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Cognitive Addition: A Short Longitudinal Study of Strategy Choice and Speed-of-Processing Differences in Normal and Mathematically Disabled Children

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This study provided a longitudinal assessment of skill development in addition for 26 normal and 12 mathematically disabled first- or second-grade children. At the first time of measurement, the children solved 40 simple addition problems. Ten months later, all subjects were readministered the addition task and a measure of working memory resources. Across times of measurement, the normal group showed increased reliance on memory retrieval and decreased reliance on counting to solve the addition problems, as well as an increase in speed of counting and of retrieving addition facts from long-term memory. The math-disabled group showed no reliable change in the mix of problem-solving strategies or in the rate of executing the counting or memory retrieval strategies. Finally, reliable differences, favoring the normal group, were found for the index of working memory resources.

A recent large-scale study found that more school-age children showed some form of mathematical disability than some form of reading disability (Badian, 1983). Yet, "comparatively, disorders of mathematics among the learning disabled remain relatively neglected" (Sutaria, 1985, p. 359). In fact, few empirical studies of the cognitive mechanisms potentially contributing to a mathematical learning disability have been conducted, even though much has been learned about the acquisition of basic mathematical concepts and procedures in academically normal children (Fuson, 1988; Ginsburg, 1983; Schoenfeld, 1987; Siegler, 1986). Moreover, the majority of studies that have been conducted in the mathematical disabilities area have been atheoretical; that is, such studies have not been guided by theoretical and conceptual models of the normal development of numerical cognition (e.g., Fleischner, Garnett, & Shepherd, 1982).

This study reports a theory-driven longitudinal assessment of skill development in addition for groups of normal and mathematically disabled children. Performance in simple addition provides useful information about factors potentially contributing to a mathematical learning problem on two dimensions. The first is *strategy development.* Although any single child will likely use multiple strategies to solve a set of addition problems, such as counting on fingers or retrieving facts directly from long-term memory, the developmental maturity of the child's strategy mix can be inferred on the basis of theoretical models and adult performance (Siegler & Jenkins, 1989). The second dimension is *speed of information processing.* That is, problem-solving solution times can be decomposed into basic information processes that might index, for example, the rate of retrieving arithmetic facts from long-term memory (Ashcraft, 1982; Geary & Burlingham-Dubree, 1989; Siegler & Jenkins, 1989).

Finally, the cognitive parameters underlying normal skill development in addition are well-delineated within the context of a more general theoretical model of cognitive development, that is, the distributions of associations model of strategy choices. Thus, the strategy choice model, described below, was followed in this study and provided the basis for experimental design and data interpretation (Siegler, 1986, 1988; Siegler & Shrager, 1984). In all, performance in simple addition will allow for theory-based inferences, contrasting normal and math-disabled children, to be drawn regarding the parameters underlying the developmental maturity of the child's mix of problem-solving strategies and the rate with which elementary information processes are executed. The description of the strategy choice model is followed by a brief overview of relevant research in the mathematical disabilities area.

Strategy Choice Model

A primary source of cognitive abilities in children is skill in the use of alternative problem-solving strategies (Geary & Burlingham-Dubree, 1989; Siegler, 1983). That is, for any given cognitive task, the child can often use one of several strategies to

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solve the presented problem. These alternative strategies will differ in the probability of producing the correct answer, the duration of the problem-solving processes, and in the demands the strategy places on working memory resources (Brainerd, 1983; Kaye, 1986; Siegler, 1986). An adaptive strategy choice is likely based on a weighted combination of these factors. Geary and Burlingham-Dubree, for instance, found that the adaptive use of alternative problem-solving strategies in addition was moderately to strongly correlated with performance on more complex mathematical achievement and ability measures but was uncorrelated with measures of verbal ability. Thus, strategy choices in addition would appear to contribute to mathematical skill. The strategy choice model specifies the mechanisms governing the adaptive use of alternative problem-solving strategies.

With regard to addition, the majority of problems are solved with one of five basic strategies (Baroody, 1987; Carpenter & Moser, 1984; Siegler, 1986; Siegler & Robinson, 1982; Siegler & Shrager, 1984; Svenson & Sjöberg, 1983). Three are visible or audible, overt strategies, and are termed (a) *counting fingers*—children use their fingers to physically represent the problem integers and then count their fingers to reach a sum; (b) *fingers*—children use their fingers to represent the integers but do not visibly count them before giving an answer; and (c) *verbal counting*—children count audibly or move their lips as if counting implicitly. The fourth strategy involves the *decomposition* of the problem into more simple problems (Siegler, 1987). For example, the problem 9 + 8 might be solved by subtracting 1 from the 8, then adding the 1 to the 9, and finally adding 7 and 10. The fifth strategy is a *retrieval* strategy, whereby an addition answer is retrieved directly from long-term memory (Siegler & Shrager, 1984). In terms of skill development in addition, memory retrieval is the preferred strategy choice because retrieval requires less time to execute and places fewer demands on working memory resources than alternative strategies (Kaye, 1986; Siegler, 1986).

In fact, the facility with which answers can be directly retrieved from long-term memory strongly influences children's strategy choices in addition. More precisely, according to the strategy choice model, the specific strategy chosen for problem solving is influenced by the peakedness of the distribution of associations between a problem and all potential answers to that problem. With a peaked distribution, "the preponderance of associative strength is concentrated on a single answer, (the peak of the distribution)" (Siegler, 1988, p. 834), which is usually the correct answer. For a flat distribution, the associative strength is distributed among several potential answers. The more peaked the distribution of associations, the more readily an answer can be retrieved from long-term memory, and therefore, the more likely retrieval processes will be used to solve the presented problem. In addition to the peakedness of the distribution of associations, the actual strategy chosen for problem solving is also influenced by a confidence criterion. The confidence criterion represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer, and the rigor of this criterion appears to vary from child to child (Siegler, 1988).

