Simple and Complex Mental Subtraction: Strategy Choice and Speed-of-Processing Differences in Younger and Older Adults

David C. Geary, Peter A. Frensch, and Judith G. Wiley

Thirty-six younger adults (10 male, 26 female; ages 18 to 38 years) and 36 older adults (14 male, 22 female; ages 61 to 80 years) completed simple and complex paper-and-pencil subtraction tests and solved a series of simple and complex computer-presented subtraction problems. For the computer task, strategies and solution times were recorded on a trial-by-trial basis. Older Ss used a developmentally more mature mix of problem-solving strategies to solve both simple and complex subtraction problems. Analyses of component scores derived from the solution times suggest that the older Ss are slower at number encoding and number production but faster at executing the borrow procedure. In contrast, groups did not appear to differ in the speed of subtraction fact retrieval. Results from a computational simulation are consistent with the interpretation that older adults' advantage for strategy choices and for the speed of executing the borrow procedure might result from more practice solving subtraction problems.

Relatively little is known about the basic numerical skills of older individuals, despite the importance of these skills in many real-world settings. The use of tasks and procedures developed for the study of arithmetic development in children appears to provide a useful tool for studying age-related changes in basic cognitive functions, as well as to provide information about the arithmetic skills of older adults (Allen, Ashcraft, & Weber, 1992; Charness & Campbell, 1988; Geary & Wiley, 1991). In particular, procedures used in the mental arithmetic area enable assessments of age-related changes during adulthood in problem-solving strategies and in the speed of information processing (see Ashcraft, 1992). The utility of these procedures has been demonstrated in several recent studies that showed that older adults, relative to college students, appear to use a developmentally more mature mix of strategies to solve simple addition problems (Geary & Wiley, 1991). Moreover, although the younger adults showed an overall advantage for solution times, component analyses suggested no age difference in the speed of retrieving addition and multiplication facts from long-term memory (Allen et al., 1992; Geary & Wiley, 1991).

In this study, younger and older adults were compared on the mix of strategies and the associated solution times used to solve simple and complex subtraction problems. This comparison provided information on the arithmetic skills of older individuals and allowed for an empirical test of several important models of age-related change in cognitive function. First, the study allowed for an assessment of age-related differences in problem-solving strategies for both simple and complex tasks (Charness, 1981; Hartley & Anderson, 1983; Salthouse, 1984). By using both simple and complex subtraction problems, we could determine if increased task complexity leads to different strategy shifts in younger and older adults (Hartley & Anderson, 1983). Second, because solution times were garnered for both simple and complex subtraction, the study tested the hypothesis that age differences in the speed of information processing, typically favoring young adults, increase with increasing task complexity (Cerella, 1985, 1990; Myerson, Hale, Wagstaff, Poon, & Smith, 1990). Finally, the solution of complex subtraction problems often requires the execution of a borrow procedure. The execution of this procedure likely increases the working memory demands of the task (Hitch, 1978). Thus, the complex subtraction task also provided a means to examine the relationship between increased working memory demands and strategic and speed-of-processing differences between younger and older adults (Salthouse, 1991, 1992). A more detailed presentation of these goals follows an overview of strategy development in mental subtraction.

Strategy Choices in Mental Subtraction

For many cognitive tasks, people use a mix of strategies, rather than a single strategy, for problem solving. Strategy choices appear to be adaptive, that is, people tend to use the strategy that is most efficient, in terms of speed and accuracy, for solving each individual problem (Geary & Burlingham-Dubre, 1989; Siegler, 1989). Siegler's (1986) strategy-choice model, developed to explain skill development in children, describes two mechanisms, namely associative strength and a confidence criterion, that appear to govern adaptive strategy...
choices. Associative strength represents the facility with which task relevant facts or procedures can be retrieved from long-term memory (Siegel & Jenkins, 1989). The confidence criterion represents an internal standard against which confidence in the accuracy or utility of the retrieved information is gauged (Siegel, 1988). If task relevant information or procedures can be retrieved from long-term memory and if this information is judged to be accurate or useful for the task, then it is used in problem solving. If task relevant information cannot be retrieved from long-term memory, or if the retrieved information is judged to be of low quality (e.g., the retrieved answer is likely to be incorrect), then a backup strategy is used to complete problem solving. The use of backup strategies, in turn, appears to lead to the development of task-relevant memory representations. Thus, with repeated use, backup strategies should eventually be usurped by memory retrieval processes for problem solving (see Siegel, 1986, for details).

For the domain of simple subtraction (e.g., 9 - 4), backup strategies typically involve counting or reference to a related addition problem (Carpenter & Moser, 1984; Siegel, 1989; Suppes, Hynan, & Jerman, 1967; Svenson & Hedenborg, 1979; Woods, Resnick, & Groen, 1975). Counting can be used in two ways, "counting down" or "counting up." For instance, to solve the problem 8 - 2, counting down would involve the sequence, "eight, seven, six." When the difference between the minuend (the first number) and subtrahend (second number) is small, then the counting up procedure is more typically used; for example, counting up one time to solve the problem 9 - 8. The "addition reference" strategy involves, for instance, referring to the problem 4 + 5 = 9, to solve the problem 9 - 4. For the solution of somewhat more complex problems, such as 15 - 3, children sometimes resort to the use of a "delete 10s rule" (Siegel, 1989). The use of this rule involves "a type of decomposition in which children [treat] the 10s value separately from the 1s value. For example, on 15 - 3, they might explain their answer by saying, '5 - 3 = 2, and you put back the 1, so 12'" (Siegel, 1989, p. 500). A summary of the most commonly used strategies for simple and complex subtraction is presented in Table 1.

Little information is available on the strategies used to solve subtraction problems with minuends > 20. Nevertheless, it seems reasonable to expect that the same types of strategies would be used, that is, retrieval, counting, addition reference, and the deleting 10s rule. Moreover, on the basis of the finding that adults solve complex addition and multiplication problems in a columnwise fashion, it is likely that retrieval in complex subtraction would also involve processing the units-column minuend and subtrahend first and then the tens-column information (Geary, Widaman, & Little, 1986). For instance, to solve the problem 53 - 8, after executing the borrow procedure, the answer to 13 - 8 would be retrieved from long-term memory. The resulting units-column difference (i.e., 5) would then be combined with the decremented tens-column value to produce the answer of 45. Finally, on the basis of the finding that adults often solve simple addition problems by using a decomposition strategy, it is possible that complex subtraction problems might also be solved by means of "decomposition." For instance, to solve the problem 75 - 8, the subtrahend could be decomposed into 5 and 3, followed by the operation 75 - 5 = 70, and finally 70 - 3 = 67.

