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Learning Disabilities in Arithmetic: Problem-Solving Differences and Cognitive Deficits

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The complexity of the field of mathematics makes the study of any associated learning disability daunting. In theory, a mathematical learning disability can result from deficits in the ability to represent or process information used in one or all of the many areas of mathematics (e.g., arithmetic and geometry), or in one or a set of individual domains (e.g., theorems vs. graphing) within each of these areas (Russell & Ginsburg, 1984). One approach that can be used to focus the search for any such learning disability (LD) is to apply the models and methods used to study mathematical development in academically normal children to the study of children with poor achievement in mathematics (e.g., Geary & Brown, 1991). Unfortunately, for most mathematical domains, such as geometry and algebra, not enough is known about the normal development of the associated competencies to provide a systematic framework for the study of LD. Theoretical models and experimental methods are, however, sufficiently well developed in the areas of number, counting, and simple arithmetic to provide such a framework {Briars & Siegler, 1984; Geary, 1994; Gelman & Meck, 1983; Siegler, 1996; Siegler & Shrager,1984).

The use of these models and methods to guide the study of children with LD has revealed a consistent pattern of cognitive strengths and weaknesses. These studies suggest that most children with LD are normal (i.e., performance is similar to academically normal peers) or only slightly delayed in the development of number concepts (Geary, Hamson, & Hoard, 2000; Gross-Tsur, Manor, & Shalev, 1996). At the same time, several studies have shown that many children with LD do not understand certain counting concepts (Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, & Hamson, 1999), and many studies have revealed that these children have a variety of deficits in simple arithmetic (Ackerman & Dykman, 1995; Barrouillet, Fayol, & Lathuli (re, 1997; Bull & Johnston, 1997; Garnett, & Fleischner, 1983; Geary, Brown, & Samaranayake, 1991; Geary, Widaman, Little, & Cormier, 1987; Jordan & Hanich, 2000; Jordan, Levine, & Huttenlocher, 1995; Jordan & Montani, 1997; Ostad, 1997, 1998a; Rasanen & Ahonen, 1995; Rourke, 1993; Svenson & Broquist, 1975). The deficits in the basic arithmetical competencies of children with LD (hereafter, arithmetical disability, or AD) have been found in studies conducted in the United States (e.g., Garnett & Fleischner, 1983), Israel (Shalev, Manor, & Gross-Tsur, 1993), and several European nations (Ostad, 2000; Svenson & Broquist, 1975). The difficulties children with AD have solving simple arithmetic problems are the focus of the next section. The second section provides an overview of research on the cognitive and potential neural mechanisms contributing to these deficits.

Arithmetical Learning Disability

The first part presents background information on the diagnosis, prevalence, and etiology of AD; the second provides an overview of our research program in AD.

Background

DIAGNOSIS

Unfortunately, measures that are specifically designed to diagnose AD are not available. As a result, most researchers rely on standardized achievement tests, often in combination with measures of intelligence (IQ). A score lower than the 20th or 25th percentile on a mathematics achievement test combined with a low-average or higher IQ score are typical criteria for diagnosing AD (e.g., Geary, Hamson, & Hoard, 2000; Gross-Tsur et al., 1996). There are, however, two difficulties with these criteria.

First, if applied in only a single academic year, the criteria often lead to a number of false positives, that is identifying children as AD who in fact have no cognitive deficits and typically show improved achievement scores in later grades (Geary, 1990; Geary et al., 1991): We have found that most children who meet these criteria across two successive grades do appear to have some form of cognitive deficit and AD. Second, the cutoff of the 25th percentile on a mathematics achievement test does not fit with the estimation, described below, that between 5 and 8% of children have some form of AD. The discrepancy results from the nature of standardized achievement tests and the often rather specific deficits of children with AD. By design, standardized achievement tests sample a broad range of arithmetical and mathematical topics, whereas children with AD often have severe deficits in some of these areas and average or better competencies in others. The result of averaging across these topics is a level of performance (e.g., at the 20th percentile) that overestimates competencies in some areas and underestimates them in others.

