Herd Behavior in Bank Runs with Partial Suspension of Convertibility*

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February 24, 2008

Abstract

I study a two-depositor, two-stage, sequential-move bank run model in an economy with aggregate consumption uncertainty and production uncertainty. I consider banking contracts that are contingent on the publicly observed withdrawal history. Depositors have private information about their consumption types and they receive noisy private signals about the quality of the bank's portfolio. In some economies, the optimal banking contract permits bank runs that bear herd effect. Some of these runs are efficient in that the bank's portfolio is liquidated before it worsens. Others are not efficient; there are cases in which the herd is misled.

JEL Classification Numbers: C73, D82, E59, G21

Keywords: Bank runs, herd, imperfect information, perfect Bayesian equilibrium, optimal bank contract, sequential-move game, fundamental-based bank runs.

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1 Introduction

Classic literature on bank runs, built upon Diamond and Dybvig (1983), assumes that the decisions to withdraw are made simultaneously. The lack of dynamics in the classic literature makes it difficult to explain some observed features of bank runs. For example, recent empirical studies suggest that some withdrawals are based on the previous withdrawals made by others.[1] In other words, some bank runs bear the feature of the herd effect (Banerjee, 1992; Bikhchandani et al., 1992). The purpose of this paper is to build a simple model with detailed dynamics to explain the herd effect in bank runs.

I build a simple two-depositor, two-stage model in which each depositor's action is observable. A depositor receives a private signal about his consumption type and a private noisy signal about the quality of the bank's portfolio. Depositors make withdrawal decisions in sequence at a given time slot. The first depositor's action to withdraw can partially reveal his private signal about the bank's portfolio, which affects the belief of the second depositor, and thus, his decision. Given a contract, there exists at least one perfect Bayesian equilibrium in which depositors' strategies are contingent on their private signals and observed withdrawals.

I study a banking contract that pays depositors contingent on the withdrawal history. The consideration of a payment scheme contingent on history has been widely discussed in the banking literature. In Diamond and Dybvig's (1983) classic work, suspension of convertibility can prevent panic-based bank runs in the economy without aggregate consumption uncertainty. Wallace (1988) proposes that partial suspension of convertibility is the best contract in a static economy with aggregate consumption uncertainty. Green and Lin (2003a, b) further the idea of partial suspension. They prove that in the economy with aggregate consumption shocks, the optimal banking contract

[1] See Schumacher (2005), Starr and Yilmaz (2007).

that pays depositors depending on their arrival time and the withdrawal history can completely eliminate panic-based bank runs.

However, the contingent payment scheme cannot eliminate the bank runs in an economy with uncertainties in both aggregate consumption and portfolio returns. Computed examples show that in some economies, a contract that permits bank runs is optimal, whereas in other economies, a run-proof contract is optimal. A depositor's decision to withdraw carries noisy information about the signals he receives – he might need to consume immediately or he might receive a low signal about the quality of the bank's portfolio. Unlike Green and Lin (2003a, b), the asymmetric information between the bank and depositors cannot be fully eliminated by depositors' simple zero-one decisions.

Besides the additional dimension of uncertainty, there are two other distinctions between Green and Lin's model and mine. First, Green and Lin employ a direct revelation mechanism in which depositors report their private information about consumption types to the bank. The direct revelation mechanism is not crucial in their model, because the contingent payment scheme induces depositors to report truthfully. It is equivalent to asking depositors to come to the bank if they need to consume immediately. However, the direct revelation mechanism is not feasible in an economy with two dimensions of uncertainties – A depositor will tell the bank he receives a high productivity signal and a consumption shock when he decides to withdraw.

Second, depositors do not observe the decision of the others in Green and Lin's economy but they do in mine. Whether a decision is observable is again not crucial to Green and Lin's model. However, it is crucial in my model because the herd effect is based on the observed withdrawals. Without this assumption, a bank run occurs solely due to the private signals of the depositors.

The rest of the paper will be organized as follows: In section 2, I introduce the model setup. In section 3 I discuss the equilibrium given a banking contract. I calculate some examples of optimal contracts in section 4. Conclusions are made in section 5.

