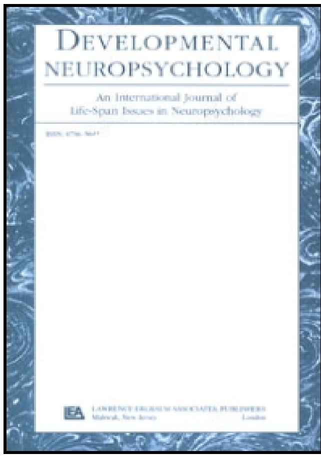


Developmental Neuropsychology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t775653638

# Mathematical Cognition in Intellectually Precocious First Graders

Mary K. Hoard [a]; David C. Geary [a]; Jennifer Byrd-Craven [a]; Lara Nugent [a]
[a] Department of Psychological Sciences, University of Missouri,

Online Publication Date: 01 May 2008

To cite this Article: Hoard, Mary K., Geary, David C., Byrd-Craven, Jennifer and Nugent, Lara (2008) 'Mathematical Cognition in Intellectually Precocious First Graders', Developmental Neuropsychology, 33:3, 251 — 276

To link to this article: DOI: 10.1080/87565640801982338
URL: http://dx.doi.org/10.1080/87565640801982338

DEVELOPMENTAL NEUROPSYCHOLOGY, *33*(3), 251–276

ISSN: 8756-5641 print / 1532-6942 online
DOI: 10.1080/87565640801982338

Psychology Press
Taylor & Francis Group

# Mathematical Cognition in Intellectually Precocious First Graders

Mary K. Hoard, David C. Geary, Jennifer Byrd-Craven, and Lara Nugent
*Department of Psychological Sciences*
*University of Missouri*

Forty-six intellectually precocious (*M* age = 74 months) and 250 intellectually typical (*M* age = 75 months) children were administered a standardized working memory battery, speed of processing measures, and tasks that assessed skill at number line estimation and strategies used to solve simple and complex addition problems. Precocious children had an advantage over same-age peers for all components of working memory, and used a more mature mix of strategies to solve addition problems and to make number line estimates; there were no group differences for speed of processing. Many of the advantages of the precocious children on the number line and addition strategy tasks were significantly reduced or eliminated when group differences in working memory were controlled. Individual differences analyses revealed that each of the three components of working memory contributed to different aspects of skilled performance on the mathematics tasks.

Strides have been made, during the past decade, in our understanding of the core quantitative and school-taught mathematical competencies of developing children (Campbell, 2005; Dowker, 2005; Geary, 2006; Royer, 2003). There have been particular gains in our understanding of the deficits of children with a learning disability in mathematics (MLD; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Jordan, Hanich, & Kaplan, 2003; Mazzocco & Delvin, in press; Murphy, Mazzocco, Hanich, & Early, 2007), the long-term scientific contributions of mathematically gifted individuals (Lubinski, Benbow, Webb, & Bleske-Rechek, 2006; Lubinski,

Jennifer Byrd-Craven is now at the Psychology Department of Oklahoma State University.
Correspondence should be addressed to Mary K. Hoard, Department of Psychological Sciences, University of Missouri, Psychology Building, 200 S. 7th St., Columbia, MO 65211-2500. E-mail: hoardm@missouri.edu

Webb, Morelock, & Benbow, 2001), and of sex differences in mathematics (Gallagher & Kaufman, 2005). Curiously absent are studies focusing on the mathematical cognition of intellectually precocious children. These are children who are typically at or above the 95th percentile on standard intelligence (IQ) tests and are common in gifted and enrichment programs throughout the United States.

Precocious children do well in academic settings (Lubinski, 2000; Walberg, 1984), will be over-represented among educated professionals in adulthood (Gottfredson, 1997), but, as a group, they differ in important ways from the mathematically gifted individuals studied by Lubinski et al. (2006); the latter group includes individuals with estimated IQ levels found in only about 1 in 10,000 individuals. The "typical" intellectually precocious child is much more common, representing about 1 in 20 to 1 in 50 children. In other words, there have been advances in our knowledge of mathematical development in unusually intelligent individuals and in low-achieving and typically achieving individuals, but little research has been conducted using samples of children that are common in school-based gifted and enrichment programs. Following the common practice for entrance into these (gifted and enrichment) programs, we identified intellectually precocious children based on IQ. Our mean IQ of 126 represents children in the top 5% of intellectual ability. Although the accelerated learning that is common in these children is often attributed to an above average IQ, this in and of itself does not shed light on the more basic mechanisms contributing to this learning.

One way to conceptualize the study of accelerated learning is in terms of a developmental difference versus developmental advance, and we put this conceptualization into practice with a study of mathematics learning. Evidence for the former would be provided if intellectually precocious children solved mathematics problems differently than intellectually typical children and if these difference were related to differential use of the three core working memory systems; the central executive, phonological loop and visuospatial sketch pad (Baddeley & Hitch, 1974). Dark and Benbow (1991) found evidence for enhanced visuospatial working memory in mathematically gifted adolescents, but it is not known whether the same advantage will be found for precocious children and, if so, whether this advantage will contribute directly to their expected advantages in mathematics (but see Bull, Espy, & Wiebe, 2008/this issue). Evidence for a developmental advance would be found if precocious children engaged the same cognitive mechanisms to solve the mathematics problems as their intellectually typical peers, but had an advantage for these mechanisms that in turn enabled the use of more sophisticated, developmentally advanced problem-solving strategies. A likely candidate is the central executive component of working memory, which is a core mechanism contributing to general fluid intelligence and contributes to mathematical learning (see Bull et al., 2008/this issue).

To test whether intellectually precocious children and intellectually typical children show differential use of working memory systems when solving mathematics problems, we administered an addition strategy assessment and a number line estimation task. Performance on the former task is related to the central executive, whereas performance on the latter appears to be related to both the central executive and visuospatial sketch pad (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). In the first section, we present an overview of general intelligence, or *g*, and how intelligence is related to working memory. In the second section, we present an overview of the mathematical tasks and related hypotheses.

## GENERAL INTELLIGENCE AND WORKING MEMORY

Nearly six decades after Spearman's (1904) discovery that strong performance on one mental test tends to be associated with strong performance on all other mental tests, and thus his term general intelligence, Cattell and Horn (Cattell, 1963; Horn & Cattell, 1966) proposed that *g* is more accurately described as being composed of two factors: crystallized intelligence (*g*C), and fluid intelligence (*g*F). Crystallized intelligence represents the knowledge base of facts, heuristics, and procedures acquired over a lifetime, and is dependent on long-term memory. Fluid intelligence represents the ability to flexibly reason, problem solve, and learn novel information, and is more strongly dependent on working memory. There are, of course, more specific abilities such as verbal, spatial, and numerical. These represent distinct knowledge bases that are components of *g*C but are often dependent on *g*F during problem solving (Carroll, 1993). The focus has shifted in recent decades to identifying the brain (Duncan et al., 2000; Kane & Engle, 2002) and cognitive (Engle, Tuholski, Laughlin, & Conway, 1999; Hunt, 1983) mechanisms that underlie *g*F and thus support the ability to learn in school (Geary, 2005, 2007). Among these cognitive mechanisms is working memory. In the first section, we review working memory and its underlying component mechanisms and in the second discuss the potential relation between these components and mathematical learning.

### Working Memory and *g*

Working memory, as described by Baddeley (1986, 2000, Baddeley & Hitch, 1974), is composed of a central executive and three representational systems; the phonological loop, visuospatial sketchpad, and episodic buffer. Nearly all of the experimental work has been conducted on the first two systems and thus the episodic buffer is not considered. The central executive is composed of the attentional and inhibitory mechanisms that control the manipulation of information activated in the echoic (phonological) and visuospatial systems. A relation between the central executive and *g*F has been firmly established: Performance on measures of *g*F are associated with individual differences in working memory (e.g., Carpenter, Just, & Shell, 1990; Engle et al., 1999). The strength of this relation ranges from moderate (*r*s ˜ 0.5; Ackerman, Beier, & Boyle, 2005) to very high (*r*s > 0.8; Conway, Cowan, Bunting, Therriault, & Minkoff, 2002). On the basis of these patterns, many scientists (Carpenter et al., 1990; Horn, 1988; Stanovich, 1999) have argued that measures of strategic problem solving and abstract reasoning define *g*F, and one of the underlying cognitive systems is attention-driven working memory.

