Mathematics and Learning Disabilities

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Abstract

Between 5% and 8% of school-age children have some form of memory or cognitive deficit that interferes with their ability to learn concepts or procedures in one or more mathematical domains. A review of the arithmetical competencies of these children is provided, along with discussion of underlying memory and cognitive deficits and potential neural correlates. The deficits are discussed in terms of three subtypes of mathematics learning disability and in terms of a more general framework for linking research in mathematical cognition to research in learning disabilities.

The breadth and complexity of the field of mathematics make the identification and study of the cognitive phenotypes that define mathematics learning disabilities (MLD) a formidable endeavor. In theory, a learning disability can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain. The goal is further complicated by the task of distinguishing poor achievement due to inadequate instruction from poor achievement due to an actual cognitive disability (Geary, Brown, & Samaranayake, 1991). Yet another complication arises from contention regarding instructional goals and approaches (Loveless, 2001), which in turn may influence whether a particular deficit would be considered a learning disability at all. Instruction that focuses on mathematics as an applied domain tends to de-emphasize the learning of procedures and mathematical facts and to emphasize conceptual understanding (National Council of Teachers of Mathematics, 2000), whereas procedures and facts are more heavily emphasized in instruction that approaches mathematics as a scientific field to be mastered (California Department of Education, 1999). With the former approach, the deficit in arithmetic fact retrieval described later in this article may not be considered a serious learning disability because of the de-emphasis on this memory-based knowledge, whereas in the latter approach it would be considered a serious disability.

One strategy that is not dependent on instructional issues involves applying the theories and methods used by cognitive psychologists to study mathematical competencies in typically achieving children to the study of children with MLD (Bull & Johnston, 1997; Garnett & Fleischner, 1983; Geary & Brown, 1991; Jordan, Levine, & Huttenlocher, 1995; Jordan & Montani, 1997; Ostad, 1997, 1998b; Russell & Ginsburg, 1984; Svenson & Broquist, 1975). When this approach is combined with studies of dyscalculia—that is, numerical and arithmetical deficits following overt brain injury (e.g., Shalev, Manor, & Gross-Tsur, 1993; Temple, 1991)—and brain imaging studies of mathematical processing (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999), a picture of the cognitive and brain systems that can contribute to MLD begins to emerge. The combination of approaches has been primarily applied to the study of numerical and arithmetical competencies and is, thus, only a first step to fully understanding the cognitive and brain systems that support mathematical competency and any associated learning disabilities. It is, nonetheless, a start, and the following sections provide an overview of what this research strategy has revealed about MLD. The first section provides a discussion of diagnostic and etiological issues, and the second provides a description of some of the performance and cognitive patterns that distinguish children with MLD from their peers. The final section presents a framework for guiding future research on mathematics and learning disabilities (LD) and reviews the basic cognitive and neural mechanisms and deficits that may underlie the performance and cognitive patterns described in the second section.

Background Characteristics of Children with MLD

Diagnosis

Unfortunately, measures that are specifically designed to diagnose MLD are not available; thus, most researchers rely on standardized achievement tests, often in combination with measures of intelligence (IQ). A score lower than the 20th or 25th percentile on a mathematics achievement test combined with a low-average or higher IQ score are typical criteria for diagnosing MLD (e.g., Geary, Hamson, & Hoard, 2000; Gross-Tsur, Manor, & Shalev, 1996). However, a lower than expected (based on IQ) mathematics achievement score does not in and of itself indicate the presence of MLD. Many children who score low on achievement tests one academic year score average or better in subsequent years. These children do not appear to have any of the underlying memory or cognitive deficits described in the next section, and thus a diagnosis of MLD is not appropriate (Geary, 1990; Geary et al., 1991; Geary et al., 2000). In contrast, children who have lower than expected achievement scores across successive academic years often have some form of memory or cognitive deficit, and a diagnosis of MLD is often warranted.

It should be noted that the cutoff of the 25th percentile on a mathematics achievement test does not fit with the estimation, described later, that between 5% and 8% of children have some form of MLD. This discrepancy results from the nature of standardized achievement tests and the often rather specific memory or cognitive deficits of children with MLD. Standardized achievement tests sample a broad range of arithmetical and mathematical topics, whereas children with MLD often have severe deficits in some of these areas and average or better competencies in others. The result of averaging across items that assess these different competencies is a level of performance (e.g., at the 20th percentile) that overestimates the competencies of children with MLD in some areas and underestimates them in others.

In addition to the development of diagnostic instruments, another issue that needs to be explored is whether treatment resistance can be used as one diagnostic criterion for MLD. As described later, many children with MLD have difficulties retrieving basic arithmetic facts from long-term memory, and these difficulties often persist despite intensive instruction on basic facts (e.g., Howell, Sidorenko, & Jurica, 1987). Although the instructional research is preliminary, it does suggest that a retrieval deficit resistant to instructional intervention might be a useful diagnostic indicator of arithmetical forms of MLD.

