, or, do you think that he counted the wrong way but still got the *right* answer?

The response was scored as either the answer and counting method were wrong, or the answer was right but the counting method was wrong.

Addition strategy assessment. Simple and complex addition problems were presented, one at a time, at the center of a 5 by 8-in.card. The integers were presented in a large font (approximately 1 in. tall) and presented at the center of the card. The simple stimuli were 14 single-digit addition problems presented horizontally (e.g., $4 + 5 =$) . The problems consisted of the integers 2 through 9, with the constraint that the same two integers (e.g., $2 + 2$, $4 + 4$) were never used in the same problem, as these appear to be solved differently than other problems (Groen & Parkman, 1972). Across stimuli, each digit was presented two to four times, and one half of the problems summed to 10 or less, inclusive. For one half of the problems, the larger-valued integer was presented in the first position, and for the remaining problems, the smaller-valued integer was presented in the first position. The order of problem presentation was determined randomly, with the constraint that no integer was presented in the same position across consecutive trials. The complex stimuli were six double-digit/single-digit problems (e.g., $16 + 7$). The double-digit integers were 14 to 19, inclusive, and the single-digit integers were 3, 4, 6, 7, 8, and 9. A double-digit integer was presented first for three problems, and a single-digit integer was presented first for the three remaining problems. The ones-column value of the double-digit integer always differed from the value of the single-digit integer, and all sums ranged between 21 ($3 + 18$) and 25 ($6 + 19$).

Following two practice problems, the 14 simple problems were presented one at a time, followed immediately by the six complex problems. The child was asked to solve each problem (without the use of paper and pencil) as quickly as possible without making too many mistakes. It was emphasized that the child could use whatever strategy was easiest for her to get the answer, and the child was instructed that as soon as she had the answer, she was to speak it aloud. Based on the child's answer and the experimenter's observations, the trial was classified into one of six strategies; specifically, counting fingers, fingers, verbal counting, retrieval, decomposition, or other/mixed strategy (Siegler & Shrager, 1984). A mixed trial was one in which the child started using one strategy, such as finger counting, but completed the problem using another strategy, such as retrieval. Counting trials were further classified based on where counting began, that is, min, sum, or max.

During problem solving, the experimenter watched for physical indications of counting, such as regular movements (e.g., fingers, mouth). For these trials, the experimenter initially classified the strategy as finger counting or verbal counting, depending on whether or not the child used her fingers to count. If the child upraised a number of fingers to represent the addends and then stated an answer without counting them, then the trial was initially classified as fingers. If the child spoke the answer quickly, without hesitation, and without obvious counting-related movements, then the trial was initially classified either as retrieval or as decomposition if this was the child's predominant retrieval-based strategy. After the child had spoken the answer, the experimenter queried the child on how she got the answer. If the child's response (e.g., "just knew it") differed from the experimenter's observations (e.g., saw the child mouthing counting), then a notation indicating disagreement between the child and the experimenter was made. If counting was overt, then the experimenter classified it as a counting strategy. If the trial was ambiguous, then the child's response was recorded as the strategy. On verbal counting trials, the experimenter probed the child as to how she counted, and the child's response was recorded on the experimental protocol. For instance, when asked how 3 + 5 was solved, the experimenter asked one child, "Can you tell me out loud what you did in your head?"; the experimenter never prompted a particular response such as asking whether the child counted the first, second, or both addends. The child answered, "I said three, four, five, six, seven, eight." This trial was coded verbal counting, max procedure.

Previous studies indicate that this method provides a useful measure of children's trial-by-trial strategy choices (e.g., Siegler, 1987). In this study, agreement between the child's description and the experimenter's observation of the molar strategy (e.g., verbal counting, but not specific procedure such as min) was found for more than 95% of trials.

Procedures

All children were tested twice during the academic year, once in fall and once in spring. Fall testing included the just-described experimental tasks, and spring testing included the IQ and achievement tests. These tests were administered in the spring because previous studies suggest that achievement test scores obtained at the end of the academic year are more predictive of achievement in later grades than are scores obtained at the beginning of the year (Geary, 1990).

For fall testing, the experimental tasks were administered in the following order: Counting Span/Working Memory, Counting Knowledge, and Strategy Assessment. For spring testing, the tests were administered in the following order: Test of Word Reading Efficiency, Mathematics Reasoning, Vocabulary, and Pattern Analysis. Testing time was approximately 30 and 40 min for the fall and spring assessments, respectively. Time elapsed between testing periods was 122 ($SD = 17$), 123 ($SD = 22$), and 137 ($SD = 15$) days, respectively, for the first-, third-, and fifth-grade children. Time elapsed did not differ significantly across groups, $F(1, 143) < 1$; grade by group interaction, $F(2, 143) = 1.39$, $p > .25$. However, the fifth-grade children were assessed approximately 2 weeks later, on average, than the younger children (ps $< .05$).

Results

We present group differences on the experimental tasks in the first two sections. Because it is not yet known if a discrepancy between IQ and achievement is useful for diagnosing MD, IQ was used as a covariate in all the reported analyses, unless noted otherwise. The result is a more conservative assessment of group differences, although in nearly all cases the results were the same with and without IQ as a covariate. In the few instances in which there were substantive differences for the analyses with and without the covariate, we report both sets of results; group means are actual means, not means adjusted for IQ. In the final section, we present analyses of the relation between counting span and counting knowledge and strategy choices.

Strategy assessment

The pattern of strategy usage across groups, grades, and for simple and complex problems is shown in Fig. 1. The corresponding error percentages are shown in Fig. 2 and use of min counting is shown in Fig. 3. The vast majority of problems were solved with finger counting, verbal counting, retrieval, or decomposition; thus, only these strategies are listed in the figures. In each of the figures, statistically significant between-group differences are noted by brackets and significant within-group differences comparing the simple and complex problems are noted by arrows. To illustrate, Fig. 1 shows that in first grade the children with MD used finger counting more frequently than did their normal peers. With the switch to complex problems, the first-grade children with MD showed a significant decrease in finger counting (down arrow) and a significant increase in retrieval/guessing (up arrow).