The other mechanism that has been proposed as underlying *g*F is speed of processing (Jensen, 1998) as faster speed of cognitive processing is related to higher scores on measures of *g*. Variability in speed of processing across trials is also related to *g*. The variability measure provides an assessment of the consistency in speed of executing the same process multiple times. Individuals who are consistently fast in executing these processes have the highest IQ scores (Deary, 2000). The relations among speed of processing, working memory, and *g*F are vigorously debated. The issues center on whether individual differences in working memory are driven by more fundamental differences in speed of neural processing (Fry & Hale, 1996), or whether the attentional focus associated with the central executive speeds information processing (Engle et al., 1999). We cannot resolve this issue, but by simultaneously assessing three working memory systems and speed of processing, we were able to assess the overlapping and independent contributions of the corresponding measures to individual and group differences on our mathematical tasks.

## Working Memory and Mathematics

Much of research on the relation between working memory and mathematical competencies has been conducted with individuals with low mathematics achievement scores, including children with MLD (Bull & Johnston, 1997; Bull, Johnston, & Roy, 1999; Geary et al., 2004; Hitch & McAuley, 1991; McLean & Hitch, 1999; Swanson, 1993; Swanson & Sachse-Lee, 2001), and adults with mathematics anxiety (Ashcraft & Kirk, 2001). In much of this and related research (Geary & Widaman, 1992; Hitch, 1978; Tronsky & Royer, 2003), the central executive has been implicated as a crucial component underlying the ability to solve problems ranging from simple addition to multi-step algebraic word problems. Logie and colleagues have demonstrated that the phonological loop is engaged in quantitative processes that involve number articulation, as in counting (Logie & Baddeley, 1987; Logie, Gilhooly, & Wynn, 1994), whereas other processes involved in estimation of quantity, as in use of a number line, appear to engage the visuospatial sketch pad (Zorzi, Priftis, & Umilta, 2002).

Given the relations among working memory, speed of processing, and IQ, it is not surprising that in comparison to intellectually typical children, intellectually precocious children process information faster (Keating & Bobbitt, 1978) and perform better on complex problem-solving tasks that require working memory resources (Steiner & Carr, 2003). At the same time, the contributions of working memory and speed of processing to the likely advantage of intellectually precocious children on mathematical tasks is not well understood. As noted, there are a few studies of highly mathematically gifted adolescents (Dark & Benbow, 1990, 1991, 1994). What is not known is if enhanced visuospatial working memory will be found for intellectually precocious children.

## MATHEMATICAL COGNITION

In nearly all of the research on the quantitative competencies of intellectually precocious children and adolescents, participants have been selected on the basis of scores on mathematical achievement tests and not IQ per se (Dark & Benbow, 1990, 1991; Geary & Brown, 1991; Mills, Ablard, & Stumpf, 1993; Robinson, Abbott, Berninger, & Busse, 1996; Swanson, 2006); the estimated IQ of these participants range from above average (˜115; Robinson et al., 1996; Swanson, 2006) to exceptional (>150; Benbow, Stanley, Kirk, Zonderman, 1983). Comparisons of children selected based on performance on measures of *g*F and their intellectually typical peers on mathematical cognition tasks are not available in the literature, to the best of our knowledge. Moreover, with the exception of Dark and Benbow and Geary and Brown, most of the studies to date have examined the performance of mathematically precocious children on psychometric-type measures, and have not provided a fine-grained assessment of quantitative competencies using cognitive measures. We redress this lack of knowledge with an assessment of the performance of intellectually precocious children on cognitive tasks in the areas of number line estimation and addition strategy choices, and relate their performance to the core working memory subsystems.

### Number Line Estimation

Children appear to rely on two forms of representation to make number line estimations, that is, to place a given numeral (e.g., 32) in the correct position on a number line. The first form of representation is based on an inherent system for estimating magnitude and the second is based on the school-taught mathematical number line (Gallistel & Gelman, 1992; Siegler & Opfer, 2003). The inherent system results in representations that conform to the natural logarithm (*ln*) of the number, that is, the representations are compressed for larger magnitudes such that the perceived distance between 2 and 3 is larger than the perceived distance between 72 and 73. Learning of the formal mathematical number line, in contrast, involves forming representations that result in an equal distance between successive numbers regardless of position on the line (Siegler & Booth, 2004).

Brain imaging and neuropsychological studies suggest a link between visuospatial abilities and skill at making number line estimates (Pinel, Piazza, Le Bihan, & Dehaene, 2004; Zorzi et al., 2002). Mathematically precocious adolescents have an advantage in visuospatial working memory (Dark & Benbow, 1991), but it is not known if young intellectually precocious children have the same advantage and if so whether any such advantage will influence skill at making number line estimates.

## Addition Strategy Choices

Individual and group differences in the mix of strategies children use to solve addition and subtraction problems have been well documented for typically achieving and low-achieving children (Geary, 1990; Siegler & Shrager, 1984), but little is known about the addition strategy choices of intellectually precocious children. Typically achieving children use a combination of counting strategies and retrieval-based processes to solve these problems. The counting strategies involve a mix of finger counting and verbal counting. When counting, the two most commonly used procedures are termed min and sum (Fuson, 1982; Groen & Parkman, 1972). The min procedure involves stating the larger valued addend and then counting a number of times equal to the value of the smaller addend, such as counting 3, 4, 5 to solve 3 + 2. The sum procedure involves counting both addends starting from 1. Sometimes the max procedure is used, which involves stating the smaller addend and then counting the larger addend. The use of counting procedures appears to result in formation of an associative link in long-term memory between the answer (correct or not) generated by means of counting and the addends (Siegler & Shrager, 1984). Once formed, these representations support direct retrieval of the fact from long-term memory and decomposition (e.g., 5 + 7 = 5 + 5 = 10 + 2 = 12).

The mix of strategies children use to solve sets of simple addition problems is initially dominated by use of counting strategies, and frequent use of the sum or max procedures (Siegler, 1996; Siegler & Shrager, 1984). With schooling, children use the min procedure more frequently and eventually the mix becomes dominated by use of decomposition and retrieval. In many domains, intellectually precocious children are similar to their intellectually typical peers but are developmentally advanced (Siegler & Kotovsky, 1986). Geary and Brown (1991) found just this pattern in a sample of fourth graders. The intellectually precocious children used more retrieval and less counting to solve simple addition problems (e.g., 4 + 7) than did their intellectually typical peers. We extend this analysis to younger children, and to the solving of more complex (e.g., 17 + 6) problems, and assess whether there are group differences in the working memory system engaged during problem solving.

## CURRENT STUDY

The current study is one of the few that compares intellectually precocious children identified using a measure of *g*F to their intellectually typical peers on experimental tasks that assess two core mathematical competencies. Moreover, the inclusion of measures of working memory and speed of processing enabled us to simultaneously assess the potential mechanisms underlying individual differences in *g*F, their potential contributions to any group differences on the mathematical tasks, and whether any such group differences are related to the same (developmental advantage) or different (developmental difference) working memory systems.

## METHOD

### Participants

All kindergarten children from 12 elementary schools were invited to participate via a letter sent home to their parents and/or guardians in a longitudinal prospective study of the development of mathematical competencies in children with MLD (Geary, et al, 2007). Parental consent and child assent were received for 311 children, 37% of the recruited population. A subset (the 211 children classified as intellectually precocious or intellectually typical) of these recruited children was examined in the current study. The 46 (27 boys) children with IQ scores >120 composed the intellectually precocious group; $M = 126$ ($SD = 5$). The comparison group included 250 (110 boys) children with IQ scores between 81 and 120, inclusive; $M = 107$ ($SD = 8$). The ethnic breakdown differed across groups, $\chi^2(6) = 14.7$, $p < .05$; 80% and 11% of the intellectually precocious sample were white and Asian, respectively, and the most of remaining children were black or of mixed race. In comparison, 69% and 15% of the intellectually typical sample were white and black, respectively, and the most of remaining children were Asian or of mixed race. The mean ages of 74 ($SD = 4$) and 75 ($SD = 4$) months for the intellectually precocious and intellectually typical groups, respectively, did not differ ($p > .10$).

