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15

Learning Disabilities in Arithmetic and Mathematics

Theoretical and Empirical Perspectives

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The breadth and complexity of the field of mathematics make the identification and study of the cognitive phenotypes that define learning disabilities in mathematics (MD) a formidable endeavor. A learning disability can result from deficits in the ability to represent or process information in one or all of the many mathematical domains (e.g., geometry) or in one or a set of individual competencies within each domain. The goal is further complicated by the task of distinguishing poor achievement due to inadequate instruction from poor achievement due to an actual cognitive disability (Geary, Brown, & Samaranayake, 1991). One approach that can be used to circumvent this instructional confound involves is to apply the theories and methods used to study mathematical competencies in normal children to the study of children with MD (Bull & Johnston, 1997; Garnett & Fleischner, 1983; Geary & Brown, 1991; Geary, Widaman, Little, & Cormier, 1987; Jordan, Levine, & Huttenlocher, 1995; Jordan, Hanich, & Kaplan, 2003a; Jordan & Montani, 1997; Ostad, 1997, 1998a; Russell & Ginsburg, 1984; Svenson & Broquist, 1975). When this approach is combined with studies of dyscalculia, that is, numerical and arithmetical deficits following overt brain injury (Shalev, Manor, & Gross-Tsur, 1993; Temple, 1991), and brain imagining studies of mathematical processing (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999), a picture of the cognitive and brain systems that can contribute to MD begins to emerge.

The combination of theoretical and empirical approaches has been primarily applied to the study of numerical and arithmetical competencies and is therefore only a first step to a complete understanding of the cognitive and brain systems that support mathematical competency, and any associated learning disabilities. It is, nonetheless, a start. We overview what this research strategy has revealed about children with MD in the second section, and discuss diagnostic, prevalence, and etiological issues in the first section. In the third section, we present a general organizational framework for approaching the study of MD in any mathematical domain and use this framework and an earlier taxonomy of MD subtypes (Geary, 1993) to better understand the cognitive deficits described in the second section.

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BACKGROUND CHARACTERISTICS OF CHILDREN WITH MD

Diagnosis

Unfortunately, measures that are specifically designed to diagnose MD are not available, and thus most researchers and practitioners rely on standardized achievement tests, often in combination with IQ scores. A score lower than the 25th or 30th percentile on a mathematics achievement test combined with a low average or higher IQ score are common criteria for diagnosing MD (Geary, Hamson, & Hoard, 2000; Gross-Tsur, Manor, & Shalev, 1996). However, a lower than expected (based on IQ) mathematics achievement score does not, in and of itself, indicate the presence of MD. Many children who score poorly on achievement tests one academic year score average or better in subsequent years. These children do not appear to have any of the underlying memory and (or) cognitive deficits described in the next section, and thus a diagnosis of MD may not be appropriate (Geary, 1990; Geary et al., 1991; Geary et al., 2000). Many children who have lower than expected achievement scores across successive academic years, in contrast, often have some form of memory and (or) cognitive deficit, and thus a diagnosis of MD is often warranted. Many of these children do show year-to-year improvements in achievement scores and on some math cognition measures, but they do not catch up to their normal peers and show more persistent deficits in some areas, such as fact retrieval (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan et al. 2003a).

It should be noted that the cutoff of the 30th percentile on a mathematics achievement test does not fit with the estimation, described below, that between 5% and 8% of children have some form of MD. The discrepancy results from the nature of standardized achievement tests and the often rather specific memory and (or) cognitive deficits of children with MD. Standardized achievement tests sample a broad range of arithmetical and mathematical topics, whereas children with MD often have severe deficits in some of these areas and average or better competencies in others. The result of averaging across items that assess different competencies is a level of performance (e.g., at the 20th percentile) that overestimates the competencies in some areas and underestimates them in others.

In addition to the development of diagnostic instruments, another issue that needs to be explored is whether treatment resistance can be used as one diagnostic criterion for MD. As described later, many children with MD have difficulties retrieving basic arithmetic facts from long-term memory and these difficulties often persist despite intensive instruction on basic facts (Howell, Sidorenko, & Jurica, 1987). Although the instructional research is preliminary, it does suggest that a retrieval deficit resistant to instructional intervention might be a useful diagnostic indicator of arithmetical forms of MD.

Prevalence and Etiology

Experimental measures that are more sensitive to MD than are standard achievement tests have been administered to samples of more than 300 children from well-defined populations (e.g., all fourth graders in an urban school district) in the United States (Badian, 1983), Europe (Kosc, 1974; Ostad, 1998b), and Israel (Gross-Tsur et al., 1996; Shalev et al., 2001). These measures have largely assessed number and arithmetic competencies and were constructed based on neuropsychological deficits associated with dyscalculia (see Geary & Hoard, 2001; Shalev et al., 1993). Performance that deviates from age-related norms and is similar to that associated with dyscalculia has been used in these studies as an indication of MD, and suggests that 5% to 8% of school-age children exhibit some form of MD. Many of these children have comorbid disorders, including reading disabilities (RD), spelling disability, attention deficit hyperactivity disorder (ADHD), or some combination of these disorders.