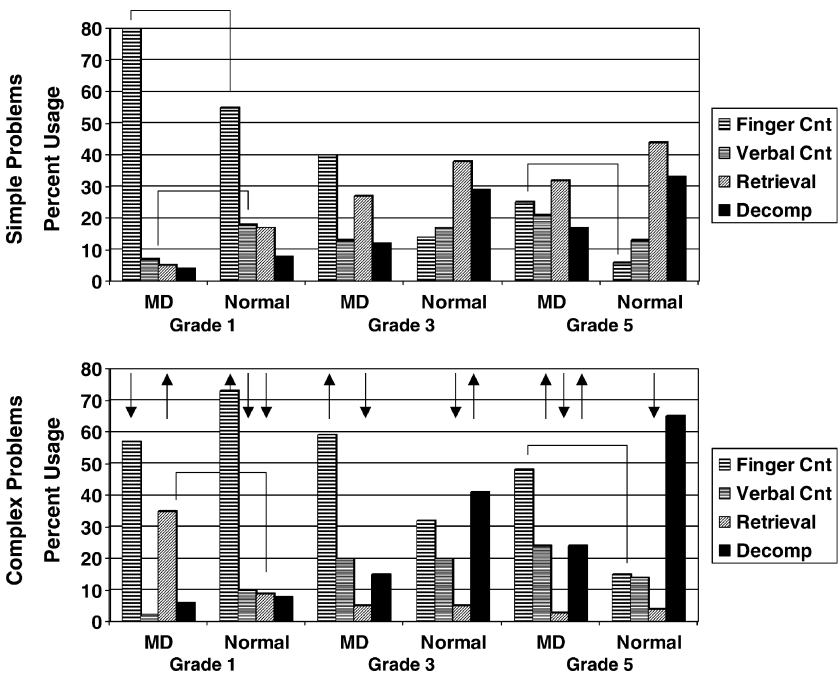

Fig. 1. The top and bottom sections show the percentage of trials on which the finger counting, verbal counting, retrieval, and decomposition strategies were used to solve simple and complex addition problems, respectively. Brackets indicate significant group differences, and within-group differences in strategy usage comparing simple to complex problems are noted by arrows. An up arrow indicates the strategy was used more frequently to solve complex problems, and a down arrow indicates the strategy was used less frequently to solve complex problems.

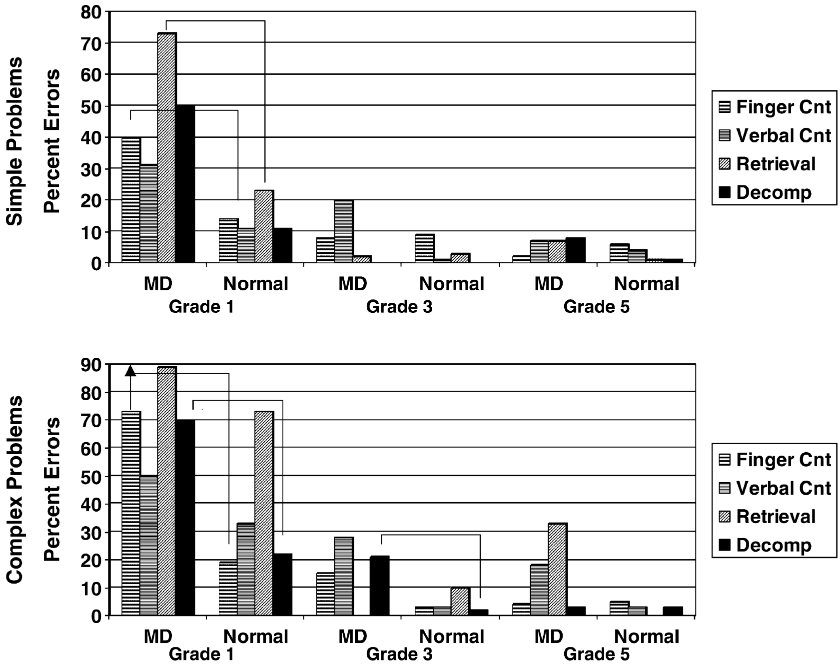

Fig. 2. The top and bottom sections show the percentage of finger counting, verbal counting, retrieval, and decomposition errors for simple and complex addition problems, respectively. Brackets indicate significant group differences. The single up arrow indicates that in first grade the children with MD committed more finger counting errors when solving complex as contrasted with simple problems.

The analyses of strategy usage was based on frequency, and the analyses of errors and min counting were based on percentages. Not all children used every strategy and, as a result, the *df* differ across some of the analyses.

Group differences

We begin with a description of overall accuracy collapsing across strategies, and then focus on between-group differences. In the latter section, the results for simple and complex problems are presented separately to facilitate comparisons to previous studies that have focused on simple problems and to highlight findings related to how the children solved the complex problems.

Accuracy. The overall problem-solving accuracy (across strategies) is shown across grade, group, and problem complexity in Table 2. A mixed ANCOVA with grade and group as between-subjects factors, problem complexity as a within-subjects factor, IQ as the covariate, and overall percentage correct (across strategies) accuracy rate as the dependent variable revealed significant main effects for grade, $F(2, 138) = 62.46$, $p < .001$, group, $F(1, 138) = 8.03$, $p < .01$, and a significant grade by group interaction, $F(2, 138) = 13.10$, $p < .001$. Complexity interacted with grade, $F(2, 138) = 17.96$, $p < .001$, and group, $F(1, 138) = 4.12$, $p < .05$, but these were