PREVALENCE AND ETIOLOGY

Large-scale epidemiological studies of the prevalence of AD have not been conducted, although several smaller-scale studies that included more than 300 children from a well-defined population have (e.g., all fourth-graders in an urban school district). Measures, designed from neuropsychological studies of number and arithmetic deficits following brain injury, are more sensitive to AD than are standard achievement tests have been used in these studies. They have been conducted in the United States (Badian, 1983), Europe (Kosc, 1974), and Israel (Gross-Tsur et al., 1996; Shalev et al., 2001). The findings across studies suggest that 5 to 7% of school-age children exhibit some form of AD. Ostad (1998a) described several related studies of elementary-school children conducted in Norway during the 1950s. These studies revealed that 8% of the children likely had some form of AD. Thus, the best estimate, at this time, is that between 5 and 8% of children have some form of AD.

Some children with AD exhibit comorbid attention-deficit/hyperactivity disorder (ADHD) or reading disability (RD). The most comprehensive of these studies indicated that 26% of the children with AD had symptoms of ADHD, and 17% had RD (Gross-Tsur et al., 1996). Badian (1983), in contrast, found that nearly half of the children with AD also showed comorbid reading difficulties, and Ostad (1998a) found that just over half of the children with AD had a comorbid spelling disability (SD). At this time, it appears that children with AD constitute at least two different subgroups, those with only difficulties in arithmetic and those with comorbid learning disabilities in other areas. The latter most typically involve language-related deficits, that is RD and (or) SD (for related discussion, see Geary, 1993; Geary & Hoard, 2001).

As with other forms of LD, twin and familial studies suggest both genetic and environmental contributions to both forms of AD. In a twin study, Light and DeFries (1995) provided evidence that the same genes may contribute to AD and RD and thus their comorbidity in many children. Shalev and her colleagues (2001) studied familial patterns of AD, excluding children with comorbid ADHD or RD. The results showed that family members (e.g., parents and siblings) of children with AD are 10 times more likely to be diagnosed with AD than are members of the general population.

Research Program

As noted, performance on standardized achievement tests does not provide information on the strengths and weaknesses of individual children within the broad domain of mathematics, only relative performance averaged across all the assessed mathematical subareas. The only means to better understand learning in mathematics, as well as learning disabilities, is to focus research efforts on circumscribed mathematical domains and to use methods that enable a fine-grained assessment of performance in each domain. To this end, our initial efforts were focused on simple addition and were guided by Ashcraft's (e.g., Ashcraft & Battaglia, 1978) information-processing studies of cognitive arithmetic and later by Siegler's (1996; Siegler & Shrager, 1984) strategy choice model of cognitive development.

INFORMATION PROCESSING

Beginning with Svenson and Broquist's (1975) study more than 25 years ago and continuing today, the information-processing approach has guided much of the cognitive research on children with AD. In our first study of children with AD (Geary et.al., 1987), we employed the reaction time (RT) techniques developed by Groen and Parkman (1972) and later elaborated by Ashcraft and his colleagues (for a review, see Ashcraft, 1995). Here, simple arithmetic problems, such as 3 + 2 = 4 or 9 + 5 = 14, are presented on a computer monitor. The child indicates by button push whether the presented answer is correct. The resulting RTs are then analyzed by means of regression equations. Here, statistical models representing the approaches potentially used while problem solving, such as counting or memory retrieval, are fit to RT patterns. As an example, if children counted both addends in the problem, starting from one, then RTs should increase linearly with the sum of the problem, and the value of the raw regression slope should be consistent with estimates of the speed with which children count implicitly (for general discussion, see Geary, Widaman, & Little, 1986; Widaman, Geary, Cormier, & Little, 1989).

In the first study in which we used these techniques, second-, fourth-, and sixth-grade children with AD were compared to their academically normal peers (Geary et al., 1987). The RT patterns suggested that children with AD differed from other children in terms of the form and frequency of counting strategies used to solve simple addition prolems. For a problem such as 9 + 5, academically normal second-grade children typically stated "nine" and then counted, "ten, eleven, twelve, thirteen, fourteen"(termed the counting-on procedure; Fuson, 1982). Children with AD tended to count, starting from one (the counting-all procedure). Cross-sectional comparisons suggested that most academically normal children gradually switched from counting to direct retrieval of the answer, whereas most children with AD did not make this transition (Geary et al., 1987). Rather, they still counted to solve addition problems, although many of these children appeared to use the more efficient counting-on procedure in later grades. The same pattern has recently been reported in a study that contrasted the subtraction competencies of children with AD with those of other children (Ostad, 2000).

