

Asymmetric Information and Bank Runs

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Abstract

In the existing literature, panic-based bank runs are triggered by a commonly acknowledged and observed (symmetric information) sunspot signal. There are only two equilibrium realizations resulting from the symmetric information structure: Either everyone runs or no one runs. I consider a more general and more realistic situation in which consumers observe noisy private sunspot signals. If the noises in signals are sufficiently small, there exists a proper correlated equilibrium for some demand deposit contracts. A full bank run, a partial bank run (in which some consumers panic whereas others do not), or no bank run occurs (i.e., no consumer panics), depending on the realization of the sunspot signals. If the probabilities of runs are small, the optimal demand deposit contract tolerates bank runs and partial bank runs.

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1 Introduction

In a panic-based bank run,^{2,3} the bank fails when a sufficiently large number of consumers withdraw their deposits. The bank may survive if the number of withdrawals is small. A bank run in which all consumers panic and withdraw is a special case, and that a bank always fails whenever there is a run is also special. However, the focus of the existing literature has been on two situations in which (i) either all consumers run or no consumer runs, and (ii) the bank always fails if a run occurs. This is because the literature has been employing a commonly observed extrinsic signal (sunspots signal) as the trigger of the panic-based runs.

Without the sunspot signals, bank runs cannot exist in equilibrium – If consumers anticipate a bank run, they will not deposit at the bank, whereas if consumers anticipate no bank run, they will deposit and will not run ex post. The assumption of the commonly observed sunspot signal in the existing literature allows consumers to always observe the same sunspot information (i.e., symmetric information) and to perfectly coordinate their actions by running on the bank in exactly the same states of nature. In this way, the fact that all agents are observing the same signal makes it easier to construct a bank run in equilibrium, and by construction, these runs are necessarily full bank runs in which all consumers withdraw.

Arguably, that all consumers observe the same sunspot signal is a strong assumption. The sunspots are purely extrinsic variables that do not affect any of the fundamentals; it is hard to identify a single sunspot that everyone agrees is important. The resulting full bank runs from the perfectly correlated information structure is also too special. Suppose instead that one assumes asymmetric sunspot information. It is important to check whether the

²The classic work on panic-based bank runs includes Diamond and Dybvig (1983) and Peck and Shell (2003).

³Some empirical studies show that many banking crises cannot be explained by macroeconomic data. See Boyd et al. (2001).

literature on panic bank runs is robust. This paper relaxes the assumption of a commonly observed sunspot signal and adopts a more general extrinsic randomizing device. By doing so, I show how much correlation in the signals is needed to generate bank runs in equilibrium and how large the set of sunspot signals that can generate bank runs is.

I consider an economy in which consumers receive different private sunspot signals. Receiving his own sunspot signal, a consumer tries to infer the signals that others observe and the actions they will take. These private sunspot signals can be viewed as different information sources of the consumers. For example, consumers reading the same newspaper or coming from a similar background share the same information source and tend to make the same decision. Kelly and O'Grada (2000) provide empirical evidence for the importance of the networks of personal contact in the financial market. Their study of the behavior of recent Irish immigrant depositors in two banking panics in the 1850s shows that depositors' social networks, more specifically, their counties of origin in Ireland, is the prime determinant of their withdrawal decisions.⁴ The present paper, by employing the private sunspot signals, provides a formal model to capture this situation: One's judgment of the economy is based on one's own information source. How does an individual consumer make his decisions, knowing others might have different information and might make different judgments, which may affect his own payoff? Can consumers hold different beliefs and make different decisions in the equilibrium? It turns out that there are economies in which the optimal banking contract tolerates full runs and partial runs depending on the realization of sunspots.

Partial runs are the more general and more realistic predictions generated by the asymmetric information structure: In a partial bank run, some but not all consumers withdraw

⁴Solomon also provides good interpretations of imperfect correlated sunspots and examples. See Solomon (2003, pp. 6-7) and Solomon (2004, p. 7) for details.

their deposits and the banks do not necessarily fail.⁵ Although partial bank runs (in particular, the partial runs based on fundamental uncertainties) are generated through other mechanisms in the literature (see Allen and Gale 1998,⁶ Ennis and Keister 2008,⁷ and Goldstein and Pauzner 2005,⁸ among others, for partial runs), this paper suggests that the imperfectly correlated information structure can be one of the convenient modeling tools.

For simplicity, there are only two groups of consumers in the model. Consumers in the same group observe the same sunspot signal, which takes a value of either 0 or 1. Signals are imperfectly correlated. They can be viewed as a public signal plus some noises. This is the minimum structure required for the analysis of imperfect coordination. I find that if the noises in the signals that consumers obtain are sufficiently small, there exists a proper correlated equilibrium for some demand deposit contracts. As the noises become larger, the set of demand deposit contracts that can generate the correlated equilibrium shrinks. In this correlated equilibrium, a full run, a partial run, or no run occurs, depending on the realization of the signals. The probabilities of full bank runs and partial bank runs are determined by the joint distribution of the sunspot signals. If the probabilities of full bank runs and partial bank runs are below some threshold, the optimal deposit demand contract will be accepted and full and partial runs will be tolerated.⁹

⁵For example, after Hurricane Katrina, several banks in the New Orleans area lost large amounts of cash due to deposit withdrawals. However, runs on these banks ceased gradually and the banks survived.

⁶The partial runs in Allen and Gale (1998) are generated due to the fact that the long-term investment is totally illiquid. Runs are necessarily partial because there are funds available in the last period regardless of whether a bank run happens in the previous period.

⁷Ennis and Keister (2008) show that the bank runs are necessarily partial in the Green and Lin (2003) model of financial intermediation with correlated consumption types – only those who are early in the line have the incentive to run because the bank would increase the payments to them due to its undue optimism about the number of future withdrawals. As the bank’s undue optimism disappears when the end of the line is approached, consumers’ incentive to run also disappears.

⁸In Goldstein and Pauzner’s (2005) work, the partial runs are generated in a global game where some consumers run on the bank as their private and imperfect information about the bank’s portfolio return suggests they do so.

⁹Postlewaite and Vives (1987) show that given the possibility that the bank runs happen ex post, the

The approach of private sunspot signals share similarities with the approach of global games (see Carlsson and van Damme 1993, Morris and Shin 1998, and Goldstein and Pauzner 2005). Strategic complementarity and noisy signals are two critical elements in both approaches. In the global games approach, consumers receive noisy signals about the fundamentals. Consumers run on the bank if their signals are above some threshold and wait otherwise. When noises become extremely small, the equilibrium outcome is uniquely determined by the fundamentals; sunspot signals do not play a role in determining the outcomes. This paper obtains a different result: In an environment with no fundamental uncertainty, if noises in the sunspot signal become extremely small, then we are back to the traditional symmetric sunspot information setup in which two equilibrium outcomes exist: a full bank run and no bank run, depending on the realization of the sunspot signal. The difference is driven by the fact that the global games approach assumes dominance regions for parameter values of the fundamentals. If the parameter values are in the upper (lower) dominance region, an individual consumer's strongly dominant strategy is to wait (withdraw) even if all others withdraw (wait) because the fundamentals are extremely strong (weak).¹⁰ Then by iterated deletion of dominated strategies, we will reach a unique equilibrium. The dominance regions for parameter values do not exist in the sunspot signals, which by definition, are purely exogenous. Extending the information structure to include a continuum of signals will not eliminate the multiplicity because it does not help introduce the dominance regions

demand deposit contract that admits the first-best allocation is usually not optimal. Cooper and Ross (1998) show that with a perfect randomizing device, if the probability of bank runs is small, then the optimal demand deposit contract admits a run outcome. Peck and Shell (2003) illustrate that within a broad class of banking mechanisms including partial suspension of convertibility, the optimal contract can tolerate bank runs if the probability is small.

¹⁰Goldstein and Pauzner (1995) create the upper dominance region by assuming that the short term return is correlated with the long term return. When returns are high, even if all other depositors withdraw, the bank survives because there are sufficient funds accrued in the short term to meet all withdrawal requests.

to the economy.¹¹

Another major line of literature on bank runs attributes the origin of the runs to the uncertainty in fundamentals. The view on fundamental-based bank runs argues that bank runs occur when consumers learn negative information about the bank's portfolio returns or about the aggregate liquidity shock.¹² The view of panic-based runs argues that the bank and bank runs are inherently intertwined because bank contracts provide short-term liquidity whereas the portfolio matures only in the long term.¹³ As a result, a panic-based run is self-fulfilling even in the absence of fundamental uncertainty. Although the runs in this paper are panic-based, the sunspot signals can be viewed as the uncertainty in the fundamentals taken to the limit. Focusing on panic-based runs serves two purposes: (i) to check the robustness of the models on panic-based bank runs and (ii) to explain the importance of consumer information networks in determining their behavior.

The remainder of the paper is organized as follows: Section 2 introduces the model setup. Section 3 discusses the equilibrium in the deposit game. Section 4 addresses the conclusions.