Prevalence and Etiology

Experimental measures that are more sensitive to MLD than are standardized achievement tests have been administered to samples of more than 300 children from well defined populations (e.g., all fourth graders in an urban school district) in the United States (Badian, 1983), Europe (Kosc, 1974; Ostad, 1998a), and Israel (Gross-Tsur et al., 1996; Shalev et al., 2001). These measures have largely assessed number and arithmetic competencies and have been constructed based on neuropsychological deficits associated with dyscalculia (for discussion, see Geary & Hoard, 2002; Shalev et al., 1993). Performance that deviates from age-related norms and is similar to that associated with dyscalculia has been used in these studies as an indication of MLD and suggests that 5% to 8% of school-age children exhibit some form of MLD. Many of these children have comorbid disorders, including reading disabilities (RD) and attention-deficit/hyperactivity disorder (ADHD; Gross-Tsur et al., 1996).

As with other forms of LD, twin and familial studies, although preliminary, suggest both genetic and environmental contributions to MLD (Light & DeFries, 1995; Shalev et al., 2001). For instance, Shalev et al. studied familial patterns of MLD, specifically, learning disabilities in number and arithmetic. The results showed that family members (e.g., parents and siblings) of children with MLD are 10 times more likely to be diagnosed with MLD than are members of the general population.

Performance and Cognitive Patterns

As noted earlier, the use of cognitive theory and its associated methodology to study children with MLD has yielded a number of insights regarding the potential sources of their learning disability. These studies have primarily focused on the number, counting, and arithmetic competencies of children with MLD (e.g., Ackerman & Dykman, 1995; Barrouillet, Fayol, & Lathulière, 1997; Bull, Johnston, & Roy, 1999; Geary, 1993; Geary, Widaman, Little, & Cormier, 1987; Hanich, Jordan, Kaplan, & Dick, 2001; Ostad, 2000; Räsänen & Ahonen, 1995; Rourke, 1993). The results have suggested that the basic numerical competencies (e.g., identifying arabic numerals, comparing the magnitudes of numbers) of most children with MLD, though often delayed, are largely intact, at least for the processing of simple numbers (e.g., 8, 12; Badian, 1983; Geary, 1993; Geary, Hoard, & Hamson, 1999; Gross-Tsur et al., 1996). Based on these findings, the numerical competencies of children with MLD are not discussed further (for discussion, see Geary & Hoard, 2002).

The following sections provide a brief overview of theoretical models of typical development in the counting and arithmetic domains, along with patterns that have been found with the comparison of children with MLD to their typically achieving peers. Unless otherwise noted, MLD refers to children with low achievement scores—relative to IQ in many of the studies—in mathematics. When studies have only focused on children with low mathematics achievement scores but average or better reading achievement scores, participants will be referred to as children with MLD only. If the study assessed children with low achievement in both mathematics and reading, participants are identified as children with MLD/RD.

Counting

Typical Development. Children's understanding of the principles associated with counting appears to emerge from a combination of inherent constraints and counting experience (Briars & Siegler, 1984; Gelman & Gallistel, 1978). Early inherent constraints can be represented by Gelman and Gallistel's (1978) five implicit principles. These principles are *one-to-one correspondence* (one and only one word tag, e.g., "one," "two," is assigned to each counted object), *stable order* (the order of the word tags must be invariant across counted sets), *cardinality* (the value of the final word tag represents the quantity of items in the counted set), *abstraction* (objects of any kind can be collected together and counted), and *order irrelevance* (items within a given set can be tagged in any sequence). The principles of one-to-one correspondence, stable order, and cardinality define the counting rules, which in turn provide the skeletal structure for children's emerging knowledge of counting (Gelman & Meck, 1983).

In addition to these inherent constraints, children make inductions about the basic characteristics of counting by observing standard counting behavior and associated outcomes (Briars & Siegler, 1984; Fuson, 1988). These inductions likely elaborate Gelman and Gallistel's (1978) counting rules and result in a belief that certain unessential features of counting are essential. These unessential features include *standard direction* (counting must start at one of the endpoints of a set of objects) and *adjacency.* The latter is the incorrect belief that items must be counted consecutively and from one contiguous item to the next—that is, jumping around during the act of counting results in an incorrect count. By 5 years of age, many children know the essential features of counting described by Gelman and Gallistel but also believe that adjacency and standard direction are essential features of counting. The latter beliefs indicate that young children's conceptual understanding of counting is rather rigid and immature and is influenced by the observation of standard counting procedures.

Children with MLD. Using the procedures developed by Gelman and Meck (1983) and Briars and Siegler (1984), Geary, Bow-Thomas, and Yao (1992) contrasted the counting knowledge of first-grade children with MLD/RD with that of their typically achieving peers. The procedure involved asking the children to watch a puppet count a set of objects. The puppet sometimes counted correctly and sometimes violated one of Gelman and Gallistel's (1978) counting principles or one of Briars and Siegler's unessential features of counting. The child's task was to determine if the puppet's count was "OK" or "not OK and wrong." In this way, the puppet performed the procedural aspect of counting (i.e., pointing at and tagging items with a number word), leaving the child's responses to be based on her or his understanding of counting principles.