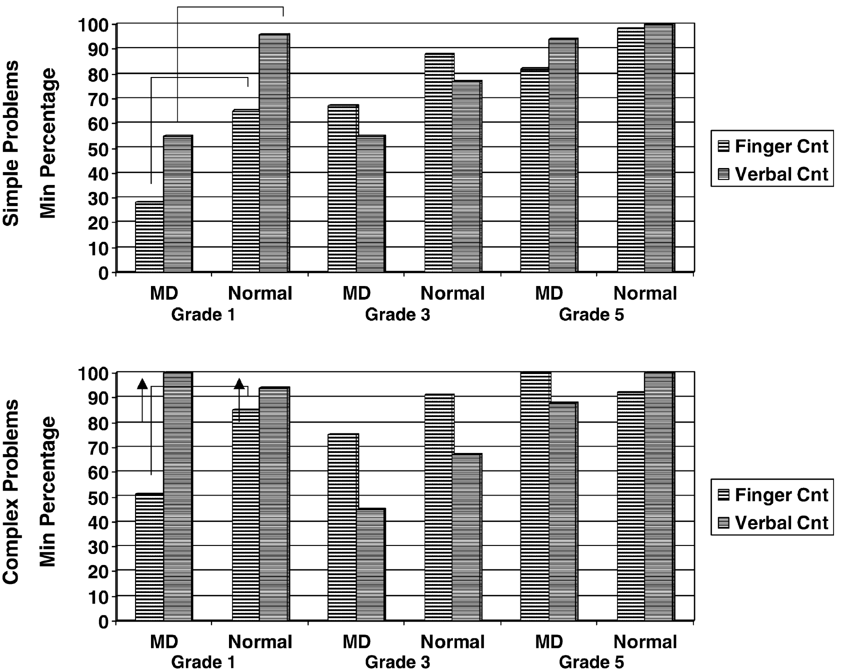

Fig. 3. The top and bottom sections show the percentage of trials on which the min procedure was executed when finger counting or verbal counting was used to solve simple and complex addition problems, respectively. Brackets indicate significant group differences in percentage of min usage, and an up arrow indicates the min procedure was used more often when solving complex as contrasted with simple problems.

Table 2
Overall problem-solving accuracy for simple and complex problems

Grade	MD		Normal	
	Simple	Complex	Simple	Complex
1	60	26	84	72
3	87	84	97	95
5	95	89	98	97

Note. Accuracy is the percentage correct across all problem-solving strategies.

qualified by a significant complexity by grade by group interaction, $F(2, 138) = 3.67$, $p < .05$. Follow-up analyses revealed a significant complexity by group interaction in first grade, $F(1, 55) = 7.03$, $p < .05$. The interaction emerged because first-grade children with MD correctly solved 60 and 26% of the simple and complex problems, respectively, compared with 84 and 72% for their normally achieving peers. The third-grade children with MD solved fewer complex problems (84%) than did the normally achieving third-grade children (95%), $F(1, 32) = 4.76$, $p < .05$. The percentage correct for simple problems in third grade and for simple and complex problems in fifth grade approached ceiling, ranging from 87 to 98, and did not differ across groups (*p*s > .10).

Simple problems. As shown in Fig. 1, the first-grade children with MD used finger counting more frequently than did their normally achieving peers (80% vs. 55%), $F(1,58) = 7.44$, $p < .01$, and verbal counting less frequently (7% vs. 18%), $F(1,58) = 4.44$, $p < .05$. The children with MD also used direct retrieval less frequently than did the normally achieving children (5% vs. 17%), but the difference did not reach conventional significance levels, $F(1,58) = 3.16$, $p = .08$. As shown in Fig. 2, in comparison to the normally achieving age-mates, first-grade children with MD committed relatively more errors when using finger counting (40% vs. 14%), $F(1,54) = 5.49$, $p < .05$, and retrieval (73% vs. 23%), $F(1,24) = 5.15$, $p < .05$. As shown in Fig. 3, the first-grade children with MD also used the min procedure less frequently when they used finger counting (28% vs. 65%), $F(1,54) = 6.34$, $p < .02$, and verbal counting (55% vs. 96%), $F(1,32) = 13.09$, $p < .001$.

For the third- and fifth-grade children, there were few significant group differences in strategy usage, error percentage, or use of min counting. The fifth-grade children with MD did, however, use finger counting more frequently (25% vs. 6%) than their normally achieving peers, $F(1,49) = 4.47$, $p < .05$ (see Fig. 1), and tended to commit more retrieval errors (7% vs. 1%), $F(1,45) = 3.54$, $p = .066$ (see Fig. 2).

Complex problems. As shown in Fig. 1, in first grade, the children with MD used retrieval more frequently than did their normally achieving peers (35% vs. 9%), $F(1,55) = 6.09$, $p < .02$, but had a very high percentage of errors when they used retrieval (89%), as did the normally achieving children (73%). In comparison to the normally achieving children, the first-grade children with MD committed more finger counting (73% vs. 19%), $F(1,47) = 20.26$, $p < .001$, and decomposition (70% vs. 22%), $F(1,8) = 5.64$, $p < .05$, errors, and used the min procedure less frequently when finger counting (51% vs. 85%), $F(1,47) = 5.62$, $p < .05$.

Again, there were few significant group differences in third and fifth grade. In third grade and in comparison to the normally achieving children, the children with MD committed more decomposition errors (21% vs. 2%), $F(1,16) = 4.71$, $p < .05$, and tended to commit more finger counting (15% vs. 3%), $F(1,22) = 3.19$, $p = .088$, and verbal counting (28% vs. 3%), $F(1,14) = 3.14$, $p = .098$, errors. In fifth grade, the children with MD used finger counting more frequently (48% vs. 15%), $F(1,49) = 3.95$, $p = .056$, tended to use decomposition less frequently (24% vs. 65%), $F(1,49) = 3.47$, $p = .069$, and tended to commit more verbal counting errors (18% vs. 3%), $F(1,23) = 3.47$, $p = .075$. In fifth grade, the group difference in retrieval errors (33% vs. 0%) was substantial but not significant; the nonsignificance might be due to the infrequent use of retrieval and thus low statistical power.

Complexity and strategy shifts

The goal of the analyses presented in the following sections was to determine if children with MD and their normal peers showed adaptive shifts in the mix of problem-solving strategies when the task shifted from simple to complex problems. For each of the strategies and associated error rates and min usage for finger and verbal counting, separate repeated measures ANCOVAs, with problem difficulty (simple vs.

complex) as the within-subjects factor and IQ as the covariate, were used to assess the shift in strategy choices comparing simple to complex problems.

Children with MD. In comparison to simple problems and as indicated by the arrows in Fig. 1, first-grade children with MD used finger counting less frequently (80% vs. 57%), $F(1, 17) = 63.76$, $p < .001$, and retrieval/guessing more frequently (5% vs. 35%), $F(1, 17) = 5.13$, $p < .05$, while solving complex problems. There was also a tendency to use verbal counting less frequently when solving complex problems (7% vs. 2%), $F(1, 17) = 4.05$, $p = .06$. The switch from simple to complex problems also resulted in a significant increase in finger counting errors (40% vs. 73%), $F(1, 12) = 31.68$, $p < .001$, and a tendency to use the min procedure more frequently during finger counting (28% vs. 51%), $F(1, 12) = 4.59$, $p = .053$.