2 The model

The basic setup of the model is standard as in Diamond and Dybvig (1983). There are three periods, $t = 0, 1, 2$, and a measure 1 of consumers in the economy. Period 0 is called *ex ante*; periods 1 and 2 are *ex post*. Each consumer is endowed with 1 unit of consumption

¹¹In a recent paper, Angeletos (2008) studies an investment problem with strategic complementarities in an economy with a continuum of noisy, private sunspot signals. He finds that there exists a cutoff value of the sunspot signals above which agents invest, below which otherwise. In other words, sunspots determine the equilibrium outcome.

¹²See Allen and Gale (1994, 1998), Gorton (1988), and Calomiris and Gorton (1991), among others, for theoretic models and empiric evidence on fundamental-based runs.

¹³The pioneering work on panic-based runs is conducted by Bryant (1980). The classic paper on this topic is Diamond and Dybvig (1983).

good in period 0 and nothing in periods 1 and 2. A consumer is an impatient type with probability α and a patient type with probability $1 - \alpha$. Denote a consumer's type by τ , $\tau \in \{\text{impatient, patient}\}$. Impatient consumers derive utility only from consumption in period 1. Their utility is described by $u(c_1)$, where c_1 is the consumption received in period 1. Patient consumers consume in the last period. If a patient consumer receives consumption in period 1, he can store it costlessly and consume it in period 2. Thus, a patient consumer's utility is described by $u(c_1 + c_2)$, where c_2 is the consumption received in period 2. The coefficient of relative risk aversion (CRRA) of the utility function, $-xu''(x)/u'(x)$, is greater than 1 for $x \geq 1$. The utility function is normalized to 0 at $x = 0$, that is, $u(0) = 0$. Whether a consumer is patient or impatient is his private information and is revealed to the individual consumer in period 1.

Consumers have two technologies for saving: investment and storage. One unit of consumption good invested in period 0 yields one unit in period 1 and R ($R > 1$) units in period 2. One unit of consumption good placed into storage yields one unit of consumption good in the following period.

The banking market is competitive. The representative bank takes deposits from the consumers and makes investment. For convenience, I assume consumers deposit all or none of their endowments at the bank. The bank offers a demand deposit contract to the consumers and faces a sequential service constraint (Wallace, 1988).¹⁴ It pays a fixed amount of consumption good to consumers who withdraw in period 1 until it runs out of resources and distributes the remaining resources equally among the consumers who wait

¹⁴Wallace (1988) explains that an environment in which consumers are isolated from each other but in contact with their bank implies the sequential service constraint. Here I follow the literature to assume that consumers move simultaneously, but the bank faces a sequential service constraint.

until the last period.¹⁵ Denote payments in periods 1 and 2 by c^1 and c^2 , respectively. Then $c^2 = \max \left\{ \frac{1-nc^1}{1-n} R, 0 \right\}$, where n is the measure of consumers who withdraw in period 1.

What differs from the standard Diamond and Dybvig setup is that consumers are grouped into two groups. Group i observes a sunspot signal, θ_i , $\theta_i \in \{0, 1\}$, $i = 1, 2$. The joint distribution of θ_1 and θ_2 is denoted by $\Pr(\theta_1, \theta_2) = p_{\theta_1\theta_2}$ (e.g., $\Pr(\theta_1 = 1, \theta_2 = 1) = p_{11}$), $\sum_{\theta_2=0}^1 \sum_{\theta_1=0}^1 p_{\theta_1\theta_2} = 1$. Consumers in the same group observe the same sunspot information and conjecture the sunspot signal that the other group observes. In the rest of the paper, I use “private signals” for “private signals to groups.” The private signals can be thought as follows: There is a public sunspot signal. However, consumers cannot observe the public sunspot perfectly. Instead, each group observes the public sunspot with a noise, or a private sunspot signal. Without loss of generality, I assume that $p_{01} = p_{10} = \rho$ to facilitate the presentation. ρ measures noise in signals. When $\rho = 0$, we have a symmetric information structure as in the existing literature. Group i has a measure of n_i consumers. By law of large numbers, the measure of impatient consumers in group i is αn_i . Consumers know ex ante that there are two groups, but they do not know which group they will be in until period 1.

The sequence of timing of the banking game is as follows.

Period 0 : Bank announces the contract.

Consumers make deposit decisions.

Period 1 : Consumers get to know their own consumption types, which groups

they are in, and observe the realizations of their sunspot signals.

Consumers make withdrawal decisions.

¹⁵The assumption of a demand deposit contract simplifies the presentation. In the appendix, I consider a banking contract that allows for partial suspension of convertibility. The results still hold. In some economies, a correlated equilibrium exists, and full bank runs and partial runs are tolerated.

Period 2 : Bank allocates the remaining resource to the rest of the consumers.

A consumer's strategy specifies a decision of whether to accept or reject the contract in period 0 as a function of the bank's contract and a decision whether to withdraw or wait in period 1 as a function of the consumer's consumption type, which group he is in, and the value of the sunspot signal he observes. The decision in period 2 is simple: A consumer withdraws if he has not done so in period 1. Denote a consumer's strategy by $y = (y_0(c^1), y_1(\tau, i, \theta_i))$, where y_0 and y_1 are the decisions in periods 0 and 1, respectively.

I look for a subgame perfect Nash-Aumann equilibrium in which (1) the sunspot signals constitute a correlated equilibrium (following Aumann, 1987) in period 1 wherever possible given a contract, and (2) the correlated equilibrium is subgame perfect in the sense that consumers accept the contract in period 0 if and only if the expected utility under the contract is higher than autarky.

3 Deposit game

3.1 Symmetric information – a benchmark case

In autarky, impatient consumers consume 1 unit of consumption good in period 1, whereas patient consumers consume R units in period 2. A consumer's expected utility in autarky is

$$\hat{W}^{AUT} = \alpha u(1) + (1 - \alpha) u(R).$$

Instead of being in autarky, consumers can deposit in a bank in period 0 and receive payments in period 1 or 2, depending on when they withdraw. A feasible demand deposit

contract satisfies the following participation incentive compatibility constraint:

$$\frac{1 - \alpha c^1}{1 - \alpha} R \geq c^1, \quad (1)$$

which says that the consumers who wait until the last period should be paid more than those who withdraw in period 1, if the impatient consumers are the only ones who withdraw their deposits in period 1. In other words, if a contract satisfies (1), the strategy $\{y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$ constitutes a Nash equilibrium in period 1. Let M denote the set that contains all contracts described by c^1 that satisfy (1). Solving (1), we get $M = [0, \frac{R}{1 - \alpha + \alpha R}]$, which is the set that includes all feasible banking contracts in the traditional bank run literature.

A banking contract in M admits bank runs if there is a Nash equilibrium in period 1 in which all consumers play $\{y_1 = \text{withdraw}\}$ regardless of their consumption types. A contract that allows for a bank run is called a run-admitting contract. Otherwise, the contract is run proof. Let M^{RA} and M^{RP} denote the sets that contain all run-admitting contracts and all run-proof contracts, respectively. Both sets are subsets of M , and they are complements to each other. In this simple model, a contract in M is run-admitting if $c^1 > 1$ and run-proof if $c^1 \leq 1$. Because CRRA of the utility function is larger than 1, the best run-proof contract provides $c^1 = 1$, which results in the same allocation as in autarky. Given a run-admitting contract, in the event that all people withdraw, each consumer receives payment with probability $1/c^1$. The ex-ante expected utility, if knowing a run will occur in period 1, is

$$W^{run}(c^1) = \frac{1}{c^1} u(c^1).$$

If no bank run occurs, the ex-ante expected utility is

$$W^{no-run}(c^1) = \alpha u(c^1) + (1 - \alpha)u\left(\frac{1 - \alpha c^1}{1 - \alpha}R\right).$$

If all consumers anticipate a bank run to occur in period 1, then they will not accept the contract in period 0 because the expected utility in a bank run, $\frac{1}{c^1}u(c^1)$, is strictly lower than \hat{W}^{AUT} for $c^1 > 1$. Alternatively, if they have accepted the contract, they all anticipate no bank run and a run will not occur in period 1. In either case, bank runs do not occur.

However, we do observe bank runs. To model bank runs in equilibrium, the existing literature relies on a sunspot signal to coordinate the actions of consumers. Whether a bank run occurs after consumers accept the contract is modeled as dependent on a publicly observed sunspot variable θ , $\theta \in \{0, 1\}$. Let $p_j = \Pr(\theta = j)$, $j = 0, 1$. The sunspot can generate bank runs if there is a subgame perfect Nash equilibrium in which consumers play the following strategy: $\{y_0(c^1) = \text{accept}, y_1(\text{patient}, \theta = 0) = \text{wait}, y_1(\text{patient}, \theta = 1) = \text{withdraw}, y_1(\text{impatient}, \theta = 0, 1) = \text{withdraw}\}$ and the contract is optimal in the sense that it maximizes consumers' expected utility.