### Table 1

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
<th>Example</th>
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<tbody>
<tr>
<td>Addition reference</td>
<td>Reference to related addition problem to help retrieve subtraction fact</td>
<td>2 + 5 = 7, so 7 - 2 = 5</td>
</tr>
<tr>
<td>Count down</td>
<td>Counting down from the minuend a number of times equal to the value of the subtrahend</td>
<td>Counting implicitly &quot;seven, six, five&quot; to solve 7 - 2</td>
</tr>
<tr>
<td>Retrieval</td>
<td>Direct retrieval of basic facts from long-term memory</td>
<td>Retriving &quot;5&quot; to solve 7 - 2</td>
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Complex subtraction

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count down</td>
<td>Same as above</td>
<td>Counting implicitly &quot;thirty-two,&quot; &quot;thirty-one,&quot; &quot;thirty,&quot; &quot;twenty-nine&quot; to solve 32 - 3</td>
</tr>
</tbody>
</table>
| Decomposition  | Breaking the subtrahend into two smaller numbers and then successively subtracting these from the minuend | To solve 32 - 3:  
Step 1: 32 - 2 = 30;  
Step 2: 30 - 1 = 29 |
| Rule           | Increasing the value of the subtrahend to 10, then subtracting 10 from the minuend, and then adding the difference between 10 and the subtrahend to the provisional answer | To solve 32 - 9:  
Step 1: 32 - 10 = 22;  
Step 3: 22 - 9 = 13;  
Step 4: 13 + 1 = 23 |
| Columnar retrieval | Retrieving basic facts from long-term memory in a column-wise fashion | To solve 32 - 9:  
Step 1: 32 - 10 = 22 (borrow);  
Step 2: 10 + 2 = 13;  
Step 4: 13 - 9 = 4;  
Step 5: 4 + 3 = 7 |

For children, the development of subtraction skills involves a gradual shift in the mix of problem-solving strategies, in particular, an increase in the use of retrieval processes and a decrease in the use of backup strategies for problem solving (Carpenter & Moser, 1984; Siegel, 1989). To the best of our knowledge, there have been no adulthood studies of strategy development in subtraction. Studies of mental addition and multiplication that have compared younger and older adults for solution times and strategy choices, however, indicate two important trends (Allen et al., 1992; Birren & Botwinick, 1951; Charness & Campbell,
1988; Geary & Wiley, 1991). The first is that the speed of fact retrieval appears to asymptote during the high-school years and does not appear to change through old age, at least for healthy adults (Allen et al., 1992; Geary & Wiley, 1991; Kail, 1988). The second is that older adults use a more mature mix of problem-solving strategies than younger adults to solve simple addition problems, as indexed by a greater proportion of direct retrieval (Geary & Wiley, 1991). This last result is consistent with Schaie's (1983, 1989) finding of considerable cohort differences for basic arithmetic skills: the later the cohort, the poorer the basic skills.

Cognitive Aging

The results of the present study bear on three issues in cognitive aging research. The first issue concerns age-related differences in the use of problem-solving strategies (Charness, 1981). The second issue addresses age-related differences in speed of processing (e.g., Cerella, 1990; Myerson et al., 1990), and the final issue concerns age-related change in working memory resources, as related to speed-of-processing and strategic differences in younger and older adults (Salthouse & Babcock, 1991). We briefly review each of these issues in turn.

Problem-Solving Strategies

Age-related differences in problem-solving approaches appear to vary with the familiarity of the task. For novel tasks, young adults often show more efficient problem solving than older adults (e.g., Hartley & Anderson, 1983). For more familiar tasks, however, older adults are sometimes more efficient problem solvers than younger adults (e.g., Charness, 1981; Salthouse, 1984). In fact, more efficient problem solving can compensate for age-related slowing, resulting in equivalent molar levels of performance for younger and older adults (Charness, 1987). We have argued that the earlier described strategic differences in mental addition reflect cohort differences in early mathematics education, rather than age-related changes per se (Geary & Wiley, 1991). In this study, we examine age-related differences in arithmetic strategies for simple and complex subtraction.

Speed of Processing

A consistent finding in cognitive aging research is that older adults tend to process information more slowly than younger adults (cf. Cerella, 1985, 1990; Myerson et al., 1990). To explain this finding, it has been argued that aging is associated with general changes in the central nervous system, such as degeneration of axonal sheaths, that lead to a general decline in processing speed (e.g., Cerella, 1990; Myerson et al., 1990). Myerson et al. have argued further that the locus of this generalized slowing is best conceptualized at the level of a neural network. In their model, it is assumed that processing units are neural ensembles, that processing occurs in discrete stages from one ensemble to the next, and that the processing time at each step is inversely related to the amount of information available at that step. It is further assumed that some information is lost as information is transferred from one step to the next. Thus, processing necessarily slows with each successive stage. Finally, it has been argued that aging results in a loss in the amount of information that each neural ensemble can capture at each step. This final argument leads to two predictions. The first is that older adults will process information more slowly than younger adults in all domains. Second, as the number of steps required to solve a task increases, overall solution time differences between younger and older adults should also increase. In other words, for timed tasks, an Age × Complexity interaction should be found, with larger differences between younger and older adults for complex than for simple tasks. The use of simple and complex subtraction problems in the current study enabled us to test this prediction with both psychometric and experimental tasks.

Other explanations for age-related change in speed of processing include disuse (i.e., lack of practice), speed-accuracy trade-offs, and strategy shifts. The finding that age differences in speed of performing simple cognitive tasks persist even after extensive practice argues against the disuse hypothesis (Salthouse & Somberg, 1982). Similarly, Cerella (1990) has argued that speed-accuracy trade-offs are not a likely explanation for age-related declines in speed of processing because accuracy rates tend to be similar across younger and older adults on most experimental tasks. Nevertheless, a group difference in speed-accuracy trade-offs for the subtraction tasks should be reflected in group differences in the proportion of retrieval errors (see Geary & Wiley, 1991; Siegler, 1988). Although it has been argued that the age-related change in speed of processing does not appear to be due to age-related change in strategic approaches to experimental tasks (e.g., Cerella, 1990), this assumption has not been rigorously tested. That is, trial-by-trial assessments of problem-solving strategies have not typically been conducted in cognitive aging research. As with our study of cognitive addition (Geary & Wiley, 1991), the procedures used in the present research enable a trial-by-trial assessment of strategy usage and thus a direct test of the hypothesis that younger and older adults tend to use the same strategies on experimental tasks.

Regardless of the source of difference, there appear to be two exceptions to the finding that younger and older adults differ in the speed of processing.Younger and older adults do not appear to differ in the speed of accessing the meaning of single words from long-term memory (Cerella & Fozard, 1984) nor do they appear to differ in the speed of retrieving addition and multiplication facts from long-term memory (Allen et al., 1992; Geary & Wiley, 1991). Geary (in press) has recently argued that word meanings and arithmetic facts might be represented in the same semantic memory system and might be supported by the same or adjacent neural systems. If this argument is correct, then the results of these previous studies converge on the conclusion that the speed of retrieving single pieces of information from semantic memory is not substantially affected by the aging process. We test this hypothesis further in this study by comparing younger and older adults on the speed of retrieving subtraction facts from long-term memory.

Working Memory

Salthouse has demonstrated that there are age-related decrements in working memory resources that negatively affect performance on many cognitive measures (e.g., Salthouse, 1992;
Mental Subtraction

Salthouse & Babcock, 1991). He has argued further that changes in working memory resources are caused, at least to some extent, by age-related changes in the speed of processing (Salthouse, 1991). Although the design of the current experiment does not enable a test of the hypothesis that speed of processing changes in adulthood contribute to age-related decrements in working memory, the experimental procedure does allow us to assess how working memory might influence the performance of younger and older adults during the solution of complex subtraction problems. More specifically, one half of the complex subtraction problems used in this study required a borrow operation (e.g., 64 – 9). Borrowing in complex subtraction represents a within-context working memory manipulation (cf. Salthouse, 1992). The borrow procedure, to solve 64 – 9 for instance, requires the subject to first decrement by 1 the tens-column minuend (i.e., 6) and retain this decremented value (i.e., 5) in working memory while performing the units-column operations. The units-column operations involve encoding the integers, adding ten to the columnar minuend (i.e., 4), and then obtaining the units-column difference (i.e., 14 – 9 = 5; VanLehn, 1990).

Borrow errors that involve the tens-column information might occur because information is lost during the processing of the units-column information. Units-column errors, in contrast, might be due to the fact that retaining the value of the decremented digit in working memory interferes with the encoding of the units-column information or disrupts the columnar retrieval process. Thus, the borrow/no borrow manipulation in this study enabled an assessment of the impact that increased working memory demands might have on the subtraction skills of both younger and older adults.