2 The Model

There are three periods ($t = 0, 1, 2$) and two depositors in the economy. $t = 0$ is ex ante, and $t = 1, 2$ are ex post. Depositors are identical ex ante, but they face liquidity shocks at $t = 1$. Each depositor has probability of α to be impatient, and probability of $1 - \alpha$ to be patient. An impatient depositor only values consumption at $t = 1$. His utility function is described by $u(c_1)$, where c_1 is the consumption at $t = 1$. A patient depositor can wait until the last period to consume. His utility is described by $u(c_1 + c_2)$, where c_2 denotes the consumption at $t = 2$. $u(x)$ is strictly increasing, strictly concave, and twice differentiable. The coefficient of relative risk aversion, $xu'(x)/u''(x) > 1$, for $x \geq 1$.

Each depositor is endowed with one unit of consumption good ex ante. The consumption good can be stored at no cost. It also can be invested in production. Production is rigid and random. The investment has to be made ex ante, and it takes two periods to mature. The return on production can be either $\overline{R} > 1$ or $\underline{R} < 1$ at $t = 2$. The ex ante probability of getting $\overline{R}$ is p_0. If the assets in production are liquidated at $t = 1$, it will pay one unit of consumption good.

Period 1 ($t = 1$) is divided into two stages. At each stage, one depositor is informed of a pair of signals. One signal tells him precisely his consumption type. The other imperfectly tells him the production status. The production signal is accurate with probability q. That is,

$$\Pr(S_n = H | R = \overline{R}) = \Pr(S_n = L | R = \underline{R}) = q.$$

Depositors have an equal chance to receive signals at stage 1. The depositor who gets the signals at the first stage is called depositor 1. The other is called depositor 2.

A representative bank offers a contract to the depositors ex ante. For convenience, the minimum amount of deposits that the bank accepts from an individual depositor is one unit of consumption good. The bank allocates the funds between storage and production and makes payments to the depositors upon withdrawals. Each depositor can withdraw his deposits when he receives signals in period 1. If he does not withdraw in period 1, he gets paid in period 2. For convenience, each depositor has only one chance to withdraw in period 1. Because there are only two depositors and two stages, allowing depositors to withdraw any time they want only adds two possible simultaneous-move games at each stage but will not change the main results. If in equilibrium, depositor 2's decision depends on the observed action by depositor 1, then there is a herd effect in the game.

The bank offers a contract that specifies the payments to withdrawals at each stage at $t = 1$ depending on the number of withdrawals that have been made and payments to withdrawals at $t = 2$ depending on the number of withdrawals at $t = 1$ and the realization of the production. Let $c^1(1)$, $c^1(0,1)$, and $c^1(1,1)$ denote the payments at stage 1, at stage 2 if depositor 1 does not withdraw, and at stage 2 if depositor 1 withdraws, respectively. Let $c^2(x_1, x_2, R)$, $x_i = 0$ or 1, $i = 1, 2$ denote the payment at $t = 2$. Note that $c^2(x_1, x_2, R)$ is increasing in R. Denote the bank contract by m, and

$$\begin{aligned} m &= \left(c^1(1), c^1(0,1), c^1(1,1), c^2\left(0,0,\overline{R}\right), c^2\left(0,0,\underline{R}\right),\right. \\ &\quad \left. c^2\left(0,1,\overline{R}\right), c^2\left(0,1,\underline{R}\right), c^2\left(1,0,\overline{R}\right), c^2\left(1,0,\underline{R}\right)\right), \end{aligned}$$

where all instances of c^1 and c^2 satisfy the resource constraints:

$$\begin{aligned}
0 &\leq c^1(1),\ c^1(0,1) \leq 2, \\
0 &\leq c^1(1,1) \leq 2 - c^1(1), \\
\left(2 - c^1(1)\right)\underline{R} &\leq c^2(1,0,\underline{R}) \leq 2 - c^1(1) \leq c^2\left(1,0,\overline{R}\right) \leq \left(2 - c^1(1)\right)\overline{R}, \\
\left(2 - c^1(0,1)\right)\underline{R} &\leq c^2(0,1,\underline{R}) \leq 2 - c^1(0,1) \leq c^2\left(0,1,\overline{R}\right) \leq \left(2 - c^1(0,1)\right)\overline{R}, \\
\underline{R} &\leq c^2(0,0,\underline{R}) \leq 1 \leq c^2\left(0,0,\overline{R}\right) \leq \overline{R}.
\end{aligned}$$

Let M denote the set that contains all contracts satisfying the resource constraints.