The results revealed that children with MLD/RD correctly identified correct counts, identified violations of most of the counting principles identified by Gelman and Gallistel (1978), and understood that counting from right to left was just as appropriate as the standard left-to-right counting (Geary et al., 1992). Many children with MLD/RD, however, did not understand Gelman and Gallistel's (1978) order irrelevance principle and believed that adjacency is an essential feature of counting. There were also group differences on trials in which either the first or the last item was counted twice. Children with MLD/RD correctly identified these counts as errors when the last item was double counted, suggesting that they understood the one-to-one correspondence principle. However, double counts were often labeled as correct when the first item was double counted, suggesting that many children with MLD/RD had difficulties holding information in working memory—in this case, noting that the first item was double counted—while monitoring the act of counting (see also Hitch & McAuley, 1991).

In a more recent study, children with IQ scores in the 80–120 range completed a series of experimental and achievement tests in first and second grade (Geary et al., 1999; Geary et al., 2000). Children who showed lower than expected (based on IQ) achievement scores in both grades were considered to have LD. Among other findings, the results were consistent with those of Geary et al. (1992); that is, the children with MLD/RD and MLD only differed from the children with RD only and the typical children on adjacency trials in first and second grade and on double-counting trials (involving the first item) in first grade. This pattern suggested that even in second grade, many children with MLD/RD and MLD only do not understand all counting principles and, in first grade, may have difficulty holding an error notation in working memory while monitoring the counting process (see also Hoard, Geary, & Hamson, 1999). In contrast, children with RD only performed as well as the typically achieving children.

In summary, many children with MLD, independent of their reading achievement levels or IQ, have a poor conceptual understanding of some aspects of counting. These children understand most of the inherent counting rules identified by Gelman and Gallistel (1978), such as stable order and cardinality, but they consistently err on tasks that assess order irrelevance or adjacency from Briars and Siegler's (1984) perspective. It is not currently known whether the poor counting knowledge of children with MLD/RD or MLD only extends beyond the second grade. In any case, the poor counting knowledge of these children appears to contribute to their delayed competencies in the use of counting to solve arithmetic problems, as described in the next section, and may result in poor skill at detecting and, thus, correcting counting errors (Ohlsson & Rees, 1991).

Arithmetic

Typical Development. The most thoroughly studied developmental and schooling-based improvement in arithmetical competency is the change in the distribution of procedures, or strategies, children use during problem solving (e.g., Ashcraft, 1982; Carpenter & Moser, 1984; Geary, 1994; Siegler, 1996; Siegler & Shrager, 1984). During the initial learning of addition, for instance, children typically count both addends (e.g., 5 + 3). These counting procedures are sometimes executed with the aid of the fingers, the *finger counting strategy,* and sometimes without them, the *verbal counting strategy* (Siegler & Shrager, 1984). The two most commonly used counting procedures, whether children use their fingers or not, are termed *counting on* and *counting all* (Fuson, 1982; Groen & Parkman, 1972). The counting-on procedure typically involves stating the larger value addend and then counting a number of times equal to the value of the smaller addend, such as counting 5, 6, 7, 8 to solve 5 + 3. Counting all involves counting both addends starting from 1. The development of procedural competencies is related in part to improvements in children's conceptual understanding of counting and is reflected in a gradual shift from the frequent use of counting all to counting on (Geary et al., 1992; Siegler, 1987).

At the same time, the use of counting procedures appears to result in the development of memory representations of basic facts (Siegler & Shrager, 1984). Once formed, these long-term memory representations support the use of memory-based problem-solving processes. The most common of these are the direct retrieval of arithmetic facts and decomposition. With *direct retrieval,* children state an answer that is associated in long-term memory with the problem presented, such as stating "/eyt/" (i.e., eight) when asked to solve 5 + 3. *Decomposition* involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem 6 + 7 might be solved by retrieving the answer to 6 + 6 (i.e., 12) and then adding 1 to this partial sum. The use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges his or her confidence in the correctness of the retrieved answer. Children with a *rigorous criterion* only state answers that they are certain are correct, whereas children with a *lenient criterion* state any retrieved answer, correct or not (Siegler, 1988).

As the strategy mix matures, children solve problems more quickly because they use more efficient memory-based strategies and because, with practice, it takes less time to execute each strategy (Delaney, Reder, Staszewski, & Ritter, 1998; Geary, Bow-Thomas, Liu, & Siegler, 1996; Lemaire & Siegler, 1995). The transition to memory-based processes results in the quick solution of individual problems and reduction of the working memory demands associated with solving these problems. The eventual automatic retrieval of basic facts and the accompanying reduction of the working memory demands in turn appear to make the solving of more complex problems in which the simple problems are embedded (e.g., word problems) less error prone (e.g., Geary & Widaman, 1992).

Children with MLD. During the solving of simple arithmetic problems (e.g., 4 + 3) and simple word problems, children with MLD/RD and MLD only use the same types of strategies (e.g., verbal counting) as typically achieving children, but they differ in the strategy mix and in the pattern of developmental change in this mix (Geary, 1990; Hanich et al., 2001). These differences have been found in the United States (Geary & Brown, 1991; Hanich et al., 2001; Jordan & Montani, 1997), Europe (Barrouillet et al., 1997; Ostad, 1997, 1998a, 1998b, 2000; Svenson & Broquist, 1975), and Israel (Gross-Tsur et al., 1996).