In early versions of the model, strategy choices were hierarchically arranged, with memory retrieval always being the first strategy attempted (Siegler, 1986). For example, to solve the problem 3 + 4, the child would first set the confidence criterion and then attempt to retrieve an answer. If no answer is readily retrievable or if the retrieved answer does not exceed the value of the confidence criterion, then the child will resort to the use of a backup strategy (Siegler, 1983). Here, most children will typically use the verbal counting strategy, although some children might occasionally use fingers, counting fingers, or decomposition as backup strategies (Siegler, 1987).

In a recent revision of the strategy choice model, it was argued that strategies are horizontally, rather than hierarchically, organized and therefore compete for expression (Siegler & Jenkins, 1989). Thus, the presentation of a problem such as 3 + 4 activates not only candidate answers, such as 7, in long-term memory but also simultaneously activates long-term memory representations of procedural knowledge, such as a schema for executing a counting algorithm (Greeno, Riley, & Gelman, 1984). If the level of activation of the counting schema exceeds the level of activation of a candidate answer, then the counting algorithm will be expressed (i.e., executed). If a counting algorithm is correctly executed, then the stated answer, 7, becomes associated with the problem. Thus, when the child must again solve 3 + 4, the level of activation of the candidate answer (i.e., 7) is higher than it was before the previous successful count.

Gradually, after many successful counts, re-presentation of the problem will lead to a stronger activation of the candidate answer, as compared to the level of activation of the counting schema. In this case, memory retrieval will be used to solve the problem. The use of counting algorithms therefore leads to their own extinction by ensuring that memory retrieval becomes the dominant problem-solving process (Siegler & Jenkins, 1989). Nevertheless, some form of confidence criterion more likely also influences whether a retrieved answer is stated or whether a backup strategy is used to complete problem solving (Geary & Brown, 1991; Siegler, 1988). In all, the normal development of skill in addition should reflect an increase in the use of the memory retrieval strategy, with fewer retrieval errors, and a decrease in the use of counting and other backup strategies (Ashcraft, 1982; Siegler, 1986). According to the strategy choice model, any such developmental shift in the distribution of strategy choices, accompanied by decreased retrieval errors, would reflect an increase in the peakedness of the underlying distributions of associations parameter, and, more likely, a more rigorous confidence criterion.

Counting Algorithms

If counting, either verbally or on fingers, is required to solve an addition problem, then first-grade children will typically use the min, or counting-on, procedure (Carpenter & Moser, 1984; Fuson, 1988; Geary, 1990; Goldman, Mertz, & Pellegrino, 1989; Groen & Parkman, 1972; Groen & Resnick, 1977; Siegler, 1987). With the min procedure, the solution of a problem, such as 3 + 4, begins with stating the cardinal value of the larger integer (i.e., 4) and then counting in a unit-by-unit fashion a number of times equal to the value of the smaller or minimum (min) integer (i.e., 3) until a sum is obtained. An alternative but developmentally less mature strategy that appears to be used by school-age, math-disabled children and younger, normal children involves counting, from zero, in a unit-by-unit fashion a number of times equal to the cardinal value of both the augend, 3, and the addend, 4 (Goldman, Pellegrino, & Mertz, 1988; Svenson & Broquist, 1975). This latter strategy has been termed the sum, or counting-all, procedure (Baroody, 1987; Carpenter & Moser, 1984; Siegler, 1987).

Numerical Cognition and Mathematical Disabilities

Previous research in the mathematical disabilities area suggests that math-disabled children differ from their normal peers on two dimensions. First, the mix of addition problem-solving strategies used by math-disabled children tends to be developmentally immature; that is, these children often use strategies more commonly used by younger, academically normal, children (Fleischner et al., 1982; Garnett & Fleischner, 1983; Geary, 1990; Geary & Brown, 1991; Geary, Widaman, Little, & Cormier, 1987; Goldman et al., 1988; Svenson & Broquist, 1975). Second, the use of less mature problem-solving strategies appears to be related, in part, to an immature or abnormal development of the long-term memory representation of basic arithmetic facts. In other words, some math-disabled children cannot retrieve many basic facts, such as 3 + 4 = 7, from long-term memory, even at the end of the elementary school years (Garnett & Fleischner, 1983; Geary et al., 1987; Goldman et al., 1988). Thus, these children have to rely on counting to solve most simple addition problems.

Especially relevant to this study's research is Geary's (1990) study that compared normal and math-disabled first- or second-grade children on the distribution of strategies used to solve a set of simple addition problems and on the rate of executing the underlying processes. Strategies used in problem solving, and their associated solution times, were recorded on a trial-by-trial basis and were classified in accordance with the strategy choice model. This task was administered at the end of the academic year. During this academic year, all math-disabled subjects received remedial instruction in mathematics. On the basis of end-of-the-year achievement test scores, the math-disabled group was divided into an improved group and a no-change group. The improved group included children who had tested out of remedial education in mathematics, whereas the no-change group included children whose test scores indicated continued need for remedial education in mathematics. Comparisons of the normal and the improved groups indicated no substantive differences in the distribution of strategy choices, strategy characteristics (e.g., error rates), or in the rate of information processing.

Although the no-change children tended to use the retrieval strategy less frequently and counting strategies more frequently than the remaining children, the overall distribution of strategy choices and the speed of information processing, such as the rate of verbal counting, did not differ reliably when comparing the no-change group with the normal and improved groups. More detailed analyses of strategy characteristics (e.g., error rates) and solution times (e.g., variability in rate of process execution), however, revealed several reliable group differences. More precisely, the performance characteristics of the no-change group, in relation to the two remaining groups, included frequent verbal counting and memory retrieval errors, relatively frequent use of the sum counting strategy, a variable rate of executing the verbal counting strategy, and solution times for retrieval trials that were largely random or unsystematic. Within the context of the strategy choice model, the performance of the no-change children was interpreted as indicating a rather lenient confidence criterion (Siegler, 1988) and relatively flat distributions of associations of addition facts. This argument was based specifically on the finding that the no-change group did not retrieve many addition facts, tended to commit an error when they did retrieve the facts, and displayed highly unsystematic solution times on their correct retrieval trials. Finally, patterns of correlation between strategy choices and problem characteristics suggested that the use of the immature sum counting algorithm by the no-change children might have been related to poor working memory resources (Geary, 1990).