This Study

In all, this study provided information on the strategies, and the speed of executing the underlying processes, that younger and older adults use to solve simple and complex subtraction problems. The study also provided potentially useful information to cognitive aging researchers. In particular, the study enabled an assessment of age-related differences in strategy approaches for solving simple and complex problems, as well as a comparison of younger and older adults on the speed of executing two important arithmetical operations, namely, fact retrieval and borrowing. Finally, the procedures enabled us to examine the relation between working memory demands and the subtraction skills of younger and older adults.

Method

Subjects

The subjects included 36 younger adults (10 male, 26 female) and 36 older adults (14 male, 22 female). The younger adults included 35 undergraduate students who received course credit for participating in the experiment and one graduate student. The older adults were recruited from the Columbia, Missouri area from a list of University faculty and staff and their spouses or were recruited from the local community. Before participating in the experimental tasks, all subjects responded to several survey questions and were administered the second half of the Vocabulary subtest of the Wechsler Adult Intelligence Scale—Revised (WAIS–R; Wechsler, 1981). The survey questions concerned the subjects’ age, self-reported health status (this was recorded using a 1 [excellent] to 5 [bad] Likert-type scale), and years of education; the associated descriptive information is displayed in Table 2. The age range for the younger subjects was 18 to 38 years and 61 to 80 years for the older subjects. The mean rating for health status ranged between good and excellent for both the younger and older samples and did not differ reliably across groups, $F(1, 70) < 1$, $M_S = 0.39$. Finally, the advantage of the older adults in years of education was reliable, $F(1, 70) = 19.2$, $M_S = 5.61$, $p < .0001$, as was their advantage on the second half of the Vocabulary subtest of the WAIS–R, $F(1, 70) = 23.81$, $M_S = 54.62$, $p < .0001$.

In terms of educational level, 2 of the elderly subjects had not completed high school, 2 were high-school graduates, 13 reported some college education but no degree, 2 were college graduates but reported no graduate education, 17 reported some graduate education, and 14 of these 17 had received an advanced degree. For the sample as a whole, years of education was not correlated with performance on either the simple subtraction test, $r(70) = .10$, $p > .25$, or the complex subtraction test, $r(70) = .11$, $p > .25$. Thus, the positive bias in years of education did not appear to substantially affect the results of this study.

Ability Tests

All subjects completed a paper-and-pencil simple subtraction test and a paper-and-pencil complex subtraction test. Both tests consisted of two parallel forms. The instructions for each form requested that the subject correctly solve as many problems as possible in 1 min. The score for each test was the number of problems solved correctly across both forms.

Simple subtraction. Each form of the simple subtraction test included 72 problems. The 72 problems consisted of the 36 pairs of minuends and subtrahends that are defined by the pairwise combinations of the integers 1 to 9 that produce a positive difference (i.e., minuend > subtrahend). Each of the resulting problems was presented twice, once within the first group of 36 problems and once within the final group of 36 problems. Within each group of 36 problems, the order of problem presentation was randomly determined for the first form. The order of problem presentation for the second form was the reverse of the presentation order for the first form.

Complex subtraction. Each form of the complex subtraction test included 56 problems. Each problem consisted of a double-digit minuend and a single-digit subtrahend (e.g., 87 – 9), excluding the integers 0 and 1. For each form, 28 of the 56 problems required a borrow operation. For the first form, the order of problem presentation was ran-

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<th>Table 2</th>
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<tr>
<td><strong>Descriptive Information for Subject Characteristics</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>Age</td>
</tr>
<tr>
<td>Health status</td>
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<tr>
<td>Years of education</td>
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<tr>
<td>WAIS-R Vocabulary</td>
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<tr>
<td>Gender (% female)</td>
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*Note. Health status was rated on a 1 [excellent] to 5 [bad] Likert-type scale, the Wechsler Adult Intelligence Scale—Revised (WAIS–R; Wechsler, 1981) Vocabulary score is based on the second half of the test.*
domly determined. The presentation order for the second form was the reverse of the presentation order for the first form.

**Experimental Tasks**

**Simple subtraction.** The simple subtraction stimuli were the same as those described for the simple subtraction ability test, although the order of problem presentation differed. Across problems, each integer, 1 to 9, appeared eight times across positions. The order of problem presentation was determined randomly with the constraint that no integer was repeated in the same position across consecutive trials.

**Complex subtraction.** The complex subtraction stimuli were the same as those used for the complex subtraction ability test, although the order of problem presentation differed. Across problems, each of the integers, 2 to 9, appeared seven times in each of the three positions. Across problems, all mnemonics were unique. The order of problem presentation was randomly determined, with the constraint that no integer was repeated in the same position across consecutive trials.

**Apparatus.** The subtraction problems were presented on a vertical display screen located at the center of a 30 × 30 cm video screen controlled by an IBM PC-XT microcomputer. A Cognitive Testing Station clocking mechanism ensured the collection of RTs with an accuracy of about ±1 ms. The timing mechanism was initiated with the presentation of the problem on the video screen and was terminated with a Gerbrands G1341T voice-operated relay. The voice-operated relay was triggered when the subject spoke the answer into a microphone connected to the relay.

For each problem, a READY prompt appeared at the center of the video screen for 1,000 ms, followed by a blank screen for 1,000 ms. Then a subtraction problem appeared on the screen and remained until the subject responded. The experimenter initiated each problem presentation sequence with a control key.

**Procedure**

Each subject was tested individually and in a quiet room. Subjects were first administered the ability measures, followed by the experimental tasks. For both the ability measures and the experimental tasks, the simple problems were administered first. For the experimental tasks, the problems were presented one at a time on the video screen. The instructions encouraged subjects to use whatever strategy made it easiest for them to obtain the answer. Equal emphasis was placed on speed and accuracy of responding. For each trial, the answer was recorded by the experimenter. Subjects were then asked to describe how they arrived at the answer and, based on their description, the trial was classified by the experimenter as one of the earlier described strategies. Several previous studies have demonstrated that children's descriptions of problem-solving strategies in arithmetic are consistent with the classifications of independent observers and with the associated reaction times (RTs) if the children are asked to describe the strategy immediately after the problem is solved (Siegel, 1987, 1989). Independent classifications are possible when children solve problems, because overt signs (e.g., finger counting or lip movements) of strategy usage are typically evident.

For adults, however, strategies cannot be easily classified by an independent observer because overt indicators of strategy usage are not always displayed. Thus, the verbal reports of the subjects were the only measure used to classify the strategies and should be interpreted with caution (Hamann & Ashcraft, 1985). Nevertheless, Geary and Wiley (1991) showed that the descriptions of the strategies used to solve addition problems were consistent with the associated RT patterns for groups of younger and older adults. Moreover, in the current study, strategy descriptions were consistent with the descriptions of children for the same strategy. For instance, for retrieval trials, subjects typically reported "remembering" or "just knowing" the answer. For the addition reference strategy, one subject reported referring to the problem 3 + 6 = 9, to solve the problem 9 − 3. For another subject, to solve the problem 8 − 2, the counting down process involved implicitly counting "eight, seven, six." For complex subtraction, the most common rule involved deleting 10s (Siegel, 1989). For example, to solve the problem 42 − 9, one subject performed the following operations: 42 − 10 = 32; 32 − 9 = 1; 32 + 1 = 33. Decomposition, for complex problems, typically involved decomposing the subtrahend into two smaller numbers, and then successively subtracting these from the minuend. For instance, to solve the problem 96 − 8, one subject performed the following operations: 96 − 6 = 90; 90 − 2 = 88. Finally, as described in the Results, solution times differed across strategies, which also appears to support the validity of the subjects' descriptions.