STRATEGY CHOICE

Soon after beginning data collection for this first study (i.e., Geary et al., 1987), I read Siegler and Shrager's (1984) strategy-choice model of arithmetical and later more general cognitive development (Siegler, 1996). The approach was based on a combination of the RT methods used by cognitive psychologists, such as Ashcraft (1995), and direct observation of problem solving used by educational researchers, such as Carpenter and Moser (1984). The goal was not only to describe the types of strategies children used to solve simple arithmetic problems but also to determine the mechanisms that governed whether a child would use one strategy (e.g., counting) or another (e.g., retrieval) to solve each particular problem. A related goal was (and still is) to understand developmental change in the mechanisms governing strategy choices. Although the model has been elaborated over the years, the basic mechanisms are the same (Siegler, 1996).

In all domains that have been studied, including arithmetic, children use a mix of strategies during problem solving. In solving arithmetic problems, children will sometimes retrieve the answer to solve one problem and count to solve the next problem. Memory retrieval is assumed to be based on an associative relationship between the presented problem and all potential answers to the problem. These associations appear to develop as children use other types of strategies during problem solving. Counting on to solving 5 + 3, for instance, appears to result in the formation of a long-term memory association between this problem and the answer generated by the count. Each time the problem is solved through counting, the strength of the association between 5 + 3 and the generated answer (typically 8) increases. Eventually children automatically retrieve 8, or whatever answer has been most frequently generated, when presented with 5 + 3. So, if an answer is not readily retrieved, due to a low associative strength between the problem and potential answers, children resort to some form of backup strategy to complete problem solving; Table 12.1 provides a description of retrieval and the primary backup strategies used to solve simple addition problems (Geary, 1994).

Our first study that followed Siegler and Shrager's (1984) method replicated and extended their basic findings (e.g., the relation between RTs and strategy choices) by demonstrating that individual differences in strategy choices were related to individual performance differences on standard arithmetical achievement and ability tests (Geary & Burlingham-Dubree, 1989). Strong performance on the strategy-choice task (e.g., fast and accurate strategy execution) was predictive of above-average performance on the achievement and ability measures (see

Table 12.1 Strategies Used to Solve Simple Addition Problems

Strategy	Description	Example
Finger counting: Counting all	A number of fingers representing the augend and addend are lifted and then counted starting from 1.	To solve 2+3, two fingers are lifted on one hand and three on the other. All uplifted fingers are then counted
Finger counting: Counting on	A number of fingers representing the augend and addend are lifted and then counted starting from the larger number.	To solve 2+3, two fingers are lifted on one hand and three on the other. The count starts with "three" and proceeds "four, five."
Verbal counting: Counting all	As above, but counting is done without the use of fingers	To solve 2+3, the child counts (explicitly or implicitly), "one, two, three, four, five."
Verbal counting: Counting on	As above, but counting is done without the use of fingers.	To solve 2+3, the child states "three" and then counts "four, five."
Retrieval	Direct retrieval of a basic fact from long-term memory.	The child states an answer quickly and without signs of counting; typically stating "just knew it."
Decomposition	Retrieval of partial sum and counting on.	To solve 2+3, the child first retrieves the answer to 2+2 and then counts up to "five."

Note. See Geary (1994) for further discussion and illustration also Siegler, 1988). The results also indicated that the methods and theoretical model proposed by Siegler and Shrager would likely provide an excellent framework for guiding the study of children with AD. The approach was followed in an initial study of first-grade children with AD (Geary, 1990), many of whom were reassessed in second grade (Geary et al., 1991), and another study of fourth-graders that included children with AD as well as gifted children (Geary & Brown, 1991).

All these studies involved the use of a variant of Siegler and Shrager's (1984) strategy assessment task for simple addition, which provides information on problem-solving strategies, accuracy of strategy use, and accompanying RTs. Here, simple addition problems, such as 5 +6, are presented one at a time on a computer monitor. The child is instructed to solve the problem using whatever means is easiest for him or her. With the completion of problem solving, the child immediately speaks the answer into a voice-activated relay which triggers an internal timing device in the computer (for recording RTs). The child's problem solving, such as whether fingers are used, is monitored and recorded by the experimenter. The child is then asked to describe how he or she got the answer. High-levels of agreement between experimenter observation and child reports (typically> 90% of trials), along with a consistency between these reports and associateed RTs patterns, attest to the utility of the method (e.g., Geary, 1990; Siegler, 1987).