### Standardized Tests

*Intelligence.* All children were administered the Raven's Coloured Progressive Matrices, a non-timed power test that is considered to be an excellent measure of *g*F for children (Jensen, 1998). A percentile ranking was obtained for each child, using age-based norms (Raven, Court, & Raven, 1993), and these were converted to scores standardized with a mean of 100 ($SD = 15$).

## Working Memory

The Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001) consists of nine subtests that assess the three core working memory systems: central executive, phonological loop, and visuospatial sketchpad. All of the subtests have six items at each span level, and the maximum span per subtest ranges from 6 to 9. Passing four items at a level moves the child to the next level. At each span level, the number of items (e.g., words) to be remembered is increased by one. Failing three items terminates the subtest. The order of subtests was designed so as not to over-tax any one component area of working memory and was generally arranged from easiest to hardest: Digit Recall, Word List Matching, Word List Recall, Nonword List Recall, Block Recall, Mazes Memory, Listening Recall, Counting Recall, and Backward Digit Recall. Each subtest generates a span score and a trials correct score. From these, standard scores are determined for each subtest and for the three working memory systems.

*Central executive.* The central executive is assessed using three dual-task subtests, Listening Recall, Counting Recall, and Backward Digit Recall. Listening Recall requires the child to determine if a sentence (or series of sentences) is true or false, and then recall the last word in each sentence. Counting Recall requires the child to count a set of 4, 5, 6, or 7 dots on a card (or series of cards), and then to recall the number of counted dots at the end of the series of cards. Backward Digit Recall is a standard format backward digit span.

*Phonological loop.* The phonological loop is assessed using four subtests, Digit Recall, Word List Matching, Word List Recall, and Nonword List Recall. Digit Recall, Word List Recall, and Nonword List Recall are standard span tasks; specifically, the child's task is to repeat words in the same order as spoken by the experimenter. In the Word List Matching task, a series of words, beginning with two words and adding one word at each successive level, is presented to the child. The same words, but possibly in a different order, are then presented again, and the child's task is to determine if the second list is in the same or different order than the first list.

*Visuospatial sketch pad.* Block Recall and Mazes Memory assess the Visuospatial Sketchpad. Block Recall is another span task, but the stimuli consist of a board with nine raised blocks in what appears to the child as a "random" arrangement. The blocks have numbers on one side that can only be seen from the experimenter's perspective. The experimenter taps a block (or series of blocks), and the child's task is to duplicate the tapping in the same order as presented by the experimenter. In the Mazes Memory task, the child is presented with a maze that has more than one solution, as well as a picture of an identical maze with a path drawn of one solution from the center to the outside. The picture is removed and the child's task is to duplicate in her response booklet the path she was shown. At each level, the mazes get larger by one wall.

## Speed of Processing

The Rapid Automatized Naming (RAN) tasks were used to assess processing speed associated with retrieval of learned items from long-term memory (Denckla & Rudel, 1976). In this task, the child is presented practice items of five letters or numbers to determine if the child recognizes and can state the name tag for each stimulus. The child is then presented with a matrix of 50 incidences (5 ×10) of these same letters (or numbers), and is asked to name them as quickly as possible without making many mistakes. Each letter (or number) occurs 9 to 11 times in the matrix with the constraint that the same letter is never adjacent (vertically or horizontally) to itself. The matrix is presented in a large font so that the 50 items take up most of a standard sized 8 1/2" × 11" paper. RT is measured via a stopwatch, and errors as well as reversals for the letters b & d and p & q, are recorded; for each type of stimulus (letters or numbers), the task generates an RT and number correct (as well as number of reversals for letters). Errors and reversals were infrequent and thus only RTs were used in the analyses.

## Mathematics Tasks

*Numerical estimation.* Stimuli for this task were twenty-four 25 cm number lines, printed across the center of a standard 8 1/2" × 11" paper in a landscape orientation. Each number line had a start point of 0 and an endpoint of 100 with a target number printed approximately 5 cm above it in a large font (72 pt). Following Siegler and Booth (2004, experiment 1), target numbers were 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96, yielding 24 trials. The numbers below 30 were over-sampled to allow for a fitting of the logarithmic model to the child's estimates. Experimental stimuli were presented in a random order for each child, following the procedures described by Siegler and Booth (2004). For each target number in this task, the child was asked to mark its appropriate location on a blank number line using a pencil. There was no time constraint for this task.

*Addition strategy assessment.* Simple and complex addition problems were horizontally presented in a large font (about 2 cm tall), one at a time, at the center of a 5" by 8" card. The simple stimuli were 14 single-digit addition problems. The problems consisted of the integers 2 through 9, with the constraint that the same two integers (e.g., 2 + 2) were never used in the same problem. Across stimuli, 1/2 of the problems summed to 10 or less. The complex stimuli were 16 + 7, 3 + 18, 9 + 15, 17 + 4, 6 + 19, and 14 + 8.

Following two practice problems, the simple problems were presented followed immediately by the complex problems. The child was asked to solve each problem (without the use of paper and pencil) as quickly as possible without making too many mistakes. It was emphasized that the child could use whatever strategy was easiest to get the answer, and was instructed to speak the answer out loud. Based on the child's answer and the experimenter's observations, the trial was classified into 1 of 6 strategies; specifically, counting fingers, fingers, verbal counting, retrieval, decomposition, or other/mixed strategy (Siegler & Shrager, 1984). A mixed trial was one in which the child started using one strategy, but completed the problem using another strategy. Counting trials were further classified as min, sum, max, or other.

During problem solving, the experimenter watched for physical indications of counting, such as regular finger or mouth movements. For these trials, the experimenter initially classified the strategy as finger counting or verbal counting, respectively. On all counting trials, the experimenter probed the child as to how she counted, and the child's response was recorded. If the child held out a number of fingers to represent the addends and then stated an answer without counting them, then the trial was initially classified as fingers. If the child spoke the answer quickly, without hesitation, and without obvious counting-related movements, then the trial was initially classified as retrieval (or as decomposition if this was the child's predominant retrieval-based strategy on previous trials). The differentiation between decomposition and direct retrieval came from the child's description of how they solved the problem. For example, for 4 + 8, the child might say "I just knew it" (retrieval) or "I broke 4 into two 2s and added one to the 8 to get 10 and then added the other to get 12." (decomposition). After the child had spoken the answer, the experimenter queried the child on how the answer was obtained. If the child's response (e.g., "just knew it") differed from the experimenter's observations (e.g., saw the child mouthing counting), then a notation indicating disagreement between the child and the experimenter was made. If counting was overt, then the experimenter classified it as a counting strategy. If the trial was ambiguous, then the child's response was recorded as the strategy. In other words, for every trial, an initial attempt to code the strategy was made based on overt behaviors and speed of answering, but the final strategy coded was dependent on the child's description of how they got the answer. Previous studies indicate this method provides a useful measure of children's trial-by-trial strategy choices (e.g., Siegler, 1987).

## Procedures

The IQ test was administered at the end of kindergarten, and the mathematical tasks in a single session in the fall of first grade. The majority of children were tested in a quiet location at their school site, and occasionally in a testing room on the university campus or in a mobile testing van if the child moved between assessments. Each session averaged 30 to 40 min. The WMTB-C was added to the study in the summer following kindergarten and for this study, we were able to assess 44 and 167 of the children in the intellectually precocious and intellectually typical groups, respectively. For the majority of children, the battery was administered in the testing van during first grade. The assessment required about 60 min and occurred when the child was not in school. The mean ages at the time of the WMTB-C assessment were 81 (*SD* = 4) and 82 (*SD* = 6) months, respectively, for the intellectually precocious and intellectually typical groups ($p > .10$); age at time of administration is not correlated with performance on any of the working memory tests (Geary et al., 2007). Geary et al. (2007) included four mathematical tasks. They were given in the following order: Numerical Estimation (number line), Counting Knowledge, Number Sets, Addition Strategy Assessment. In this study, we focus on the number line and addition strategy choice because these are the most complex of these tasks.