Given the sunspot information structure, the probability of a run is p_1 . The optimal run-admitting contract solves

$$\begin{aligned} \max_{c^1} p_1 W^{run}(c^1) + p_0 W^{no-run}(c^1) \\ \text{s.t. } c^1 \in M^{RA}. \end{aligned}$$

If the probability of bank runs is small, the optimal run-admitting contract yields higher expected utility than autarky so it will be accepted, and bank runs can occur in equilibrium

(Cooper and Ross, 1998).

3.2 Asymmetric information

The assumption in the existing literature that all consumers observe exactly the same signal makes it easier to construct a run in equilibrium: Consumers are coordinated perfectly to take the same actions by running on the bank in the same states of nature. However, we do not have a good theory to explain where the sunspot variables come from and which sunspot variables consumers use as their guidance for actions.¹⁶ Nor do we know whether all consumers agree on the same sunspot signal and if they do not, how different (measured by noises) their signals are. In this section, I relax the assumption of a commonly observed signal and show how consumers are coordinated on actions if their signals are different and how much symmetry there must be in the signals to generate bank runs in equilibrium.

3.2.1 Sets of contracts

I look for an equilibrium in which consumers are coordinated by the noisy private sunspot signals in period 1 given a contract. In particular, I look for a correlated equilibrium in period 1 that is based on the joint distribution of the sunspot signals. I exclude the degenerate cases in which either everyone runs or no one runs. In this sense, I use “correlated equilibrium” for “proper correlated equilibrium.” I focus on the case in which consumers in the same group take the same action, for I want to show how the sunspot signals coordinate people’s actions.

A contract allows for a correlated equilibrium in period 1 if consumers play the strategy

¹⁶Cass (1989, pp. 677-678) considers an economy in which “the ‘psychology’ of the market ... decides that what will really count tomorrow is some heretofore unrecognized or unimagined aspect of ‘reality’.” The changing sunspot beliefs militate against having a complete market for insuring against the extrinsic uncertainty.

$\{y_1(\text{patient}, i, \theta_i = 1) = \text{run}, y_1(\text{patient}, i, \theta_i = 0) = \text{wait}, y_1(\text{impatient}, i, \theta_i = 0, 1) = \text{run}, i = 1, 2\}$. The contract must be individually rational in the sense that it is accepted only if the expected utility from depositing in the bank is at least equal to that under autarky. The contract also must be optimal in the sense that it maximizes the consumers' ex-ante expected utility among all the feasible contracts. Because the best run-proof contract results in the same allocation as autarky, I assume throughout the rest of the paper that consumers stay in autarky if a run-proof contract is provided. So the analysis in the rest of the paper is focused on the run-admitting contracts.

Generally speaking, because sunspots do not affect the fundamentals, people can interpret the signals in any way they please. Thus, an imperfect information structure can allow for more than one correlated equilibrium. This paper focuses on the case where consumers in both groups interpret a signal of value 1 as the sign to withdraw and 0 as the sign to wait. If the exogenous uncertainty is viewed as the uncertainty in fundamentals taken to the limit, then it can be understood that people usually have common views on which signal is favorable and which is not.

For an impatient consumer, withdrawal in period 1 is his dominant strategy because he only values consumption in that period. The correlated equilibrium strategies for a patient consumer in group i in period 1 require the following conditions:

$$\Pr(\theta_{-i} = 0|\theta_i = 1) \min \left\{ \frac{1}{(n_i + \alpha n_{-i}) c^1}, 1 \right\} u(c^1) + \Pr(\theta_{-i} = 1|\theta_i = 1) \frac{1}{c^1} u(c^1) \geq \Pr(\theta_{-i} = 0|\theta_i = 1) u \left(\max \left\{ \frac{1 - (n_i + \alpha n_{-i}) c^1}{(1 - \alpha) n_{-i}} R, 0 \right\} \right), \quad (2)$$

$$\Pr(\theta_{-i} = 1|\theta_i = 0) u \left(\max \left\{ \frac{1 - (n_{-i} + \alpha n_i) c^1}{(1 - \alpha) n_i} R, 0 \right\} \right) + \Pr(\theta_{-i} = 0|\theta_i = 0) u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) \geq \\ \Pr(\theta_{-i} = 1|\theta_i = 0) \min \left\{ \frac{1}{(n_{-i} + \alpha n_i) c^1}, 1 \right\} u(c^1) + \Pr(\theta_{-i} = 0|\theta_i = 0) u(c^1). \quad (3)$$

Equation (2) is the condition that a patient consumer in group i runs on the bank given the probability that some, but not all, patient consumers wait until the last period. When a patient consumer in group i observes $\theta_i = 1$, he knows by the Bayes rule that with conditional probability $\Pr(\theta_{-i} = 1|\theta_i = 1)$, the other group (denoted by $-i$) observes $\theta_{-i} = 1$ and withdraws, and that with probability $\Pr(\theta_{-i} = 0|\theta_i = 1)$, the other group waits. The left-hand side of (2) is the expected utility if he withdraws, given his group members withdraw and the members in the other group follow the signal they observe. The right-hand side of (2) is the expected utility if he waits, whereas the others follow their signals. For a correlated equilibrium, a patient consumer in group i should find “withdraw” to be the best response given the strategies of the members of the other group and his own group. Similarly, (3) is the condition that a patient consumer in group i waits given the probability that some, but not all, patient consumers run on the bank.

Let $M^{CE}(p_{11}, \rho)$ denote the set of contracts that satisfy (2) – (3) for a given distribution of θ 's. Because a contract that satisfies (2) – (3) always satisfies (1), whereas the reverse is not true when ρ is strictly positive, $M^{CE}(p_{11}, \rho)$ is a subset of M^{RA} . If $\rho = 0$, then and we have $M^{CE}(p_{11}, 0) = M^{RA}$.

If a contract in $M^{RA} \setminus M^{CE}$ is provided, the sunspot signals fail to coordinate the actions of consumers. A consumer's strategy is not contingent on the sunspot realizations. Our equilibrium concept of correlated equilibrium does not apply to these contracts. Here, I follow Diamond and Dybvig to assume that consumers play a Nash equilibrium. Given a

contract in $M^{RA} \setminus M^{CE}$, there are two Nash equilibria in period 1 : $\{y_1 = \text{withdraw}\}$ and $\{y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$. In the main text, I assume that consumers play $\{y_1 = \text{withdraw}\}$. I leave the alternative case to the appendix.

3.2.2 Ex-ante expected utilities

In this section, I calculate the ex-ante expected utility given a contract and the equilibrium strategies. The ex-ante expected utility is critical for two reasons: (i) A competitive bank provides the contract that maximizes the consumers' ex-ante expected utility; (ii) consumers accept the contract if and only if doing so is a better choice than staying in autarky.

In a correlated equilibrium, there are three possible outcomes in period 1: If $\theta_1 = \theta_2 = 1$, all consumers withdraw deposits; if $\theta_1 = 0$ and $\theta_2 = 1$, or if $\theta_1 = 1$ and $\theta_2 = 0$, only a fraction of patient consumers run on the bank; no patient consumer runs on the bank when $\theta_1 = \theta_2 = 0$. Define a full bank run, partial bank run, and no bank run as follows:

Definition 1 In period 1, if all consumers withdraw deposits, then a full bank run occurs; if some, but not all, patient consumers withdraw, then a partial bank run occurs; if no patient consumers withdraw, then no bank run occurs.

Given the distribution of the sunspots, the probabilities of having a full run and no run are p_{11} and p_{00} , respectively, and the probabilities of having partial runs driven by groups 1 and 2, respectively, are ρ . Throughout this paper, I assume $p_{00} < 1$ to dismiss a trivial case in which bank runs never occur. Given the probabilities, the expected utility of a consumer in period 0 if he accepts contract c^1 is:

$$W^{CE}(c^1) = p_{11}W^{run}(c^1) + \rho W^{p-run-1}(c^1) + \rho W^{p-run-2}(c^1) + p_{00}W^{no-run}(c^1)$$

where

$$W^{p-run-i}(c^1) = \min \left\{ n_i + \alpha n_{-i}, \frac{1}{c^1} \right\} u(c^1) + (1-\alpha)n_{-i} u \left(\max \left\{ \frac{1 - (n_i + \alpha n_{-i})c^1}{(1-\alpha)n_{-i}} R, 0 \right\} \right)$$

is the expected utility when a partial run driven by group i occurs. The first term captures the fact that when a partial run occurs, a measure $n_i + \alpha n_{-i}$ of consumers withdraw, but at most, only a measure $\frac{1}{c^1}$ of consumers are served. The second term captures the utility of those who do not withdraw in period 1.

The bank chooses a contract to offer to the consumers. A rational consumer accepts the contract only if the expected utility under the contract is higher than autarky.

Definition 2 A contract c^1 is said to be individually rational if and only if the ex-ante expected utility from accepting the contract is higher than staying in autarky.