As an example, Geary et al. (1999; Geary et al., 2000) found consistent differences comparing the strategies used to solve simple addition problems across groups of children with MLD/RD, MLD only, and RD only, as contrasted with typically achieving children (see also Jordan & Montani, 1997). In first and second grades, children with MLD only and especially children with MLD/RD committed more counting errors and used the developmentally immature counting-all procedure more frequently than did the children in other groups. Moreover, in keeping with models of typical arithmetical development, the children in the RD-only and typically achieving groups showed a shift, from first grade to second grade, from heavy reliance on finger counting to verbal counting and retrieval. The children in the MLD/RD and MLD-only groups, in contrast, did not show this shift but instead relied heavily on finger counting in both grades. These patterns replicated previous studies of children with MLD/RD and demonstrated that many of the same deficits, although to a lesser degree, are evident for children with MLD only (e.g., Geary et al., 1991; Jordan & Montani, 1997; Ostad, 1998b). Other studies have suggested that by the end of the elementary school years, many children with MLD/RD and presumably MLD only eventually abandon finger counting for verbal counting and become increasingly skilled at executing counting strategies—that is, they do not commit as many errors (e.g., Geary & Brown, 1991).

The most consistent finding in the literature is that children with MLD/RD or MLD only differ from their typically achieving peers in the ability to use retrieval-based processes to solve simple arithmetic and simple word problems (e.g., Barrouillet et al., 1997; Garnett & Fleischner, 1983; Geary, 1990, 1993; Hanich et al., 2001; Jordan & Montani, 1997; Ostad, 1997, 2000). Unlike the use of counting strategies, it appears that the ability to retrieve basic facts does not substantively improve across the elementary school years for most children with MLD/RD and MLD only. When these children do retrieve arithmetic facts from long-term memory, they commit many more errors and often show error and reaction time (RT) patterns that differ from those found with younger, typically achieving children (Barrouillet et al., 1997; Fayol, Barrouillet, & Marinthe, 1998; Geary, 1990; Geary & Brown, 1991; Räsänen & Ahonen, 1995). Moreover, these patterns are sometimes found to be similar to the patterns evident with children who have suffered from an early (before age 8) lesion to the left hemisphere or associated subcortical regions (Ashcraft, Yamashita, & Aram, 1992). These patterns suggest that the memory retrieval deficits of children with MLD/RD or MLD only reflect a cognitive disability and not, for instance, a lack of exposure to arithmetic problems, poor motivation, a low confidence criterion, or low IQ (Geary et al., 2000).

In summary, research on the problem-solving strategies used by young children to solve simple arithmetic and word problems has consistently revealed differences in the strategic and memory-based processes used by children with MLD/RD or MLD only and their typically achieving or RD-only peers (e.g., Barrouillet et al., 1997; Geary, 1990; Geary et al., 1987; Gross-Tsur et al., 1996; Jordan & Montani, 1997; Ostad, 1997, 1998b; Svenson & Broquist, 1975). As a group, children with MLD/RD or MLD only commit more counting errors and use developmentally immature procedures (e.g., counting all rather than counting on) more frequently and for more years than do their peers. The differences are especially pronounced for children with MLD/RD, as children with MLD only appear to develop typical levels of procedural competency more quickly than do children with MLD/RD (Geary et al., 2000; Jordan & Montani, 1997). At the same time, many children with MLD/RD and MLD only do not show the shift from procedure-based problem solving to memory-based problem solving that is commonly found in typically achieving children, suggesting difficulties in storing arithmetic facts in or accessing them from long-term memory (Garnett & Fleischner, 1983; Geary et al., 1991; Jordan & Montani, 1997; Ostad, 1997, 1998a).