**Results**

The results are presented in five major sections. In the first section, results for performance on the ability tests are presented. Group differences in strategy choices for the simple and complex subtraction tasks are described in the second section, and analyses of component scores derived from the solution times from these tasks are presented in the third section. Individual differences analyses, relating component scores to performance on the ability measures, are described in section four. The final section presents a computational simulation of the solution of complex subtraction problems. The simulation provided a feasibility check for our interpretation of age differences in solution time patterns, described in section three, for complex subtraction.

**Ability Tests**

For the simple subtraction test, the mean number of problems solved correctly was 120 (SD = 22) and 101 (SD = 23) for the younger and older groups, respectively; the respective means for the complex subtraction test were 38 (SD = 12) and 34 (SD = 10). A mixed design analysis of variance (ANOVA), with group (younger vs. older) as a between-subjects factor and complexity (simple vs. complex) as a within-subjects factor, was used to analyze the ability test scores. The results indicated reliable main effects for group, $F(1, 70) = 11.18, M_S = 464.84, p < .005$, and complexity, $F(1, 70) = 1,318.50, M_S = 150.58, p < .0001$, as well as a reliable Group × Complexity interaction, $F(1, 70) = 15.04, M_S = 150.58, p < .001$. One-way ANOVAs revealed that the advantage of the young subjects for performance on the simple subtraction test was reliable, $F(1, 70) = 14.41, M_S = 496.71, p < .001$, but was not reliable for the complex subtraction test, $F(1, 70) = 2.53, M_S = 118.71, p > .10$. The pattern of a reliable group difference for the simple test and a nonreliable group difference for the complex test is inconsistent with the hypothesis that age differences on cognitive measures should increase in magnitude as the complexity of the task increases (Myerson et al., 1990).

1 The simple measures were administered first because pilot testing indicated that for the experimental tasks the simple problems provided a good "warm up" for the complex problems.
Strategy Choices

The analyses of group differences in strategy choices are described in two sections, the first for simple subtraction and the second for complex subtraction. For the associated analyses, the frequency with which each individual subject used each strategy was calculated and, therefore, the reported group means represent the mean of the individual means. The df differ across some of the analyses reported in this and following sections, because not all subjects used the same strategies. In this circumstance, the number of observations differed for many of the analyses.

Simple subtraction. As noted in Table 3, retrieval was the primary strategy choice for both the younger and older subjects, although the groups differed in the strategy mix. The older adults almost always relied on retrieval for problem solving. The younger adults, however, had to rely on some form of backup strategy, typically addition reference or counting down, to solve 29% of the simple subtraction problems. One-way ANOVAs indicated reliable differences across groups for the frequency with which the retrieval, F(1, 70) = 20.58, $M_X = 572.59$, $p < .0001$, and addition reference, $F(1, 70) = 11.75$, $M_X = 435.36$, $p < .005$, strategies were used for problem solving. A second set of ANOVAs revealed no reliable group difference in the proportion of errors for either of these strategies (ps > .50). In all, the older subjects used direct retrieval more frequently, and the backup addition reference and counting down strategies less frequently, than the younger subjects to solve simple subtraction problems.

Examination of individual protocols indicated that 12 of the 36 younger subjects used retrieval to solve 100% of the problems. For 11 of the 24 remaining subjects, retrieval was the modal strategy; addition reference was the modal strategy for 5 subjects, and counting down was the modal strategy for 1 subject. For 5 of the 7 remaining subjects, retrieval and addition reference were used in roughly equal proportions, whereas for the final 2 subjects, retrieval and counting down were used in roughly equal proportions. Thirty of the 36 older subjects used retrieval to solve 100% of the problems. For 5 of the 6 remaining subjects, retrieval was used to solve at least 31 of the 36 problems. The other subject used addition reference to solve 31 problems and retrieval to solve 5 problems.

The final set of strategy choice analyses for simple subtraction sought to determine if the use of direct retrieval, rather than of backup strategies, was related to problem difficulty. Previous studies of strategy choice in arithmetic have found that the use of retrieval decreases in frequency as the difficulty of the problem increases (Geary & Wiley, 1991; Siegler, 1989; Siegler & Shriger, 1984). To assess the relationship between strategy choice and problem difficulty, the frequency with which direct retrieval was used for problem solving was correlated with the value of the minuend, subtrahend, and difference, as the difficulty of the problem should increase as each of these values increases (Ashcraft, 1992). For both groups, an index of retrieval frequency showed the strongest zero-order correlation with the value of the minuend. The frequency of direct retrieval reliably decreased as the value of the minuend increased for the younger, $r(34) = -.49$, $p < .005$; and older, $r(34) = -.37$, $p < .05$; groups; the value of these two coefficients did not differ reliably, $Z = 0.60$, $p > .25$. For both younger and older adults, the use of direct retrieval appears to decrease in frequency as the difficulty of the simple subtraction problem increases.

Complex subtraction. The group-level characteristics of the complex subtraction strategies are displayed in Table 4. As shown in Table 4, group differences in the strategy mix are evident. Although both the younger and older subjects relied primarily on columnar retrieval for problem solving, the younger subjects showed a higher percentage of backup strategy usage than did the older subjects (33% and 7% for the younger and older groups, respectively). A one-way ANOVA revealed that the advantage of the older group in the use of columnar retrieval was reliable, $F(1, 69) = 22.88$, $M_X = 542.77$, $p < .0001$; the proportion of columnar retrieval errors did not differ reliably across groups, $F(1, 68) = 3.65$, $M_X = 37.25$, $p > .05$. Nevertheless, the frequency of columnar retrieval errors for borrow and no-borrow problems was obtained for each group. The resulting contingency table revealed that the older subjects tended to commit more borrow errors than did the young subjects. The number of errors for no-borrow and borrow problems were 24/38 and 30/105 for the younger and older groups, respectively; 89% of the younger group's errors and 80% of the older group's errors involved the units-column rather than the tens-column.

Examination of individual protocols indicated that 8 of the 36 younger subjects used columnar retrieval to solve 100% of the complex subtraction problems. For 19 of the 28 remaining subjects, columnar retrieval was used to solve at least 28 of the 56 problems. For 4 of the 9 remaining subjects, columnar retrieval and decomposition were used in roughly equal proportions. Columnar retrieval and counting were used in roughly equal proportions for one other subject. Two subjects relied primarily on counting to solve the complex subtraction problems, whereas the final 2 subjects used a combination of columnar retrieval, decomposition, and the deleting 10s rule. Twenty-six of the 35 older subjects used columnar retrieval to solve 100% of the complex subtraction problems. For 6 of the 9 remaining subjects, columnar retrieval was used to solve at least 49 of the 56 problems. For one of the older subjects, the columnar retrieval and addition reference strategies were used in roughly equal proportions. One subject reported solving 100% of the complex subtraction problems by whole retrieval (i.e., retrieving the answer to the whole problem, rather than in columnwise fashion). The final older subject used columnar retrieval to solve 33 problems, addition reference for 18 problems, counting for 2 problems, and the deleting 10s rule for 1 problem (2 trials were spoiled).

The final set of strategy choice analyses for complex subtraction sought to determine if strategy choices varied with problem difficulty. The most salient and theoretically interesting

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2 Strategy data for complex subtraction were lost for one elderly subject.

3 The $df$ differ by 1 for the strategy frequency analysis and the error percentage analysis. This is so because one elderly subject was dropped from the error percentage analysis, because he never used the columnar retrieval strategy.
Table 3
 Characteristics of Simple Subtraction Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Strategy usage (%)</th>
<th>Errors (%)</th>
<th>RT (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Younger</td>
<td>Older</td>
<td>Younger</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Addition reference</td>
<td>20</td>
<td>26</td>
<td>3</td>
</tr>
<tr>
<td>Count down</td>
<td>6</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>Retrieval</td>
<td>71</td>
<td>31</td>
<td>97</td>
</tr>
</tbody>
</table>

Note. Mean reaction time (RT) excluded error and spoiled trials (7.5% and 7.1% of the trials were spoiled for the younger and older subjects, respectively). Overall mean RTs, across strategies, were 970 and 1,061 ms for the younger and older groups, respectively.