Because the bank's contract is contingent on the number of withdrawals made, the bank always meets the payment demands. If one depositor is paid less than the other in the same period, the bank does not default on the payments, because the situation is described in the contract. For such a contract, we say a bank run happens if the depositors withdraw the deposits but do not need to consume immediately. If a run-proof contract is provided, depositors do not withdraw unless they are impatient.

Sequence of timing is as follows:

$t = 0$:

Bank announces the contract.

Depositors make deposit decision.

$t = 1$:

Stage 1 :

Depositor 1 receives signals about his consumption type and about productivity.

He decides whether to withdraw or not.

Stage 2 :

Depositor 2 receives signals about his consumption type and about productivity.

He decides whether to withdraw or not.

$t = 2$:

Bank allocates the remaining resource to the depositors who do not withdraw in period 1.

The postdeposit game starts after depositors make deposits at the bank. An individual depositor decides when to withdraw from the bank. Knowing what depositors will be doing in the postdeposit game, the competitive bank offers a contract that maximizes the ex-ante expected utility of the depositors at $t = 0$. Depositors determine whether to deposit at the bank or stay in autarky. Starting at $t = 0$, the entire game is called the predeposit game. Following Peck and Shell (2003), I first assume a banking contract and describe the equilibrium. I start with the postdeposit game, then I compare the expected utilities in autarky and in a banking economy to complete the predeposit game.

3 Postdeposit Game

Given the contract announced at $t = 0$, each depositor decides whether he wants to withdraw when he receives the signals. I look for a perfect Bayesian equilibrium in the postdeposit game in which the strategies of the depositors are optimal given their beliefs about productivity and the beliefs are by the Bayes rule whenever possible.

The public history records withdrawals that have occurred at each stage. Depositor i's private history includes the public history and the signals he receives. For example, when depositor 1 makes a decision at stage 1, his private history is $(\varnothing, S_1)$, which means there is no previously made withdrawals and the productivity signal he receives is S_1. When depositor 2 makes a decision at stage 2, his private history is (x_1, S_2). At the end of stage 2, each depositor's privately history is (x_1, x_2, S_i).

Depositor 1's strategies are: withdraw if impatient or withdraw with probability

θ_L^1 if a low signal is received;

θ_H^1 if a high signal is received.

Depositor 2's strategies are: withdraw if impatient or withdraw with probability

$\theta_{1,L}^2$ if a low signal is received and depositor 1 withdraws;

$\theta_{0,L}^2$ if a low signal is received and depositor 1 waits;

$\theta_{1,H}^2$ if a high signal is received and depositor 1 withdraws;

$\theta_{0,H}^2$ if a high signal is received and depositor 1 waits.

When a depositor receives his signal about productivity, he updates his belief about the productivity being high by the following Bayes rule:

$$p_n^i(p) = \begin{cases} P_L(p) = \dfrac{p(1-q)}{p(1-q)+(1-p)q}, & \text{if } S_n = L; \\ P_H(p) = \dfrac{pq}{pq+(1-p)(1-q)}, & \text{if } S_n = H. \end{cases} \tag{1}$$

where p is the prior at the stage n. p_n^i represents depositor i's posterior belief at stage n.