Very little is currently known about the etiology of MD, although preliminary twin and familial studies suggest genetic and environmental contributions (Light & DeFries, 1995; Shalev et al., 2001). For instance, Shalev and her colleagues studied familial patterns of MD, specifically learning disabilities in number and arithmetic. The results showed that family members (e.g., parents and siblings) of children with MD are 10 times more likely to be diagnosed with MD than are members of the general population, suggesting a heritable risk for the development of MD. It is possible that this risk is only expressed under certain environmental conditions, but these are not yet understood.

COGNITIVE PHENOTYPE OF CHILDREN WITH MD

In the respective sections below, we provide a brief overview of theoretical models of normal development in the number, counting, and arithmetic domains, along with patterns that have been found with the comparison of children with MD to their normal peers. Unless otherwise noted, MD refers to children with low achievement scores—relative to IQ in many of the studies—in mathematics. When studies have only focused on children with low mathematics achievement scores but average or better reading achievement scores, they will be referred to as children with MD only. If the study assessed children with low achievement in mathematics and reading, they will be identified as children with MD/RD.

Number Representation

The comprehension and production of number requires an understanding of and the ability to access representations of the associated magnitudes (Gallistel & Gelman, 1992). In addition, children must learn to process verbal (e.g., "three hundred forty two") and Arabic representations (e.g., "342") of numbers and to translate numerals from one representation to another (e.g., "three hundred forty two" to "342"; Dehaene, 1992; Seron & Fayol, 1994). It appears that young children with MD and MD/RD have a normal, or only a slightly delayed understanding of small quantities, and can associate these with corresponding Arabic and number-word representations (Geary et al., 2000; Gross-Tsur et al., 1996; Temple & Sherwood, 2002).

One area in which number representation has not been studied in children with MD involves their ability to form a spatially-based mental number line and then use this line to make estimates of numerical magnitude. Jordan and her colleagues (Hanich et al., 2001; Jordan et al., 2003a) found that children with MD only and MD/RD were not as skilled as other children, including RD children, at estimating whether the answer to problems such as 9 + 8 is closer to 20 or 30. As noted by Jordan et al. it is not clear how these children were making these estimates, although use of some form of spatially-based mental number line is one possibility (Dehaene et al., 1999). Implications are discussed in a later section.

Counting

Normal Development

Children's understanding of the principles that constrain counting behavior appear to emerge from a combination of inherent constraints and counting experience (Briars & Siegler, 1984; Geary, 1995; Gelman & Gallistel, 1978). Early inherent constraints can be represented by Gelman and Gallistel's five implicit principles. These principles are one-one correspondence (one and only one word tag, e.g., "one," "two," is assigned to each counted object); stable order (the order of the word tags must be invariant across counted sets); cardinality (the value of the final word tag represents the quantity of items in the counted set); abstraction (objects of any kind can be collected together and counted); and order-irrelevance (items within a given set can be tagged in any sequence). The principles of one-one correspondence, stable order, and cardinality define the "how to count" rules, which, in turn, provide the skeletal structure for children's emerging knowledge of counting (Gelman & Meck, 1983).

In addition to inherent constraints, children make inductions about the basic characteristics of counting by observing standard counting behavior and associated outcomes (Briars & Siegler, 1984; Fuson, 1988). These inductions appear to elaborate and add to Gelman and Gallistel's counting rules (1978). One result is a belief that certain unessential features of counting are essential. These unessential features include standard direction (counting must start at one of the end points of a set of objects), and adjacency. The latter is the incorrect belief that items must be counted consecutively and from one contiguous item to the next, that is, "jumping around" during the act of counting results in an incorrect count. By five years of age, many children know most of the essential features of counting described by Gelman and Gallistel but also believe that adjacency and start at an end are also essential features of counting. The latter beliefs indicate that young children's understanding of counting is rather rigid and immature and influenced by the observation of standard counting procedures.

Children with MD

Using the procedures developed by Gelman and Meck (1983) and Briars and Siegler (1984), Geary, Bow-Thomas, and Yao (1992) compared the counting knowledge of first grade children with MD/RD with that of their normal peers. The procedure involved asking the children to watch a puppet count a set of objects. The puppet sometimes counted correctly and sometimes violated one of Gelman and Gallistel's (1978) counting principles or Briars and Siegler's unessential features of counting. The child's task was to determine if the puppet's count was "OK" or "not OK and wrong." In this way, the puppet performed the procedural aspect of counting (i.e., pointing at and tagging items), leaving the child's responses to be based on her or his understanding of counting principles.