Because the performance characteristics of the previously described groups provided potentially important insights into the factors contributing to a math disability, we decided to conduct a follow-up assessment of the children evaluated in this earlier experiment (Geary, 1990). The follow-up study enabled a direct assessment of the pattern of skill development in addition by comparing a group of cognitively normal children with a group of children with an apparent cognitive deficit (Goldman et al., 1988). As with the first time of measurement, the groups were compared on the distribution of strategies, and the associated solution times, used to solve a set of simple addition problems. For this second time of measurement, all subjects were also administered an index of working memory resources to test the hypothesis that the no-change children have a relatively poor working memory capacity (Geary, 1990).

Method

Subjects

The subjects were selected from a single elementary school that served a rural working-class population. The sample at the first time of measurement included 52 first- or second-grade, academically normal or math-disabled, students.[1] The original math-disabled group consisted of 11 male and 18 female first- (n = 22) or second-grade (n = 7) children, and the original normal group consisted of 13 male and 10 female first-grade children (Geary, 1990). Moreover, as previously mentioned, the original math-disabled sample was divided into an improved group (n = 13) and a no-change group (n = 16). For the second time of measurement, 15 of the 23 normal, 11 of the 13 improved, and 12 of the 16 no-change subjects were evaluated. The remaining 14 subjects had either moved (n = 9), did not return the consent form (n = 3), or were not available (e.g., absent) to participate in the experiment (n = 2).

After 1 year of remedial services, all of the improved children, and

[1] In Geary (1990, Table 2), achievement test data are presented for a Time 1 and a Time 2. Time 1 refers to scores from achievement testing in the spring of their kindergarten year or first grade for some of the math-disabled children. Time 2 refers to scores from the following academic year. The addition task was only administered at Time 2. In this study, Time 1, or the first time of measurement, refers to the first administration of the addition task (Time 2 in Geary, 1990), and Time 2, or the second time of measurement, refers to the second administration of the addition task.

none of the no-change children had tested out of remedial education in mathematics. In fact, after 1 year of remedial services (i.e., after the first measurement), the achievement test scores in mathematics for the improved group were quite similar to those of the normal children, with the mean rankings, for children assessed at the second measurement, being 63.6 (SD = 10.9) and 69.9 (SD = 13.7) for the improved and normal groups, respectively. Thus, at the time of the second measurement, none of the improved children were receiving, or had received during the academic year, remedial services in mathematics and should therefore be considered normal on this dimension. Moreover, as was found for the first time of measurement, the performance of the normal and improved groups on the experimental tasks did not differ reliably for the second time of measurement.

Thus, on the basis of these findings and to improve the power of the subsequently described statistical tests, we combined these two groups and hereinafter refer to them as the *normal* group, whereas the no-change group will be termed the *math-disabled* group. The normal group consisted of 12 boys and 14 girls, and the math-disabled group consisted of 5 boys and 7 girls. For the first time of measurement, the mean age of the normal and math-disabled groups was 87.4 (SD = 6.7) and 88.7 (SD = 8.1) months, respectively. The difference in mean age was not reliable ($p > .50$). The first and second times of measurement were separated by an average of 10.2 months (SD = 9.5 days).

Subjects included in the math-disabled group were, at both times of measurement, receiving Chapter 1 remedial education services in mathematics (a federally funded program for low-achieving children). Inclusion in this program required a score on a standard mathematics achievement test that was below the 46th national percentile ranking (in fact, scores tend to be below the 30th percentile). Chapter 1 services were provided for the entire academic year and involved 20 min per day, 5 days per week, of specialized instruction in number concepts and mathematical procedures. For example, the children were instructed in the use of the min counting algorithm. All of the math-disabled subjects attended general education courses for most of their school day, although many of these children also received remedial services in reading. Children who were placed in special day classes (i.e., those who spent the entire school day in remedial classes) were not included in the sample. These children were excluded because children in such classes often show an array of social–behavioral, cognitive, and sometimes neuropsychological deficits (Sutaria, 1985), which would make data interpretation difficult.

Descriptive information for performance on the Science Research Associates Survey of basic skills, for both times of measurement, was obtained from school files and is presented in Table 1. Here, the normal group showed a reliably higher mean percentile ranking than did the math-disabled group for both the mathematics and the reading measures and for both times of measurement ($ps < .05$). Dependent t tests indicated no reliable change for either the mathematics measure or the reading measure across times of measurement for the normal group ($ps > .10$), but the math-disabled group showed a reliably higher mean percentile ranking on both measures for the second time of measurement ($ps < .05$).

Table 1
Descriptive Information for Performance on Achievement Measures

	Normal		Math disabled	
Subject	Time 1	Time 2	Time 1	Time 2
Mathematics				
M	67.2	65.7	31.0	48.7
SD	12.9	20.9	11.3	20.0
Reading				
M	65.4	64.0	28.3	42.4
SD	24.6	22.7	18.8	14.4
n	26		12	

Experimental Tasks

The experimental session for both times of measurement required the subjects to solve an identical set of simple addition problems. At the time of the second measurement a working memory task was also administered.

Addition stimuli. The experimental stimuli consisted of 40 pairs of vertically placed single-digit integers. Stimuli were constructed from the 56 possible nontie, pairwise combinations of the integers 2–9 (e.g., a tie problem is 2 + 2). The frequency and placement of all integers were counterbalanced. That is, each integer appeared 5 times as the augend and 5 times as the addend, and the smaller value integer appeared 20 times as the augend and 20 times as the addend. No repetition of either the augend or the addend was allowed across consecutive problems.