## RESULTS

### Working Memory and Speed of Processing

For the age range assessed in this study, our sample of 211 children was larger than the standardization sub-sample of 86 children (Pickering & Gathercole, 2001). We thus standardized ($M = 100$, $SD = 15$) component scores for the Phonological Loop, Visuospatial Sketch Pad, and Central Executive scales for our sample and used these for subsequent analyses. A mixed ANOVA with group as the between-subjects variable and working memory system as the within-subjects variable revealed a significant effect for group, $F(1, 209) = 49.66$, $p < .0001$, but non-significant effects for working memory system, $F(2, 418) < 1$, and the interaction, $F(2, 418) < 1$. Follow-up ANOVAs confirmed the intellectually precocious children had higher standard scores for all working memory systems (*p*s < .0001), as shown in Table 1.

For letter naming, the mean RTs were 38 and 41 sec, respectively, for the intellectually-precocious and intellectually-typical groups, and the respective means for number naming were 41 and 43 sec. A 2 (group) by 2 (type: letters, numbers) mixed ANOVA revealed no significant effects (*p*s > .10) for group, $F(1, 279) = 1.76$, or for the type by group interaction, $F(1, 279) < 1$, but the effect for type was significant, $F(1, 279) = 13.09$, $p > .001$. The children were significantly faster at reading letters than numbers. Mean RTs (averaged across letter and number naming) were significantly (*p*s < .0001) correlated with working memory scores, such that faster speed of processing was associated with better performance on the central executive, $r(210) = 0.49$, visuospatial sketch pad, $r(210) = 0.30$, and phonological loop, $r(210) = 0.39$, measures. To determine the relations between speed of processing and working memory, we simultaneously regressed the mean RAN RT on the three working memory variables. Higher central executive scores were asso-

TABLE 1
Working Memory Scores

| | *Working Memory System* | | | | | |
|---|---|---|---|---|---|---|
| | *CE* | | *VSSP* | | *PL* | |
| | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| IT | 97 | 14 | 97 | 14 | 97 | 15 |
| IP | 112 | 14 | 110 | 15 | 110 | 11 |

*Note*. Note. IT = intellectually typical; IP = intellectually precocious; Phonological loop (PL), visuospatial sketchpad (VSSP), and central executive (CE).

ciated with faster speed of processing, $\beta = .36$, $t(206) = 4.42$, $p < .0001$, but scores on the phonological loop and visuospatial sketch pad variables were unrelated to RAN RT (*p*s > .10) when performance on the central executive was controlled.

## Mathematics Tasks

We first explored group differences in task performance and then individual and group differences in the cognitive mechanisms potentially related to this performance. The mechanisms we explored were the three working memory systems and speed of processing; the latter was indexed by RAN RT. For the mechanism analyses, all variables were standardized ($M = 100$, $SD = 15$) and parameters from the mathematics tasks (e.g., % correct) were regressed on RAN RT and working memory scores. If one or several of these variables predicted individual differences in mathematics-task performance and there were significant group differences in this performance, then a group variable was added to the regression equation. This allowed us to determine the extent to which group differences in working memory or speed of processing influenced group differences in mathematics task performance. Although the group difference for RAN RT was not significant, it was in the expected direction and thus was included in these analyses.

## Numerical Estimation

*Group differences.* Using the techniques described by Siegler and Opfer (2003), median estimates for each group were fitted to the logarithmic and linear models, as shown in Figure 1. For the intellectually typical group, both the logarithmic ($R^2 = .95$) and linear models ($R^2 = .92$) provided a good fit to the medians; the difference in fit was not significant, $t(23) = 0.92$, $p > .50$. For the intellectually precocious group, the linear model ($R^2 = .97$) fitted the medians better than did the logarithmic one ($R^2 = .91$), $t(23) = -2.29$, $p < .05$.

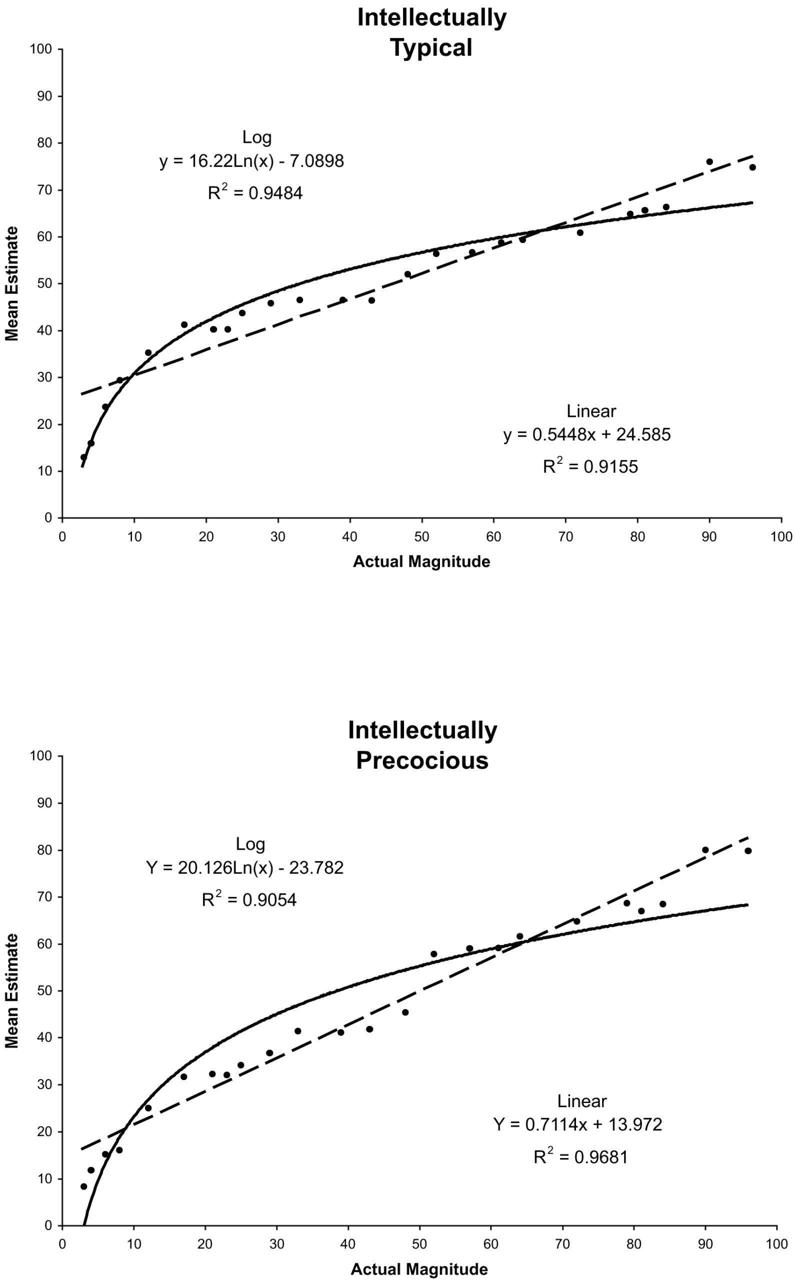


FIGURE 1 Logarithmic and linear fits to median number line estimates.