In the first study based on this approach, first-grade children with AD were divided into two groups: improved and no change (Geary, 1990). The children in the improved group had below-average mathematics achievement scores at the end of kindergarten but average or better scores at the end of first grade. Children in the no-change group had below-average scores at both assessments. There were no differences comparing children in the improved group to children in an academically normal control group in terms of strategy choices, error rates, or RTs. This pattern was subsequently replicated (Geary, Hamson, & Hoard, 2000), which bolstered the conclusion that the initial low mathematics achievement of children in the improved group was not likely to be due to any form of cognitive deficit or AD. Their initial poor performance may have been due to inattention during test taking or poor early math instruction. In any case, the improved group is not considered further.

An unexpected finding was that children with AD (i.e., the no-change group) did not differ from their academically normal peers in terms of the mix of strategies used to solve simple addition problems, as shown in Table 12.2. Differences were, however, found in percentage of retrieval and counting errors and in the use of the counting-on procedure, all favoring the academically normal group. Error and RT patterns for problems on which an answer was retrieved also differed comparing the academically normal and AD groups. For children with AD, the distribution of RTs was unusual. The pattern was not similar to that found in younger, academically normal children and seemed to reflect a highly variable speed of fact retrieval. The interpretation of this pat-

TABLE 12.2. Addition Strategy Characteristics Comparing Academically Normal Children and Children with AD

	Trials on which Strategy used (%)		Error (%)		Counting on (%)	
Strategy	Normal	AD	Normal	AD	Normal	AD
Counting fingers	5	6	21	50	100	69
Verbal counting	60	64	7	31	100	86
Retrieval	35	26	5	22	-	-

Note. Data based on Geary (1990)

tern was that it suggested “an anomalous long-term memory representation of addition facts” (Geary, 1990, p. 379). Further analyses revealed greater variability in the speed with which children with AD executed other numerical processes, such as number articulation, in comparison to their academically normal peers.

A year later, many of these children were reassessed on the strategy-choice task and were administered a numerical digit span task (Geary et al., 1991). In keeping with models of arithmetical development (Ashcraft, 1982), the academically normal children showed an across-grade shift from reliance on verbal counting (56% to 44% across years) to retrieval (39% to 51 % across years). The academically normal children also showed faster retrieval times and fewer retrieval errors (6% to 2%), comparing grade 1 to grade 2 performance. As with the previously described cross-sectional study (Geary et al., 1987), the children with AD showed no developmental change in the mix of problem-solving strategies (e.g., 26% to 25% retrieval across years) or in retrieval accuracy (e.g., 18% to 16% retrieval errors).

The children with AD did, however, show improvement in how effectively they used counting to solve addition problems. In grade 2, they almost always used the counting-on procedure when using a counting strategy to problem solve and showed a marked reduction in the proportion of counting errors (e.g., from 49% to 10% for finger counting). Analyses of RTs indicated that the academically normal children showed faster counting comparing grade 2 to grade 1, but the children with AD showed no change in counting speed. Again, the distribution of retrieval RTs of children with AD differed from that of their academically normal peers. An important discovery was that the pattern of retrieval RTs of children with AD was similar to that found with children who had suffered from an early (before 8 years of age) lesion to the left hemisphere or subcortical regions (Ashcraft, Yamashita, & Aram, 1992).

This pattern of developmental change suggested that the children with AD were developmentally delayed in terms of their ability to use counting to solve arithmetic problems and were fundamentally different from normal children in the mechanisms supporting fact retrieval (see also Garnett & Fleischner, 1983). Subsequent studies using the same model and methods have been conducted in the United States by Jordan and her colleagues (Jordan & Hanich, 2000; Jordan et al., 1995; Jordan & Montani, 1997) and by Ostad and others in Europe (e.g., Barrouillet et al., 1997; Ostad, 1997, 1998b, 2000). These studies have confirmed the differences in counting-strategy use and retrieval deficit and extended the domain of study to subtraction, multiplication, and word problems, among others (e.g., Hanich, Jordan, Kaplan, & Dick, 2001; Ostad, 1998b). Subsequent research has also led to the discovery of at least two different forms of AD (Jordan & Montani, 1997)-that is, AD with no comorbid forms of LD and AD with comorbid RD or other forms of language-related disorder (e.g., SD). Subsequent research has further demonstrated that the differences comparing children with AD to other children cannot be attributed to differences in IQ (Geary et al., 1999; Geary, Hamson, & Hoard, 2000; McLean & Hitch, 1999).