Apparatus. The addition problems were presented at the center of a 30 × 30 cm video screen controlled by an IBM PC-XT microcomputer. A Cognitive Testing Station clocking mechanism ensured the collection of reaction times (RT) with ±1-ms accuracy. The timing mechanism was initiated with the presentation of the problem on the video screen and was terminated using a Gerbrands G1341T voice-operated relay. The voice-operated relay was triggered when the subject spoke the answer into a microphone connected to the relay.

For each problem, a ready prompt appeared at the center of the video screen for a 1000-ms duration, followed by a 1000-ms blank screen. Then, an addition problem appeared on the screen and remained until the subject responded. The experimenter initiated each problem presentation sequence using a control key.

Working memory task. Both the forward and backward sections of the Digit Span subtest of the Wechsler Intelligence Scale for Children—Revised (Wechsler, 1974) were used as indices of working memory resources (Case, 1985; Sattler, 1974). After the completion of each section, subjects were questioned regarding what, if anything, they had done (e.g., rehearsal) to help remember the digit string.

Procedure

Each subject was tested individually and in a quiet room. The order with which the subjects were administered the Digit Span and arithmetic tasks was counterbalanced, but both tasks were administered in a single session. For the arithmetic task, the subjects were asked to solve 40 addition problems, preceded by 8 practice problems, presented one at a time on the video screen. Subjects were encouraged to use whatever strategy made it easiest for them to obtain the answer, although equal emphasis was placed on speed and accuracy. The achievement measures for the second time of measurement were administered about 1 month after participation in the experimental session.

After each trial in the addition task, subjects were asked to describe how they got the answer, and the answer and strategy used to solve each problem were recorded by the experimenter and classified as one of the earlier described strategies: (a) counting fingers, (b) fingers, (c) verbal counting, (d) decomposition, or (e) memory retrieval. Several previous studies have demonstrated that children can accurately describe problem-solving strategies in arithmetic, if they are asked immediately after the problem is solved (Siegler, 1987, 1989). On the basis of subject descriptions, the counting fingers and verbal counting trials were further classified in accordance with the specific algorithm used for problem solving. That is, the trials were classified as min, if they counted only the smaller value integer, or sum, if they counted both integers. Finally, the child's description was compared with the experimenter's initial classification (e.g., verbal counting or retrieval), and there was indicated agreement between the experimenter and subject on 94% of the trials. Disagreements typically occurred for trials on which there was no indication of verbal counting (e.g., no lip movements). For these trials, the experimenter scored the trial as retrieval but the subject described a counting or decomposition process. For those trials on which the experimenter and the subject disagreed, the strategy was classified on the basis of the child's description.

Results

For clarity of presentation, the results, with brief discussion, are presented in four major sections, followed by a more general discussion of the results and their implications. In the first section, analyses of group differences in strategy choices are presented. The second section presents a componential analysis of the RT data designed to assess potential group differences in the rate of executing various arithmetical processes, (e.g., rate of verbal counting). Results for the working memory task are presented in the third section. The final section presents results for the relationship between the experimental tasks and mathematics achievement.

Strategy Choices

Longitudinal analyses. Table 2 presents group-level characteristics of addition strategies for both times of measurement. Inspection of Table 2 reveals a shift across times of measurement in the distribution of problem-solving strategies for the normal group but little change for the math-disabled group. For the normal group, dependent t tests revealed a reliable increase in the use of the retrieval strategy, $t(25) = 2.54$, $p < .05$, and a reliable decrease in the use of the verbal counting strategy, $t(25) = -2.12$, $p < .05$. In contrast, the math-disabled group showed no reliable change across times of measurement in the frequency with which the verbal counting, $t(11) = 0.85$, $p > .10$, or the retrieval, $t(11) = -0.19$, $p > .10$, strategies were used for problem solving. There was no reliable change in the frequency of usage of the counting fingers strategy for either the normal children, $t(25) = -0.56$, $p > .10$, or the math-disabled children, $t(11) = -0.08$, $p > .10$. The frequency with which the sum counting algorithm was used did, however, decrease reliably for the math-disabled children, $t(11) = -3.00$, $p < .05$. Finally, to determine if the math-disabled group was correctly retrieving answers to the same problems, across times of measurement, the frequency of correct retrieval trials (across problems) was correlated for the first and second time of measurement. The magnitude of the resulting coefficient, $r(38) = .78$, $p < .0001$, is consistent with the argument that, as a group, the math-disabled children correctly retrieved facts for the same problems across the times of measurement. Examination of individual protocols indicated that the math-disabled children correctly retrieved, on average, an additional 0.4 problem from the first to the second time of measurement, whereas the normal children correctly retrieved, on average, an additional 5.4 problems from the first to the second time of measurement.

Analyses of change in the frequency of errors across times of measurement indicated a reliable decrease in retrieval errors for the normal group ($p < .05$) but no reliable change in the frequency of counting fingers or verbal counting errors ($ps > .10$). For the math-disabled group, a reliable decrease in the proportion of counting fingers errors was found ($p < .01$), as well as a marginally reliable decrease in the frequency of verbal counting errors ($p < .10$). However, there was no reliable change in the frequency of retrieval errors ($p > .10$).