The results revealed that children with MD/RD identified correct counts, as well as violations of most of the principles identified by Gelman and Gallistel (1978). They also understood that counting from right to left was just as appropriate as the standard left to right counting (Geary et al. 1992). As the same time, many children with MD/RD did not understand Gelman and Gallistel's (1978) order-irrelevance principle, and believed that adjacency is an essential feature of counting. There were also group differences on trials in which either the first or the last item was counted twice. Children with MD/RD correctly identified these counts as errors when the last item was double counted, suggesting that they understood the one-one correspondence principle. Double counts were often labeled as correct when the first item was counted, suggesting that many children with MD/RD had difficulties holding information in working memory—in this case noting that the first item was double counted—while monitoring the act of counting (see also Hitch & McAuley, 1991).

In a more recent study, children with IQ scores in the 80–120 range were administered a series of cognitive and achievement tests in first and second grade (Geary, Hoard, & Hampson, 1999; Geary et al., 2000). Children with low average or better IQ scores and poor achievement scores in reading and/or math in both grades were considered LD. Among other findings, the results were consistent with those of Geary et al. (1992), that is, children with MD/RD and MD only differed from the children with RD only and normal children on adjacency trials in first and second grade and on double counting trials (the first item) in first grade. The pattern suggests that even in second grade, many children with MD/RD and MD only do not understand all counting principles, and in first grade may have difficulty holding an error notation in working memory while monitoring a short counting sequence (see also Hoard, Geary, & Hampson, 1999). Children with RD only performed as well as the normal children.

In summary, many children with MD, independent of reading achievement levels or IQ, have a poor conceptual understanding of some aspects of counting. These children understand most of the inherent counting rules identified by Gelman and Gallistel (1978), but consistently err on tasks that assess order-irrelevance or adjacency from Briars and Siegler's (1984) perspective. The poor counting knowledge of these children appears to contribute to their delayed competencies in the use of counting to solve arithmetic problems and may result in poor skill at detecting and thus correcting counting errors (Ohlsson & Rees, 1991).

Arithmetic

Normal Development

The most thoroughly studied developmental and schooling-based improvement in arithmetical competency is change in the distribution of procedures, or strategies, children use during problem solving (Ashcraft, 1982; Carpenter & Moser, 1984; Geary, 1994; Siegler, 1996). During the early learning of how to solve simple addition problems, for instance, children typically count both addends (e.g., 5 + 3). These counting procedures are sometimes executed with the aid of fingers (finger counting strategy), and sometimes without them (verbal counting strategy; Siegler & Shrager, 1984). The two most commonly used counting procedures, whether children use their fingers or not, are termed counting-on or counting-all (Fuson, 1982; Groen & Parkman, 1972). The counting-on procedure typically involves stating the larger valued addend and then counting a number of times equal to the value of the smaller addend, such as counting 5, 6, 7, 8 to solve 5 + 3. Counting all involves counting both addends starting from 1. The development of procedural competencies is related, in part, to improvements in children's conceptual understanding of counting and is reflected in a gradual shift from frequent use of counting-all to counting-on (Geary et al., 1992; Siegler, 1987).

The use of counting procedures appears to result in the development of memory representations of basic facts (Siegler & Shrager, 1984; but see Temple & Sherwood, 2002). Once formed, these long-term memory representations support the use of memory-based problem-solving, specifically direct retrieval of arithmetic facts and decomposition. With direct retrieval, children state an answer that is associated in long-term memory with the presented problem, such as stating "eight" when asked to solve 5 + 3. Decomposition involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem 6 + 7 might be solved by retrieving the answer to 6 + 6 (i.e., 12) and then adding 1 to this partial sum. The use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer. Children with a rigorous criterion only state answers that they are certain are correct, whereas children with a lenient criterion state any retrieved answer, correct or not (Siegler, 1988).

Figure15.1 shows the common developmental and schooling-based changes in the strategy mix. As the mix matures, children solve problems more quickly because they use more efficient memory-based strategies and because, with practice, it takes less time to execute each strategy (Delaney, Reder, Staszewski, & Ritter, 1998; Geary, Bow-Thomas, Fan, & Siegler, 1996; Lemaire & Siegler, 1995). The transition to memory-based processes results in the quick solution of individual problems and reductions in the working memory demands associated with solving these problems. The eventual automatic retrieval of basic facts and the accompanying reduction in working memory demands, in turn, facilitate the solving of more complex problems (Geary & Widaman, 1992; Geary, Liu, Chen, Saults, & Hoard, 1999).