When depositor 1 makes a decision at stage 1, his decision carries noisy information about the signals he has received. Depositor 2 updates his belief about the productivity being high by the Bayes rule. Given depositor 1's strategies, depositor 2's belief at stage 1 is

$$p_1^2(p) = \begin{cases} P_{\widetilde{L}}^2(p) = \dfrac{p\left[(1-\alpha)\left(\theta_L^1(1-q)+\theta_H^1 q\right)+\alpha\right]}{\alpha+(1-\alpha)\left\{\theta_L^1\left[p(1-q)+(1-p)q\right]+\theta_H^1\left[pq+(1-p)(1-q)\right]\right\}}, \\ \qquad \text{if depositor 1 withdraws;} \\ \\ P_{\widetilde{H}}^2(p) = \dfrac{p\left[\left(1-\theta_L^1\right)(1-q)+\left(1-\theta_H^1\right)q\right]}{\left(1-\theta_L^1\right)\left[(1-p)q+p(1-q)\right]+\left(1-\theta_H^1\right)\left[pq+(1-p)(1-q)\right]}, \\ \qquad \text{if depositor 1 waits.} \end{cases} \tag{2}$$

After depositor 2 makes his decision, depositor 1's updated belief (although he has no chance to change his decision) is:

$$p_2^1(p) = \begin{cases} P_{x_1,\widetilde{L}}^1(p) = \dfrac{p\left[(1-\alpha)\left(\theta_{x_1,L}^2(1-q)+\theta_{x_1,H}^2 q\right)+\alpha\right]}{\alpha+(1-\alpha)\left\{\theta_{x_1,L}^2\left[p(1-q)+(1-p)q\right]+\theta_{x_1,H}^2\left[pq+(1-p)(1-q)\right]\right\}}, \\ \qquad \text{if depositor 2 withdraws;} \\ \\ P_{x_1,\widetilde{H}}^1(p) = \dfrac{p\left[\left(1-\theta_{x_1,L}^2\right)(1-q)+\left(1-\theta_{x_1,H}^2\right)q\right]}{\left(1-\theta_{x_1,L}^2\right)\left[(1-p)q+p(1-q)\right]+\left(1-\theta_{x_1,H}^2\right)\left[pq+(1-p)(1-q)\right]}, \\ \qquad \text{if depositor 2 waits.} \end{cases} \tag{3}$$

If depositor 1 does not withdraw at stage 1, his expected utility at the end of stage 2 is

$$\hat{w}^1(0,1,S_1) = P_{0,\widetilde{L}}^1 P_{S_1}(p_0)\, u\left(c^2\left(0,1,\overline{R}\right)\right) + \left(1 - P_{0,\widetilde{L}}^1 P_{S_1}(p_0)\right) u\left(c^2\left(0,1,\underline{R}\right)\right); \tag{4}$$

$$\hat{w}^1(0,0,S_1) = P_{0,\widetilde{H}}^1 P_{S_1}(p_0)\, u\left(c^2\left(0,0,\overline{R}\right)\right) + \left(1 - P_{0,\widetilde{H}}^1 P_{S_1}(p_0)\right) u\left(c^2\left(0,0,\underline{R}\right)\right). \tag{5}$$

The equilibrium strategies are a vector of $\theta = \left(\theta_L^1, \theta_H^1, \theta_{1,L}^2, \theta_{0,L}^2, \theta_{1,H}^2, \theta_{0,H}^2\right)$ that solves

$$\hat{w}^2(1, S_2) = \max_{\theta_{1,S_2}^2 \in [0,1]} \theta_{1,S_2}^2 u\left(c^1(1,1)\right) + \tag{6}$$

$$+\left(1-\theta_{1,S_2}^2\right)\left[P_{S_2} P_{\widetilde{L}}^2(p_0) u\left(c^2\left(1,0,\overline{R}\right)\right) + \left(1 - P_{S_2} P_{\widetilde{L}}^2(p_0)\right) u\left(c^2\left(1,0,\underline{R}\right)\right)\right],$$

$$\hat{w}^2(0, S_2) = \max_{\theta_{0,S_2}^2 \in [0,1]} \theta_{0,S_2}^2 u\left(c^1(0,1)\right) + \tag{7}$$

$$+\left(1-\theta_{0,S_2}^2\right)\left[P_{S_2} P_{\widetilde{H}}^2(p_0) u\left(c^2\left(0,0,\overline{R}\right)\right) + \left(1 - P_{S_2} P_{\widetilde{H}}^2(p_0)\right) u\left(c^2\left(0,0,\underline{R}\right)\right)\right],$$