In contrast, third-grade children with MD used finger counting more frequently when solving complex problems (40% vs. 59%), $F(1, 15) = 10.53$, $p < .01$, and retrieval less frequently (27% vs. 5%), $F(1, 15) = 26.73$, $p < .001$. Similar to the pattern found for the third-grade children, in fifth grade, the switch from simple to complex problems resulted in a substantial drop in retrieval frequency (32% vs. 3%), $F(1, 20) = 69.95$, and increased use of all other strategies, although the differences were only significant for verbal counting (21% vs. 24%), $F(1, 20) = 13.27$, $p < .01$, and decomposition (17% vs. 24%), $F(1, 20) = 5.68$, $p < .05$. In fifth grade, the increase in the use of the min procedure during finger counting (82% vs. 100%) approached conventional significance levels, $F(1, 12) = 4.28$, $p = .068$.

Normally achieving children. In first grade, the shift to complex problems resulted in a significant increase in finger counting (55% vs. 73%), $F(1, 39) = 32.60$, $p < .001$, and a significant decrease in verbal counting (18% vs. 10%), $F(1, 39) = 24.62$, $p < .001$, and retrieval (17% vs. 9%), $F(1, 39) = 17.73$, $p < .001$. The min procedure was also used more frequently in finger counting (65% vs. 85%), $F(1, 35) = 9.21$, $p < .01$, and there was a tendency for verbal counting (11% vs. 33%), $F(1, 9) = 4.33$, $p = .067$, and retrieval (23% vs. 73%), $F(1, 7) = 4.44$, $p = .073$, errors to increase.

A similar pattern of less retrieval and more frequent use of other strategies was evident in third and fifth grade. In third grade, the shift from simple to complex problems resulted in less retrieval (38% vs. 5%), $F(1, 18) = 78.69$, $p < .001$, and more decomposition (29% vs. 41%), $F(1, 18) = 9.19$, $p < .01$. The same shift was evident in fifth grade; for retrieval (44% vs. 4%), $F(1, 30) = 101.79$, $p < .001$, and decomposition (33% vs. 65%), $F(1, 30) = 2.96$, $p = .096$.

Between-group differences. To assess between-group differences in the pattern of strategy shifts, a mixed ANOVA was carried out at each grade, with group as the between-subjects factor, problem difficulty (simple vs. complex) and problem type strategy (e.g., finger counting and retrieval) as within-subjects factors, and IQ as the covariate. In first grade, the majority of problems were solved using finger counting or retrieval; thus, the analysis was restricted to these two strategies. The only significant effect was for the group by type by difficulty interaction, $F(1, 55) = 16.95$, $p < .01$. The interaction emerged because, as noted above, with the switch from simple to complex problems, children with MD used finger counting less frequently, whereas their normally achieving peers used it more frequently. The children with MD also used retrieval more frequently, but largely guessed (89% error rate), and the normally achieving children used it less frequently.

The same procedure was used for third and fifth grade, except the set of finger counting, verbal counting, retrieval, and decomposition strategies was used in one analysis, and each of these strategies was assessed separately in follow-up analyses. There were no significant group by type by difficulty or group by difficulty effects in either third or fifth grade (ps $> .10$). When IQ was dropped as the covariate, the group by difficulty effect remained nonsignificant in both grades (ps $< .10$), but the group by type by difficulty effect was marginally significant in both third, $F(3, 99) = 2.67$, $p = .052$, and fifth, $F(3, 150) = 2.22$, $p = .089$, grade. For third grade, the interaction emerged because with the switch from simple to complex problems, the children with MD used finger counting more frequently, $F(1, 33) = 5.23$, $p < .05$, and the normally achieving children used retrieval less frequently, $F(1, 33) = 3.13$, $p = .086$. As shown in Fig. 1, the normally achieving third graders also showed a significant increase in decomposition, but the group by difficulty interaction was not significant for this strategy ($p > .25$). For fifth grade, the three-way interaction emerged because with the shift to more complex problems, the drop in the frequency of retrieval was larger for the normally achieving children than for the children with MD, $F(1, 50) = 3.94$, $p = .053$, combined with no significant group by difficulty effects for any of the other strategies (ps $> .10$).

Summary

Differences in the strategies used by children with MD and their normally achieving peers were found largely in first grade. In this grade, children with MD primarily used finger counting to solve simple addition problems, compared with a mix of finger counting and direct retrieval for normally achieving children. With the use of counting, the children with MD committed more errors and used the min procedure less frequently. Even in fifth grade, children with MD relied on finger counting to solve 25% of the simple problems, compared with 6% for the normally achieving fifth graders. With the switch from simple to complex problems, normally achieving children in each grade appeared to make an adaptive change in the strategy mix. As an example, in first grade they used direct retrieval less often to solve complex problems, in favor of the more accurate finger and verbal counting strategies, whereas the same-grade children with MD guessed more and counted less comparing complex to simple problems. The children with MD did make adaptive changes in the strategy mix, but not until several grades after the normally achieving children showed the same adaptive change.