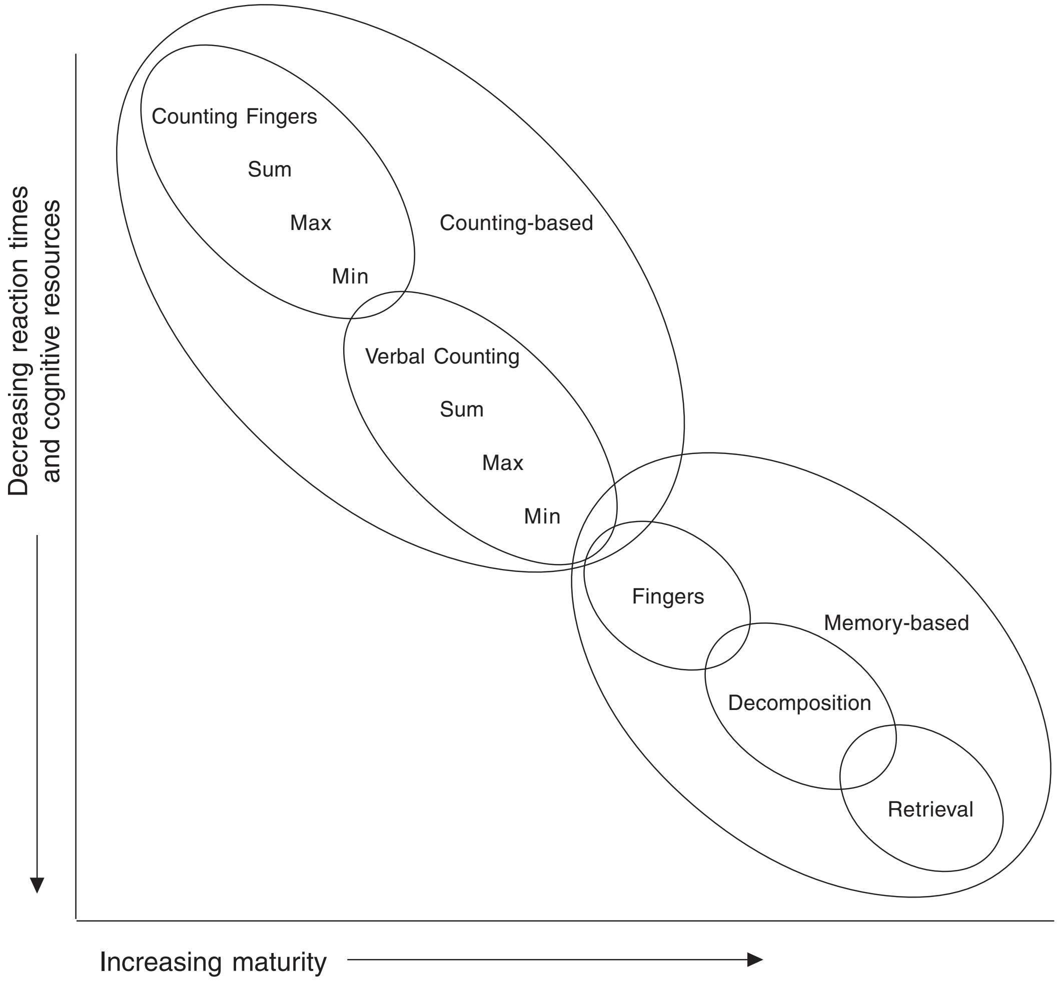

Figure 15.1. With development and schooling, children shift from the use of counting-based to memory-based processes to solve simple arithmetic problems.

Children with MD

When they solve simple arithmetic problems (4 + 3) and simple word problems, children with MD/RD and MD only use the same types of strategies (e.g., verbal counting) as normal children, but differ in the strategy mix and in the pattern of developmental change in this mix (Geary, 1990; Hanich et al., 2001; Jordan et al., 2003a; Jordan, Hanich, & Kaplan, 2003b). In comparison to their normal peers, children with MD often: (a) rely on developmentally immature strategies, such as finger counting, (b) frequently commit counting errors, (c) use immature counting procedures [they often use counting-all rather than counting-on], and (d) have difficulties retrieving basic facts from long-term memory. These differences have been found in the United States (Geary & Brown, 1991; Hanich et al., 2001; Jordan & Montani, 1997), in several European nations (Barrouillet, Fayol, & Lathuliere, 1997; Ostad, 1997, 1998a, 1998b, 2000; Svenson, & Broquist, 1975), and in Israel (Gross-Tsur et al., 1996).