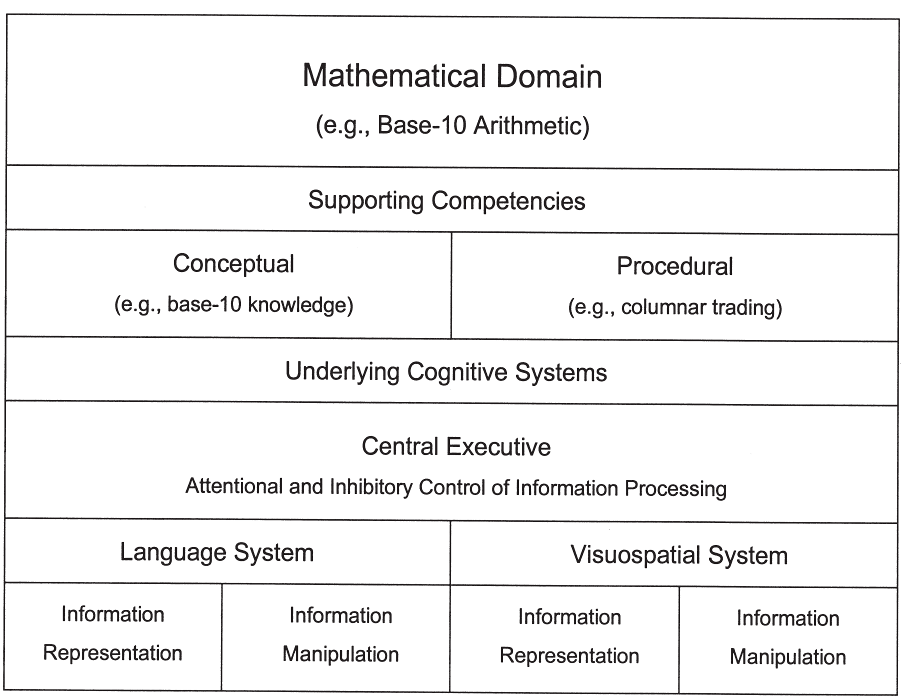

FIGURE 1. Framework for the identification and study of potential learning disabilities in mathematics.

Cognitive Mechanisms and Deficits

As noted in the introduction, the complexity of the field of mathematics makes the search for any associated learning disabilities daunting. Figure 1 shows a conceptual scheme for approaching the task. Competencies in any given area of mathematics will depend on a conceptual understanding of the domain and procedural knowledge that supports actual problem solving (Geary, 1994). Base-10 arithmetic is one example in which instruction focuses on teaching the conceptual foundation (i.e., the repeating number system based on sequences of 10) and its related procedural skills, such as trading from the tens column to the units column while solving complex arithmetic problems (e.g., subtracting 129 from 243; Fuson & Kwon, 1992). Conceptual and procedural competencies, in turn, are supported by an array of cognitive systems, as shown in the bottom sections of Figure 1.

The central executive controls the attentional and inhibitory processes needed to use procedures during problem solving, and much of the information supporting conceptual and procedural competencies is likely to be represented in the language or visuospatial systems (Baddeley, 1986). The language systems are important for certain forms of information representation, as in articulating number words, and information manipulation in working memory, as during the act of counting. The visuospatial system appears to be involved in representing some forms of conceptual knowledge, such as number magnitudes (Dehaene & Cohen, 1991), and in representing and manipulating mathematical information that is cast in a spatial form, as in a mental number line (Zorzi, Priftis, & Umiltá, 2002). A mathematics learning disability would be manifest as a deficit in conceptual or procedural competencies that define the mathematical domain, and these, in theory, would be due to underlying deficits in the central executive or in the information representation or manipulation (i.e., working memory) systems of the language or visuospatial domains.

The organizational frame shown in Figure 1 is a guide for future research. For now, our understanding of MLD is largely limited to arithmetic and is based on empirical studies of only a few of the potential sources of MLD outlined in Figure 1. What we do understand is outlined in Table 1 as a preliminary taxonomy of the three subtypes of MLD, specifically, procedural, semantic memory, and visuospatial. This taxonomy was developed based on an earlier review of the cognitive deficits of children with MLD and the related dyscalculia and behavioral genetic literatures (Geary, 1993). Eventually, the taxonomy will need to be expanded to include all the features shown in Figure 1 and extended to other mathematical domains (e.g., algebra). The goal here is to try to understand the aforementioned performance and cognitive patterns of children with MLD/RD and MLD only—hereafter children with MLD—in terms of the procedural and semantic memory subtypes, as described in the next two sections and, where possible, to provide links to the framework outlined in Figure 1. The following section provides discussion of how visuospatial deficits may contribute to learning disabilities in other mathematical domains.

Procedural Deficits

As described earlier, children with MLD commit many counting errors while solving simple arithmetic problems, and they tend to use developmentally immature strategies (e.g., finger counting) and problem-solving procedures (e.g., counting all). A few studies have assessed the procedural competencies of children with MLD during the solving of multistep arithmetic problems, such as 45 × 12 or 126 + 537. Russell and Ginsburg (1984) found that fourth-grade children with MLD committed more errors than their IQ-matched typically achieving peers did when solving such problems. These errors involved the misalignment of numbers while writing down partial answers or errors while carrying or borrowing from one column to the next. The following subsections discuss these procedural characteristics of children with MLD in terms of working memory, conceptual knowledge, and neural correlates.

Working Memory. Although the relation between working memory and difficulties in executing arithmetical procedures is not yet fully understood, it is clear that children with MLD have some form of working memory deficit (Hitch & McAuley, 1991; McLean & Hitch, 1999; Siegel & Ryan, 1989: Swanson, 1993). Based on the framework shown in Figure 1, this deficit appears to involve information representation and manipulation in the language system—that is, the systems that support the representation and articulation of number words and that support associated procedural competencies, such as counting. As with all competencies that engage working memory, deficits in the central executive, such as poor attentional control, can also disrupt the execution of mathematical procedures (Hitch, 1978).