Working memory and counting knowledge

Working memory/Counting span

Mean counting span scores are shown in Fig. 4. Regression analyses were used to estimate mean change in counting span across grades and group, while controlling

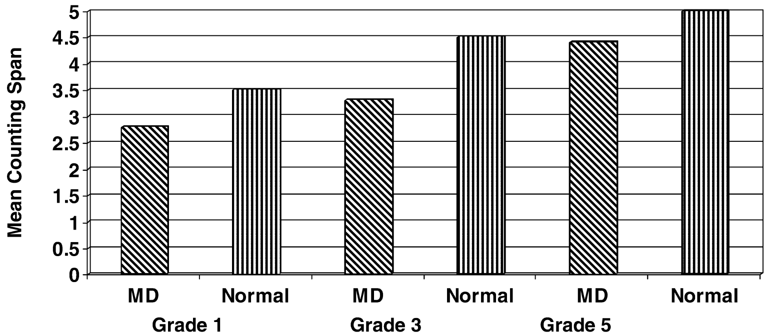

Fig. 4. Mean counting span scores.

for IQ. For the first contrast, first grade was coded 0, and third and fifth grade were coded 2 and 4, respectively. In this way, counting span in first grade served as the baseline and the contrast estimated mean change across grade level, controlling for IQ and group differences. For the second contrast, the normally achieving group was coded 0 and the MD group was coded 1. In this way, the counting span of the normally achieving children served as the baseline, and the contrast estimated the mean counting span deficit of the children with MD, controlling for grade level and IQ. The contrasts yielded an estimated .39 increase in counting span per grade level, $t(149) = 8.02$, $p < .001$, and a .36 counting span deficit for the children with MD, $t(149) = -2.11$, $p < .05$. A separate analysis yielded a nonsignificant grade by group interaction ($p > .25$), suggesting that children with MD have a counting span deficit that is constant across grades and equivalent to about one grade-level of growth in normally achieving children.

Counting knowledge

Recall that each child was asked to identify whether the puppet performed various types of counts correctly; thus, the variable of interest was whether the child correctly identified correct counts that varied in procedure (i.e., correct, right-left, and pseudoerror trials) as correct and error counts (i.e., error trials) as incorrect. If the latter was identified as "wrong," then the child correctly identified the error. The mean percentage of correct identifications across the four types of counting trial is shown in Fig. 5. A 2 (group) by 3(grade) by 4 (counting type) mixed ANCOVA, with group and grade as between-subjects factors and type as a within-subjects factor, revealed that all of the between subjects (ps $> .10$) and within subjects (ps $> .25$) main effects and interactions were nonsignificant. However, the pattern in the figure suggests significant differences across count type, and this was confirmed when IQ was dropped as the covariate, $F(1, 429) = 17.36$, $p < .001$, although all main effects and interactions involving grade and group remained nonsignificant (ps $> .10$). Across grade and group, correct trials were correctly identified 99% of the time, right-left and error trials were correctly identified 94% of the time, and pseudoerror trails were correctly identified 81% of the time. Follow-up analyses confirmed that correct trials were correctly identified more often than right-left and error trials,

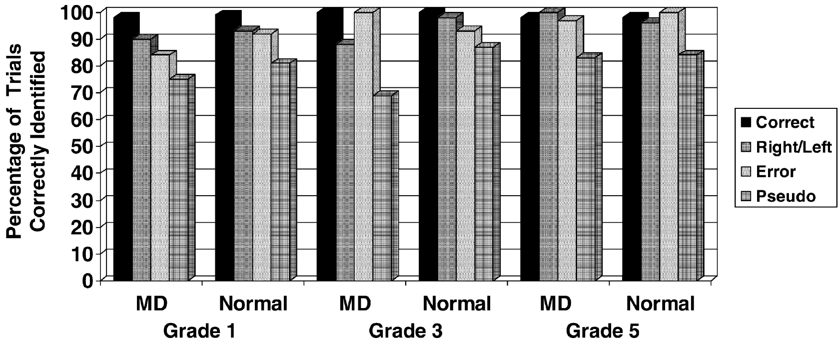

Fig. 5. The values are the percentage of trials that were correctly identified for correct, right/left, error, and pseudoerror counting trials. With an adjustment for guessing, children who stated the pseudoerror method was wrong but the count was correct (see Method section) were given credit for pseudoerror trials they initially identified as incorrect.

and both of the latter trial types were correctly identified more often than pseudoerror trials (*p*s $< .01$).

A post hoc analysis with IQ as a covariate revealed that normally achieving children outperformed the children with MD for performance combined across the error and pseudoerror trials in first grade, $F(1,57) = 6.82$, $p < .02$, but not third, $F(1,27) < 1$, or fifth, $F(1,36) < 1$, grade. Finally, recall that children who answered that the last pseudoerror count was "not OK" were explicitly asked if the method was wrong and the answer was right, or if both were wrong. Across grades, 7 of the 24 children with MD who missed the last pseudoerror item indicated both the method and answer were wrong compared to 4 of 41 normally achieving children, $\chi^2(1) = 4.06$, $p < .05$.

Working memory, counting knowledge, and strategy choices

As noted earlier, one of our goals was to determine if working memory (i.e., counting span scores) and counting knowledge contribute to individual and group differences in strategy choices. Preliminary analyses of the within-grade correlations between counting span and the four counting knowledge variables revealed *r*s ranging from $-.14$ to $.22$ (*p*s $> .05$), indicating that the counting span and counting knowledge tasks are assessing independent competencies; therefore these were used as independent predictors. Accordingly, raw strategy frequency and error percentage for all strategies, and min usage for finger counting and verbal counting were individually regressed on IQ, working memory (i.e., counting span), and the error and pseudoerror variables from the counting knowledge task. The result was three regression equations per grade for the finger counting and verbal counting strategies (frequency, percentage error, and percentage min usage) and two per grade for the retrieval and decomposition strategies (frequency and percentage error).

In the next section, we present individual differences results. These results are of interest because they yield information on the relation between counting knowledge, working memory, and children's strategy choices, independent of group status. These results were then used to narrow the focus of the group differences analyses reported under Group differences. In this section, we determine if any significant relations uncovered with the individual differences analyses contribute to the group differences in strategy choices. For all analyses, the variables were first standardized with a mean of 0 and a *SD* of 1, and all predictors were entered simultaneously. All four predictor variables (i.e., IQ, counting span, and counting knowledge error and pseudoerror) were used for the analyses of first grade strategy choices, but the error variable from the counting knowledge task was dropped for the third and fifth grade analyses, due to ceiling effects.

Individual differences

The β coefficients and associated *t*-values and significance levels for the finger counting and verbal counting strategies for the set of simple addition problems can be seen in Table 3. The first and fourth set of results indicate that the working memory and counting knowledge variables predicted when and how first graders used the finger counting and verbal counting strategies to solve simple addition problems. As working memory increased, the frequency of finger counting decreased, as did the percentage of finger counting errors. As scores on the counting knowledge pseudoerror variable increased, the frequency of min usage increased for finger counting. Turning to first graders use of verbal counting, as working memory increased, use of verbal counting increased, and as performance on the error-counting task improved, verbal counting errors decreased. At the same time, and again for first graders, the working memory and counting knowledge variables were not related to frequency of retrieval or decomposition, or to retrieval or decomposition error rates (ps $> .05$); these are not shown in the table. For complex problems, as working memory increased, the frequency of verbal counting errors decreased, $\beta = -.60$, $t(9) = -3.33$, $p < .01$, but there were no other significant relations involving the working memory or counting knowledge variables for first graders.