To estimate trial-by-trial variation in the representation used to make the estimate, we calculated the absolute difference between each child's estimate for each trial—where on the number line they placed the mark—and an expected estimate if they were using a linear or log representation. For the linear representation, we used the actual magnitude for the trial, and for the log representation we used $16Ln(x) - 7$, where $x$ = the value to be estimated. This equation is based on the log model fitted to the medians of the intellectually typical group. As an example, for the numeral 23, the expected estimate if the child is using a linear representation is 23, whereas the expected estimate if the child is using a log representation is 43. If the estimate was closer to 23 than to 43, the trial was classified as linear, and if the estimate was closer to 43, the trial was classified as log. When the expected value for the linear and log models differed by less than $\pm 5$ or the child's estimate was not clearly better fitted by the linear or log model (i.e., the deviation from the log and linear fits was $< \pm 5$) then the trial was classified as ambiguous.

The percentages of trials classified as linear, log, or ambiguous are shown in Table 2. A mixed ANOVA with group as the between-subjects variable and strategy (linear or log) as the within-subjects variable revealed significant ($p$s < .001), strategy, $F(2, 558) = 30.44$, and group by strategy, $F(2, 558) = 8.41$, effects. Follow-up ANOVAs revealed significant group effects ($p$s < .05) for each strategy. The interaction emerged because the magnitude of the group difference varied across strategy, with the largest difference for frequency of log trials. The four rightmost columns of Table 2 show the absolute degree of fit for linear and log trials. For example, when the children in the intellectually typical group used a linear representation, their estimate deviated from the actual linear value by an average of 7, as compared to a deviation of 5 for the intellectually precocious children. A mixed ANOVA with group as the between-subjects variable and strategy (linear, log) as the within-subjects variable revealed significant effects ($p$s < .002) for group, $F(1, 258) = 14.47$, strategy, $F(1, 258) = 34.92$, and group by strategy, $F(1, 258) = 10.51$. Follow-up ANOVAs confirmed an advantage for the intellectually precocious group for both linear, $F(1, 258) = 5.19$, $p < .05$, and log, $F(1, 258) = 15.39$, $p <$

TABLE 2
Number Line Strategy and Fit Accuracy

| | *Strategy %* | | | *Fit Difference* | | | |
|---|---|---|---|---|---|---|---|
| | *Linear* | *Log* | *Amb* | *Linear* | | *Log* | |
| | *M* | *M* | *M* | *M* | *SD* | *M* | *SD* |
| IT | 33 | 48 | 19 | 7 | 4 | 12 | 8 |
| IP | 40 | 35 | 24 | 5 | 3 | 7 | 4 |

*Note*. IT = intellectually typical; IP = intellectually precocious. Amb = ambiguous trial; strategy % may not = 100 due to rounding.

.0001, trials. The intellectually precocious children were more accurate than the intellectually normal children on this task, independent of whether they used a linear or logarithmic strategy. The interaction emerged because the group difference in fit was larger for log than for linear trials.

*Cognitive mechanisms.* RAN RT and the three working memory variables were used as predictors of percent use of the linear and log strategies and mean difference in accuracy of the estimates for these strategies. Higher central executive, $\beta = .19$, $t(205) = 1.82$, $p = .07$, and higher visuospatial sketch pad, $\beta = .15$, $t(205) = 1.75$, $p = .08$, scores were associated with more frequent use of the linear strategy, whereas lower central executive, $\beta = -.18$, $t(205) = -1.79$, $p = .07$, and lower visuospatial sketch pad, $\beta = -.17$, $t(205) = -2.04$, $p < .05$, scores were associated with more frequent use of the log strategy. In the final equation, the group difference in use of the linear strategy was reduced to non-significance, $F(1, 204) = 1.27$, $p > .25$, as was the group difference in use of the log strategy, $F(1, 204) = 3.15$, $p > .05$.

None of the variables emerged as a significant predictor of accuracy of linear strategy estimates, but higher central executive, $\beta = -.23$, $t(199) = -2.40$, $p < .02$, and phonological loop, $\beta = -.15$, $t(199) = -1.75$, $p = .08$, scores were associated with more accurate estimates (i.e., a smaller difference comparing the predicted value and the child's estimate) when using the log strategy. The group difference in accuracy on log trials remained significant, $F(1, 198) = 4.90$, $p < .05$, but the variance explained by group was reduced by 73%.

## Addition Strategy Choices

*Group differences.* For children in both groups, most of the simple and complex problems were solved using the counting fingers, verbal counting, retrieval, or decomposition strategies, and thus only these were analyzed. The strategy mix, error percentages, and use of the min procedure for the finger and verbal counting strategies are presented in Figure 2.

When solving simple problems, intellectually precocious children used decomposition more frequently their peers, $F(1, 279) = 15.43$, $p < .0001$, and used the min procedure more frequently when finger $F(1, 251) = 10.27$, $p < .01$, and verbal $F(1, 200) = 7.31$, $p < .01$ counting. The intellectually precocious children committed fewer errors than the intellectually typical children for counting fingers, $F(1, 251) = 4.41$, $p < .05$, verbal counting, $F(1,200) = 4.62$, $p < .05$, and retrieval, $F(1, 177) = 12.69$, $p < .001$, but not for decomposition $F(1, 68) < 1$.

When solving complex problems, the intellectually typical children used retrieval more frequently than the intellectually precocious children, $F(1, 279) = 4.61$, $p < .05$, but their mean error rate of 79% indicated they were largely guessing; the intellectually precocious children committed fewer retrieval errors, $F(1,$

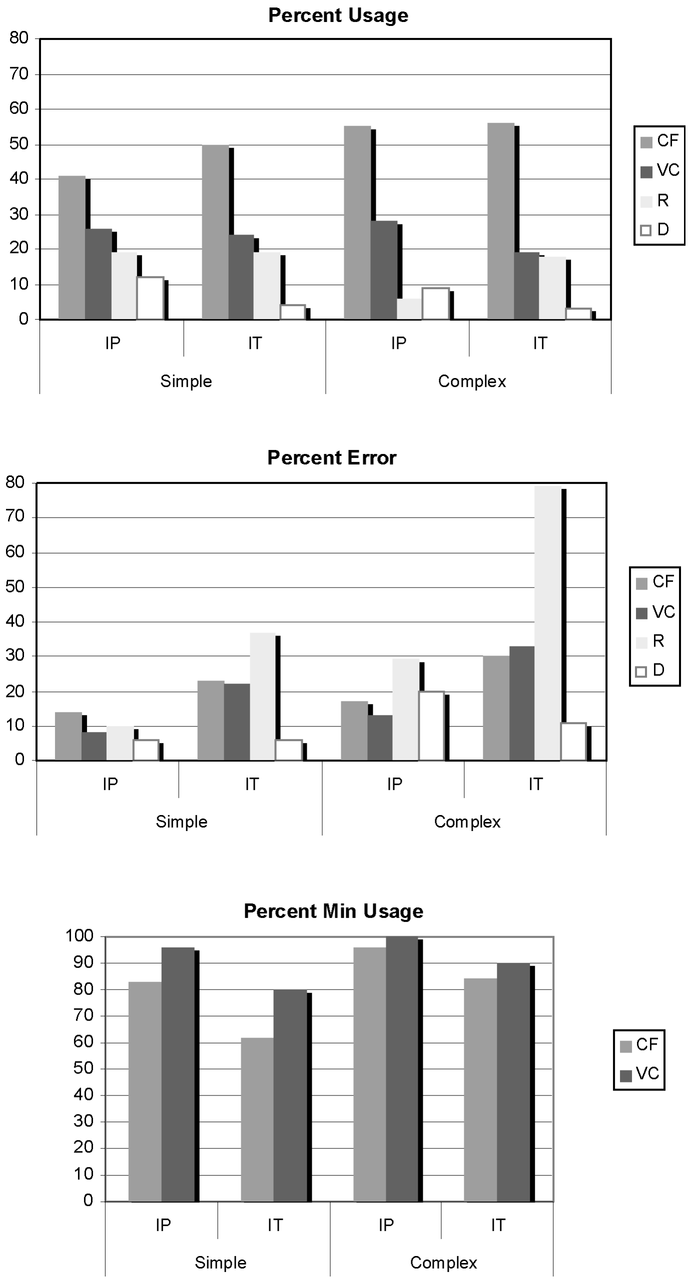


FIGURE 2 Strategy choices, error percentage, and use of the min procedure for solving simple and complex addition problems.