Summary. Consistent with theories of skill development in arithmetic (Ashcraft, 1982; Ashcraft & Fierman, 1982; Hamann & Ashcraft, 1985; Siegler & Jenkins, 1989), the change in the strategy mix for the normal group across times of measurement reflected increased reliance on memory retrieval, a decrease in associated error rates, and decreased reliance on counting to solve addition problems. The math-disabled group, however, showed no change across times of measurement in their strategy mix, although these children were more proficient counters

Table 2
Characteristics of Addition Strategies for Both Times of Measurement

	Mean % of trials[a]		Mean RT (s)[b]		Mean % of errors		Mean % of counting trials[c]	
Strategy	Normal	Math disabled	Normal	Math disabled	Normal	Math disabled	Normal	Math disabled
Time 1								
Counting fingers	6	7	6.1	6.5	14	49	100	60
Fingers	0	4	—	5.2	—	54	—	—
Verbal counting	56	62	4.2	4.6	9	31	95	86
Retrieval	39	26	2.6	1.9	6	18	—	—
Time 2								
Counting fingers	4	6	4.5	4.9	18	10	100	98
Fingers	0	0	—	—	—	—	—	—
Verbal counting	44	69	3.3	4.3	8	16	99	98
Retrieval	51	25	2.0	1.9	2	16	—	—

Note. Columnar totals may not sum to 100 because of rounding. RT = reaction time.
[a] Mean percentage of trials on which the strategy was used.
[b] Mean RT was based on min trials for the counting fingers and verbal counting strategies and excluded error and spoiled trials for all strategies.
[c] Mean percentage of counting trials on which a min strategy was used.

at the second time of measurement. The improvement in counting skills of the math-disabled children is consistent with their remedial instruction and was reflected in fewer counting errors and a lessened reliance on the sum algorithm.

Second time of measurement. In addition to the just described longitudinal assessment for the normal and math-disabled children, additional analyses were carried out to assess differences between the two groups just at the second time of measurement. (Results for the first time of measurement are described in Geary, 1990.) Inspection of Table 2 reveals that for the second time of measurement, verbal counting and memory retrieval were the primary strategy choices of both groups, although the counting fingers strategy was used occasionally.[2] Univariate F tests revealed reliable differences between groups for the frequency with which both the verbal counting, $F(1, 36) = 6.34$, $p < .05$, and memory retrieval, $F(1, 36) = 5.89$, $p < .05$, strategies were used for problem solving. The frequency with which the counting fingers strategy was used for problem solving, however, did not differ reliably between groups, $F(1, 36) < 1$. In all, the normal children used the memory retrieval strategy more frequently and the verbal counting strategy less frequently than did the math-disabled children.

A second set of F tests revealed that the normal group committed reliably fewer memory retrieval, $F(1, 35) = 22.82$, $p < .01$, and verbal counting, $F(1, 33) = 3.06$, $p < .10$, errors than did the math-disabled group, but the groups did not differ reliably in the frequency of counting fingers errors, $F(1, 7) < 1$.

Counting trials. For both groups, the frequency of counting trials (across the counting fingers and verbal counting strategies) was correlated with the value of the problem's correct sum. The value of the correct sum provides a reliable indicator of the difficulty of the problem (Siegler & Shrager, 1984; Washburne & Vogel, 1928). The frequency of counting errors was correlated with the value of the smaller (i.e., min) integer. Because nearly all counting trials involved the use of the min algorithm, this min value represents the number of incrementations required to solve the problem. These analyses indicated that the frequency of counting trials increased with an increase in the value of the correct sum for both the normal, $r(38) = .40$, $p < .05$, and math-disabled, $r(38) = .66$, $p < .001$, groups. The frequency of counting errors increased with an increase in the min value for both the normal, $r(38) = .32$, $p < .05$, and math-disabled, $r(38) = .51$, $p < .001$, groups. The value of the correlation coefficients did not differ reliably comparing the normal and math-disabled groups ($ps > .10$). The results indicated that the normal and math-disabled subjects tended to use counting to solve more difficult problems and that the probability of committing a counting error increased as the number of required incrementations increased (Siegler & Shrager, 1984).

Retrieval trials. For both groups, the frequency of retrieval trials and the associated error rates were correlated with the value of the problem's correct sum. These analyses mirrored the counting data and indicated that the frequency of retrieval trials decreased with an increase in the value of the correct sum for both the normal, $r(38) = -.48$, $p < .05$, and math-disabled, $r(38) = -.65$, $p < .001$, groups.[3] The value of these two correlation coefficients did not differ reliably ($p > .10$). The frequency of retrieval errors, however, was not correlated with the value of the correct sum for either group ($ps > .10$). In all, this pattern of results combined with the previously described findings indicated that the normal and math-disabled subjects used the retrieval strategy to solve less difficult problems and tended to use a backup counting strategy to solve more difficult problems.

Summary. Unlike the first time of measurement, when the distribution of strategy choices did not differ reliably across groups (Geary, 1990), reliable group differences, favoring the normal children, in the developmental maturity of the strategy mix were evident for the second time of measurement. Here, the normal group used the retrieval strategy more frequently, with fewer retrieval errors, and the verbal counting strategy less frequently than did the math-disabled group, although both the normal and math-disabled children tended to use counting strategies to solve more difficult problems.

Componential Analysis

The componential analyses were designed to determine whether the rate of executing the various problem-solving processes, such as verbal counting, differed across groups and times of measurement. All of these analyses were based on correct trials but excluded trials on which the voice-operated relay was triggered by responses other than the answer (e.g., coughing) or if the initial response did not trigger the relay.[4] Process models for addition were fit to average RT data using regression techniques for both groups. Here, RTs were analyzed separately for verbal counting trials (when the min procedure was used) and retrieval trials. Because not all subjects used the same strategy to solve all problems, the matrix of RTs from which these averages were computed necessarily contained missing data. In this circumstance, the resulting regression equation could be biased in some way. This bias would likely result in some increase in the variability of the regression estimates and attenuate the goodness-of-fit of the overall model. Nevertheless, this procedure seems preferable to the alternative of averaging all RTs across different strategies (Siegler, 1987, 1989).

For both groups, the min variable was used as the independent measure in the analysis of verbal counting trial RTs (Geary, 1990; Siegler, 1987). To be consistent with the analyses for the first time of measurement (Geary, 1990), we used the correct product (prod) of the augend and addend as the independent measure in the analysis of retrieval trial RTs. The product variable can be used to represent the long-term memory network of

[2] The fingers and decomposition strategies were used to solve less than 1% of the problems. These trials were therefore excluded from any of the analyses.