Consumers only accept individually rational contracts. A contract in M^{CE} is individually rational if $W^{CE}(c^1) > \hat{W}^{AUT}$. A contract in $M^{RA} \setminus M^{CE}$ is not individually rational because by the assumed equilibrium strategy $\{y_1 = \text{withdraw}\}$, the ex-ante expected utility is $W^{run}(c^1)$, which is lower than \hat{W}^{AUT} for $c^1 > 1$ by concavity of the utility function. The bank thus has choices only in set M^{CE} .

The best contract in M^{CE} solves

$$\begin{aligned} \hat{W}^{CE} &= \max_{c^1} W^{CE}(c^1) && \text{(PCE)} \\ &s.t. \ c^1 \in M^{CE}. \end{aligned}$$

The noisy private sunspot signals can generate bank runs if (i) there exists at least one contract in M^{CE} that yields higher ex-ante utility than autarky so it will be accepted, and (ii) it is the optimal contract in terms of ex-ante expected utility, given the equilibrium

strategies that consumers play if a contract in $M^{RA} \setminus M^{CE}$ is provided. Formally, we have the following definition.

Definition 3 Given the equilibrium strategies of the consumers if a contract in $M^{RA} \setminus M^{CE}$ is provided, the distribution of the sunspot variables $\Pr(\theta_1, \theta_2)$ can generate bank runs if there exists an individually rational contract $c^1 \in M^{CE}$, under which (1) there is a correlated equilibrium in period 1 in which consumers play the strategy $\{y_1(\text{patient}, i, \theta_i = 1) = \text{withdraw}, y_1(\text{patient}, i, \theta_i = 0) = \text{wait}, y_1(\text{impatient}, i, \theta_i = 0, 1) = \text{withdraw}, i = 1, 2\}$, and (2) the contract is optimal.

The next step is to show the conditions of the information structure that are needed for the sunspot signals to generate bank runs. We care about two features of the information structure: (i) the size of noises in the signals, which determines the size of the set of M^{CE} , and (ii) the probabilities of bank runs, which determine the consumers' ex-ante expected utility given a contract. I will first derive the properties of M^{CE} and of the value function of \hat{W}^{CE} .

3.2.3 Properties of M^{CE} and \hat{W}^{CE}

Lemma 1 shows that $M^{CE}(p_{11}, \rho)$ shrinks when the signals become less correlated.

Lemma 1 Given p_{11} , let $\rho' > \rho$. We have $M^{CE}(p_{11}, \rho') \subseteq M^{CE}(p_{11}, \rho)$. $M^{CE}(p_{11}, \rho') = M^{CE}(p_{11}, \rho)$ if and only if $M^{CE}(p_{11}, \rho) = \emptyset$.

The strategies of the consumers are complementary. If the probability that other consumers will take a certain action increases, then an individual consumer's incentive to take a

different action falls. With the increases in ρ , it is more difficult to generate a run as well as no run, because when an individual consumer receives a signal that tells him to take a certain action, he knows that the other group is more likely to take a different action. Hence, as ρ increases, there are fewer contracts consistent with the definition of a correlated equilibrium. In extreme cases, no contract allows for a correlated equilibrium. In the example below, $M^{CE}(p_{11}, \rho)$ becomes empty when ρ gets large.

Example 1 $u(c) = \frac{(c+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}$, $b = 0.5$, $\gamma = 2$. $n_1 = 0.45$, $n_2 = 0.55$, $\alpha = 0.4$, $R = 1.5$. $p_{11} = 0.001$. Let ρ vary between 0 and 0.4995. Figure 1 shows how the set of $M^{CE}(p_{11}, \rho)$ varies with ρ . In this example, $M^{CE}(p_{11}, \rho)$ is a convex set: For given ρ , there exists an interval of c^1 . All contracts in that interval allow for a correlated equilibrium in period 1. The points on the dashed line and the solid line are the lowest and the highest c^1 in $M^{CE}(p_{11}, \rho)$ for the corresponding ρ , respectively. The set of $M^{CE}(p_{11}, \rho)$ shrinks in ρ . When $\rho > 0.39$, no contract allows for a correlated equilibrium.

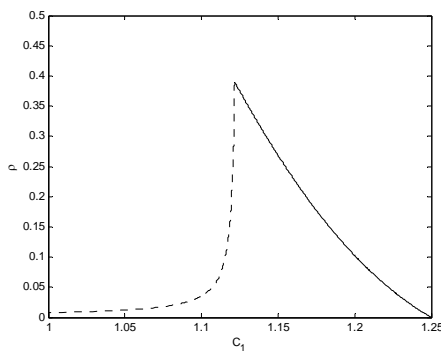


Figure 1: An example of M^{CE} .

Another way to show that consumers can be coordinated by their private signals in period 1 is to demonstrate that for any run-admitting contract, there is an upper bound of ρ below

which the contract permits a correlated equilibrium in period 1. In most of the cases, the upper bound is strictly positive.

Lemma 2 Given a run-admitting contract, there exists $\varepsilon(p_{11}, c^1) \geq 0$ such that if $\rho \leq \varepsilon(p_{11}, c^1)$, then $c^1 \in M^{CE}(p_{11}, \rho)$. We have $\varepsilon(p_{11}, c^1) = 0$ if and only if (1) $c^1 = \frac{R}{1 - \alpha + \alpha R}$, or (2) $p_{11} = 0$, and $c^1 < \max \left\{ \frac{1 - (n_i + \alpha n_{-i}) c^1}{(1 - \alpha) n_{-i}} R, i = 1, 2 \right\}$.

When the participation incentive compatibility constraint is binding (i.e., $c^1 = \frac{R}{1 - \alpha + \alpha R}$), only the minimum incentive for a patient consumer to wait is satisfied. Any increase in the measure of consumers running on the bank, or any increase in the probability of more than α measure of consumers running on the bank, breaks down the participation incentive compatibility constraint. Thus, the contract that offers $c^1 = \frac{R}{1 - \alpha + \alpha R}$ does not allow for a correlated equilibrium unless $\rho = 0$.

If $p_{11} = 0$, when a consumer in group i receives a signal to withdraw, he knows for sure that the other group receives the signal to wait. The incentive for him to withdraw is that if all of his group members run, the resource remaining at the bank for period 2 will yield a lower payment than what he can get from immediate withdrawal. However, suppose all other consumers in group i run on the bank, yet the bank still survives and pays a higher amount to those who wait (i.e., $c^1 < \frac{1 - (n_i + \alpha n_{-i}) c^1}{(1 - \alpha) n_{-i}} R$), then a consumer in group i will not withdraw even though his signal tells him to do so. So a correlated equilibrium breaks down. Therefore, for these contracts, the probability that group i runs on the bank whereas the other group does not has to be zero.¹⁷ Also note that lemma 2 implies that given $p_{11} = 0$ if

¹⁷A special case occurs when $p_{11} = p_{00} = 0$ and $n_1 = n_2 = 0.5$. The only contract that allows for a correlated equilibrium requires $c^1 = \frac{1 - (0.5 + 0.5\alpha)c^1}{0.5(1 - \alpha)} R$. In the equilibrium, depositors can perfectly infer the sunspot signal the other network observes, but they will always make different decisions. Put more generally, there exists a rational equilibrium in some economies in which depositors with the same information can have different opinions regarding the consequences of their information.

the difference in group sizes becomes larger, then more contracts do not allow for a correlated equilibrium.

Given $p_{11} = 0$, define the maximization problem PNR as

$$\begin{aligned} \hat{W}^{CNR} &= \max_{c^1} W^{no-run}(c^1) & (\text{PNR}) \\ \text{s.t. } \max \left\{ \frac{1 - (n_i + \alpha n_{-i}) c^1}{n_i (1 - \alpha)} R, i = 1, 2 \right\} &\leq c^1 \leq \frac{R}{1 - \alpha + \alpha R}. \end{aligned}$$

The value of problem PNR, \hat{W}^{CNR} , is the highest expected utility under no bank run if the bank provides a contract that permits a correlated equilibrium given $p_{11} = 0$. So it is the upper bound of W^{CE} when $p_{11} = 0$. Although no run is assumed, the value of \hat{W}^{CNR} is not necessarily higher than \hat{W}^{AUT} , because the choice set of contracts is more limited. Example 2 shows the case.

Example 2 $u(c) = \frac{c}{1 + 1.2c}$. $R = 1.5$, $\alpha = 0.2$, $n_1 = n_2 = 0.5$. In this example, $\hat{W}^{AUT} = 0.5195$, whereas $\hat{W}^{CNR} = 0.5193$ with $c^1 = 1.1538$.

Given p_{11} , the value function of problem PCE strictly decreases in ρ for two reasons: First, the set of contracts that allow for a correlated equilibrium shrinks when noises increase, so the choice set is smaller. Second, because the expected utility under a partial run is lower than that under no run, the same contract yields lower expected utility if the probabilities of partial runs are larger and the probability of no run is smaller.

Lemma 3 Given p_{11} , the value function of problem PCE is strictly decreasing in ρ for $M^{CE}(p_{11}, \rho) \neq \emptyset$.

Note that given ρ , the value function of PCE is not necessarily decreasing in p_{11} . Although the probability of having a full bank run is lowered when p_{11} decreases, the choice set of M^{CE} may shrink.