As an example, Geary et al. (1999, 2000) and Jordan et al. (2003a; Jordan & Montani, 1997) found consistent differences in the strategy mix across groups of MD/RD, MD only, RD only children, and normal children. In first through third grade, children with MD only and children with MD/RD committed more counting errors and used the developmentally immature counting-all procedure more frequently than did the children in these other groups. The children in the RD only and normal groups showed a shift, across grades, from heavy reliance on finger counting to verbal counting and retrieval (see Figure 15.1). The children in the MD/RD and MD only groups, in contrast, did not show this shift, but instead relied heavily on finger counting, even in third grade (Jordan et al., 2003a). These patterns replicated previous studies of children with MD/RD and demonstrated that many of the same deficits, although to a lesser degree, are evident for children with MD only (Geary et al., 2000; Ostad, 1998a). There were, however, a few differences comparing MD only and MD/RD children: MD only children they made fewer counting procedure errors (Geary et al. 2000), and were more accurate at solving simple word problems, presumably because of better reading comprehension (Jordan et al., 2003a).

The most consistent finding in this literature is that children with MD/RD and MD only differ from their normal peers in the ability to use retrieval-based processes to solve simple arithmetic and simple word problems (Barrouillet et al.,1997; Garnett, & Fleischner, 1983; Geary, 1990, 1993; Hanich, et al., 2001; Jordan et al., 2003a; Jordan & Montani, 1997; Ostad, 1997, 2000; Temple & Sherwood, 2002). Unlike the use of counting strategies, it appears that the ability to retrieve basic facts does not substantively improve across the elementary-school years for many children with MD/RD and MD only. When these children do retrieve arithmetic facts from long-term memory, they commit many more errors and sometimes show error and reaction time (RT) patterns that differ from those found with younger, normal children (Barrouillet et al., 1997; Fayol, Barrouillet, and Marinthe, 1998; Geary, 1990; Geary & Brown, 1991; Räsänen & Ahonen, 1995). Moreover, these patterns are sometimes found to be similar to the patterns evident with children who have suffered from an early (before age 8 years) lesion to the left-hemisphere or associated subcortical regions (Ashcraft, Yamashita, & Aram, 1992). The overall pattern suggests that the memory retrieval deficits of children with MD/RD or MD only reflect a cognitive disability, and not, for instance, a lack of exposure to arithmetic problems, poor motivation, a low confidence criterion, or low IQ.

There are, at least, two potential sources of these retrieval difficulties, a deficit in the ability to represent phonetic/semantic information in long-term memory (Geary, 1993) or (and) a deficit in the ability to inhibit irrelevant associations from entering working memory during problem solving (Barrouillet et al., 1997). The latter form of retrieval deficit was first discovered by Barrouillet et al. (1997), based on the memory model of Conway and Engle (1994), and was recently confirmed in our laboratory (Geary et al. 2000; see also Koontz & Berch, 1996). In the Geary et al. study, one of the arithmetic tasks required children to only use retrieval—the children were instructed not to use counting strategies—to solve simple addition problems (see also Jordan & Montani, 1997). Children with MD/RD, MD only, as well as children with RD, committed more retrieval errors than did their normal peers, even after controlling for IQ. The most common error was a counting-string associate of one of the addends. For instance, common retrieval errors for the problem 6 + 2 were 7 and 3, the numbers following 6 and 2, respectively, in the counting sequence. Hanich et al. (2001) found a similar pattern, although the proportion of retrieval errors that were counting-string associates was lower than that found by Geary et al. (2000). A third potential source of the retrieval deficit is a disruption in the development or functioning of a more modularized—independent of phonetic/semantic memory and working memory—cognitive system for the representation and retrieval of arithmetical knowledge, including arithmetic facts (Butterworth, 1999; Temple & Sherwood, 2002).

Finally, research results on more complex forms of arithmetic are beginning to emerge (Fuchs & Fuchs, 2002, Jordan & Hanich, 2000), and appear to support the separation of MD only and MD/RD groups. In comparison to MD only children, children with MD/RD show more pervasive deficits as the problem complexity increases from simple operations to complex, multi-step story problems, although children in both groups demonstrate performance below “normal” peers.

FRAMEWORK FOR MD RESEARCH

As noted earlier, the complexity of the field of mathematics results in a very large number of potential sources of MD. In Figure 15.2 we present a conceptual scheme for focusing future MD research and for better understanding the number, counting, and arithmetic deficits we just described. As noted in the figure, competencies in any given area of mathematics will depend on a conceptual understanding of the domain and procedural knowledge that supports actual problem solving (Geary, 1994). Base-10 arithmetic is one example, whereby instruction focuses on teaching the conceptual foundation (i.e., the repeating number system based on sequences of 10) and related procedural skills, such as trading from the tens column to the units column while solving complex arithmetic problems (e.g., 243–129; Fuson & Kwon, 1992). Conceptual and procedural competencies, in turn, are supported by an array of cognitive systems, as shown in the bottom sections of Figure 15.2.

The central executive controls the attentional and inhibitory processes needed to use procedures during problem solving, and much of the information supporting conceptual and procedural competencies is likely to be represented in the language or visuospatial systems (Baddeley, 1986), although a distinct modular system for arithmetic has also been proposed (Butterworth, 1999; Temple & Sherwood, 2002). Geary (1995), in fact, proposed an evolved system for processing simple quantities, but this is more circumscribed than the system proposed by Butterworth and Temple and Sherwood, and would not include arithmetic facts.