* Two percent of the trials for the younger subjects involved either counting up or the use of a rule.

index of problem difficulty for complex subtraction should be the presence or absence of the borrow operation (Hitch, 1978). Thus, an index of the frequency with which columnar retrieval was used for problem solving was correlated with a variable representing the presence or absence of the borrow procedure. The results indicated that the use of columnar retrieval decreased reliably with the presence of the borrow operation for both the younger, r(54) = -.98, p < .0001, and older, r(54) = -.70, p < .0001, groups; the value of these two coefficients did not differ reliably, Z = 1.44, p > .10. An examination of the relation between strategy usage and borrowing indicated that the younger and older adults resorted to a backup strategy to solve 59% and 13%, respectively, of the problems that required borrowing. In contrast, backup strategies were used to solve 11% and 6% of the problems that did not require borrowing for the younger and older groups, respectively. This pattern indicates that backup strategies were more likely to be used when the problem required the execution of the borrow procedure.

Summary. The same pattern of group differences in strategy choices emerged for both simple and complex subtraction. The older adults used retrieval-based processes to solve nearly all of the simple and complex subtraction problems, whereas the younger adults resorted to the use of backup strategies to solve 29% and 33% of the simple and complex subtraction problems, respectively. When the problem required borrowing, the use of backup strategies was particularly pronounced for the younger group, enabling the subjects to avoid having to execute the borrow procedure. If one assumes that the mastery of basic arithmetic involves the automatic retrieval of basic facts from long-term memory (columnar facts for complex problems; Siegler, 1986), then in this study nearly all of the older subjects, but only a small minority of the younger subjects, appear to have mastered simple and complex subtraction.

Solution Times

The analyses of the solution times are described in three sections. The first section examines solution time patterns across the various strategies to assess the validity of the strategy descriptions. The second section presents group-level analyses of mean solution times. The final section presents analyses of component scores for solution times for the retrieval strategy and columnar retrieval strategy, for simple and complex subtraction, respectively.

Validity of self-reports. The validity of the strategy self-reports can be assessed by examining the pattern of correlation between solution times and variables representing subtraction operations for the different strategies (Geary & Wiley, 1991; Suppes et al., 1967). For simple subtraction, mean solution times, averaged across subjects, were correlated with the value of the minuend, subtrahend, and the difference for the retrieval, addition reference, and count down strategies. For retrieval, solution times showed the strongest zero-order correlation with the difference; the rs were .68 (df = 34) and .58 (df = 34) for the younger and older groups, respectively (ps < .0002). Solution times for retrieval trials were not reliably correlated with the value of the subtrahend; the respective rs were -.23 (df = 34) and -.22 (df = 34) for the younger and older groups (ps > .10). Inspection of these same correlations for individual subjects indicated that individual RTs showed the strongest correlation with the difference variable for 33 of the 59 subjects who had three or more correct retrieval trials; the strongest correlation was with the minuend for 14 subjects and the subtrahend for 12 subjects.

In contrast, for the count down strategy, solution times showed the strongest zero-order correlation with the subtrahend, that is, the number of times counted to solve the problem, r(28) = .54, p < .005, for the younger group. Moreover, count down solution times were not correlated with the difference, r(28) = -.03, p > .50. For individual RTs, the strongest correlation was with the subtrahend for five of the nine subjects who counted correctly on more than three trials; only one subject showed a pattern of a stronger correlation between RT and the difference variable. Finally, for the addition reference strategy, solution times were not reliably correlated with any of the variables for the older group (p < .10), and were only correlated with the value of the minuend for the younger group, r(34) = .36, p < .05. This pattern of no reliable correlation between addition reference RTs and the subtraction variables was found for 15 of the 17 subjects who correctly used addition reference on more than three trials.

* Because of a disk failure, RT data were lost for 13 (7 younger, 6 older) subjects.
Table 4  
Characteristics of Complex Subtraction Strategies*  

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Younger M</th>
<th>Younger SD</th>
<th>Older M</th>
<th>Older SD</th>
<th>Younger M</th>
<th>Younger SD</th>
<th>Older M</th>
<th>Older SD</th>
<th>RT (ms) M</th>
<th>RT (ms) SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count down</td>
<td>6</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>4,834</td>
<td>3,311</td>
</tr>
<tr>
<td>Decomposition</td>
<td>18</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>3,075</td>
<td>1,044</td>
</tr>
<tr>
<td>Rule</td>
<td>6</td>
<td>9</td>
<td>1</td>
<td>2</td>
<td>13</td>
<td>27</td>
<td>22</td>
<td>25</td>
<td>3,194</td>
<td>885</td>
</tr>
<tr>
<td>Columnar retrieval</td>
<td>67</td>
<td>26</td>
<td>93</td>
<td>19</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>7</td>
<td>1,874</td>
<td>634</td>
</tr>
</tbody>
</table>

Note. Mean reaction time (RT) excluded error and spoiled trials (11.7% and 16.5% of the trials were spoiled for the younger and older subjects, respectively). Overall mean RTs, across strategies, were 2,436 and 2,044 ms for the younger and older groups, respectively.

* Strategies that occurred on less than 4% of the trials for both groups are not reported. These strategies include addition reference (1% and 2% for the younger and older samples, respectively); count up (2% and 0%); and whole retrieval (0% and 3%).

For complex subtraction, mean solution times were correlated with the minuend, subtrahend, units-column difference, and the presence or absence of the borrow operation. Solution times for columnar retrieval showed a very high zero-order correlation with the borrow variable, the most time-consuming process in columnar retrieval, for both the younger, r(54) = .92, and older, r(54) = .89, groups (ps < .001). Inspection of correlations for individual subjects revealed that 52 of the 57 subjects who correctly solved both borrow and no-borrow problems using columnar retrieval showed the same pattern; that is, the strongest correlation was between RTs and the borrow variable. In contrast, the borrow operation should not be required when the decomposition or rule strategies are used for problem solving. Indeed, solution times for the use of a rule were not correlated with the borrow variable for either the younger, r(20) = .17, group or the older, r(2) = .00, group (ps > .25). Similarly, for the younger group, solution times for the use of decomposition were not reliably correlated with the borrow variable, r(27) = -.13, p > .25. Inspection of correlations for individual subjects indicated that this pattern was due to the infrequent use of the decomposition and rule strategies to solve no-borrow problems. Finally, for the younger group, solution times for the count down strategy showed the strongest zero-order correlation with the value of the subtrahend, r(48) = .71, p < .001, that is, the number of times counted to solve the problem; the finding of a strong zero-order correlation between the subtrahend and count down RTs was obtained for four of the five subjects who correctly counted on three or more trials. In all, the results show a different pattern of correlation, across strategies, between solution times and variables representing subtraction operations and thereby appear to support the validity of the self-reports for both simple and complex subtraction.

Mean solution times. Mean solution times for the simple subtraction strategies are presented in the rightmost columns of Table 3. Inspection of these data indicate that the fastest mean solution times were for retrieval, followed by addition reference and counting. The finding that the fastest mean RTs were for retrieval and the slowest were for counting, with addition reference in between, is consistent with RT patterns found for children across the same strategies (Siegler, 1987) and provides further support for the validity of the strategy descriptions. One-way ANOVAs revealed that mean RTs differed reliably across groups for retrieval, F(1, 57) = 11.63, MSq = 28,670.54, p < .005, but did not differ reliably for addition reference, F(1, 18) < 1, MSq = 72,023.15. In all, the younger adults had a 151 ms advantage over the older adults when using direct retrieval to solve simple subtraction problems. The failure to find a reliable mean difference in RTs, across groups, for the addition reference strategy needs to be interpreted with caution given the small number of addition reference trials for the older subjects.