Cognitive Mechanisms and Deficits

The aforementioned studies led to attempts to discern the nature of the cognitive deficits underlying some children’s developmental delay in the use of counting procedures and their difficulties in representing and/or retrieving basic facts from long-term memory. Cognitive studies combined with research on arithmetical difficulties associated with brain injury (i.e., dysca1culia) and with behavioral genetic studies of individual differences in mathematical abilities provided dues as to possible sources of the problem-solving characteristics of children with AD. The integration of these literatures resulted in a taxonomy of three general subtypes of mathematical disability (MD), procedural, semantic memory, and visuospatial (Geary, 1993). Table 12.3 shows the defining characteristics of these subtypes.

The development delay in the use of counting procedures while solving arithmetic problems is subsumed under the more general procedural subtype of MD. Deficits in the retrieval of basic arithmetic facts is the defining feature of the semantic memory

TABLE 12.3. Subtypes of Learning Disabilities in Mathematics

<u>Procedural subtype</u>

Cognitive and performance features

A. Relatively frequent use of developmentally immature procedures (i.e., the use of procedures that are more commonly used by younger, academically normal children)
B. Frequent errors in the execution of procedures
C. Poor understanding of the concepts underlying procedural use
D. Difficulties sequencing the multiple steps in complex procedures

Neuropsychological features

Unclear, although some data suggest an association with left-hemispheric dysfunction and in some cases (especially for feature D above) a prefrontal dysfunction

Genetic features

Unclear

Developmental features

Appears, in many cases, to represent a developmental delay (i.e., performance is similar to that of younger, academically normal children, and often improves across age and grade)

Relation to RD

Unclear

<u>Semantic memory subtype</u>

Cognitive and performance features

A. Difficulties retrieving mathematical facts, such as answers to simple arithmetic problems
B. What facts are retrieved, there is a high error rate
C. For arithmetic, retrieval errors are often associates of numbers in the problem (e.g., retrieving 4 to 2 + 3 = ?; 4 is the counting-string associate that follows 2,3)
D. RTs for correct retrieval are unsystematic

Neuropsychological features

A. Appears to be associated with left-hemispheric dysfunction, possibly the posterior regions for one form of retrieval deficit and the prefrontal regions for another
B. Possible subcortical involvement, such as the basal ganglia

Genetic features

Appears to be a heritable deficit

Developmental features

Appears to represent a developmental difference (i.e., cognitive and performance features differ from that of younger, academically normal children, and do not change substantively across age or grade)

Relation to RD

Appears to occur with phonetic forms of RD

<u>Visuospatial subtype</u>

Cognitive and performance features

A. Difficulties in spatially representing numerical and other forms of mathematical information and relationships
B. Frequent misinterpretation or misunderstanding of spatially represented information

Neuropsychological features

Appears to be associated with right-hemispheric dysfunction, in particular, posterior regions of the right hemisphere, although the parietal cortex of the left hemisphere may be implicated as well

Genetic features

Unclear, although the cognitive and performance features are common with certain genetic disorders (e.g., Turner's syndrome)

Developmental features

Unclear

Relation to RD

Does not appear to be related

Note. Adapted from Geary (1993,2000).

subtype. Still, semantic memory deficits should affect other mathematical competencies that are based on the retrieval of facts, such as recalling prime numbers. In any case, the respective sections that follow describe research on the arithmetical problem solving of children with AD in terms of the three forms of MD subtype, and as related to the cognitive and neural systems that may underlie their problem-solving characteristics (e.g., the retrieval deficit).

Procedural Deficits

Much of the research on children with AD has focused on their use of counting procedures to solve simple arithmetic problems. As described, when they solve such problems, children with AD often commit more errors than do their academically normal peers, and they often use problem-solving procedures, such as counting all, that are more commonly used by younger children (Geary, 1990; Jordan et al., 1995; Jordan & Montani, 1997). The errors result when these children miscount, typically undercounting or overcounting by 1 (Geary, 1990). As a group, children with AD also rely on finger counting, as contrasted with verbal counting, more frequently and use this strategy for more years than do academically normal children. A few studies have assessed the procedural competencies of children with AD during the solving of multistep arithmetic problems, such as 45 x 12 or 126 + 537. Russell and Ginsburg (1984) found that fourth-grade children with AD committed more errors than did their IQ-matched academically normal peers when solving such problems. These errors involved (1) the misalignment of numbers while writing down partial answers or (2) errors while carrying or borrowing from one column to the next. The following sections discuss these procedural characteristics of children with AD in terms of working memory, conceptual knowledge, and neural correlates.