In any case, Engle (2002) has argued and empirically demonstrated (Conway & Engle, 1994) that Baddeley's central executive is synonymous with the working memory components of attentional and inhibitory control, and thus working memory and central executive will be used interchangeably. The language systems are important for certain forms of information representation, as in articulating number words during the act of counting. These same systems may support the formation of associations between arithmetic problems and answers generated by counting, and thus a poor ability to represent information in these systems would, in theory, result in a fact retrieval deficit (Geary, 1993), although Temple and Sherwood (2002)

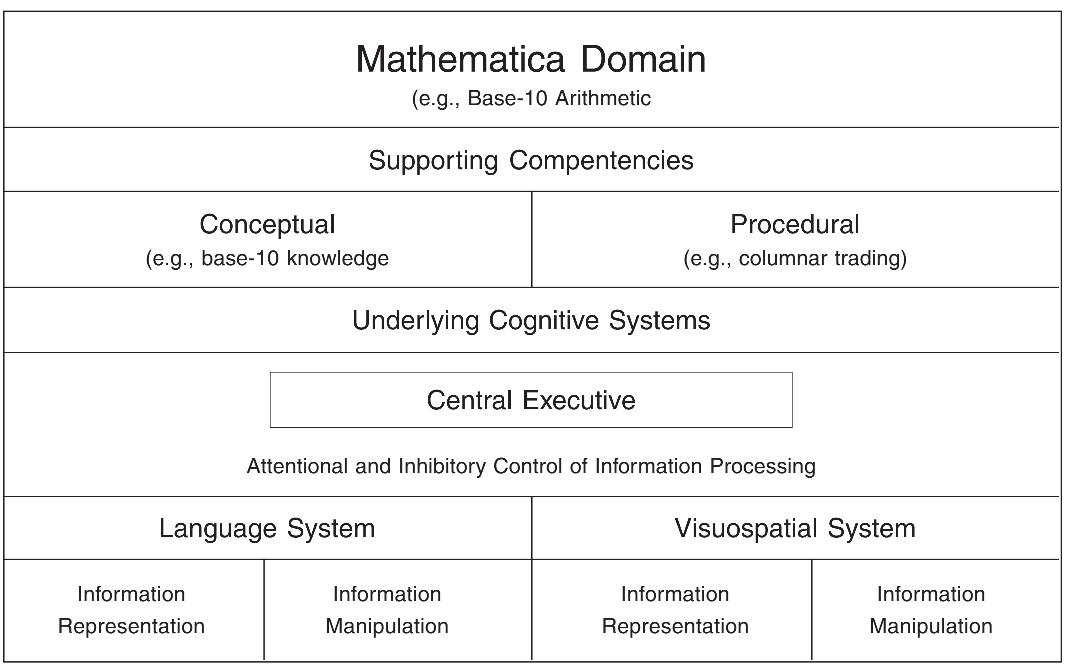

Figure 15.2. A conceptual framework for approaching the study of mathematical disabilities.

argued that the retrieval deficits are specific to the arithmetic module. The visuospatial system appears to be involved in representing some forms of conceptual knowledge, such as number magnitudes (Dehaene & Cohen, 1991), and for representing and manipulating mathematical information that is cast in a spatial form, as in a mental number line (Zorzi, Priftis, & Umiltá, 2002). On the basis of this framework, MD would be manifest—be evident while solving mathematics problems—as a deficit in conceptual and/or procedural competencies that define the mathematical domain. These conceptual or procedural deficits, in theory, could be due to underlying deficits in (1) the central executive and (or) (2) in the information representation or manipulation (i.e., changing the way the information is represented) systems of the language or visuospatial domains.

The organizational framework shown in Figure 15.2 can also be used to understand MD in the number, counting, and arithmetic domains we described earlier. What we do understand in these domains is outlined in Table 15.1 as a preliminary taxonomy of three subtypes of MD, specifically, procedural, semantic memory, and visuospatial. The taxonomy was developed based on an earlier review of the cognitive deficits of children with MD and related dyscalculia and behavioral genetic literatures (Geary, 1993). Eventually the taxonomy will need to be expanded to include all of the features shown in Figure 15.2 and for additional mathematical domains (e.g., algebra). The goal here is to try to understand the above described performance and cognitive patterns of children with MD/RD and MD only in terms of the procedural, semantic memory, and spatial subtypes in Table 15.1, and in terms of the systems shown in Figure 15.2. For ease of presentation, children with MD will refer to both MD only and MD/RD children, unless specific differences were found.