For retrieval, research in mental addition and multiplication has revealed a problem difficulty effect (Ashcraft, 1992; Geary et al., 1986). That is, answers to problems with smaller valued sums or products are retrieved from memory more quickly than answers to problems with larger valued sums or products. The finding that RTs for the retrieval strategy were positively correlated with the difference suggests that a problem difficulty effect is also evident for simple subtraction; the larger the difference, the longer the solution time. Following analyses for other arithmetic operations (see Ashcraft, 1992), simple subtraction problems were divided into smaller valued answers (< 4 in this analysis) and larger valued answers. The slope between means for RTs from smaller and larger valued answers represents the amount of extra time needed to locate answers to larger problems, and thus provides an index for retrieval speed (Ashcraft, 1992; Ashcraft & Battaglia, 1978). Thus, mean RTs for smaller and larger valued differences were obtained for both groups. A mixed design ANOVA, with group (younger vs. older) as a between-subjects factor and size (small vs. large) as a within-subjects factor, revealed a reliable main effect for group, F(1, 55) = 8.98, MSq = 63,110.98, p < .005, and size, F(1, 55) = 23.97, MSq = 6,988.95, p < .001, but a nonreliable Group × Size interaction, F(1, 55) < 1, MSq = 6,988.95. Consistent with studies of mental addition and multiplication (Allen et al., 1992; Geary & Wiley, 1991), the nonreliable interaction suggests that the younger and older groups did not differ in the speed of retrieving subtraction facts from long-term memory.

Mean solution times for complex subtraction problems are presented in the rightmost columns of Table 4. Inspection of these data reveals that the fastest mean RTs were obtained for
columnar retrieval, the slowest were for counting, with mean RTs for the use of a rule and the decomposition strategy falling in between the retrieval and counting RTs. A one-way ANOVA revealed no reliable difference across groups in mean RTs for the columnar retrieval strategy, \( F(1, 55) < 1, MS_r = 365.911.31 \). Nevertheless, mean RTs from problems requiring the borrow operation and problems not requiring the borrow operation were obtained for subjects in both groups. The resulting RTs are displayed in Figure 1 and were analyzed by means of a mixed design ANOVA, with group (younger vs. older) as a between-subjects factor and borrow (presence vs. absence) as a within-subjects factor. The results indicated a nonreliable main effect for group, \( F(1, 46) = 3.47, MS_r = 978.630.91, p > .05 \), but a reliable main effect for borrow, \( F(1, 46) = 106.08, MS_r = 418.241.09, p < .001 \), and a reliable Group × Borrow interaction, \( F(1, 46) = 11.29, MS_r = 418.241.09, p < .05 \). One-way ANOVAs confirmed that the groups did not differ reliably for no-borrow mean RTs, \( F(1, 46) < 1, MS_r = 134.453.50 \), but the speed advantage of the older subjects for borrow problems was reliable, \( F(1, 46) = 6.39, MS_r = 1,262,418.30, p < .02 \). In all, the pattern of results for columnar retrieval suggests that older adults were faster at executing the borrow operation than younger adults. The difference in mean RTs for borrow problems across the younger (\( M = 3356 \) ms) and older (\( M = 2524 \) ms) groups was 832 ms.

Finally, to test the hypothesis that age differences in solution times increase as the complexity of the task increases (Myerson et al., 1990), a mixed design ANOVA was performed on the solution times for the retrieval and columnar retrieval strategies from simple and complex subtraction, respectively. In this analysis, group was a between-subjects factor, and complexity (simple subtraction vs. complex subtraction) was a within-subjects factor. The results revealed a nonreliable main effect for group, \( F(1, 55) = 2.00, MS_r = 248.996.05, p > .10 \), a reliable main effect for complexity, \( F(1, 55) = 176.91, MS_r = 146.014.24, p < .001 \), and a nonreliable Group × Complexity interaction, \( F(1, 55) < 1, MS_r = 146.014.24 \). The finding of no reliable interaction fails to support the hypothesis that age differences in processing speed increase with increases in task complexity (Cerella, 1990; Myerson et al., 1990).

Analysis of component scores. The component score analyses were conducted to assess further the two main findings that emerged from the analyses of the mean RTs; namely, that there was no group difference in speed of retrieving simple subtraction facts from long-term memory and a faster speed of executing the borrow procedure for the older group. All analyses were based on correct retrieval trials (columnar retrieval for complex subtraction) and excluded trials on which the voice-operated relay was triggered by responses other than the answer (e.g., coughing) or trials on which the initial response did not trigger the relay (see Tables 3 and 4).

Process models for subtraction were fitted to individual RT data using regression techniques. On the basis of the finding that the difference variable showed the strongest zero-order correlation with retrieval trial solution times for simple subtraction, the difference variable was used to model RTs for simple subtraction. Across subjects, the difference variable was correlated - .26 to .70 (\( Mdn = .26 \)) with individual correct retrieval trial RTs. The raw regression weight, which is termed a component score, for the difference variable provided an estimate for retrieval speed. The intercept term provided an estimate of the speed of executing processes other than fact retrieval, such as encoding the numbers and verbally producing the answer (Ashcraft, 1982; Geary et al., 1986; Geary & Wiley, 1991).

The analyses of RTs from complex subtraction problems followed the same procedure as described for simple subtraction, except that the difference variable was replaced by a borrow variable (coded 0 and 1 for the absence or presence of the borrow operation, respectively). Across subjects, the borrow variable was correlated -.47 to .96 (\( Mdn = .71 \); only three values

![Figure 1](image-url)
MENTAL SUBTRACTION

were < .50 with individual columnar retrieval RTs. The raw regression weight for the borrow variable provided an estimate for the speed of executing the borrow operation. The intercept term provided a combined estimate for the duration of all other processes (i.e., number encoding, answer production, and columnar fact retrieval). To illustrate, the component score for the borrow variable for one of our younger subjects was 2,122 and 2,865 for another subject. These values indicate that the first of these subjects required 2,122 ms to execute the borrow procedure, whereas the other subject required 2,865 ms to execute the same procedure.

Mean component scores across group and problem complexity are displayed in Table 5. For simple subtraction, the advantage of the younger group was reliable for the intercept term, F(1, 57) = 15.21, MSEM = 32,797.95, p < .001, but was not reliable for the difference variable, F(1, 57) < 1, MSEM = 484.21. For complex subtraction, the advantage of the younger group for the intercept term was marginally reliable, F(1, 56) = 2.98, MSEM = 122,780.60, p < .10, whereas the advantage of the older group on the borrow variable was highly reliable, F(1, 46) = 12.57, MSEM = 820,422.33, p < .001.8

Finally, given the unusual finding of considerably faster process speeds favoring the older subjects for the borrow procedure, combined with the reliable educational difference, also favoring the older subjects, further zero-order correlations and multiple regression equations were computed, using the borrow, age, education, and health variables. The zero-order correlations indicated that component scores for the borrow variable were reliably correlated with age, r(46) = -.46, p < .001, and education, r(46) = -.29, p < .05, but not with reported health status, r(46) = -.02, p > .50. Next, a hierarchical regression equation using the borrow variable as the dependent measure and age, education, and their interaction (entered in that order) as independent measures was computed. The results revealed a reliable effect for age, F(1, 46) = 12.42, MSEM = 822,474.65, p < .0001, but nonreliable education, F(1, 45) < 1, MSEM = 829,468.78, and interaction, F(1, 44) < 1, MSEM = 834,332.02, effects.