For third and fifth graders, working memory was not a significant predictor of the use of finger counting, verbal counting, retrieval, or decomposition to solve simple or complex problems (ps $> .05$) when IQ was used as a covariate. However, when IQ was dropped as a covariate, high working memory capacity (i.e., high counting span scores) was associated with less finger counting to solve simple problems in third grade, $\beta = -.31$, $t(22) = -2.06$, $p < .05$, and more frequent use of decomposition to solve complex problems in both third, $\beta = .41$, $t(32) = 2.67$, $p < .02$, and fifth, $\beta = .39$, $t(49) = 2.94$, $p < .01$, grade.

At the same time, the pseudoerror variable from the counting knowledge task was a consistent predictor of min usage in both third and fifth grades and for both simple and complex problems, whether or not IQ was used as a covariate. As shown in Table 3 for third graders, as performance on the pseudoerror trails improved, the frequency of min usage increased for finger counting, and verbal counting. The same pattern was found when third graders solved complex problems; as pseudoerror scores increased, min usage increased for finger counting, $\beta = .68$, $t(21) = 4.42$, $p < .001$, and for verbal counting, $\beta = .61$, $t(13) = 3.89$, $p < .005$. For fifth graders,

Table 3
Working memory and counting knowledge predictors of finger and verbal counting

Predictor	Counting trials			Counting errors (%)			Min counting (%)		
	β	t-value	p	β	t-value	p	β	t-value	p
First grade finger counting									
IQ	−.01	<1	NS	−.44	−3.67	.001	.38	3.02	.005
Count error	−.05	<1	NS	−.05	<1	NS	.04	<1	NS
Count pseudo	−.13	1.09	NS	.14	1.31	NS	.26	2.24	.05
Working mem	−.46	−3.74	.001	−.30	−2.28	.05	.22	1.56	NS
Third grade finger counting									
IQ	−.32	−2.00	.055	.21	<1	NS	.18	<1	NS
Count pseudo	−.38	−2.66	.02	−.10	<1	NS	.52	3.28	.005
Working mem	−.17	−1.08	NS	−.41	−1.76	.09	−.14	<1	NS
Fifth grade finger counting									
IQ	−.49	−3.24	.005	.45	1.70	NS	.28	1.07	NS
Count pseudo	.18	1.32	NS	−.22	<1	NS	.24	1.07	NS
Working mem	.00	<1	NS	−.35	−1.42	NS	.19	<1	NS
First grade verbal counting									
IQ	−.08	<1	NS	−.27	−1.75	NS	.52	3.54	.002
Count error	.03	<1	NS	−.42	−2.51	.02	−.03	<1	NS
Count pseudo	.15	1.23	NS	−.08	<1	NS	.23	1.20	NS
Working mem	.39	3.00	.005	−.06	<1	NS	−.20	−1.36	NS
Third grade verbal counting									
IQ	.06	<1	NS	−.41	−1.79	.09	.21	1.04	NS
Count pseudo	.09	<1	NS	−.01	<1	NS	.53	3.05	.01
Working mem	−.09	<1	NS	−.18	<1	NS	−.10	<1	NS
Fifth grade verbal counting									
IQ	−.37	−2.29	.05	−.14	<1	NS	.14	<1	NS
Count pseudo	−.04	<1	NS	−.38	−2.42	.05	.29	1.80	.08
Working mem	.00	<1	NS	−.04	<1	NS	.08	<1	NS

Note. Counting trials refers to the raw frequency with which finger counting or verbal counting was used to solve the simple addition problems. β = standardized ($M = 1; SD = 0$) regression coefficient. Count error is the counting error score on the Counting Knowledge task; Count pseudo is the pseudoerror score on the Counting Knowledge task; Working mem is working memory as indexed by counting span; NS = not significant ($p > .10$). The estimates for grades 3 and 5 are conservative, due to fewer overall finger and verbal counting trials.

the pseudoerror variable was a significant predictor of min usage while finger counting to solve complex problems, $\beta = .55$, $t(22) = 2.54$, $p < .02$, and it was a marginally significant predictor while verbal counting to solve simple problems. The latter relation was significant when IQ was dropped as a covariate, $\beta = .32$, $t(32) = 2.11$, $p < .05$. The two nonsignificant relations (ps $> .10$) between the pseudoerror variable and min usage were in the same direction; $\beta = .24$, $t(19) = 1.07$, for the use of finger counting to solve simple problems; and, $\beta = .30$, $t(22) = 1.61$, for use of verbal counting to solve complex problems. Again, the latter relation was significant when IQ was dropped as a covariate, $\beta = .35$, $t(23) = 2.08$, $p < .05$.

The only other significant pattern to emerge involved decomposition and the pseudoerror variable. High performance on the pseudoerror variable was associated with a greater frequency of decomposition in third grade, $\beta = .37$, $t(31) = 2.40$, $p < .05$, and fewer decomposition errors in fifth grade, $\beta = -.40$, $t(40) = -2.52$, $p < .02$.

Group differences

For first graders, significant group differences were found for frequency of finger counting and verbal counting to solve simple problems, and use of both of these counting strategies was related to working memory, as described previously. Based on these two sets of findings, it is possible that the group differences in the use of finger counting and verbal counting may be related to the group difference in working memory capacity. To test this possibility, the group difference in use of these strategies was re-examined in an ANCOVA, with group as a between-subjects factor and IQ and working memory (i.e., counting span) as covariates.

The group difference in the use of finger counting remained significant, $F(1,57) = 4.11$, $p < .05$. However, by covarying the working memory variable, the sums of squares associated with the group variable was reduced by 55%. Using procedures described by Geary et al. (1999), we determined that the magnitude of the reduction in the variance associated with the group variable was marginally significant, $F(1,56) = 3.81$, $p < .10$. When the group difference in working memory was controlled, the advantage of normally achieving children in the use of verbal counting was no longer significant, $F(1,57) = 2.11$, $p > .10$.