For instance, children with MLD appear to use finger counting as a strategy for solving arithmetic problems because representing the addends on fingers and then using fingers to note the counting sequence appears to greatly reduce the working memory demands of the counting process (Geary, 1990). Working memory may also contribute to the tendency of children with MLD to undercount or overcount—the source of their counting procedure errors (Geary, 1990; Hanich et al., 2001)—during the problem-solving process. Such miscounting can occur if the child loses track of where he or she is in the counting process—that is, how many fingers he or she has counted and how many remain to be counted. These deficits could be due to difficulties with information representation in the language, specifically the phonetic–articulatory system, or from a deficit in accompanying executive processes, such as attentional control (see McLean & Hitch, 1999). If the phonetic representations of number words fade more quickly or do not achieve typical levels of acoustical fidelity, then manipulating these representations in working memory, as with counting, will be difficult for children with MLD (Geary, 1993).

The procedural errors committed by children with MLD while solving more complex arithmetic problems, as described by Russell and Ginsburg (1984), may result from difficulties in monitoring and coordinating the sequence of problem-solving steps, which in turn suggest that functions of the central executive are compromised.

Conceptual Knowledge. A poor understanding of the concepts underlying a procedure can contribute to a developmental delay in the adoption of more sophisticated procedures and reduce the ability to detect procedural errors (e.g., Ohlsson & Rees, 1991). In other words, the delayed use of the counting-on procedure and the frequent counting errors of children with MLD appear to be related in part to immature counting knowledge. As mentioned earlier, many children with MLD who do not understand the order irrelevance concept or who believe that adjacency is an essential feature of counting use the counting-all procedure while solving simple addition problems more frequently than do

TABLE 1
Subtypes of Learning Disabilities in Mathematics

Cognitive and performance features	Neuropsychological features	Genetic features	Developmental features	Relation to RD
Procedural Subtype				
Relatively frequent use of developmentally immature procedures (i.e., the use of procedures that are more commonly used by younger, typically achieving children) Frequent errors in the execution of procedures Poor understanding of the concepts underlying procedural use Difficulties sequencing the multiple steps in complex procedures	Unclear, although some data suggest an association with left hemispheric dysfunction and, in some cases, (especially for sequencing problems) a prefrontal dysfunction	Unclear	Appears, in many cases, to represent a developmental delay (i.e., performance is similar to that of younger, typically achieving children and often improves across age and grade)	Unclear
Semantic Memory Subtype				
Difficulties retrieving mathematical facts, such as answers to simple arithmetic problems For facts that are retrieved, there is a high error rate For arithmetic, retrieval errors are often associates of numbers in the problem (e.g., retrieving 4 to 2 + 3 = ?; 4 is the counting-string associate that follows 2, 3) RTs for correct retrieval are unsystematic	Appears to be associated with left hemispheric dysfunction, possibly the posterior regions for one form of retrieval deficit and the prefrontal regions for another Possible subcortical involvement, such as the basal ganglia	Appears to be a heritable deficit	Appears to represent a developmental difference (i.e., cognitive and performance features differ from those of younger, typically achieving children and do not change substantively across age or grade)	Appears to occur with phonetic forms of RD
Visuospatial Subtype				
Difficulties in spatially representing numerical and other forms of mathematical information and relationships Frequent misinterpretation or misunderstanding of spatially represented information	Appears to be associated with right hemispheric dysfunction, in particular, posterior regions of the right hemisphere, although the parietal cortex of the left hemisphere may be implicated as well	Unclear, although the cognitive and performance features are common with certain genetic disorders (e.g., Turner's syndrome)	Unclear	Does not appear to be related

Note. From "Mathematical Disabilities: Cognitive, Neuropsychological, and Genetic Components," by D. C. Geary, 1993, *Psychological Bulletin, 114*, p. 362. Copyright 1993 by the American Psychological Association. Adapted with permission. RD = reading disabilities; RT = reaction time.

other children (Geary et al., 1992; Geary et al., 2000). It is possible that the switch from the use of counting all to the use of counting on requires an understanding that counting does not need to start from 1 and proceed in the standard sequential order (i.e., 1, 2, 3, etc.). The immature counting knowledge of children with MLD may also contribute to their frequent counting errors, in particular to a failure to detect and, thus, self-correct these errors.

Neural Correlates. Based on the similarity between the deficits associated with MLD and those associated with acquired dyscalculia, neuropsychological studies of dyscalculia provide insights into the potential neural systems contributing to the procedural deficits of children with MLD (Geary, 1993; Geary & Hoard, 2001). As is found with children with MLD, individuals with acquired or developmental dyscalculia are generally able to count arrays of objects, to recite the correct sequence of number words during the act of counting (e.g., counting from 1 to 20), and to understand many basic counting concepts, such as cardinality (Hittmair-Delazer, Sailer, & Benke, 1995; Seron et al., 1991; Temple, 1989). Individuals with dyscalculia caused by damage to the right hemisphere sometimes show difficulties with the procedural component of counting—specifically, difficulties with systematically pointing to successive objects as they are enumerated (Seron et al., 1991). However, the relation between this feature of dyscalculia and the procedural deficits of children with MLD is not clear.