where $S_2 = L$, H, and

$$\hat{w}^1(S_1) = \max_{\theta_{S_1}^1 \in [0,1]} \theta_{S_1}^1 u\left(c^1(1)\right) + \tag{8}$$

$$+\left(1-\theta_{S_1}^1\right)\left[\begin{array}{c} \left(\alpha + (1-\alpha)\left(\begin{array}{c} \theta_{0,L}^2\left(P_{S_1}(p_0)(1-q) + (1-P_{S_1}(p_0))q\right) + \\ \theta_{0,H}^2\left(P_{S_1}(p_0)\, q + (1-P_{S_1}(p_0))(1-q)\right) \end{array}\right)\right) \hat{w}^1(0,1,S_1) + \\ (1-\alpha)\left(\begin{array}{c} \left(1-\theta_{0,L}^2\right)\left(P_{S_1}(p_0)(1-q) + (1-P_{S_1}(p_0))q\right) + \\ \left(1-\theta_{0,H}^2\right)\left(P_{S_1}(p)\, q + (1-P_{S_1}(p_0))(1-q)\right) \end{array}\right) \hat{w}^1(0,0,S_1) \end{array}\right],$$

where $S_1 = L$, H.

Depositor 2's expected utility at the end of stage 1 is

$$\hat{w}^2(1) = \alpha u\left(c^1(1,1)\right) + (1-\alpha)\left\{\begin{array}{c} \left[P_{\widetilde{L}}^2(p_0)(1-q) + \left(1-P_{\widetilde{L}}^2(p_0)\right) q\right] \hat{w}^2(1,L) + \\ \left[P_{\widetilde{L}}^2(p_0) q + \left(1-P_{\widetilde{L}}^2(p_0)\right)(1-q)\right] \hat{w}^2(1,H) \end{array}\right\}, \tag{9}$$

$$\hat{w}^2(0) = \alpha u\left(c^1(0,1)\right) + (1-\alpha)\left\{\begin{array}{c} \left[P_{\widetilde{H}}^2(p_0)(1-q) + \left(1-P_{\widetilde{H}}^2(p_0)\right) q\right] \hat{w}^2(0,L) + \\ \left[P_{\widetilde{H}}^2(p_0) q + \left(1-P_{\widetilde{H}}^2(p_0)\right)(1-q)\right] \hat{w}^2(0,H) \end{array}\right\}. \tag{10}$$

Knowing each of them has probability $\frac{1}{2}$ to be the first to receive the signals and make a decision, a depositor calculates his expected utility at the beginning of period 1 as follows:

$$w_0 = \frac{1}{2}\left\{\alpha u\left(c^1(1)\right) + (1-\alpha)\left[\left(p_0(1-q)+(1-p_0)q\right)\hat{w}^1(L) + \left(p_0 q + (1-p_0)(1-q)\right)\hat{w}^1(H)\right]\right\} \quad (11)$$

$$+\frac{1}{2}\left\{\begin{array}{c} \left[\alpha + (1-\alpha)\left[\left(p_0(1-q)+(1-p_0)q\right)\theta_L^1 + \left(p_0 q + (1-p_0)(1-q)\right)\theta_H^1\right]\right]\hat{w}^2(1) + \\ (1-\alpha)\left[\left(p_0(1-q)+(1-p_0)q\right)\left(1-\theta_L^1\right) + \left(p_0 q + (1-p_0)(1-q)\right)\left(1-\theta_H^1\right)\right]\hat{w}^2(0) \end{array}\right\}.$$

Because there are a finite number of players and a finite number of strategies, the postdeposit game has at least one perfect Bayesian equilibrium. In an equilibrium, depositor 2 will make an inference of the productivity status by watching depositor 1's action. His belief is updated by his private signal and depositor 1's action. When depositor 1 makes a decision, he also knows his decision will affect depositor 2's belief and decision, and thus, depositor 1's payoff.