[3] The correlations between the sum variable and (a) the frequency of counting trials and (b) the frequency of retrieval trials are not symmetrical because of missing data. Strategy choice data were missing because either the fingers or decomposition strategies were used on the trial or the voice-operated relay was triggered (e.g., by a cough) before a strategy could be executed. In all, this missing data represented less than 3.5% of the trials.

[4] With the use of the verbal counting strategy, some children audibly counted. During this count, if the voice-operated relay was triggered before the child stated the final answer, then the solution time was spoiled and not used in any of the RT analyses. Less than 2% of the verbal counting trials were spoiled in this manner.

addition facts (Geary, Widaman, & Little, 1986; Miller, Perlmutter, & Keating, 1984). The raw regression weight for the prod variable multiplied by the product of the augend and addend provides an estimate for the rate of retrieving the correct sum from long-term memory. Regression equations for both the verbal counting and the retrieval trial data also included a dummy coded variable, time (coded 0 and 1 for the first and second times of measurement, respectively). The partial F ratio for the time variable tested the intercept difference between times of measurement. The Time × Process Variable (e.g., min) interaction tested the slope difference between the first and second time of measurement for the process variable.

Verbal counting strategy. For the second time of measurement, the min variable showed a strong zero-order correlation with verbal counting trial RTs for both the normal, $r(38) = .94$, $p < .001$, and math-disabled, $r(38) = .92$, $p < .001$, groups. The overall regression equations, displayed in the top half of Table 3, which include the min and time variables along with the interaction, showed a modest to strong level of fit across groups. Here, the intercept terms (e.g., 1,692 for the normal group) theoretically represent a combined estimate for the rate of encoding digits and for the rate of the strategy selection and the answer production processes for the first time of measurement (Ashcraft & Battaglia, 1978; Campbell & Clark, 1988; McCloskey, Caramazza, & Basili, 1985). The regression weight for the time variable represents the change in the intercept term from the first to the second time of measurement. The regression weight for the min variable provides an estimate of the counting rate per incrementation for the first time of measurement, whereas the regression weight for the Time × Min interaction provides an estimate of the change in counting rate from the first to the second time of measurement.

Inspection of the first two equations presented in Table 3 reveals reliable partial F ratios for the min variable for both groups and reliable F ratios for both the time variable and Time × Min interaction for the normal group ($ps < .05$), but nonreliable partial F ratios for the time variable and the Time × Min interaction for the math-disabled group ($ps > .10$). Additional analyses using the same statistical technique revealed that for the second time of measurement the intercept term and the regression estimate for the min variable were reliably lower for the normal group when compared with the math-disabled group ($ps < .05$); recall that slope differences comparing the normal and math-disabled groups for the min variable did not differ reliably for the first time of measurement. In all, the results indicate that across times of measurement the normal group showed a reliable increase in the speed with which the verbal counting strategy could be executed but the math-disabled group showed no reliable change in the speed of executing this strategy. Thus, for the second time of measurement the normal children, in comparison with the math-disabled children, showed a faster verbal counting rate and a shorter duration of the processes subsumed by the intercept term.

Memory retrieval strategy. Consistent with findings for the first time of measurement, the correct retrieval trial RTs for the math-disabled group were largely unsystematic and not reliably correlated with the prod variable. In fact, the retrieval trial RTs for the math-disabled group were not reliably correlated with any of the variables, such as an index of associative strength (Siegler, 1986), commonly used to represent retrieval trial RTs ($ps > .50$). Finally, to ensure that the math-disabled subjects were not counting on these trials, retrieval trial RTs were correlated with the min variable. Unlike verbal counting trial RTs, retrieval trial RTs showed no relationship to the min variable, $r(37) = -.08$, $p > .50$, indicating that the math-disabled subjects were not counting on these trials. Thus, no equation for the retrieval trial data is presented for the math-disabled group.

The regression equation, which included the time and prod variables along with the interaction, for the retrieval trial data for the normal group is the final equation in Table 3. Inspection of this equation reveals a nonreliable estimate for the time variable ($p > .10$) but reliable estimates for the prod variable and the Time × Prod interaction ($ps < .05$). The reliable prod variable indicates that time required to retrieve a fact from long-

Table 3
Statistical Summaries of Regression Analyses

Equation	Partial Fs	R^2	F	df	RMSe
Verbal counting strategy					
Normal					
RT = 1692 − 378 (Time)* + 647 (Min)* − 154 (Time × Min)*	4.02, 429.48, 12.16	.918	284.78	3, 76	332
Math disabled					
RT = 2592 − 662 (Time) + 505 (Min)* + 96 (Time × Min)	2.21, 46.92, 0.85	.604	38.68	3, 76	782
Retrieval strategy					
Normal					
RT = 1869 − 203 (Time) + 25 (Prod)* − 14 (Time × Prod)*	1.10, 37.45, 6.04	.532	28.82	3, 76	426

Note. Min = the cardinal value of the smaller integer; Prod = the product of the augend and the addend; RMSe = regression analysis mean square error. Time was coded 0 for the first time of measurement and 1 for the second time of measurement.
* $p < .05$.

term memory increases with increases in the value of augend and addend (see Ashcraft, 1990). The reliable Time × Prod interaction indicates that across times of measurement the normal children showed a faster rate of retrieving addition facts from long-term memory, but the decrease in the duration of the processes subsumed by the intercept term was not reliable.

Working Memory Task

Results for performance on the Digit Span subtest are displayed in Table 4. The rows termed *Forward* and *Backward* give, for the normal and math-disabled groups, the mean scores (i.e., the number of digit strings, or items, recalled correctly) for the forward and backward sections, respectively, of the Digit Span measure. The rows termed *High forward* and *High backward* give the means for the highest number of digits (on a single trial) correctly recalled on the forward and backward sections, respectively, of the Digit Span subtest. Inspection of these results indicates a reliable difference, favoring the normal group, on all measures ($ps < .05$). Except for one math-disabled subject, all children reported rehearsing the digits in order to help them to remember the digit string. These results are consistent with the hypothesis that the math-disabled children have relatively poor working memory resources (Geary, 1990) and that the group difference in working memory capacity does not appear to be related to different strategy approaches to the digit span task.