3.2.4 Equilibrium bank runs

Now we are ready to state the condition under which the noisy private signals can generate bank runs. Again, I consider the case in which consumers play $\{y_0 = \text{reject}, y_1 = \text{withdraw}\}$ if a contract in $M^{RA} \setminus M^{CE}$ is provided. Given the strategies, the bank will not choose to offer such a contract ex ante. Hence, if the best contract in M^{CE} can generate higher utility than autarky, it will be provided by the bank and be accepted by the consumers, and the consumers will be coordinated by the sunspots ex post. Proposition 1 shows that the imperfectly correlated sunspot signals can generate bank runs, if the probabilities of full bank runs and partial bank runs are below certain thresholds. Similar arguments apply to and similar results obtain in the alternative case in the referee's appendix.

Proposition 1 Let consumers play the strategy

$$\{y_0(c^1) = \text{reject}, y_1 = \text{withdraw}\}$$

if a contract in $M^{RA} \setminus M^{CE}$ is provided. There exist $\bar{p}_{11} > 0$ and $\bar{\rho}(p_{11}) \geq 0$ such that for $p_{11} \leq \bar{p}_{11}$ and $\rho \leq \bar{\rho}(p_{11})$, the distribution of the sunspot variables can generate bank runs. $\bar{\rho}(p_{11}) > 0$ for $0 < p_{11} < \bar{p}_{11}$. $\bar{\rho}(0) > 0$ if $\hat{W}^{CNR} > \hat{W}^{AUT}$.

Proof. Given p_{11} , if $\rho = 0$, the problem is the same as the traditional public sunspot equilibrium problem. There is a unique cutoff level of $p_{11} \in (0, 1)$, above which autarky is

better than the banking economy and below which the optimal contract tolerates bank runs (see Cooper and Ross, 1998). Denote this cutoff level by \bar{p}_{11} .

Given p_{11} , the value function of PCE is strictly decreasing in ρ by lemma 3. Hence, $\rho(\bar{p}_{11}) = 0$.

Now consider $p_{11} > 0$. The value of PCE given $0 < p_{11} < \bar{p}_{11}$ and $\rho = 0$ is strictly higher than that in autarky. Holding p_{11} constant, by the monotonicity of the value function in ρ , we can find a cutoff probability $\bar{\rho}(p_{11})$, below which the contract allowing for a correlated equilibrium is better than autarky and above which otherwise.

The value function of PCE is not necessarily continuous, because the choice set can be nonconvex. In the next step, we prove that $\bar{\rho}(p_{11})$ is not equal to 0 for $0 < p_{11} < \bar{p}_{11}$. Let \hat{c}^1 denote the solution to problem PCE with $0 < p_{11} < \bar{p}_{11}$ and $\rho = 0$. \hat{c}^1 cannot be $\frac{R}{1-\alpha+\alpha R}$ (see Ennis and Keister, 2006). By lemma 2, \hat{c}^1 allows for a correlated equilibrium for $\rho \leq \varepsilon(p_{11}, \hat{c}^1)$, where $\varepsilon(p_{11}, \hat{c}^1) > 0$. The expected utility at \hat{c}^1 for $\rho \leq \varepsilon(p_{11}, \hat{c}^1)$ is continuous in ρ . So ρ can be increased at least to $\min\{\bar{p}_{11} - p_{11}, \varepsilon(p_{11}, \hat{c}^1)\}$ and \hat{c}^1 can still be better than autarky. Thus, when $0 < p_{11} < \bar{p}_{11}$, $\bar{\rho}(p_{11}) > 0$.

Now consider $p_{11} = 0$. The upper bound of W^{CE} is \hat{W}^{CNR} . If $\hat{W}^{CNR} > \hat{W}^{AUT}$, we can apply the above argument to show there exists a $\bar{\rho}(0) > 0$ such that if $\rho < \bar{\rho}(0)$, the distribution of the sunspot variables can generate bank runs. ■

The following example draws $\bar{\rho}$ as a function of p_{11} .

Example 3 Continue example 1. In this example, $\bar{p}_{11} = 0.0064$. For $p_{11} < \bar{p}_{11}$, the upper bound of the noises in signal, $\bar{\rho}$, is drawn as a function of p_{11} in Figure 2. $\bar{\rho}$ is not monotone in p_{11} . When p_{11} increases, on one hand, the probability of having a full bank run increases, which lowers the value of PCE, so the probability of partial runs needs to be lowered to

keep the value of PCE above \hat{W}^{AUT} . On the other hand, the conditional probability of $\Pr(\theta_{-i} = 1|\theta_i = 1)$ also increases, which means the signals are more correlated and the size of the choice set of contracts becomes larger. So the value of PCE is increased, and a higher probability of partial runs will be allowed. Whether \bar{p} is increasing or decreasing in p_{11} depends on the strength of these two forces. In this example, there is a critical p_{11} , below which \bar{p} is increasing in p_{11} and above which \bar{p} is decreasing in p_{11} .

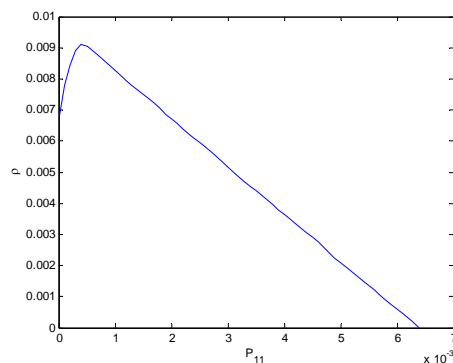


Figure 2: An example of $\bar{p}(p_{11})$.

Proposition 1 states similar results to those of Cooper and Ross (1998) and Peck and Shell (2003). If the probabilities of runs (both full and partial) are small, then bank runs can be equilibrium phenomena. There are two reasons behind it. First, consumers are willing to accept a run-admitting contract if the chance of having bad outcomes is small. This is common in the literature. Second, the probabilities of partial runs also measures the noises in the signals. The smaller the noises are, the better the consumers are coordinated and the more room the bank has in choosing the optimal contract. This mechanism is unique to our asymmetric information structure.

Proposition 1 also implies that partial runs can exist in an economy in which full bank runs never occur. In this sense, partial runs are more general phenomena of financial distress. Banks do not necessarily close in a partial run. One way to understand the partial runs is to think about runs on several banks in a coalition of many banks or on several branches of a bank with nationwide branches. Studies show that historically, branch-banking systems (e.g., the Canadian banks) tend to be less prone to the effects of panics than unit-banking systems (e.g., the U.S. banks) (Calomiris and Gorton, 1991). The common view is that the branch-banking system can better absorb the risk imposed by the runs on individual banks. This paper provides another plausible explanation: Depositors of a bank with nationwide branches come from more diversified backgrounds. The coordination among these depositors is more difficult due to, for example, geographic separation or different news sources. As a result, the probability of a full run is reduced and the bank is more likely to survive.

3.2.5 Discussions

A partial bank run here results from the asymmetry information structure. In the traditional Diamond-Dybvig setup with symmetric information structure, a partial run can happen if we allow for mixed strategies. In a mixed strategy Nash equilibrium, the fraction of patient consumers who run on the bank is uniquely determined by the contract.¹⁸ In contrast, in a correlated equilibrium, the fraction of consumers who run on the bank is determined by the information structure. A partial run can occur with different probabilities due to different information structures for the same contract. In this sense, a correlated equilibrium is more general than a Nash equilibrium.

¹⁸In a mixed strategy equilibrium, the probability that a patient consumer runs on the bank, denoted by p , solves $\frac{1 - [\alpha + (1 - \alpha)p]c^1}{(1 - \alpha)(1 - p)} = c^1$. By the law of large numbers, the fraction of the patient consumers who run on the bank is $(1 - \alpha)p$.

A more general way to analyze a consumer's withdrawal decision is to model it as dependent on the withdrawals made by others as well as the consumer's private information. Gu (2008) studies fundamental-based bank runs as a result of herd behavior.¹⁹ The withdrawal decisions are assumed to be made simultaneously in this paper. However, the correlated equilibrium (and partial runs) can survive even if we allow consumers to withdraw after they observe withdrawals by others. Let us consider a simple two-stage game in period 1. Depositors observe the sunspots at stage 1 and make withdrawal decisions. At stage 2, those who have not withdrawn have another chance to withdraw. Will a partial run at stage 1 develop into a full run at stage 2? The answer is not necessarily. Think about the following case. Suppose after a partial run initiated by group i , the funds left for the other group in period 2 are higher than c^1 , that is, $\frac{1-(n_i+\alpha n_{-i})c^1}{n_{-i}(1-\alpha)}R \geq c^1$. Then no run at stage 2 is a subgame perfect Nash equilibrium. Bank run will stop at stage 2, and the correlated equilibrium at stage 1 is part of the overall equilibrium. Because the correlated equilibrium in a sequential-move game has no substantial difference from the one in a static game, I study a static game in period 1 for simplicity.