Difficulties in solving complex arithmetic problems are also common with acquired and developmental dyscalculia (Semenza, Miceli, & Girelli, 1997; Temple, 1991). For example, in an extensive assessment of the counting, number, and arithmetic competencies of a 17-year-old (M. M.) with severe congenital damage to the right frontal and parietal cortices, Semenza et al. reported deficits very similar to those reported by Russell and Ginsburg (1984) for children with MLD. Basic number and counting skills were intact, as was the ability to retrieve basic facts (such as 8 for 5 + 3) from long-term memory. However, M. M. had difficulty solving complex division and multiplication problems, such as 32 × 67. Of particular difficulty was the tracking of the sequence of partial products. Once the first step was completed (2 × 7), difficulties placing the partial product (4) in the correct position and carrying 10 to the next column were evident. Thus, the primary deficit of M. M. appeared to involve difficulties sequencing the order of operations and monitoring the problem-solving process, suggesting deficits in the central executive, as are often found with damage to the frontal cortex (Luria, 1980). Temple (1991) reported a similar pattern of procedural difficulties for an individual with neurodevelopmental abnormalities in the right frontal cortex. It remains to be seen if a compromised right frontal cortex and a compromised central executive contribute to aspects of the procedural deficits of children with MLD.

Semantic Memory Deficits

Unlike that of other children, the arithmetical development of children with MLD does not always entail a shift from procedure-based problem solving to memory-based problem solving (Geary et al., 1987; Ostad, 1997). The implication is that children with MLD have difficulties storing arithmetic facts in or accessing them from long-term memory (Barrouillet et al., 1997; Bull & Johnston, 1997; Garnett & Fleischner, 1983; Geary, 1993; Geary & Brown, 1991; Geary et al., 1987; Jordan & Montani, 1997; Ostad, 1997). Disruptions in the ability to retrieve basic facts from long-term memory might, in fact, be considered a defining feature of arithmetical forms of MLD, thus the inclusion of a semantic memory subtype in Table 1. However, most of these individuals can retrieve some facts, and disruptions in the ability to retrieve facts associated with one operation (e.g., multiplication) are sometimes found together with intact retrieval of facts associated with another operation (e.g., subtraction), at least when retrieval deficits are associated with overt brain injury (Pesenti, Seron, & Van Der Linden, 1994).

As described in Table 1, when they retrieve arithmetic facts from long-term memory, children with MLD commit many more errors than do their typically achieving peers, and they show error and reaction time (RT) patterns that often differ from the patterns found with younger, typically achieving children (Geary, 1993; Geary et al., 2000). Although the results are not conclusive, the RT patterns are sometimes found to be similar to the RT patterns found with children who have suffered from an early lesion to the left hemisphere or associated subcortical regions (Ashcraft et al., 1992), as noted earlier. Of course, this pattern does not indicate that children with MLD have suffered from some form of overt brain injury, but it does suggest that the memory-based deficits of many of these children may reflect the same mechanisms underlying the retrieval deficits associated with dyscalculia (Geary, 1993; Rourke, 1993).

The cognitive and neural mechanisms underlying these deficits are not completely understood but, based on the framework shown in Figure 1, are hypothesized to involve the information representation mechanisms of the language system. This hypothesis is based on the cognitive mechanisms involved in forming long-term memory representations of arithmetic facts. The solving of arithmetic problems by means of counting should eventually result in associations forming between problems and generated answers (Siegler, 1996; Siegler & Shrager, 1984). Because counting typically engages the phonetic and semantic (e.g., understanding the quantity associated with number words) representational systems of the language domain, any disruption in the ability to represent or retrieve information from these systems should, in theory, result in difficulties in forming problem–answer associations during counting (Geary, 1993; Geary, Bow-Thomas, Fan, & Siegler, 1993). The consequences would include difficulties in learning arithmetic facts and in retrieving those facts that do become represented in long-term memory.

Although not definitive with respect to this hypothesis, the work of Dehaene and his colleagues suggested that the retrieval of arithmetic facts is indeed supported by a system of neural structures that appear to support phonetic and semantic representations and are engaged during incrementing processes (e.g., counting). These areas include the left basal ganglia and the left parieto-occipito-temporal areas (Dehaene & Cohen, 1995, 1997). Damage to either the subcortical or cortical structures in this network is associated with difficulties in accessing previously known arithmetic facts (Dehaene & Cohen, 1991, 1997). However, it is not currently known if the retrieval deficits of children with MLD are the result of damage to or neurodevelopmental abnormalities in the regions identified by Dehaene and Cohen (1995, 1997).

More recent studies of children with MLD have suggested a second form of retrieval deficit—specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. This form of retrieval deficit was first discovered by Barrouillet et al. (1997), based on the memory model of Conway and Engle (1994), and was recently confirmed in our laboratory (Geary et al., 2000; see also Koontz & Berch, 1996). In the Geary et al. (2000) study, one of the arithmetic tasks required children to use only retrieval—the children were instructed not to use counting strategies—to solve simple addition problems (see also Jordan & Montani, 1997). Children with MLD and children with RD committed more retrieval errors than did their typically achieving peers, even after controlling for IQ. The most common of these errors was a counting-string associate of one of the addends. For instance, common retrieval errors for the problem 6 + 2 were 7 and 3, the numbers following 6 and 2, respectively, in the counting sequence. Hanich et al. (2001) found a similar pattern, although the proportion of retrieval errors that were counting-string associates was lower than that found by Geary et al. (2000).