77) = 10.86, $p < .01$. The intellectually precocious children again used decomposition more frequently than did the intellectually typical children, $F(1, 279) = 3.97$, $p < .05$. During finger counting, the intellectually precocious children used the min procedure more often, $F(1, 205) = 4.20$, $p < .05$, and with fewer errors, $F(1, 205) = 4.08$, $p < .05$. The same was found for verbal counting; $F(1, 115) = 3.37$, $p < .08$, for min usage, and $F(1,115) = 6.09$, $p < .02$, for error percent.

Shifts in the strategy mix comparing the simple and complex problems were examined by means of multiple repeated measures ANOVAs for each strategy (Geary et al., 2004). Specifically, 2 (complexity) by 2 (group) ANOVAs with complexity as the within-subjects variable and group as the between-subjects variable confirmed significant ($p$s < .0001) changes in strategies usage across the simple and complex problems for all four strategies: finger counting, $F(1, 279) = 90.79$, verbal counting, $F(1, 279) = 92.37$, retrieval, $F(1, 279) = 72.53$, and decomposition, $F(1, 279) = 60.35$. Group differences emerged for use of decomposition, $F(1, 279) = 12.30$, $p < .001$, and the complexity by group interaction for finger counting, $F(1, 279) = p < .06$, and decomposition $F(1, 279) = 14.38$, $p <.001$. In short, the shift from simple to complex problems resulted in less decomposition and more finger counting and the magnitude of this shift was larger for the intellectually precocious children than for their intellectually typical peers.

*Cognitive mechanisms.* For simple problems, higher visuospatial scores were associated with more frequent use of verbal counting, $\beta = .18$, $t(205) = 2.09$, $p < .05$, and decomposition, $\beta = .18$, $t(205) = 2.14$, $p < .05$. Higher phonological loop scores were associated with fewer finger counting errors, $\beta = -.21$, $t(182) = -2.42$, $p < .02$, and more frequent use of the min procedure when finger counting, $\beta = .19$, $t(182) = 1.94$, $p = .054$. Slower speed of processing (i.e., higher RT values) was associated with more frequent errors when children used finger counting, $\beta = .18$, $t(182) = 2.13$, $p < .05$, verbal counting, $\beta = .45$, $t(142) = 4.58$, $p < .001$, and decomposition, $\beta = .43$, $t(45) = 2.20$, $p < .05$. Slower speed of processing was also associated with less frequent use of the min procedure when verbal counting, $\beta = -.25$, $t(142) = -2.32$, $p < .05$. The only effects for the central executive were two trends ($p$s < .08); higher scores were associated with less frequent verbal counting ($\beta = -.20$) and fewer retrieval errors ($\beta = -.22$). Controlling for working memory and speed of processing eliminated the group differences for finger counting, verbal counting, and retrieval errors ($p$s > .15), but the group differences in use of decomposition and the min procedure for finger counting remained significant ($p$s < .01). There was a trend for the group difference in use of the min procedure when counting verbally ($p = .08$).

The central executive emerged as a significant predictor of strategy choices for complex addition problems; higher scores were associated with less frequent use of retrieval, $\beta = -.20$, $t(205) = -2.01$, $p < .05$, and fewer retrieval errors, $\beta = -.58$, $t(56) = -2.85$, $p < .01$. The visuospatial sketch pad did not emerge as a significant predictor of any strategy variable, but higher phonological loop scores were associated with more frequent verbal counting, $\beta = .18$, $t(205) = 1.92$, $p = .057$, fewer finger counting errors, $\beta = -.18$, $t(147) = -1.92$, $p = .057$, and more frequent use of the min procedure when finger counting, $\beta = .23$, $t(147) = 2.07$, $p < .05$. Slower speed of processing was associated with more frequent finger counting errors, $\beta = .55$, $t(147) = 5.99$, $p < .001$, and less frequent use of the min procedure when finger counting, $\beta = -.29$, $t(147) = -2.71$, $p < .01$. Controlling for working memory and speed of processing eliminated the group differences in use of retrieval, and frequency of finger counting errors and min usage when finger counting ($p$s > .15). The group difference in use of retrieval errors remained significant ($p < .005$).

## DISCUSSION

The research provides one of the few studies of the mix of strategies used by intellectually precocious children to solve addition problems, the first study of their abilities to estimate on a number line, and one of the few studies of the potential mechanisms underlying their advantages in these areas. The current study also complements research on precocious children and adolescents that have been identified on the basis of mathematics achievement and ability scores rather than IQ per se (Dark & Benbow, 1990, 1991, 1994; Geary & Brown, 1991; Swanson, 2006). In the first section, we review group and individual differences in the basic mechanisms—working memory and speed of processing—that appear to contribute to variation in *g*F. In the second section, we focus on the mathematics tasks and the potential relation between working memory, speed of processing, and group and individual differences for performance on these tasks.

### Working Memory and Speed of Processing

In keeping with studies of the relation between working memory and *g*F (Carpenter et al., 1990; Engle et al., 1999), our intellectually precocious children had higher scores on the central executive component of working memory than did their intellectually typical peers. As found by Dark and Benbow (1991) for mathematically precocious adolescents, our intellectually precocious sample also had an advantage in visuospatial working memory. We did not, however, find evidence for differential enhancement of visuospatial working memory relative to verbal working memory (i.e., the phonological loop), as implicated in Dark and Benbow's (1991, 1994) contrasts of mathematically precocious and verbally precocious adolescents. Rather, our intellectually precocious children had a roughly 1 *SD* advantage for all three working memory systems. The different findings for our study as compared to those of Dark and Benbow are not surprising given that the individuals studied by Benbow and colleagues are profoundly intellectually precocious and fall well above the top 1% in intelligence, whereas our group of intellectually precocious children includes those in the top 5% to 10% of cognitive ability.

On the basis of the relation between speed of processing and *g*F (Hunt, 1978; Jensen, 1998), we expected and found that the intellectually precocious children were slightly faster than their intellectually typical peers for speed of accessing letter and number names from long-term memory, but the group differences did not reach statistical significance. Even so, the often found correlation between speed of processing measures and measures of *g*F (e.g., Fry & Hale, 1996) was confirmed for our sample. Zero order correlations indicated that faster speed of processing was associated with enhanced performance for each of the three working memory systems. However, the results for the simultaneous regression equation indicated that once performance on the central executive variable was controlled, the relation between speed of processing, as measured by RAN RT, and performance on the phonological loop and visuospatial sketch pad variables were no longer significant. Although cause and effect cannot be determined from these data, the pattern of results suggests that the attentional and inhibitory control associated with the central executive may influence the speed of information representation and manipulation in the echoic (phonological) and visuospatial representational systems, in keeping with the working memory model of Engle and colleagues (Engle, 2002; Engle et al., 1999; Kane & Engle, 2002).

## Mathematical Cognition

*Number line estimation.* Siegler and colleagues have found that first graders rely on the logarithmic representation to make number line placements and rely on the linear mathematical representation in later grades (Siegler & Booth, 2004, Siegler & Opfer, 2003). The grade-related pattern was mirrored by differences between our intellectually typical children and their intellectually precocious peers. Although we found evidence in both groups for use of a mixture of log and linear strategies in making number line placements, as a group and as individuals the intellectually precocious children showed more frequent use of the linear than the log representations, whereas the intellectually typical children showed the opposite pattern. Even when the groups used the same strategy, the intellectually precocious children were more accurate in their estimates when using both the log and linear representations.

The finding that individual differences in visuospatial working memory support use of linear and log representations to make the number line estimates is consistent with neuroimaging and neuropsychological studies that have implicated areas of the parietal cortex and visuospatial system for making magnitude estimates, including number line estimates (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Zorzi et al., 2002). The intellectually typical children's heavy reliance on the log representation to make these estimates is consistent with use of an inherent but inexact magnitude estimation system (Gallistel & Gelman, 1992). The finding that individual differences for the phonological loop were associated with accuracy of the log estimates is consistent with our observation that some of the children adjusted their initial estimates through counting. In other words, the intellectually typical children appear to be making their estimates using an analogue representation of relative magnitude, in conjunction with newly acquired rote counting skills, but have yet to map a one-to-one correspondence of the "space" each digit represents on the number line with the digit itself and in relation to the string of natural numbers.