This paper focuses on a simple demand deposit contract, which is widely used in the banking industry. However, the results in the paper obtain in a broad class of banking mechanisms, including partial suspension of convertibility (Wallace, 1990) in the Peck and Shell (2003) framework. An example of the correlated equilibrium with partial suspension of convertibility is shown in the referee's appendix.

¹⁹In Gu (2008), a full bank run occurs as a result of a herd of withdrawals when all depositors withdraw due to unfavorable signals and/or unfavorable observations on withdrawals. Likewise, a non-run occurs as a result of a herd of non-withdrawals. Before either herd begins, depositors follow their private signals. Thus, there are cases in which some depositors withdraw even though a full run does not occur later.

4 Conclusion

In this paper, I extend the analysis of panic-based runs to include an asymmetric-information, extrinsic randomizing device. I explain how individual consumers make withdrawal decisions when there are uncertainties about the information and decisions of other consumers. I show that in an economy with an asymmetric sunspot information structure, there exists a correlated equilibrium in a class of demand deposit contracts for the case in which the noises in sunspots are sufficiently small. In this equilibrium, a full bank run, a partial bank run, or no bank run occurs, depending on the realization of the sunspot signals. If the probabilities of full bank runs and partial bank runs are below certain thresholds, the optimal banking contract tolerates full runs and partial runs. This correlated equilibrium (and the partial runs) can survive in a game with a longer time horizon in which consumers are allowed to make their decisions dependent on the observed withdrawals by others.

This paper is also a robustness test of the sunspot-triggered, panic-based bank runs. Not all sunspot signals can generate bank runs in equilibrium. The set of contracts that allow for the correlated equilibrium diminishes if the signals become more noisy. The set can be empty if the noises in the signals are high enough.

The model I consider in this paper assumes that the asymmetric randomizing device and the division of groups are given exogenously. What happens if consumers can choose groups? It is obvious that there is an equilibrium such that all consumers choose the same group so they are coordinated perfectly. However, it is possible that consumers choose different groups in the equilibrium. It happens when the measures of the consumers in two groups are adjusted to the point that the ex-ante expected utilities of two groups are equal. In this sense, an asymmetric randomizing device can be an equilibrium choice, and

the heterogeneous beliefs about the bank's stability can also be an endogenous choice.

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5 Referee's appendix

5.1 Consumers do not run on the bank given a contract in $M^{RA} \setminus M^{CE}$

Now let us consider consumers' alternative equilibrium strategy given a contract in $M^{RA} \setminus M^{CE}$. Let consumers play $\{y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$ in period 1. Let consumers accept an individually rational contract and do not run if a contract in $M^{RA} \setminus M^{CE}$ is provided. Consumers' expected utility is $W^{no-run}(c^1)$. The best contract in $M^{RA} \setminus M^{CE}$ solves

$$\begin{aligned} \hat{W}^{NR} &= \max_{c^1} W^{no-run}(c^1) \\ \text{s.t. } c^1 &\in M^{RA} \setminus M^{CE}. \end{aligned}$$

In this scenario, in order for a contract in M^{CE} to be chosen by the bank, the expected utility generated by such a contract needs to be higher than not only autarky, but also the best contract in $M^{RA} \setminus M^{CE}$. That is, $\hat{W}^{CE} > \max\{\hat{W}^{AUT}, \hat{W}^{NR}\}$. The way to show the distribution of sunspots still can generate bank runs with this additional condition is similar to that in section 3.2.4. I will start with a lemma that describes the value of \hat{W}^{NR} as a function of the distribution of the signals.

Lemma 4 Given p_{11} , let $\rho' > \rho$. Then $\hat{W}^{NR}(p_{11}, \rho') \geq \hat{W}^{NR}(p_{11}, \rho)$.

The intuition behind Lemma 4 is straightforward. If noises become larger, by lemma 1, the set of M^{CE} becomes smaller, so the set of $M^{RA} \setminus M^{CE}$ becomes larger. The value of the maximum thus increases because the choice set becomes larger.

Proposition 2 Let consumers play the strategy

$$\{y_0(W^{no-run}(c^1) \geq \hat{W}^{AUT}) = \text{accept}, y_0(W^{no-run}(c^1) < \hat{W}^{AUT}) = \text{reject}, \\ y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$$

if a contract in $M^{RA} \setminus M^{CE}$ is provided. There exist $\hat{p}_{11} > 0$ and $\hat{\rho}(p_{11}) \geq 0$ such that if $p_{11} \leq \hat{p}_{11}$ and $\rho \leq \hat{\rho}(p_{11})$ the distribution of the sunspot variables can generate bank runs. In some economies, $\hat{\rho}(p_{11})$ is strictly positive for $p_{11} < \hat{p}_{11}$. Given $p_{11} = 0$, if $\hat{W}^{CNR} > \hat{W}^{AUT}$, then there exist $\hat{\rho}(0) \geq 0$ such that if $\rho \leq \hat{\rho}(0)$, the distribution of the sunspot variables can generate bank runs. In some economies, $\hat{\rho}(0)$ is strictly positive.

Proof. The proof of the proposition is similar to that of proposition 1. Note that $\hat{p}_{11} = \bar{p}_{11}$, because when $\rho = 0$, the set of $M^{RA} \setminus M^{CE}$ is empty.

Holding $0 < p_{11} < \hat{p}_{11}$ constant, the value function of \hat{W}^{CE} is decreasing in ρ , whereas the value function of \hat{W}^{NR} is increasing in ρ . So there exists a cutoff $\hat{\rho}(p_{11}) \geq 0$ below which the expected utility from the best contract in M^{CE} is higher than $\max\{\hat{W}^{AUT}, \hat{W}^{NR}\}$, and above otherwise. However, because the set of $M^{RA} \setminus M^{CE}$ is nonconvex, \hat{W}^{NR} is not necessarily continuous. Therefore, $\hat{\rho}(p_{11})$ can be 0 if there is a little bit of noise in the signals.

If $p_{11} = 0$, then $\hat{W}^{CNR} > \hat{W}^{AUT}$ is the necessary condition for $\hat{\rho}(0)$ to be positive. However, because \hat{W}^{NR} is not necessarily continuous, the cutoffs can be zeros even though $\hat{W}^{CNR} > \hat{W}^{AUT}$. Given the parameters in example 3, we have $\hat{\rho}(0) = 0$, although $\hat{W}^{CNR} > \hat{W}^{AUT}$. However, if $R = 5$ whereas other parameters remain the same, we have $\hat{\rho}(0) = 0.0031$.

Example 4 shows $\hat{\rho}(p_{11})$ is strictly positive in some economies.

Example 4 Continue example 3. $\hat{\rho}$ is pictured as a function of p_{11} in Figure 3. Comparing Figure 2 and Figure 3, we find that the $\hat{\rho}(p_{11})$ is always below $\bar{\rho}(p_{11})$, which means that the set of the sunspots that can generate bank runs is larger if consumers refuse rather than accept the contracts that fail to allow consumers to coordinate on the sunspots.

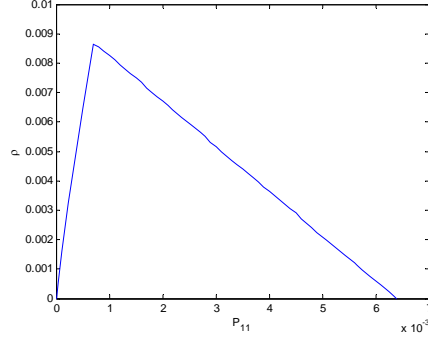


Figure 3: An example of $\hat{\rho}(p_{11})$.

■

Proposition 2 implies that if consumers do not run given a contract in $M^{RA} \setminus M^{CE}$, the correlated equilibrium is more difficult to emerge because the bank has alternative contracts that can possibly achieve higher expected utility. In particular, $\hat{\rho}(p_{11})$ can be zero for $p_{11} < \hat{p}_{11}$. Consider the following situation: Let p_{11} be very close to \hat{p}_{11} . For small noises in the signals, \hat{W}^{CE} is close to \hat{W}^{AUT} . By lemma 1, the contract $c^1 = \frac{R}{1-\alpha+\alpha R}$ is in $M^{RA} \setminus M^{CE}$ for any positive ρ . If $W^{no-run}(\frac{R}{1-\alpha+\alpha R})$ is far above \hat{W}^{AUT} , then the optimal contract will be in the set of $M^{RA} \setminus M^{CE}$, and a contract in M^{CE} will not be provided.