WORKING MEMORY

Although the relationship between working memory and difficulties in executing arithmetical procedures is not yet fully understood, it is clear that children with AD have some form of working-memory deficit (Hitch & McAuley, 1991; McLean &Hitch, 1999; Siegel & Ryan, 1989: Swanson, 1993). There are several ways in which a working-memory deficit could affect the procedural competencies of children with AD.

As an example, children with AD (and younger, academically normal children) appear to use finger counting as a working-memory aid, in that fingers appear to help these children to keep track of the counting process (Geary, 1990). In particular, representing the problem addends on fingers and then using fingers to note the counting sequence should greatly reduce the working-memory demands of the counting process. Working-memory may also contribute to the tendency of children with AD to undercount or overcount during the problem-solving process. Such miscounting can occur if the child loses track of where he or she is in the counting process, that is, how many fingers he or she has counted and how many remain to be counted. Working memory is also implicated in the difficulties that children with AD have during the solving of more complex arithmetic problems. The procedural errors for the children with AD assessed by Russell and Ginsburg (1984) appeared to result from difficulties monitoring and coordinating the sequence of problem-solving steps, which, in turn, suggest compromised executive functions.

At a more basic level, a working-memory deficit could result from difficulties with representing information in the basic phonetic/articulatory or visuospatial working memory systems, or from a deficit in accompanying executive processes, such as attentional or inhibitory control (see McLean & Hitch, 1999). In theory, difficulties with representing and manipulating information in the phonetic buffer could disrupt the representation of number words and their articulation during the counting process. Attentional and other executive difficulties could result in problems keeping track of the counting process and sequencing the multiple steps involved in executing complex procedures (Geary, 1993).

CONCEPTUAL KNOWLEDGE

In addition to working memory, a poor understanding of the concepts underlying a procedure can also contribute to a developmental delay in the adoption of more sophisticated procedures and reduce the ability to detect procedural errors.

For instance, delayed use of the counting-on procedure and frequent counting errors of children with AD appear to be related, in part, to immature counting knowledge. In our first study in the area of counting, we found that first-grade children with AD and RD understood most of the essential features of counting, such as cardinality-that is, that the last stated number word represents the total number of items in the counted set (Geary et al., 1992; for discussion of counting, see Briars & Siegler, 1984; Gelman & Gallistel, 1978). However, these children consistently made errors on tasks that assessed other features of counting, in particular order irrelevance. Many of the children with AD believed that a correct but nonsequential counting of items (e.g., skipping items and then coming back to count them) resulted in an incorrect count. The pattern suggests that although most children with AD know the standard counting sequence and understand some counting concepts, they nonetheless appear to view counting as a rote, mechanical activity. Children, including many children with AD, who do not understand the order-irrelevance concept use the counting-on procedure during problem solving much less frequently than do other children (Geary et al., 1992; Geary, Hamson, & Hoard, 2000). It is possible that the switch from use of the counting-all procedure to the counting-on procedure requires an understanding that counting does not have to start from one and proceed in the standard sequential order. The immature counting knowledge of children with AD may also contribute to their frequent counting errors, in particular a failure to detect and thus self-correct these errors. In other words, conceptual knowledge not only guides procedural use, it may also provide a frame for evaluating the accuracy with which procedures are executed.

NEURAL CORRELATES

Given the similarity between the deficits associated with AD and those associated with acquired dyscalculia, neuropsychological studies of dyscalculia provide insights as to the potential neural systems contributing to the procedural deficits of children with AD (Geary, 1993; Geary & Hoard, 2001). As is found with children with AD, individuals with acquired or developmental dyscalculia are generally able to count arrays of objects, recite the correct sequence of number words during the act of counting (e.g., counting from 1 to 20), and understand many basic counting concepts (e.g., cardinality; Hittmair-Delazer, Sailer, & Benke, 1995; Seron et al., 1991; Temple, 1989). Individuals with dyscalculia caused by damage to the right hemisphere sometimes show difficulties with the procedural component of counting, specifically, difficulties with systematically pointing to successive objects as they are enumerated (Seron et al., 1991). However, the relation between this feature of dyscalculia and the procedural deficits of children with AD is not clear.