The schooling-based mechanisms that contribute to the development and use of the linear mathematical representation of the number line are not known, but have been predicted to be dependent, in part, on the attentional control systems of the central executive (Geary, 2005, 2007). The current data do not allow for a direct assessment of this hypothesis, but are nonetheless consistent with it: Higher central executive scores were associated with more frequent use of linear representations and less frequent use of log representations. We suggest that competencies associated with the central executive are important for inhibition of the natural and presumably automatically invoked log representation and explicit construction and later use of the linear representation. In any case, the advantages of the intellectually precocious children on this task were related to their advantages in the central executive and visuospatial sketch pad.

*Addition strategy choices.* This study replicated and extended the strategy use findings reported by Geary and Brown (1991) for fourth graders. In both studies, typical and precocious children used the same mix of problem solving strategies—finger counting, verbal counting, retrieval, and decomposition—but the intellectually precocious children used a more mature strategy mix. However, in contrast to Geary and Brown's finding that the advantage of mathematically precocious children was largely reflected in their frequent use of direct retrieval, we found that decomposition emerged as a favored strategy among the intellectually precocious children; Geary and Brown reported a very low frequency of decomposition. Whether the difference across studies is due to changes in curriculum emphasis, comparisons across different grades, or refinement of the task itself and associated improvement in sensitivity to the use of decomposition is not known.

In any case, intellectually precocious children in the current study not only accurately used a more mature strategy mix than did their peers to solve simple and more complex addition problems, they more successfully shifted in the use of memory-based processes to counting-based strategies in response to the shift from simple to complex problems. Siegler (1996) termed this process adaptive strategy choice; that is, when complexity of the problem changes, adaptive choices involve using strategies that are most likely to produce the correct answer. In this case, we demonstrated that intellectually precocious children used relatively more memory-based processes to solve simple problems and relatively more counting to solve complex problems. The shift is adaptive, because use of counting is more likely to yield a correct answer when solving complex problems than is the use of memory-based processes. The intellectually typical children did not make this adaptive shift and as a result committed many more errors than their precocious peers when attempting to solve the complex problems and in comparison to their own performance when they solved the simple problems.

The relations among individual and group differences in speed of processing, working memory, and the sophistication of the strategy mix and accuracy in strategy execution differed in some ways from patterns that have emerged with the study of children with a learning disability in mathematics (MLD). Contrasts of children with MLD, low achieving, and typically achieving children have revealed the central executive to be an important contributor to group differences in strategies, and associated error rates, used to solve simple addition problems (Geary et al., 2007), but this relation did not emerge in the current study. Rather, speed of processing and the phonological loop were more consistently related to strategy characteristics and to group differences in error rates for simple addition. For these problems, the consistent relation between individual and group differences in the phonological loop and use of counting strategies is in keeping with the findings of Logie and colleagues (Logie & Baddeley, 1987; Logie et al., 1994).

Speed of processing and the phonological loop remained consistent predictors of individual and group differences in use of finger counting and frequency of finger counting errors when the children were solving more complex addition problems; the phonological loop also contributed to individual differences in use of verbal counting. The central executive emerged as important for the inhibition of the largely inaccurate retrieval process and thereby likely contributed to the use of more accurate counting processes. Skilled use of counting to solve complex problems, in turn, was dependent on speed of processing and the phonological loop.

## SUMMARY AND CONCLUSION

It has been known for some time that intellectually precocious children learn academic material at a faster pace than their more typical peers (Siegler & Kotovsky, 1986). Less is known about the cognitive processes that underlie this advantage in specific academic areas and the related cognitive processes. We found that intellectually precocious children use the formal mathematical and linear representation to make number line placements more frequently than their more intellectually typical peers. The precocious children also use more sophisticated strategies to solve addition problems and more easily make adaptive shifts in the mix of problem-solving strategies with changes in problem complexity. The group differences in strategic approaches to the number line and addition tasks are consistent with a developmental advance and not a developmental difference. This is because children in both groups used the same types of strategies to solve the problems, but differed in the maturity and accuracy of the strategy mix; the strategy mix and accuracy of strategy execution were several years ahead of the intellectually typical children, but did not differ in kind.

Unlike the findings of Dark and Benbow (1991) for mathematically gifted adolescents, we did not find an especially advanced visuospatial working memory system for our precocious sample. Rather, our sample showed an across the board advantage for all three core working memory systems, in keeping with a developmental advance and not a difference. At the same time, and unlike studies of children with MLD—where the central executive is a core deficit across mathematical areas—the advantages of the intellectually precocious children on the mathematics tasks were differentially related to each of the three components of working memory. The precocious children not only had an advantage for each working memory system they also appeared to be better at differentially allocating these resources across the mathematical tasks, that is, they appeared to have an advantage in the ability to engage one system or another contingent on task demands. Whether contingent use of different constellations of working memory systems represents a developmental advance or a developmental difference remains to be determined.

## ACKNOWLEDGMENTS

This study was supported by grants R01 HD38283 from the National Institute of Child Health and Human Development (NICHD), and R37 HD045914 co-funded by NICHD and the Institute for Education Sciences. We thank the Columbia Public Schools, and the many teachers, students, parents, and administrators. Linda Coutts, the K–5 mathematics coordinator, has been instrumental in encouraging and coordinating our research in the schools. We also thank staff and research assistants of the Cognitive Development Lab: Kendra Andersen, Catherine Desoto, Sara Ensenberger, Matthew Felton, Larissa Haggard, Rebecca Hale, Mary Lemp, Patrick Maloney, Chatavee Numtee, Catherine Ford O'Connor, Amanda Shocklee, Jennifer Smith, and Ashley Stickney.

## REFERENCES

Ackerman, P. L., Beier, M. E., & Boyle, M. O. (2005). Working memory and intelligence: The same or different constructs? *Psychological Bulletin, 131*, 30–60.

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General, 130*, 224–237.

Baddeley, A. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Sciences, 4,* 417–423.

Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. H. Bower (ed.), *The psychology of learning and motivation: Vol. 8. Advances in research and theory* (pp. 47–89). New York: Academic Press.

Baddeley, A. D. (1986). *Working memory.* Oxford: Oxford University Press.

Benbow, C. P., Stanley, J. C., Kirk, M. K., & Zonderman, A. B. (1983). Structure of intelligence in intellectually precocious children and their parents. *Intelligence, 7*, 129–152.

Bull, R., Espy, K. A., & Wiebe, S. A. (2008/this issue). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology, 33,* 205–228.

Bull, R. & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65*, 1–24.

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology, 15*, 421–442.

Campbell, J. I. D. (ed.) (2005). *Handbook of mathematical cognition*. New York: Psychology Press.

Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review, 97,* 404–431.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor analytic studies.* New York: Cambridge University Press.

Cattell, R. B. (1963). Theory of fluid and crystallized intelligence: A critical experiment. *Journal of Educational Psychology, 54*, 1–22.

Conway, A. R. A., Cowan, N., Bunting, M. F., Therriault, D. J., & Minkoff, S. R. B. (2002). A latent variable analysis of working memory capacity, short-term memory capacity, processing speed, and general fluid intelligence. *Intelligence, 30,* 163–183.

Dark, V. J., & Benbow, C. P. (1990). Enhanced problem translation and short-term memory: Components of mathematical talent. *Journal of Educational Psychology, 82*, 420–429.

Dark, V. J., & Benbow, C. P. (1991). Differential enhancement of working memory with mathematical versus verbal precocity. *Journal of Educational Psychology, 83*, 48–60.

Dark, V. J., & Benbow, C. P. (1994). Type of stimulus mediates the relationship between working-memory performance and type of precocity. *Intelligence, 19*, 337–357.