Organization of Cognitive Deficits

Procedural Deficits

As described earlier, a common procedural deficit of children with MD involves use of developmentally immature strategies, such as counting-all and finger counting, and miscounting when using these procedures to solve simple arithmetic problems. Potential sources of these procedural deficits include: (a) a poor conceptual understanding of counting concepts (Geary et al., 1992, 2000) and (or), (b) poor working memory/central executive resources (Hitch & McAuley, 1991). Geary et al. (1992) found that children with MD who have a delayed understanding of counting concepts rarely used the counting-on procedure. They seemed to believe that counting always had to start from 1 and thus relied on counting-all. Our current work indicates a strong relation between accuracy of procedural execution and performance on a measure of working memory/central executive functioning (Byrd-Craven, Hoard, & Geary, 2002), and confirms the relation between conceptual knowledge and procedural competency.

Semantic Memory Deficits

If a general deficit in the ability to retrieve information from long-term memory contributes to arithmetic fact retrieval deficits of children with MD, then these children should also show deficits on measures that assess skill at accessing other types of semantic information, such as words, from long-term memory (Geary, 1993). In fact, Geary argued that the comorbidity of MD and RD is related, in part, to difficulties in accessing both words and arithmetic facts from semantic memory, although the data on this are mixed (e.g., Jordan et al., in press, a). In terms of the organizing framework shown in Figure 15.2, the deficit would reside in the systems that support information representation in the language system. It seems that some MD children with fact retrieval deficits do have a language-representation deficit (Geary et al., 2000), but others may not (Jordan et al., 2003a).

Table 15.1
Subtypes of Learning Disabilities in Mathematics

	Cognitive and performance features	Neuropsychological features	Genetic features	Developmental features	Relation to RD
Procedural	Relatively frequent use of developmentally immature procedures (i.e., the use of procedures that are more commonly used by younger, normal children) Frequent errors in dysfunction procedural steps, a the execution of procedures Poor understanding of the concepts underlying procedural use Difficulties sequencing the multiple steps in complex procedures	Unclear, although some data suggest an association with left hemispheric dysfunction. In some cases, especially difficulties sequencing multiple improves across age prefrontal and grade)	Unclear	Appears, in many cases, to represent a developmental delay (i.e., performance is similar to that of younger, normal children, and often	Unclear
Semantic Memory	Difficulties retrieving mathematical facts, such as answers to simple arithmetic problems When facts are retrieved, there is a high error rate For arithmetic, retrieval errors are often associates of numbers in the problems (e.g., retrieving 4 to 2+3=?; 4 is the counting-string associate that follows 2, 3) RTs for correct retrieval are often unsystematic	Appears to be associated with left-hemispheric dysfunction, possibly the posterior regions for one form of retrieval deficit and the prefrontal regions for another Possible subcortical involvement, such as the basal ganglia	Appears to to be a heritable deficit	Appears to represent a developmental difference (i.e., cognitive and performance features differ from that of younger, normal children, and do not change substantively across age or grade)	Appears to occur with phonetic forms of RD
Visuospatial	Difficulties in spatially representing numerical and other forms of mathematical information and relationships Frequent misinterpretation of misunderstanding of spatially represented information	Appears to be associated with right hemispheric dysfunction, in cognitive particular, posterior regions of the right hemisphere, although the parietal cortex of the left hemisphere may be implicated as well	Unclear, although the and performance features are common with certain genetic disorders (e.g. Turner's syndrome)	Unclear	Does not appear to be related

As we described earlier, Barrouillet et al. (1997) argued that the retrieval deficit may result from response competition during the retrieval process (Conway & Engle, 1994; Engle, 2002; Koontz & Berch, 1996). As an example, presentation of the problem 4 × 5 not only prompts retrieval of 20, it also prompts retrieval of related, but irrelevant to this problem, numbers, such as 9 (4 + 5) and 25 (5 × 5; Campbell, 1995). There is now strong evidence that individuals with poor working memory/central executive resources have difficulties inhibiting these irrelevant associations (Engle, Conway, Tuholski, & Shisler, 1995). For these individuals, poor information retrieval is more strongly related to the central executive than to the language system per se. There is evidence that some children with MD do not inhibit irrelevant associations during fact retrieval (Geary et al., 2000), but evidence to date is not conclusive (Hanich et al., 2001). We are currently conducting studies that should help to clarify whether response competition, semantic retrieval, or some combination of these potential forms of cognitive deficit are the primary source or sources of the difficulties children with MD have with retrieving arithmetic facts. As noted, an alternative view is that the retrieval deficits are specific to an arithmetic module (Temple & Sherwood, 2002).