The pattern in these more recent studies (e.g., Geary et al., 2000) and Barrouillet et al.'s (1997) study is in keeping with Conway and Engle's (1994) position that individual differences in working memory and retrieval efficiency are related, in part, to the ability to inhibit irrelevant associations. In this model, the presentation of a problem to be solved results in the activation of relevant information in working memory, including problem features—such as the addends in a simple addition problem—and information associated with these features. Problem solving is efficient when irrelevant associations are inhibited and prevented from entering working memory. Inefficient inhibition results in the activation of irrelevant information, which functionally lowers working memory capacity. In this view, some children with MLD make retrieval errors in part because they cannot inhibit irrelevant associations from entering working memory. Once in working memory, these associations either suppress or compete with the correct association for expression. These results suggest that the retrieval deficits of some children with MLD may result from deficits in the central executive and associated areas of the prefrontal cortex that support inhibitory mechanisms (Bull et al., 1999; Welsh & Pennington, 1988). The results also suggest that inhibitory mechanisms should be considered as potential contributors to the comorbidity of MLD and ADHD in some children (Gross-Tsur et al., 1996).

Visuospatial Deficits

The relation between visuospatial competencies and MLD has not been systematically explored. Nonetheless, visuospatial systems support many mathematical competencies, such as certain areas of geometry and the solving of complex word problems (e.g., Dehaene et al., 1999; Geary, 1996), and thus, any deficits in these visuospatial systems could result in a corresponding learning disability. Indeed, Zorzi et al. (2002) found that individuals with an injury to the right parietal cortex showed a deficit in spatial orientation and a deficit in the ability to generate and use a mental number line. McLean and Hitch (1999) found that children with MLD showed a performance deficit on a spatial working memory task, although it is not clear if the difference resulted from an actual deficit in the ability to represent information in visuospatial systems or from a deficit in executive function (e.g., ability to maintain attention on the spatial task). Hanich et al. (2001) found that children with MLD differed from their peers on an estimation task and in the ability to solve complex word problems. Although performance on both of these tasks is supported by spatial abilities (Dehaene et al., 1999; Geary, 1996; Geary et al., 2000), it is not clear if the results of Hanich et al. were due to a spatial deficit in the children with MLD assessed in this study.

At the same time, many children with the procedural or semantic memory forms of MLD, at least as related to simple arithmetic, do not appear to differ from other children in basic visuospatial competencies (Geary et al., 2000; Morris et al., 1998). This is presumably because many of the conceptual and procedural competencies that support simple arithmetic are more dependent on the language system than on the visuospatial system.

Conclusion

Through the use of cognitive theory and experimental methods, we now have a reasonable understanding of the number, counting, and arithmetic competencies and deficits of children with MLD (Geary et al., 2000; Hanich et al., 2001; Ostad, 2000). Most of these children appear to have nearly average number processing skills, at least for the processing of simple numbers (e.g., 3, 6), but they show persistent deficits in some areas of arithmetic and counting knowledge. Many of these children have an immature understanding of certain counting principles and, with respect to arithmetic, use problem-solving procedures that are more commonly used by younger, typically achieving children. They also frequently commit procedural errors. For some of these children, procedural skills, at least as related to simple arithmetic, improve over the course of the elementary school years, and thus, the early deficit may not be due to a permanent cognitive disability. At the same time, many children with MLD also have difficulties retrieving basic arithmetic facts from long-term memory, a deficit that often does not improve.

On the basis of the framework shown in Figure 1, these developmental delays and deficits appear to be related to a combination of disrupted functions of the central executive, including attentional control and poor inhibition of irrelevant associations, and difficulties with information representation and manipulation in the language system. In theory, MLD can also result from compromised visuospatial systems, although these potential forms of MLD are not well understood. Some insights have also been gained regarding the potential neural mechanisms contributing to these procedural and retrieval characteristics of children with MLD, although definitive conclusions must await brain imaging studies of these children.

Despite some advances over the past 10 years, much remains to be accomplished. In comparison to simple arithmetic, relatively little research has been conducted on the ability of children with MLD to solve more complex arithmetic problems (but see Russell & Ginsburg, 1984), and even less has been conducted in other mathematical domains. Even in the area of simple arithmetic, the cognitive and neural mechanisms that contribute to the problem-solving characteristics of children with MLD are not fully understood. Other areas that are in need of attention include the development of diagnostic instruments for MLD, cognitive and behavioral genetic research on the comorbidity of MLD and other forms of LD and ADHD, and of course, the development of remedial techniques. If the progress over the past 10 years is any indication, then we should see significant advances in many of these areas in the years to come.

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The preparation of this article was supported by Grant R01 HD38283 from the National Institute of Child Health and Human Development.

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