Summary: The component score analyses suggest that younger and older adults do not differ in the speed of executing a substantive arithmetical operation—retrieving subtraction facts from long-term memory. This result is consistent with the finding of no retrieval speed differences, across younger and older adults, for addition (Geary & Wiley, 1991) and multiplication (Allen et al., 1992). The younger adults, however, appeared to have an advantage over the older adults in the speed of executing some other operations, perhaps speed of encoding numbers or verbally producing the answer, as indicated by the reliable and marginally reliable intercept differences for simple and complex subtraction, respectively (Allen et al., 1992; Charness, 1987; Geary & Wiley, 1991). In contrast, for complex subtraction, older adults appeared to have a 947 ms advantage over younger adults in the speed of executing the borrow procedure. This latter finding was unexpected and runs counter to previous cognitive aging theory and empirical research (Cerella, 1985, 1990; Myerson et al., 1990). Thus, in the final section we explore one potential explanation for this finding, namely, that this result reflects differential practice of complex subtraction for younger and older adults. Finally, it should be noted that the finding of no reliable group difference in the proportion of retrieval and columnar retrieval errors, for simple and complex problems, respectively, suggests that the solution time results were not substantively influenced by speed-accuracy trade-off differences across groups (Cerella, 1990; Geary & Wiley, 1991).

Individual Differences

In this section, we explore the relation between performance on the ability measures and component scores from the experimental tasks. Component scores for the intercept and difference variable from simple subtraction retrieval trials were reliably correlated with performance on the simple subtraction ability measure, R = .77, F(2, 44) = 31.87, MSEM = 254.67, p < .0001. Partialing mean intercept values from performance on the simple subtraction test reduced the advantage of the younger group on this test to nonsignificance, F(1, 56) = 1.18, MSEM = 300.93, p > .25. Scores for the intercept and borrow variables from the complex subtraction task were highly correlated with performance on the complex subtraction ability measure, R = .73, F(2, 43) = 24.61, MSEM = 48.59, p < .0001. We did not partial these variables from the group difference on the complex subtraction test because the group difference on this test was not reliable.

Consistent with previous studies, parameters derived from the experimental tasks were reliably related to performance on the paper-and-pencil ability measures (Geary & Birmahambubree, 1989; Geary & Widaman, 1987, 1992; Siegler, 1988; Sternberg & Gardner, 1983). The advantage of the younger adults on the simple subtraction test appeared to be related to their speed advantage for executing processes subsumed by the intercept term, such as number encoding and verbally producing the answer (Charness, 1987). It also seems reasonable to suspect that the speed of writing the answers also differed across groups, and likely favored the younger adults. Thus, writing speed differences might have also contributed to the younger subjects’ advantage on the simple subtraction test.

Computational Analysis

As noted in the Analysis of component scores section, the finding of a substantial group difference, favoring the older subjects, in the speed of executing the borrow procedure was unexpected. In fact, we had expected that the older subjects would execute the borrow procedure more slowly than the younger subjects because of the complexity and working memory requirements for executing this procedure (Frensch &

3 The component score analyses were redone for simple subtraction after deleting individual subjects with a correlation between the difference variable and RT that was less than the median r (i.e., .26). For complex subtraction, the analyses were redone after deleting the three subjects with a correlation between the borrow variable and RT of < .50. In both cases, the pattern of results was identical to that presented in Table 5.

4 If a subject never executed the borrow procedure but did use the columnar retrieval strategy to solve some no-borrow problems, then there was no estimate for the borrow variable but there was an estimate for the intercept term. As a result, the df differ across the intercept and borrow analyses.
Table 5

<table>
<thead>
<tr>
<th>Component Scores for Simple and Complex Subtraction (in ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Younger</td>
</tr>
<tr>
<td>Older</td>
</tr>
</tbody>
</table>

Geary, 1993; Hitch, 1978; VanLehn, 1990). In this section, we explore the possibility that the working memory requirements of the borrow procedure might actually have been less demanding for the older than for the younger adults. This is possible if the younger and older subjects differed in the level of practice of the borrow operation and if it can be assumed that the borrow procedure requires several steps, or rules, to complete (see Frensch & Geary, 1993). If so, then one can argue that, with practice, the rules required to execute the borrow procedure become collapsed, or composed, into a set of simpler rules (Charness & Campbell, 1988). The composition of the rules required to execute the borrow procedure would reduce both the working memory demands, as well as the amount of time needed to execute the procedure.

To test this hypothesis, we developed a computational model of the composition process that might occur with the repeated solution of complex subtraction problems. The computational model was implemented as a production system and provided a feasibility check (Simon, 1969) of our interpretation of the age difference for the speed of borrowing. Our description of the model is deliberately kept brief. The reader interested in the details of the model may consult Frensch (1991, 1992) or Frensch and Geary (1993).

**Modeling performance in the subtraction task.** In general, a production system consists of two data bases that are linked through a processing cycle. The data bases include a short-term working memory and a long-term production memory. The production memory contains a collection of IF-THEN rules; that is, condition—action rules (productions) of the following form: IF (condition) THEN (action). With the repeated serial execution of production rules, these rules can be collapsed, or composed, into a smaller set of rules. The computational model of complex subtraction simulated the columnar retrieval strategy. The simulation initially required 25 and 19 production rules for borrow and no.borrow problems, respectively; the borrow procedure itself required six production rules to execute. If one assumes that solution time is a function of the number of recognize—act cycles and the number of operators needed to complete problem-solving, then the present system can be used to estimate changes in solution times, for borrow and no-borrow problems, across practice. The number of recognize—act cycles in the system is equal to the number of production rules that are executed to solve a particular problem. Operators are the elementary actions contained in the THEN parts of the production rules.

To predict the effects of practice on speed of processing and working memory requirements, we compared the original, un-composed, production system with its composed version. To develop the composed version, the simulation was run for 3,000 trials for 10 subjects each for borrow and no-borrow problems. We ran more than one subject for each condition, because the composition mechanism is probabilistic and therefore differs somewhat for each subject. Mean changes, across practice trials, for the number of recognize—act cycles and operations needed to solve borrow and no-borrow problems are displayed in Figure 2. Inspection of the figure indicates that the rate of change across practice in the number of recognize—act cycles and operations is accelerated for borrow relative to no-borrow problems. This result suggests that with continued practice the mean solution times for borrow and no-borrow problems should converge. If so, then the mean solution time difference between borrow and no-borrow problems should provide an index for level of practice, and if our assumption of cohort differences is correct, then this difference should be smaller for older than for younger adults. This is exactly the pattern we found; the respective differences were 929 and 1829 ms for the older and younger groups, respectively, $F(1, 46) = 11.29, MS_e = 836,482.18, p < .005$.

Finally, to assess the working memory demands of borrow and no-borrow problems across levels of practice, the minimum number of working memory slots required for problem solving was compared for the uncomposed and composed models. For the uncomposed system, the minimum number of slots was 11 and 7 for borrow and no-borrow problems, respectively. For the completely composed model, the number of slots for both borrow and no-borrow problems was 2. In other words, the completely composed model suggested that RIs for borrow and no-borrow problems would be the same. That is, after extensive practice the subjects would solve all problems by retrieving the answer to the entire problem (i.e., whole retrieval) rather than retrieving columnar answers. In all, the results indicate that the working memory demands for solving both borrow and no-borrow problems appear to decrease across levels of practice, although the reduction in working memory demands is more pronounced for borrow than for no-borrow problems. Thus, if the younger and older subjects differed in the level of practice of complex subtraction, then the complexity of the task likely also differed across groups.