Difficulties solving complex arithmetic problems are also common with acquired and developmental dyscalculia (Semenza, Miceli, & Girelli, 1997; Temple, 1991). As an example, in an extensive assessment of the counting, number, and arithmetic competencies of a 17-year-old--M.M.-- with severe congenital damage to the right frontal and parietal cortices, Semenza and his colleagues reported deficits similar to those reported by Russell and Ginsburg (1984) for children with AD. Basic number and counting skills were intact, as was the ability to retrieve basic facts (such as 8 for 5 + 3) from memory. However, M.M. had difficulty solving complex division and multiplication problems, such as 32 x 67. Of particular difficulty was tracking the sequence of partial products. Once the first step was completed (2 x 7), difficulties placing the partial product (4) in the correct position and carrying to the next column were evident. Thus, the primary deficit of M.M. appeared to involve difficulties sequencing the order of operations and monitoring the problem-solving process, as is often found with damage to the frontal cortex (Luria, 1980); Temple (1991) reported a similar pattern of procedural difficulties for an individual with neurodevelopmental abnormalities in the right frontal cortex. It remains to be seen if a compromised right frontal cortex contributes to aspects of the procedural deficits of children with AD.

Semantic Memory Deficits

As described earlier, many children with AD do not show the shift from procedural-based problem solving to memory-based problem solving that is commonly found in academically normal children (Geary et al.1987; Ostad, 1997). The pattern suggests that children with AD have difficulties storing or accessing arithmetic facts in or from long-term memory. Indeed, disrupted memory-based processes are consistently found with comparisons of children with AD and other children (Barrouillet et al., 1997; Bull & Johnston, 1997; Garnett & Fleischner, 1983; Geary, 1993; Geary & Brown, 1991; Geary et al., 1987; Jordan & Montani,1997; Ostad, 1997). Disruptions in the ability to retrieve basic facts from long-term memory might, in fact, be considered a defining feature of AD (Geary, 1993). Most of these individuals can, however, retrieve some facts, and disruptions in the ability to retrieve facts associated with one operation (e.g., multiplication) are sometimes found with intact retrieval of facts associated with another operation (e.g., subtraction), at least when retrieval deficits are associated with overt brain injury (Pesenti, Seron, & Van Der Linden, 1994).

As described in Table 12.3, when they retrieve arithmetic facts from long-term memory, children with AD commit many more errors than do their academically normal peers and show error and RT patterns that often differ from the patterns found with younger, academically normal children (Geary, 1993; Geary, Hamson, & Hoard, 2000). The RT patterns are similar to the patterns found with children who have suffered from an early (before age 8 years) lesion to the left hemisphere or associated subcortical regions (Ashcraft et al., 1992), as noted earlier. Although this pattern does not indicate that children with AD have suffered from some form of overt brain injury, it does suggest that the memory-based deficits of many of these children may reflect the same mechanisms underlying the retrieval deficits associated with dyscalculia (Geary, 1993; Rourke, 1993).

However, the cognitive and neural mechanisms underlying these deficits are not completely understood. On the basis of Siegler's strategy-choice model, solving arithmetic problems by means of counting should eventually result in associations forming between problems and generated answers (Siegler, 1996; Siegler & Shrager, 1984). Because counting typically engages the phonetic and semantic (e.g., understanding the quantity associated with number words) memory systems, any disruption in the ability to represent or retrieve information from these systems should, in theory, result in difficulties in forming problem/answer associations during counting (Geary, 1993; Geary, Bow-Thomas, Fan, & Siegler, 1993). Although not definitive with respect to this hypothesis, the work of Dehaene and his colleagues suggests that the retrieval of arithmetic facts is indeed supported by a system of neural structures that appear to support phonetic and semantic representations and are engaged during incrementing processes, such as counting. These areas include the left basal ganglia and the left pari-eto-occipito-temporal areas (Dehaene & Cohen, 1995, 1997). Damage to either the subcortical or cortical structures in this network is associated with difficulties accessing previously known arithmetic facts (Dehaene & Cohen, 1991, 1997). However, it is not currently known if the retrieval deficits of children with AD are the result of damage to or neurodevelopmental abnormalities in the regions identified by Dehaene and Co-hen (1995, 1997).

More recent studies of children with AD suggest a second form of retrieval deficit, specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. This form of retrieval deficit was first discovered by Barrouillet and colleagues (1997), based on the memory model of Conway and Engle (1994), and was recently confirmed in our laboratory (Geary, Hamson, & Hoard, 2000; see also Koontz & Berch, 1996). In the Geary and colleagues (2000) study, one of the arithmetic tasks required children to use only retrieval-the children were instructed not to use counting strategies to solve simple addition problems (see also Jordan & Montani, 1997). Children with AD, as well as children with RD, committed more retrieval errors than did their academically normal peers, even after controlling for IQ. The most common of these errors was acounting-string associate of one of the addends. For instance, common retrieval errors for the problem 6 + 2 were 7 and 3, the numbers following 6 and 2, respectively, in the counting sequence. Hanich and colleagues (2Q.01) found a similar pattern, although the proportion of retrieval errors that were counting-string associates was lower than that found by Geary and colleagues.