In first grade, there were also significant group differences in min usage for finger counting and verbal counting while solving simple problems, and for verbal counting while solving complex problems. As shown in Table 3, min usage was related to the pseudoerror variable, but the children with MD differed from their normally achieving peers only for a variable that summed pseudo and error-trial performance on the counting knowledge task. Thus, the ANOVAs included IQ and the combined (pseudo + error) variable. The results revealed that controlling for performance on the combined counting-knowledge variable eliminated the group difference in min usage while finger counting to solve simple, $F(1,53) = 2.26$, $p > .10$, and complex, $F(1,45) = 1.63$, $p > .20$, problems. For min usage while verbal counting to solve simple problems, controlling for the combined variable reduced (by 9%), but did not eliminate the group difference, $F(1,31) = 12.49$, $p < .002$.

We did not extend these analyses to third and fifth grade, because in these grades the group differences in strategy choices did not overlap the individual differences relations between working memory capacity, counting knowledge, and strategy choices described in the previous section.

Strategy shifts

Children with MD

Recall, in first grade the shift from simple to complex problems was associated with a decrease in the use of finger counting, an increase in guessing (i.e., retrieval with 89% error rate), and an increase in finger counting errors. Using the same procedures described in the previous section, we determined that when working memory (i.e., counting span scores) was controlled, there was a 75% reduction in the variance associated with problem complexity, $F(1, 15) = 36.36$, $p < .001$. In other words, 75% of the variance in the use of finger counting comparing simple to complex problems was explained by working memory capacity. Still, significant differences in use of finger counting across simple and complex problems remained, $F(1, 16) = 18.43$, $p < .001$, indicating that some factor other than working memory was related to the greater use of finger counting to solve simple compared to complex problems.

With regard to finger counting errors, controlling for working memory resulted in an 84% reduction in the variance associated with problem complexity, $F(1, 10) = 24.11$, $p < .001$, but the difference in the frequency of finger counting errors remained significant, $F(1, 11) = 4.74$, $p = .052$. In other words, the increase in finger counting errors with the switch from simple to complex problems can largely be attributed to increased working memory demands associated with the use of finger counting to solve complex problems. Finally, a similar analysis revealed that in first grade the pseudoerror variable from the counting knowledge task was not related to the increase in use of the min procedure comparing simple to complex problems.

Normally achieving children

In first grade, controlling for working memory eliminated the difference in use of verbal counting across simple and complex problems, $F(1, 38) < 1$, and reduced the difference in the use of finger counting by 55%, $F(1, 37) = 23.93$, $p < .001$; the difference in the use of finger counting across simple and complex problems remained significant, $F(1, 38) = 16.87$, $p < .001$. Stated differently, normally achieving children with high working memory capacity were better able to switch to finger counting effectively to solve the complex problems than were their peers with low working memory capacity.

In first grade, controlling for the pseudoerror variable eliminated the difference in min usage across simple and complex problems while finger counting, $F(1, 34) = 3.26$, $p > .05$. In third grade, controlling for the pseudoerror variable eliminated the difference in use of decomposition across simple and complex problems, $F(1, 17) < 1$. In other words, normally achieving children with low scores on the pseudoerror variable were less likely to switch to min counting and decomposition when confronted with the complex problems.

Discussion

We examined the strategies used by children with MD and their normally achieving peers to solve simple and more complex addition problems across three grade levels, and sought to determine whether individual and group differences in strategy usage were related to working memory capacity, as measured by the counting span task, and counting knowledge. Group and individual differences were evident in each of these areas, as were relations between working memory and counting knowledge and the pattern of strategy choices. Of course, the data are cross-sectional and thus inferences about developmental or grade-related changes need to be interpreted with caution, but nonetheless, this study does allow qualified inferences to be drawn in these areas.

Strategy choices

In first grade and in comparison to their normally achieving peers, children with MD relied heavily on finger counting to solve both simple and complex addition problems, committed many counting errors, and did not use the min procedure as often. For simple problems, the group differences for frequency of counting errors and use of the min procedure are in keeping with previous findings (Geary, 1990; Hanich et al., 2001; Jordan & Montani, 1997), but the group difference in reliance on finger counting is larger than is typical with such comparisons. The difference across studies is due to more finger counting and less guessing for the current sample of children with MD compared to samples assessed in previous studies (e.g., Geary et al., 2000).

Although less is known about strategy choices in older children with MD, the current results are consistent with findings from the comparatively fewer studies that have assessed these groups (Geary & Brown, 1991; Geary, Widaman, Little, & Cormier, 1987; Jordan et al., 2003a; Ostad, 1997, 1999). Consistent with the findings of Jordan et al. (2003a) and Ostad (1999), children with MD used finger counting more often than their peers in third and fifth grade, although the difference was not statistically significant for the current third-grade sample. In keeping with studies of third (Jordan et al., 2003a) and fourth graders (Geary & Brown, 1991), the children with MD in this study were just as accurate as their normally achieving peers in the use of counting strategies and had closed the gap in use of the min procedure.

With the switch from simple to complex problems, the children with MD as well as their normally achieving peers showed a shift in the strategy mix at all three grade levels. The shift did not appear to be particularly adaptive for the first-grade children with MD, as they abandoned finger counting in favor of guessing. However, for the complex problems the error rates for finger counting (73%) and retrieval/guessing (89%) did not differ substantively; thus, guessing might be considered an adaptive choice, as it is quicker and requires less effort. Either way, the normally achieving children showed clearly adaptive shifts in all three grades. In first grade, the switch from simple to complex problems was associated with a significant drop in use of the error-prone retrieval strategy and a significant increase in use of the more accurate finger and verbal counting strategies. Both groups of children also used the min procedure more frequently while solving complex compared to simple problems, possibly because the smaller-valued integer is easily identified with the complex problems used in this study. For the normally achieving children, the strategy shift was from less retrieval to more decomposition in third and fifth grade. The children with MD also showed adaptive shifts in the strategy mix in these grades. However, in third grade the shift was from retrieval to finger counting, but in fifth grade the shift was similar to that of the normally achieving children, that is, from retrieval to decomposition.