**Conclusion.** The pattern of results from the computational simulation is consistent with the argument that the older subjects have practiced complex subtraction more than the college students (Schaie, 1989). As predicted by the simulation, the difference between mean solution times for borrow and no-borrow problems was considerably smaller for the older than
Figure 2. Practice-related change in the number of recognize-act cycles and operations needed to solve complex subtraction problems.
the younger group. Given this convergence in borrow and no-borrow solution times for the older group, it is likely that their actual level of skill for solving complex subtraction problems is much closer to the composed model, relative to the younger group. In this circumstance, the working memory demands of borrowing would actually be lower for older than for younger adults because the number of production rules that would need to be executed in working memory is smaller. Moreover, given that the borrow procedure would require fewer production rules and consequently fewer recognize-act cycles and operations for the older individuals, the overall borrow speeds should be faster for older adults than for younger adults. This is not to say that younger and older adults do not differ in the speed of executing more basic operations, such as cycle time (Saithouse & Babcock, 1991), nor is it the only explanation for our findings. Regardless, the pattern of results from the simulation are consistent with the empirical findings and provide a feasible explanation for the unexpected results for complex subtraction.

Discussion

The results of this study are applicable to applied and theoretical issues in human aging. One practical finding is that the basic subtraction skills of many older adults appear to be well developed, given good health (Manton, Siegler, & Woodbury, 1986). In fact, in terms of strategy choices and speed of borrowing, the subtraction skills of the older adults assessed in this study were better developed than those of the college students. The better strategic skills of the older adults were reflected in the greater reliance on retrieval-based processes, rather than backup strategies, for problem solving. This finding is consistent with the results for mental addition (Geary & Wiley, 1991), as well as with the finding that recent cohorts perform more poorly on arithmetic tests than earlier cohorts (Schaie, 1983). Given the latter, the most parsimonious explanation for the advantage of older adults in strategy choices and for the speed of borrowing is that their early education in basic mathematics was superior to that of the college students, although the effect of practice throughout adulthood might have also contributed to the advantage of the older group.

The argument for practice differences across groups was also supported by the computational simulation. The results of the simulation suggested that one method that can be used to assess the level of practice is to compare the difference in mean RTs for borrow and no-borrow problems, the difference should decrease with increased practice. The finding of smaller borrow–no-borrow differences for the older adults than for the younger adults is consistent with considerable practice differences across groups, favoring the older adults. As noted earlier, increased practice should affect solution times and should also lead to increased use of retrieval-based processes, rather than backup strategies, for problem solving (see Siegler, 1986). Although the use of direct retrieval is not problem solving per se, it does make the use of more time consuming and effortful problem-solving strategies unnecessary (Charness, 1981). The advantage in the use of retrieval-based processes was especially pronounced for complex subtraction, where overall solution times, averaged across all strategies, were faster for the older adults than for the younger adults; the respective means were 2,044 and 2,436 ms (p < .05).

Consistent with studies that have used children as subjects (e.g., Siegler, 1987), the use of backup strategies increased in frequency as the difficulty of the problem increased, for both simple and complex problems and for both the younger and older groups. Given that problem difficulty appears to be an index for the ease of retrieval (Ashcraft, 1992; Siegler, 1986), this result suggests that backup strategies were used to solve simple subtraction problems when facts were not readily retrievable from long-term memory. For complex subtraction, the most salient index of problem difficulty was the presence or absence of the borrow operation. Indeed, the presence of the borrow operation appeared to contribute to the use of backup strategies, especially for the younger group. The use of backup strategies, apparently to avoid borrowing, suggests that the borrow operation might have been more effortful for the younger adults than for the older adults. This interpretation is also consistent with the computational simulation, which suggested that the borrow operation might have been more demanding, in terms of working memory resources, for the younger than for the older group. For the older adults, the reduced working memory demands of borrowing presumably made this procedure less effortful and therefore lessened the need to resort to backup strategies.

The results for the solution time data were perhaps the most interesting, in terms of theoretical issues in cognitive aging. In particular, the results for the central arithmetical processes, namely fact retrieval and borrowing, were inconsistent with the argument that processing speed slows in all domains with aging and that age differences in processing speed increase with increases in task complexity (Cerella, 1990; Myerson et al., 1990). Consistent with recent findings for addition and multiplication (Allen et al., 1992; Geary & Wiley, 1991), there did not appear to be a group difference in the speed of retrieving subtraction facts from long-term memory. This result provides further support for the hypothesis that aging does not substantially affect the speed of retrieving single pieces of information from semantic memory (Cerella & Foard, 1984; Geary, in press). However, given the apparent difference in level of practice favoring the older group, any more fundamental changes that might have affected the speed of information access from semantic memory might have been compensated for by practice. Indeed, given the apparent group difference in the level of practice, the advantage of older adults in speed of borrowing should not be taken as strong evidence against the position that age differences in processing speed increase with increases in task complexity (Myerson et al., 1990).

Despite the apparent differences in level of practice favoring the older adults, the younger adults appeared to execute the processes subsumed by the intercept terms faster than the older adults (Allen et al., 1992; Geary & Wiley, 1991). Although it cannot be argued with certainty, these intercept differences might reflect age-related change in the speed of encoding and verbally producing numbers (Charness, 1987; Geary & Wiley, 1991). This argument is based on the finding of slower verbal production times for older adults as compared with younger adults (Charness, 1987; but see Nebes, 1978) and age-related changes in encoding speeds (Hunt, 1978). This pattern, in com-
bination with the finding that the younger adults did not have an advantage for retrieval and borrowing speeds, appears to support Charness’s (1987) position that “much of the age-related slowing is attributable to encoding and response processes” (pp. 229-230) and is not primarily due to changes in more central processes. Again, for the retrieval and borrow findings, this argument needs to be tempered by the potential practice differences across the younger and older groups.

Finally, consider the working memory manipulation. The potential collapse of the production rules for the borrow procedure across practice suggests that the within-context working-memory demands of this procedure might have differed for the younger and older groups. That is, although the borrow procedure represents a working memory manipulation (Hitch, 1978), the finding that the older adults were faster at this procedure than the younger adults should not be taken as evidence against the argument that aging is associated with decrements in working memory resources (e.g., Salthouse, 1991). This is so because the apparent group difference in the level of practice suggests that the working memory demands of the task might have been higher for the younger group than for the older group. Even though the older adults executed the borrow procedure more quickly than the younger adults, they were more likely to commit an error while executing this procedure. The majority of these errors appeared to occur during the processing of the units-column rather than the tens-column information. Recall that tens-column errors would suggest that information is lost during the processing of the units-column information, whereas units-column errors could be due to retrieval or encoding problems. The finding of nearly perfect retrieval for simple subtraction facts suggests that holding the value of the decremented tens-column minuend in working memory interfered with the encoding of the units-column information.

In summary, the results of this study appear to reflect substantial cohort differences, favoring older adults, in basic arithmetic skills. The apparent cohort difference was most sharply reflected in the advantage of the older adults in strategy choices and for speed of borrowing. Regardless of the source of the difference, the present study suggests that the basic arithmetic skills of at least well-educated older adults are intact and argues against the position that nearly all high-school graduates in the United States are proficient in arithmetic (Mullis, Dossey, Owen, & Phillips, 1991). In fact, less than one half of the college students assessed in this study and a comparable study of addition skills (Geary & Wiley, 1991) appeared to have mastered basic arithmetic, with mastery defined as the automatic retrieval of basic facts and columnar facts from long-term memory (Siegler, 1986). Finally, we believe that this study and related studies (Allen et al., 1992; Geary & Wiley, 1991) argue for the utility of mental arithmetic procedures in cognitive aging research. The procedures allow for reliable trial-by-trial classifications of problem-solving strategies, componential analyses of the associated solution times, and for rather easy manipulations of task complexity (e.g., borrow vs. no-borrow problems). Moreover, given that the basic arithmetic skills of many older adults appear to be well developed, the arithmetic tasks might prove to be useful for studying changes in cognitive functioning associated with pathology, such as Alzheimer’s disease. The procedures, for instance, might be used to assess changes in strategy choices, retrieval speeds, and retrieval errors that occur during the course of the disease.

References


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