Deary, I. J. (2000). *Looking down on human intelligence: From psychophysics to the brain*. Oxford, UK: Oxford University Press.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284*, 970–974.

Denckla, M. B., & Rudel, R. (1976). Rapid automatized naming (RAN): Dyslexia differentiated from other learning disabilities. *Neuropsychologia, 14*, 471–479.

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. Hove, UK: Psychology Press.

Duncan, J., Rüdiger, J. S., Kolodny, J., Bor, D., Herzog, H., Ahmed, A., et al. (2000). A neural basis for general intelligence. *Science, 289*, 457–460.

Engle, R. W. (2002). Working memory capacity as executive attention. *Current Directions in Psychological Science, 11*, 19–23.

Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). Working memory, short-term memory, and general fluid intelligence: A latent-variable approach. *Journal of Experimental Psychology: General, 128*, 309–331.

Fry, A. F., & Hale, S. (1996). Processing speed, working memory, and fluid intelligence: Evidence for a developmental cascade. *Psychological Science, 7*, 237–241.

Fuson, K. C. (1982). An analysis of the counting-on solution procedure in addition. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (eds.), *Addition and subtraction: A cognitive perspective* (pp. 67–81). Hillsdale, NJ: Erlbaum.

Gallagher, A. M., & Kaufman, J. C. (eds.) (2005). *Gender differences in mathematics: An integrative psychological approach.* Cambridge UK: Cambridge University Press.

Gallistel, C. R., & Gelman, R. (1992), Preverbal and verbal cognition and computation*. Cognition, 44*, 43–74.

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49*, 363–383.

Geary, D. C. (2005). *The origin of mind: Evolution of brain, cognition, and general intelligence*. Washington, DC: American Psychological Association.

Geary, D. C. (2006). Development of mathematical understanding. In D. Kuhl & R. S. Siegler (vol. eds.), *Cognition, perception, and language, vol. 2* (pp. 777–810). W. Damon (gen. ed.), *Handbook of child psychology* (6th ed.). New York: John Wiley & Sons.

Geary, D. C. (2007). An evolutionary perspective on learning disability in mathematics. *Developmental Neuropsychology, 32*, 471–519.

Geary, D. C., & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in intellectually-precocious, normal, and mathematically disabled children. *Developmental Psychology, 27*, 398–406.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & Desoto, M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88*, 121–151.

Geary, D. C., & Widaman, K. F. (1992). Numerical cognition: On the convergence of componential and psychometric models. *Intelligence, 16*, 47–80.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, *78*, 1343–1359.

Gottfredson, L. (1997). Why *g* matters: The complexity of everyday life. *Intelligence, 24*, 79–132.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79*, 329–343.

Hitch, G. J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology, 10*, 302–323.

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology, 82,* 375–386.

Horn, J. L. (1988). Thinking about human abilities. In J. R. Nesselroade & R. B. Cattell (eds.), *Handbook of multivariate experimental psychology* (2nd ed., pp. 645–685). New York: Plenum.

Horn, J. L., & Cattell, R. B. (1966). Refinement and test of the theory of fluid and crystallized general intelligence. *Journal of Educational Psychology, 57*, 253–270.

Hunt, E. (1983). On the nature of intelligence. *Science, 219*, 141–146.

Hunt, E. (1978). Mechanisms of verbal ability. *Psychological Review, 85*, 109–130.

Jensen, A. R. (1998) *The g factor: The science of mental ability.* Westport, CT: Praeger.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology, 85*, 103–119.

Kane, M. J., & Engle, R. W. (2002). The role of prefrontal cortex in working-memory capacity, executive attention, and general fluid intelligence: An individual-differences perspective. *Psychonomic Bulletin & Review, 9*, 637–671.

Keating, D. P., & Bobbitt, B. L. (1978). Individual and developmental differences in cognitive-processing components of mental ability. *Child Development, 49*, 155–167.

Logie, R. H., & Baddeley, A. D. (1987). Cognitive processing in counting. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 13*, 310–326.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition, 22*, 395–410.

Lubinski, D. (2000). Scientific and social significance of assessing individual differences: "Sinking shafts at a few critical points." *Annual Review of Psychology, 51*, 405–444.

Lubinski, D., Benbow, C. P., Webb, R. M., & Bleske-Rechek, A. (2006). Tracking exceptional human capital over two decades. *Psychological Science, 17*, 194–199.

Lubinski, D., Webb, R. M., Morelock, M. J., & Benbow, C. P. (2001) Top 1 in 10,000: A 10-year follow-up of the profoundly intellectually-precocious. *Journal of Applied Psychology, 86*, 718–729.

Mazzocco, M. M. M., & Devlin, K. T. (in press). Parts and "holes": Gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science*.

McLean, J. F., & Hitch, G. H. (1999). Working memory impairments in children with specific mathematics learning difficulties. *Journal of Experimental Child Psychology, 74,* 240–260.

Mills, C. J., Ablard, K. E., & Stumpf, H. (1993). Gender differences in academically talented young students' mathematical reasoning: Patterns across age and subskills. *Journal of Educational Psychology, 85*, 340–346.

Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities, 40*, 458–478.

Pickering, S., & Gathercole, S. (2001). *Working Memory Test Battery for Children (WMTB-C) manual*. London: Psychological Corporation Ltd.

Pinel, P., Piazza, D., Le Bihan, D., & Dehaene, S. (2004). Distributed and overlapping cerebral representations of number, size, and luminance during comparative judgments. *Neuron, 41*, 1–20.

Raven, J. C., Court, J. H., & Raven, J. (1993). *Manual for Raven's Progressive Matrices and Vocabulary Scales*. London: H. K. Lewis & Co.

Robinson, N. M., Abbott, R. D., Berninger, V. W., & Busse, J. (1996). The structure of abilities in math-precocious young children: Gender similarities and differences. *Journal of Educational Psychology, 88*, 341–352.

Royer, J. M. (ed.) (2003). *Mathematical cognition*. Greenwich, CT: Information Age Publishing.

Siegler, R. S. (1987). Some general conclusions about children's strategy choice procedures. Special Issue: The neo-Piagetian theories of cognitive development: Toward an integration. *International Journal of Psychology, 22*, 729–749.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428–444.

Siegler, R. S., & Kotovsky, K. (1986). Two levels of intellectually-precociousness: Shall ever the twain meet? In R. J. Sternberg and J. E. Davidson (eds.), *Conceptions of intellectually-precociousness* (pp. 417–435). Cambridge, England: Cambridge University Press.

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*, 234–243.

Siegler, R. S., & Shrager, J. (1984). Strategy choice in addition and subtraction: How do children know what to do? In C. Sophian (ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Erlbaum.

Spearman, C. (1904). General intelligence, objectively determined and measured. *American Journal of Psychology, 15*, 201–293.

Stanovich, K. E. (1999). The sociopsychometrics of learning disabilities. *Journal of Learning Disabilities, 32*, 350–361.

Steiner, H. H., & Carr, M. (2003). Cognitive development in intellectually-precocious children: Toward a more precise understanding of emerging differences in intelligence. *Educational Psychology Review, 15*, 215–246.

Swanson, H. L. (1993). Working memory in learning disability subgroups. *Journal of Experimental Child Psychology, 56*, 87–114.

Swanson, H. L. (2006). Cognitive processes that underlie mathematical precociousness in young children. *Journal of Experimental Child Psychology, 93*, 239–264.

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology, 79*, 294–321.

Tronsky, L. N., & Royer, J. M. (2003). Relationships among basic computational automaticity, working memory, and complex mathematical problem solving: What we know and what we need to know. In J. M. Royer (ed.), *Mathematical cognition* (pp. 117–145). Greenwich, CT: Information Age Publishing.

Walberg, H. J. (1984). Improving the productivity of America's schools. *Educational Leadership, 41*, 19–27.

Zorzi, M., Priftis, K., & Umiltá, C. (2002, May 9). Neglect disrupts the mental number line. *Nature, 417*, 138.