Memory Span, Strategy Choices, and Mathematical Achievement

The final set of analyses were conducted in order to determine the relationships among strategy choices (for the second time of measurement) and performance on the memory span and mathematical achievement measures. For these analyses, the high forward measure from the Digit Span subtest was used as the index of working memory resources. This measure was used because it provides an easily interpretable index of the maximum quantity of information that can be retained in working memory (Case, 1985). These analyses excluded data from one normal child because of missing achievement test scores. In the first set of analyses, the memory span measure was correlated with strategy characteristics (i.e., the frequency, and the associated error rates, with which each of the strategies was used to solve the presented problems). The resulting matrix of correlations produced a single reliable result; the memory span measure was related to the frequency of verbal counting errors, $r(35) = -.35$, $p < .05$.

In the next set of analyses, performance on the mathematics achievement measure was correlated with strategy characteristics and the memory span index. The resulting matrix produced three reliable relationships. Performance on the mathematics test was linearly related to both the memory span index, $r(35) = .50$, $p < .001$, and to the frequency of verbal counting errors, $r(35) = -.57$, $p < .001$. Finally, a quadratic relationship between the frequency of correct memory retrieval trials and performance on the mathematics test was found, $R = .47$, $F(2, 34) = 4.84$, $p < .05$. Here, children who had performed poorly on the mathematics test (i.e., the math-disabled children) had relatively few correct retrieval trials. Children who had intermediate mathematics scores (averaging at about the 60th percentile) tended to have the highest frequency of correct retrieval trials, whereas children who had high mathematics scores (greater than the 80th percentile) tended to have an intermediate number of correct retrieval trials. Additional analyses indicated that this quadratic function was due to the fact that the higher mathematical ability children often reverted to a backup counting strategy to solve the presented problems; that is, the intermediate number of correct retrieval trials for these high-achieving children was not due to a high frequency of retrieval errors (Siegler, 1988).

In the final set of analyses, the memory span and strategy choice variables (i.e., the frequency of verbal counting errors and correct retrieval trials) were used as potential predictors of performance on the mathematics test, which, as previously mentioned, was administered about 1 month after the experimental session. The resulting regression equation revealed that the strategy choice (i.e., the frequency of verbal counting errors and correct retrieval trials) and memory span indices were moderately predictive of later mathematical achievement, $R^2 = .53$, $F(4, 32) = 9.16$, $p < .0001$. Each of these variables—memory span, verbal counting errors, and correct retrieval trials (fitting the quadratic relationship)—independently contributed to the prediction of later mathematical achievement test scores. Finally, after partialing scores for these three variables, the difference between normal and the math-disabled group on the mathematics measure was no longer reliable, $F(1, 31) < 1$. This same equation, however, was also moderately predictive of reading achievement test scores, $R^2 = .45$, $F(4, 32) = 6.46$, $p < .001$, although the partial F ratio for the verbal counting errors variable was not reliable, $F(1, 32) = 1.41$, $p > .10$.

Table 4
Descriptive Information for Performance on the Digit Span Test

	Normal		Math disabled		
Measure	*M*	*SD*	*M*	*SD*	*F*
Forward	5.3	1.5	4.2	1.2	5.08*
High forward	5.2	1.1	4.3	0.7	6.67*
Backward	4.1	1.1	3.0	1.3	7.16*
High backward	3.5	0.8	2.8	1.1	5.98*

Note. Forward and backward refer to the mean number of items correctly recalled on the corresponding forward and backward sections, respectively, of the Digit Span test. High forward and high backward refer to the mean for the highest number of digits correctly recalled on the forward and backward sections, respectively, of the Digit Span test.
* $p < .05$.

Discussion

This study provided a theory-based longitudinal assessment of skill development in addition for groups of normal and math-disabled children. Across times of measurement the normal children showed an increased reliance on memory retrieval, with fewer retrieval errors, and a decreased reliance on counting to solve addition problems. This change in the distribution of strategy choices and in the frequency of memory retrieval errors is consistent with conceptual models for the maturational development of basic arithmetic skills (Ashcraft, 1982; Goldman et al., 1988; Siegler, 1986). In contrast to the normal children, the math-disabled children did not show a developmental shift in the mix of problem-solving strategies. For both times of measurement, the math-disabled children relied on counting to solve the majority of addition problems, and when the memory retrieval strategy was used, it often resulted in an error.

Within the context of the strategy choice model, developmental change in the mix of problem-solving strategies for addition and in the proportion of retrieval errors should reflect changes in the distributions of associations and confidence criterion parameters (Siegler, 1986, 1988). Accordingly, for the normal children, the developmental change in the frequency with which memory retrieval was used for problem solving and the associated decrease in error rate can be parsimoniously explained by an increase, across times of measurement, in the peakedness of the distributions of associations parameter. For the normal group, the retrieval trial and retrieval error data, in themselves, do not allow for strong inferences to be made about the rigor of the confidence criterion, because relatively peaked distributions of associations could result in both more retrieval trials and fewer retrieval errors (Siegler, 1986, 1988).

Nevertheless, the finding of a quadratic relationship between the frequency of correct retrieval trials and performance on the mathematics achievement test suggests a rather rigorous confidence criterion for the higher achieving normal children. This result mirrors a finding by Siegler (1988), who demonstrated that children he termed "perfectionists" were just as knowledgeable in mathematics as peers who were described as "good" students but used counting to solve simple arithmetic problems more frequently than the good students and a group of "not-so-good" students. When the perfectionists did state a retrieved answer, it was almost always the correct answer. Siegler argued that this pattern of individual differences could be explained if the knowledge base for the perfectionists and the good students was similar and if the perfectionists had a much more stringent confidence criterion. Thus, the perfectionists stated a retrieved answer only if they were absolutely certain that this retrieved answer was correct; otherwise they reverted to a backup strategy. In this study, the quadratic relationship between the frequency of correct retrieval trials and mathematics achievement could be explained by a rigorous confidence criterion for the higher ability children (Geary & Brown, 1991). For these children, backup counting strategies were more likely used to verify the accuracy of retrieved, but not stated, answers.