5.2 A banking contract with partial suspension of convertibility

5.2.1 Model Setup

Banking contracts will be generalized in this section. I discuss an economy that bears aggregate uncertainty and let the contract be contingent on the positions of the depositors in the queue. To keep the illustration simple, a discrete case will be analyzed here. I follow the notation and the definitions in Peck and Shell (2003) as much as possible. The preferences of the consumers and technology are exactly the same as in Peck-Shell. There are N depositors in the economy, among whom there are α number of impatient consumers, where $\alpha \leq N$ is a random number with probability density function $f(\alpha)$. Each consumer is endowed with 1 unit of consumption good at $t = 0$. Following Peck-Shell, the utility functions of patient and impatient consumers can be different. The utility function of the impatient consumers is denoted by $u(c^1)$, and the utility function of the patient depositors is by $v(c^1 + c^2)$. u and v are strictly increasing, strictly concave, and twice continuously differentiable. The coefficients of relative risk aversion of u and v are greater than 1. The assumption of different utility functions helps generate a correlated equilibrium with fewer consumers²⁰ thus reduces the dimension of the contract (which depends on the total number of consumers).

The specification of the information structure is the same as in the previous framework. Consumers do not know which network they will be in ex ante. Group i has N_i number of consumers, where $i = 1, 2$, and $N_1 + N_2 = N$. N_1 and N_2 are known ex ante. Consumers have probability N_1/N to be in group 1, and probability N_2/N to be in group 2. For each α ,

²⁰Let us call $v(x) = u(x)$ for $x > 0$ a benchmark case. Now suppose $u'(x) > v'(x)$ for $x > 0$. Because the impatient consumer has higher marginal utility, consumers prefer to have more consumption goods (compared to the benchmark case) allocated to the state where they turn out to be impatient. Thus, payments in period 1 are raised in general. This gives a patient consumer an incentive to run even though not many consumers run the bank or the probability of runs is small.

let α_i ($0 \leq \alpha_i \leq \min\{N_i, \alpha\}$) be the number of impatient consumers in group i , $i = 1, 2$, and $\alpha_1 + \alpha_2 = \alpha$. Denote the ex-ante conditional probability of having α_1 impatient consumers in group 1 and α_2 in group 2 conditional on α by $g(\alpha_1, \alpha_2|\alpha)$. The *ex-ante* probability that there are α number of impatient consumers, α_1 of them in group 1 and α_2 of them in group 2 is:

$$h(\alpha_1, \alpha_2, \alpha) = f(\alpha)g(\alpha_1, \alpha_2|\alpha).$$

After the consumption shock and information shock are realized, a patient consumer updates the probability of α by the Bayes' rule conditional on his consumption type and information type (which group he is in). The ex-post probability of α , contingent on a consumer being patient is denoted by $f_p(\alpha)$. The ex-post probability that there are α_i number of patient consumers in group i contingent on α and on a patient consumer is in group i is denoted by $g_p^i(\alpha_1, \alpha_2|\alpha)$. Hence, the ex-post probability that there are α number of impatient consumers and among them α_1 are in group 1 and α_2 are in group 2 for a patient consumer in group i is:

$$h_p^i(\alpha_1, \alpha_2, \alpha) = f_p(\alpha)g_p^i(\alpha_1, \alpha_2|\alpha)$$

The technology is the same as in the demand deposit case. Following Peck-Shell, let $c^1(z)$ denote the period 1 withdrawal of consumption by the consumer in arrival position z . The resource constraint is

$$c^2(\alpha^1) = \frac{N - \sum_{z=1}^{\alpha^1} c^1(z)}{N - \alpha^1} R, \quad c^1(N) = N - \sum_{z=1}^{N-1} c^1(z). \quad (4)$$

Consumers do not know their positions in the line when they make withdrawals. They have equal chance to be in any position in the line. If there are α^1 depositors withdrawing the deposits, then the probability of getting $c^1(z)$ will be $\frac{1}{\alpha^1}$ for $z = 1, 2, \dots, \alpha^1$. Therefore, the expected utility for a patient consumer if he withdraws the deposit in period 1 is $\frac{1}{\alpha^1} \sum_{z=1}^{\alpha^1} v(c^1(z))$.

5.2.2 Sets of contracts

A banking contract ψ that allows for partial suspension in the postdeposit game is described by the vector

$$\begin{aligned} \psi = & (c^1(1), \dots, c^1(z), \dots, c^1(N), c^2(0), \dots, c^2(N-1)) \text{ and} \\ & (c^1(1), \dots, c^1(z), \dots, c^1(N), c^2(0), \dots, c^2(N-1)) \text{ satisfies (4).} \end{aligned}$$

The participation incentive compatibility constraint requires

$$\sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{N - \sum_{z=1}^{\alpha} c^1(z)}{N - \alpha} R\right) \geq \sum_{\alpha=0}^{N-1} f_p(\alpha) \left[\frac{1}{\alpha + 1} \sum_{z=1}^{\alpha+1} v(c^1(z)) \right]. \quad (5)$$

The set of feasible banking contracts Ψ is defined as:

$$\Psi = \left\{ \psi \in R_+^{2N} : (4) - (5) \text{ hold for all } \alpha \right\}.$$

A run-proof contract requires

$$v\left(\left[N - \sum_{z=1}^{N-1} c^1(z)\right] R\right) \geq \frac{1}{N} \sum_{z=1}^N v(c^1(z)). \quad (6)$$

The set of run-proof banking contracts Ψ^{RP} is defined as:

$$\Psi^{RP} = \{\psi \in \Psi : (6) \text{ hold for all } \alpha\}.$$

I continue to use the definition of a correlated equilibrium as in the main text. The set of the contracts that the bank can choose from given the sunspot signals must satisfy the following conditions: For a patient consumer in group i ,

$$\begin{aligned} \Pr(\theta_{-i} = 0 | \theta_i = 1) \sum_{\alpha=0}^{N-1} \sum_{\alpha_i=0}^{\min\{N_i-1, \alpha\}} h_p^i(\alpha_1, \alpha_2, \alpha) \frac{\sum_{z=1}^{N_i+\alpha-i} v(c^1(z))}{N_i + \alpha - i} + & (7) \\ \Pr(\theta_{-i} = 1 | \theta_i = 1) \frac{1}{N} \sum_{z=1}^N v(c^1(z)) \geq \\ \Pr(\theta_{-i} = 0 | \theta_i = 1) \sum_{\alpha=0}^{N-1} \sum_{\alpha_i=0}^{\min\{N_i-1, \alpha\}} h_p^i(\alpha_1, \alpha_2, \alpha) v \left(\frac{N - \sum_{z=1}^{N_i+\alpha-i-1} c^1(z)}{\alpha - i + 1} R \right) + \\ \Pr(\theta_{-i} = 1 | \theta_i = 1) v \left(\frac{N - \sum_{z=1}^{N-1} c^1(z)}{N} R \right) \end{aligned}$$

$$\begin{aligned} \Pr(\theta_{-i} = 1 | \theta_i = 0) \sum_{\alpha=0}^{N-1} \sum_{\alpha_i=0}^{\min\{N_i-1, \alpha\}} h_p^i(\alpha_1, \alpha_2, \alpha) v \left(\frac{N - \sum_{z=1}^{N-i+\alpha} c^1(z)}{N_i - \alpha} R \right) + & (8) \\ \Pr(\theta_{-i} = 0 | \theta_i = 0) \sum_{\alpha=0}^{N-1} f_p(\alpha) v \left(\frac{Ny - \sum_{z=1}^{\alpha} c^1(z)}{N - \alpha} R \right) \geq \\ \Pr(\theta_{-i} = 1 | \theta_i = 0) \sum_{\alpha=0}^{N-1} \sum_{\alpha_i=0}^{\min\{N_i-1, \alpha\}} h_p^i(\alpha_1, \alpha_2, \alpha) \frac{\sum_{z=1}^{N-i+\alpha+1} v(c^1(z))}{N - i + \alpha + 1} + \\ \Pr(\theta_{-i} = 0 | \theta_i = 0) \sum_{\alpha=0}^{N-1} f_p(\alpha) \frac{\sum_{z=1}^{\alpha+1} v(c^1(z))}{\alpha + 1} \end{aligned}$$

The set of banking contracts that are consistent with a correlated equilibrium is defined as:

$$\Psi^{CE} = \{\psi \in \Psi : (7) - (8) \text{ hold for all } \alpha \text{ and } i = 1, 2\}.$$

Example 5 The parameters in the following example are similar to that in Peck and Shell (2003). There are two consumers, one in each group. The probability of being in either group is equal for both of them ex ante. Let $u(x) = \frac{Ax^{1-a}}{1-a}$, $v(x) = \frac{x^{1-b}}{1-b}$, $A = 7$, $a = b = 1.01$, and $R = 1.1$, $y = 3$. A depositor is impatient with probability p , $p = 0.4$. In this simple example, there is only one choice variable, which is $c^1(1)$. Let $p_{11} = 0.001$, $p_{10} = 0$, $p_{01} = 0.009$. Sets of banking contracts are described as follows:

Table 1: An example of partitions of feasible banking contracts
(partial suspension)

ψ in	$c^1(1)$
Ψ	$[0, 3.2937]$
Ψ^{RP}	$[0, 3.2852]$
Ψ^{CE}	$[3.2928, 3.2936]$
$\Psi^{RA} \setminus \Psi^{CE}$	$(3.2852, 3.2928) \cup (3.2936, 3.2937]$

5.2.3 Equilibrium bank runs

The bank provides the best contract. Consumers compare the expected utility under autarky with the ex-ante expected utility the contract yields. The optimal contract that allows for a correlated equilibrium can be calculated in the same way as in the previous section. The example below show that in some economies, the optimal banking contract with partial suspension of convertibility allows for a correlated equilibrium.