Spatial Deficits

As noted in Table 15.1, spatial deficits have been associated with misalignment of numbers when setting up arithmetic problems (e.g., writing 45 × 68 out horizontally) and in interpreting the positional, base-10 meaning of the numbers (Geary, 1993; Russell & Ginsburg, 1984). There are many other areas of mathematics that are dependent on spatial abilities (e.g., many subareas of geometry), including some basic competencies in the areas of number representation and comparison. We suggested earlier that children with MD might have difficulties spatially representing number magnitude, but this hypothesis has not yet been tested. Using the framework shown in Figure 15.2, difficulties might, in theory, reside in the ability represent number information in a spatial form or (and) in the ability to easily change spatial representations.

Dehaene and Cohen (1997), for instance, argued that magnitude representations of numbers are based on a spatial representation, specifically, a logarithmic number line. One way to represent this number line is in terms of an exponential increase in the distance between consecutive pairs of numbers, as contrasted with the equal distances between numbers in the formal Arabic system. The formal and learned relations are shown as the first number line in Figure 15.3, and the logarithmic number line is below this. In a cross sectional study of second, fourth, and sixth graders as well as college students, Siegler and Opfer (2003) showed that children's estimates of where numbers fit on a number line ranging from 1 to 100 or 1 to ,1000 fit either the formal or logarithmic model, but tended to shift toward the formal model

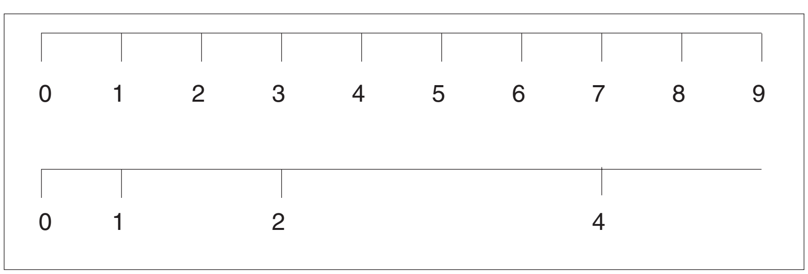

Figure 15.3. The top section shows spatial-based number representations that conform to the formal and school-taught Arabic system. The bottom section shows the spatial-based logarithmic form of number representation that is common before schooling.

in higher grades. The shift reflects a change in the representational format in the spatial domain. The results support Dehaene's and Cohen's model and suggest that magnitude representations can be spatially based.

These results are in keeping with the distinction in Figure15. 2 between information representation and the ability to change the form of this representation. The results also suggest two potential spatial-based forms of MD, as they might affect magnitude representation. The first would, in theory, manifest as a non-logarithmic form of spatial-based number representations. If children with MD show the normal—before schooling—logarithmic representations, then the second potential deficit would manifest as difficulties in changing the representational form to match the formal Arabic system. We are currently conducting pilot research to explore the likelihood of these two forms of spatial-based MD.

CONCLUSION

Over the past decade, a reasonable understanding of the number, counting, and arithmetical competencies and deficits of children with MD has emerged (Geary et al., 2000; Hanich et al., 2001; Ostad, 2000). Most of these children appear to have near-normal number-processing skills, at least for the processing of simple numbers (e.g., 3, 6), but their representational and processing skills for larger numbers (e.g., 345) remains largely unexplored, as does their ability to form spatial number lines. Whatever future studies might reveal in these areas, it is clear that children with MD have persistent deficits in some areas of arithmetic and counting knowledge. Many of these children have an immature understanding of certain counting principles. They often use problem-solving procedures in arithmetic that are more commonly used by younger, academically-normal children, and frequently commit procedural errors. For some of these children, procedural skills, at least as related to simple arithmetic, improve over the course of the elementary-school years and thus the early deficit may not be due to a permanent cognitive disability. At the same time, many children with MD also have difficulties retrieving basic arithmetic facts from long-term memory, a deficit that often does not improve for many of these children.

On the bases of the framework shown in Figure 15.2, these developmental delays and deficits can be understood as being related to a combination of disrupted functions of the central executive, including attentional control and poor inhibition of irrelevant associations, and (or) difficulties with information representation and manipulation in the language system. In theory, MD can also result from compromised visuospatial systems, although these potential forms of MD are not well understood. Some insights have also been gained regarding the potential neural mechanisms contributing to these procedural and retrieval characteristics of children with MD (see Geary & Hoard, 2001), although definitive conclusions must await brain imaging studies of these children.

Despite important advances during the past decade, much remains to be accomplished. In comparison to simple arithmetic, relatively little research has been conducted on ability of children with MD to solve more complex arithmetic problems (but see Russell & Ginsburg, 1984), and even less has been conducted in other mathematical domains. Even in the area of simple arithmetic, the cognitive and neural mechanisms that contribute to the problem-solving characteristics of children with MD are not fully understood. Other areas in need of attention include the development of diagnostic instruments for MD; cognitive and behavioral genetic research on the comorbidity of MD and other forms of LD and ADHD; and, of course, the development of remedial techniques. If progress over the past decade is any indication, then we should see significant advances in many of these areas in the years to come.

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