The pattern in these more recent studies (e.g., Geary, Hamson, & Hoard, 2000) and that of Barrouillet and colleagues (1997) is in keeping with Conway and Engle's (1994) position that individual differences in working memory and retrieval efficiency are related, in part, to the ability to inhibit irrelevant associations. In this model, the presentation of a to-be-solved problem results in the activation of relevant information in working memory, including problem features such as the addends in a simple addition problem and information associated with these features. Problem solving is efficient when irrelevant associations are inhibited and prevented from entering working memory. Inefficient inhibition results in activation of irrelevant information, which functionally lowers working-memory capacity. In this view, children with AD make retrieval errors, in part because they cannot inhibit irrelevant associations from entering working memory. Once in working memory, these associations either suppress or compete with the correct association for expression. Whatever the cognitive mechanism, these results suggest that the retrieval deficits of some children with AD may spring from either delayed development of those areas of the prefrontal cortex that support inhibitory mechanisms, or neurodevelopmental abnormalities in these regions (Bull, Johnston, & Roy, 1999; Welsh & Pennington, 1988). The results also suggest that inhibitory mechanisms should be considered potential contributors to the comorbidity of AD and ADHD in some children.

Visuospatial Deficits

The relation between visuospatial competencies and AD has not been fully explored. In theory, visuospatial deficits should affect performance in some mathematical domains, such as certain areas of geometry and the solving of complex word problems, but not other domains, such as fact retrieval or knowledge of geometric theorems (e.g. Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Geary, 1993, 1996). Many children with the procedural and/or semantic memory forms of AD, at least as related to simple arithmetic, do not appear to differ from other children in basic visuospatial competencies (Geary, Hamson, & Hoard, 2000; Morris et al., 1998). There is, however, evidence that some children with AD who show broader performance deficits in mathematics may have a deficit in visuospatial competencies.

McLean and Hitch (1999) found that children with AD showed a performance deficit on a spatial working memory task, although it is not clear if the difference resulted from an actual spatial deficit or from a deficit in executive functions (e.g., the ability to maintain attention on the spatial task). In any case, Hanich and her colleagues (2001) found that children with AD differed from their peers on an estimation task and in the ability to solve complex word problems. Although performance on both of these tasks is supported by spatial abilities (Dehaene et al.,1999; Geary, 1996; Geary, Saults, Liu, & Hoard, 2000), it is not clear whether the results of Hanich and colleagues were due to a spatial deficit in children with AD.

Conclusion

The theoretical models and experimental methods used to study the development of number, counting, and arithmetical competencies in academically normal children have provided a much needed framework for guiding the study of children with AD. We now understand the problem-solving functions and deficits of children with AD, at least as related to the solving of simple arithmetic problems (e.g., 4 + 7) and simple word problems (Geary, Hamson, & Hoard, 2000; Hanich et al., 2001; Ostad, 2000). Most of these children use problem-solving procedures that are more commonly used by younger, academically normal children, and tend to commit more procedural errors. Over the course of the elementary-school years, the procedural competencies of many children with AD tend to improve, and thus their early deficits seem to represent a developmental delay and not a fundamental cognitive deficit. At the same time, many children with AD have difficulties retrieving basic arithmetic facts from long-term memory, a deficit that often does not improve and thus may represent a developmental difference. Some insights have also been gained regarding the cognitive and neural mechanisms contributing to the procedural and retrieval characteristics of children with AD, including compromised working memory and executive functions.

Much remains to be accomplished, however. In comparison to simple arithmetic, relatively little research has been conducted on the ability of children with AD to solve more complex arithmetic problems (but see Russell & Ginsburg, 1984), and even less has been conducted in other mathematical domains. Even in the area of simple arithmetic, the cognitive and neural mechanisms that contribute to the problem-solving characteristics of children with AD are not fully understood and are thus in need of further study. Other areas that are in need of attention include the development of diagnostic instruments for AD; cognitive and behavioral genetic research on the comorbidity of AD and other forms of LD and ADHD; and, of course, the development of remedial techniques. If progress over the past 10 years is any indication, we should see significant advances in many of these areas over the next 10 years.

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