Example 6 The economy has three consumers: Group 1 has 1 consumer and group 2 has 2 consumers. $\alpha = 0.5$. $u(x) = \frac{Ax^{1-a}}{1-a}$, $v(x) = \frac{x^{1-b}}{1-b}$, $A = 10$, $a = b = 1.01$, $R = 2$. $y = 3$. $p_{11} = 0.0001$, $p_{10} = 0$, $p_{01} = 0.0009$. There are two choice variables here: $c^1(1)$ and $c^1(2)$. Welfare is normalized to be $W + 1629$. In autarky, $W^{aut} = -1.9473$. The highest ex-ante

expected utility that the best contracts in each subset can achieve are:

Table 2: An example of an optimal contract that allows for a correlated equilibrium (partial suspension)

ψ in	$c^1(1)$	$c^1(2)$	$\hat{W}(m)$
Ψ^{RP}	4.7340	2.9479	0.3280
Ψ^{CE}	4.8681	3.1101	0.3930
$\Psi^{RA} \setminus \Psi^{CE}$	4.7475	3.0718	0.3756

In this economy, full bank runs and partial bank runs are allowed in the predeposit game.

5.3 Proofs of lemmas

Proof of Lemma 1. Let $M^{CE}(p_{11}, \rho) \neq \emptyset$. Prove the proposition in two steps. First, I illustrate that for any c^1 in $M^{CE}(\rho'; p_{11})$, it is also in $M^{CE}(p_{11}, \rho)$. Second, I determine that there exists some c^1 in $M^{CE}(p_{11}, \rho)$ but not in $M^{CE}(p_{11}, \rho')$.

Discuss cases by parameters. Let $(\alpha n_1 + n_2)c^1 \leq 1$ and $(\alpha n_2 + n_1)c^1 \leq 1$. Rewrite inequalities (2) – (3) for both groups. We have

$$\rho \left[u(c^1) - u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right) \right] \geq -p_{11} \frac{1}{c^1} u(c^1), \quad (2')$$

$$-\rho \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(\alpha n_1+n_2)c^1}{(1-\alpha)n_1}R\right) \right] - \rho \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1) \right] \geq -(1-p_{11}) \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1) \right]; \quad (3')$$

$$\rho \left[u(c^1) - u\left(\frac{1-(\alpha n_1+n_2)c^1}{(1-\alpha)n_1}R\right) \right] \geq -p_{11} \frac{1}{c^1} u(c^1), \quad (2'')$$

$$-\rho \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right) \right] - \rho \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1) \right] \geq -(1-p_{11}) \left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1) \right]. \quad (3'')$$

We focus on (2') – (3'). Same reasoning applies to (2'') and (3''). The RHS of (2') is negative. If $u(c^1) - u(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R) \geq 0$, any change in ρ does not affect the sign of (2'). If $u(c^1) - u(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R) < 0$, a decrease in ρ raises the LHS. Therefore, the inequality in (2') still holds for $\rho' \geq \rho$. In (3'), each term in the brackets is positive. The LHS is decreasing in ρ . Therefore, the contract satisfies (3') if $\rho' \geq \rho$.

In the second step, I show that for $\rho' \geq \rho$, there exists a contract c^1 that is in $M^{CE}(p_{11}, \rho)$ but not in $M^{CE}(p_{11}, \rho')$. Suppose that there is $c^1 \in M^{CE}(p_{11}, \rho)$ such that at least one of (2') – (3'') is binding. Change in the probabilities from ρ to ρ' breaks down the inequality sign, such that at least one of (2') – (3'') is no longer satisfied. Such a contract is not in $M^{CE}(p_{11}, \rho')$. Next, I prove that there exists at least one contract in $M^{CE}(p_{11}, \rho)$ such that at least one of (2'') – (3'') is binding. If $\rho = 0$, (2') and (2'') are always satisfied, and (3') and (3'') are binding at $c^1 = \frac{R}{1-\alpha+\alpha R}$. If $\rho > 0$, then (3') and (3'') are violated at $c^1 = \frac{R}{1-\alpha+\alpha R}$. By the continuity of the utility function, the inequality must be binding at some $c^1 < \frac{R}{1-\alpha+\alpha R}$ because $M^{CE}(p_{11}, \rho)$ is not empty.

If $M^{CE}(p_{11}, \rho) = \emptyset$, at least one of the inequalities is not satisfied for any feasible c^1 . It is easy to see that increasing noises (ρ) will not restore the inequalities.

In the same way, we can prove the other three cases in which (1) $(\alpha n_1 + n_2)c^1 > 1$ and $(n_1 + \alpha n_2)c^1 \leq 1$, or (2) $(\alpha n_1 + n_2)c^1 \leq 1$ and $(n_1 + \alpha n_2)c^1 > 1$, or (3) $(\alpha n_1 + n_2)c^1 < 1$ and $(n_1 + \alpha n_2)c^1 < 1$. ■

Proof of Lemma 2. Suppose $(\alpha n_1 + n_2)c^1 \leq 1$ and $(\alpha n_2 + n_1)c^1 \leq 1$. Let $\bar{v}(c^1) = \max \left\{ u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right), u\left(\frac{1-(n_2+\alpha n_1)c^1}{(1-\alpha)n_1}R\right) \right\}$, and $\underline{v}(c^1) = \min \left\{ u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right), u\left(\frac{1-(n_2+\alpha n_1)c^1}{(1-\alpha)n_1}R\right) \right\}$.

Rewrite (2') – (3''), by simple algebra, we have $\varepsilon(p_{11}, c^1)$, the upper bound of ρ , as follows:

$$\varepsilon(p_{11}, c^1) = \begin{cases} \min \left\{ \begin{array}{l} \frac{p_{11} \frac{1}{c^1} u(c^1)}{\bar{v}(c^1) - u(c^1)}, \\ (1 - p_{11}) \left[u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) - u(c^1) \right] \\ \frac{2u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) - \underline{v}(c^1) - u(c^1)}{(1 - p_{11}) \left[u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) - u(c^1) \right]} \end{array} \right\}, & \text{if } \bar{v}(c^1) > u(c^1) \\ \frac{(1 - p_{11}) \left[u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) - u(c^1) \right]}{2u \left(\frac{1 - \alpha c^1}{1 - \alpha} R \right) - \underline{v}(c^1) - u(c^1)}, & \text{if } \bar{v}(c^1) \leq u(c^1). \end{cases}$$

It is easy to see that $\rho = 0$ if and only if (1) $c^1 = \frac{R}{1 - \alpha + \alpha R}$, or (2) $p_{11} = 0$ and $\bar{v}(c^1) > u(c^1)$ (i.e. $c^1 < \max \left\{ u \left(\frac{1 - (n_1 + \alpha n_2) c^1}{(1 - \alpha) n_2} R \right), u \left(\frac{1 - (n_2 + \alpha n_1) c^1}{(1 - \alpha) n_1} R \right) \right\}$).

In the same way, we can get the upper bounds of ρ for the other three cases in which (1) $(\alpha n_1 + n_2) c^1 > 1$ and $(n_1 + \alpha n_2) c^1 \leq 1$, or (2) $(\alpha n_1 + n_2) c^1 \leq 1$ and $(n_1 + \alpha n_2) c^1 > 1$, or (3) $(\alpha n_1 + n_2) c^1 < 1$ and $(n_1 + \alpha n_2) c^1 < 1$. ■

Proof of Lemma 3. By lemma 1, given p_{11} , the set of M^{CE} diminishes when ρ increase. Let $\rho' > \rho$. The solution to PCE, given (p_{11}, ρ') , $c^{1'}$, is in $M^{CE}(p_{11}, \rho)$. Plug $c^{1'}$ into the objective function of PCE given (p_{11}, ρ) . Denote the expected utility achieved by $W(c^{1'}; p_{11}, \rho)$. The expected utility under partial runs is lower than under no run, so $W(c^{1'}; p_{11}, \rho) > \hat{W}(c^{1'}; p_{11}, \rho')$. Because $\hat{W}(c^1; p_{11}, \rho)$ is (weakly) greater than $W(c^{1'}; p_{11}, \rho)$, we have $\hat{W}(c^1; p_{11}, \rho) > \hat{W}(c^{1'}; p_{11}, \rho')$. ■

Proof of Lemma 4. By lemma 1, $M^{CE}(p_{11}, \rho') \subset M^{CE}(p_{11}, \rho)$. As $M^{RA} \setminus M^{CE}$ is a complement to M^{CE} , we have $M^{RA} \setminus M^{CE}(p_{11}, \rho) \subset M^{RA} \setminus M^{CE}(p_{11}, \rho')$. The value function of \hat{W}^{NR} decreases in ρ because the choice set shrinks when ρ gets larger. ■