In all, the pattern of change across times of measurement for the normal group suggests both an increase in the peakedness of the distributions of associations parameter and a somewhat more stringent confidence criterion. Moreover, for the normal group the shift in the mix of problem-solving strategies was accompanied by a developmental change in the rate of executing two substantive addition processes; across times of measurement these children were reliably faster for the rate of verbal counting and for the rate of retrieving addition facts directly from long-term memory. For the math-disabled group, however, the finding of no reliable change in either the distribution of strategy choices or the proportion of retrieval errors suggests little if any developmental change in the peakedness of the distributions of associations or the confidence criterion parameters. Moreover, these children showed no reliable improvement in the rate of executing either the verbal counting or the memory retrieval strategies. Nevertheless, the math-disabled children were more proficient counters for the second time of measurement, as reflected in fewer counting errors and a reduced reliance on the sum algorithm. This improvement in computational skills is consistent with their remedial instruction and might have contributed to the improvement in achievement test scores across times of measurement.

Overall, the pattern of change across times of measurement, combined with the group difference, in the distribution of strategy choices suggests that one factor contributing to mathematical achievement resides in the development of the long-term memory organization of basic facts and that a mathematical learning disability is related in part to a delayed or perhaps anomalous development of the long-term memory organization of these facts (Geary, 1990; Geary & Brown, 1991; Goldman et al., 1988, 1989). For the math-disabled children, the failure to develop an adequate long-term memory representation of basic facts did not appear to be due to inadequate experience, because the distribution of associations between a problem and its correct answer appears to develop automatically, at least with normal children, with the use of counting algorithms (Siegler, 1986). The math-disabled children for the second time of measurement used these counting strategies more frequently than did the normal children, but the execution of these strategies apparently did not produce strong changes in the associative strength between the problem and its correct answer.

Moreover, the finding of reliable group differences, favoring the normal children, on the Digit Span subtest suggests that relatively poor working memory resources is a second factor contributing to a mathematical disability (Geary, 1990). In fact, for the math-disabled children, relatively poor working memory resources may have contributed to the apparent failure to develop adequate long-term memory representations of basic facts. More precisely, with the execution of a counting algorithm, if the original representation of the problem's integers decays quickly in working memory, then the answer generated by this count and the original representation of the problem would not become associated (R. S. Siegler, personal communication, May 26, 1990). In this circumstance, the distribution of associations between the problem and its correct answer would remain flat. In other words, associations between the problem and the answer would not develop in long-term memory. The failure to develop associative relationships in long-term memory would most likely occur for problems with large-value digits, because the duration of the counting process would be longer for these problems than for problems with smaller value digits. The longer the counting process, the greater the probability the requisite information will decay in working memory (Baddeley, 1986). Consistent with this view was the finding that the frequency of retrieval trials decreased as the value of the correct sum increased.[5]

[5] The argument for the relationship between working memory and the development of arithmetic skill, as pointed out by an anonymous

Finally, the pattern of relations between strategy choices, memory span, and achievement in mathematics suggests that the development of mathematical skills requires (a) the acquisition of an adequate knowledge base, (b) the development of computational abilities (e.g., the error-free execution of counting algorithms), and (c) adequate working memory resources. The finding that the frequency with which addition facts were retrieved from long-term memory was also predictive of achievement in reading does not necessarily indicate that knowledge of arithmetic facts contributes to reading skills. It is more likely the case that a failure to represent addition facts in long-term memory parallels phonological processing deficits associated with some forms of reading disorders. This argument is based on findings that indicate that addition facts are more likely represented in a semantic network and that deficits in arithmetic fact retrieval also accompany certain forms of dyslexia and aphasia (Boller & Grafman, 1983; Richman, 1983). The finding that the frequency of verbal counting errors was not predictive of later reading skills is consistent with this argument. In other words, computational skill in arithmetic would appear to be domain specific, whereas the apparent long-term memory deficit more likely extends to other semantic networks and would therefore influence not only fact retrieval in arithmetic but also language-related skills such as reading (Luria, 1980).

In conclusion, this study suggests that a primary factor contributing to an early learning problem in mathematics is difficulty in the retrieval of basic information from long-term memory (Geary et al., 1987; Goldman et al., 1988; Svenson & Broquist, 1975). The failure to readily retrieve basic facts from long-term memory appears to underlie the frequent use of counting strategies by math-disabled children to solve addition problems. Although we have argued that the failure to retrieve facts from long-term memory is likely due to the failure to represent these facts, further studies are needed to determine if this retrieval deficit involves a failure to represent basic information in long-term memory or a difficulty in the accessing of this information. Finally, math-disabled children also appear to have relatively poor working memory resources (Richman, 1983), which may lead to frequent computational (i.e., verbal counting) errors. Further studies will also be needed to determine if relatively poor working memory resources contribute to the apparent failure to develop associative connections in long-term memory between an arithmetic problem and its correct answer.

reviewer, would be bolstered if the performance of the improved group on the memory span measure exceeded the performance of the no-change group at the first measurement. Unfortunately, an index of working memory resources was not obtained at the first measurement. Nevertheless, we did examine group differences for the second measurement. Here, the improved group showed better performance on both the high forward ($M = 4.8$ and 4.3 for the improved and no-change groups, respectively) and high backward ($M = 3.4$ and 2.8 for the improved and no-change groups, respectively) measures. Because of the small n, however, these differences were only marginally reliable ($p <$ .08 and $p < .09$, one-tailed, for the forward and backward measures, respectively). Regardless, the hypothesis that working memory is a core deficit underlying a math disability merits serious consideration in future studies.

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