In all, the pattern for both groups is that of improving arithmetical competencies, as recently found by Jordan and her colleagues (Jordan et al., 2003a), but also of developmental delays for the children with MD. In terms of the actual strategies used during problem solving, the children with MD did not show the same level of strategy sophistication, even in fifth grade. Comparing fifth-grade children with MD to normally achieving third-grade children, there are many similarities in the strategy mix. In fact, a post hoc comparison of these groups revealed only two significant differences in strategy usage and accuracy: The fifth-grade children with MD used the min procedure more frequently than the normally achieving third graders while verbal counting to solve simple and complex problems ($ps < .05$). The same differences emerged with a comparison of the third- and fifth-grade normally achieving children ($ps < .05$), but, there were also significant reductions in finger counting while solving simple and complex problems, and a significant increase in the use of decomposition in fifth grade ($ps < .05$). The comparison suggests about a two grade-level delay in the sophistication of the mix of strategies children with MD use to solve addition problems.

Working memory and counting knowledge

Consistent with previous studies, the results for the working memory (i.e., counting span) task indicate that children with MD have a working memory deficit (Hitch & McAuley, 1991; McLean & Hitch, 1999; Swanson, 1993). Moreover, the current results suggest that this working memory deficit, at least as assessed by counting span, is equivalent to about one grade level of growth in normally achieving children, and improves at about the same rate as that of normally achieving children. Inconsistent with previous studies (Geary et al., 1992; Geary et al., 2000), we did not find substantial group differences in counting knowledge, although there was a tendency for the children with MD to perform more poorly than their normally achieving peers on the pseudoerror trails in first and third grade. The reasons for the difference in this study compared to our previous studies are not clear, but there is the possibility that group differences may emerge with more sophisticated counting-knowledge measures.

Regardless, the relations between the counting knowledge and working memory and the pattern of individual and group differences in strategy choices were largely consistent with predictions (Geary, 1990; Geary et al., 1992; Ohlsson & Rees, 1991). As predicted, strong performance on the working memory measure was associated with less finger counting, more verbal counting, and fewer finger counting errors in first grade, and less finger counting in third grade. In first grade, the group difference in working memory capacity appeared to mediate the greater use of verbal counting by the normally achieving children and contributed to the greater use of finger counting by the children with MD. In third and fifth grade, working memory capacity contributed to individual differences in the use of decomposition to solve complex problems.

Within both the MD and normally achieving groups, individual differences in working memory capacity were related to the pattern of strategy shifts comparing simple to complex problems, especially as related to the use of finger counting and in first grade. For normally achieving children, a high working memory capacity also appeared to facilitate the shift to decomposition when solving complex problems. The finding that working memory was not related to retrieval in third and fifth grade is consistent with Ackerman's (1988) and Siegler's (1996) findings. Specifically, working memory is most important during the initial phase of learning and it declines in importance as procedures are used less frequently and facts become represented in long-term memory and thus automatically and effortlessly executed during problem solving.

Performance on the counting knowledge task—pseudoerror trials in particular—was consistently related to use of the min procedure while using finger and verbal counting across grades, and to verbal counting errors in first grade. The latter was predicted by the error-task and is consistent with the prediction of Ohlsson and Rees (1991); specifically, sensitivity to counting errors will enable children to better detect and self correct these errors and thus facilitate the accurate use of counting strategies to solve arithmetic problems. However, this predicted pattern was not found for finger counting errors in first grade. The most consistent relation was between performance on pseudoerror trails and use of the min procedure.

The current findings confirm our earlier results for first grade (Geary et al., 1992) and extend the findings to third and fifth grade. Also in keeping with this earlier study, counting knowledge (combined pseudoerror and error performance) in first grade contributed to the tendency of children with MD to use the min procedure less often than did their normally achieving peers. An unexpected finding was that strong performance on pseudoerror trials was associated with greater use of decomposition to solve simple problems in third grade, to solve complex problems in third and fifth grade, and to commit fewer decomposition errors while solving complex problems in fifth grade.

The common features across pseudoerror counting, use of the min procedure, and decomposition are the manipulation of number sets. The pseudoerror counting trials require the child at least to implicitly understand that the combined sets of red and blue chips yield a number that is the same as the cardinal value of the total set. Standard, contiguous counting, in contrast, does not require an implicit understanding of number sets. By definition, decomposition requires breaking a larger-valued number into two smaller sets, and then the mental manipulation (addition in this study) of each set. Similarly, min counting requires the child to understand that stating the cardinal value of the larger addend yields the same result as counting this set starting from one (Fuson, 1988). In other words, we are suggesting that the pattern of results in this study may reflect a more general understanding of relations among numbers and sets within each number. Facility at conceiving numbers as being composed of sets of smaller-valued numbers, and ease of manipulating these sets and subsets of numbers should be related to ease of determining that pseudoerror counting, though unconventional, yields the correct answer; that min counting yields the same answer as counting from one; and, numbers can be decomposed and combined or subtracted using the rules of arithmetic. Two implications are that an understanding of number sets contributes to many features of arithmetical development, and that children with MD may be delayed in this regard.

Conclusion

The common finding that children with MD use finger counting strategies more often than their normally achieving peers and commit more counting errors appears to be related, in part, to a deficit in working memory capacity, at least in first grade. In the early elementary school years, children with MD also use the developmentally sophisticated min procedure less frequently than do normally achieving children, and this difference appears to be influenced by an immature understanding of counting, or perhaps number sets. For children with MD and normally achieving children, poor working memory is associated with more finger counting and more finger counting errors in first grade, and a poor conceptual understanding of counting, or perhaps number sets, is associated with less frequent use of the min procedure across grades. In all, the current results suggest that the strategies used by children with MD to solve simple and complex addition problems are about two grade levels behind that of their peers, and their working memory capacity is about one grade level behind that of their peers, but all of these competencies improve from one grade to the next (Jordan et al., 2003a). This does not mean that children with MD are simply delayed in the development of arithmetic skills. Many studies have found more persistent grade-to-grade deficits in the ability to represent arithmetic facts in or retrieve them from long-term memory (e.g., Geary, 1993; Geary et al., 1991; Jordan et al., 2003b), but we did not assess these skills in this study.

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