The solution to maximization problems $(6)-(8)$ given a contract is not necessarily unique. Example 1 shows a case in which a contract can allow for more than one perfect Bayesian equilibrium.

Example 1 The utility function and other parameters in the economy are: $u(c) = \frac{(c+b)^\gamma - b^\gamma}{1-\gamma}$, $b = 0.01$, $\gamma = 1.5$. $\alpha = 0.5$. $\overline{R} = 1.3$, $\underline{R} = 0.1$, $p_0 = 0.5$. $q = 0.7$.

Let the contract be $c^1(1) = 0.9998$, $c^1(1,1) = 1.0002$, $c^1(0,1) = 1.0000$, $c^2\left(0,0,\overline{R}\right) = 1.0000$, $c^2\left(0,1,\overline{R}\right) = 1.0001$, $c^2\left(1,0,\overline{R}\right) = 1.0002$, $c^2\left(0,0,\underline{R}\right) = 1.0000$, $c^2\left(0,1,\underline{R}\right) = 0.9997$, and $c^2\left(1,0,\underline{R}\right) = 1.0002$.

The contract has two pure strategy perfect Bayesian equilibria. They are

(1) $\left(\theta_L^1 = 0,\ \theta_H^1 = 0,\ \theta_{1,L}^2 = 1,\ \theta_{0,L}^2 = 1,\ \theta_{1,H}^2 = 0,\ \theta_{0,H}^2 = 0\right)$, and

(2) $\left(\theta_L^1 = 1,\ \theta_H^1 = 0,\ \theta_{1,L}^2 = 1,\ \theta_{0,L}^2 = 0,\ \theta_{1,H}^2 = 0,\ \theta_{0,H}^2 = 0\right)$.

In the first equilibrium, depositor 1's productivity signal does not affect his decision. Depositor 2 cannot infer any information from depositor 1's action. Thus, depositor 2's decisions are based solely on his private signals, not the withdrawal history.

In the second equilibrium, depositor 1 reacts differently to different productivity signals. His action partially reveals the productivity signal he has received, which affects depositor 2's decision. Depositor 2's decision is dependent on the withdrawal history.

A banking contract is run-proof if $\theta = (0,0,0,0,0,0)$ is the unique solution to maximization problems $(6) - (8)$. All other contracts are called run-admitting.

4 Predeposit Game

The ex-ante expected utility of the depositors are determined by their strategies, which in turn, are determined by the contract. Therefore, w_0 is a function of $\theta(m)$. Knowing the strategies of the depositors in the postdeposit game given a contract, the representative bank offers a contract that maximizes the ex-ante expected utility of the depositors. That is,

$$\hat{w}_0 = \max_{m \in M} w_0 (\theta(m)). \tag{12}$$

To avoid the discussion about which equilibrium actually takes place in the postdeposit game when a contract allows for multiple equilibria, I focus on the cases in which the optimal banking contract allows for a unique equilibrium in the postdeposit game. The analytical solution to the optimal contract is complicated to solve and to present. Hence, I use numerical examples to illustrate that in some economies the optimal contract allows for bank runs that resemble the herd effect.

Given the contract, depositors decide whether to stay in autarky or to deposit at the bank at $t = 0$. In autarky, each depositor receives private signals about consumption type and productivity status. A depositor adjusts his investment portfolio after he receives the signals at $t = 1$. If the ex-ante expected utility in autarky is higher than that under the banking contract, the contract will be accepted, and the postdeposit game will be played. Otherwise, depositors prefer to stay in autarky.

In autarky, if the depositor is revealed to be impatient, he consumes immediately all of his available asset and gets utility of $u(1)$. A patient depositor's expected utility in period 1 after receiving the productivity signal is solved by

$$\hat{w}_1^{aut}(S) = \max_{\lambda_S \in [0,1]} P_S(p_0)\, u\left(\lambda_S + (1-\lambda_S)\overline{R}\right) + (1 - P_S(p_0))\, u\left(\lambda_S + (1-\lambda_S)\underline{R}\right),$$

where $S = H, L$ denotes the realization of the productivity signal and λ_S denotes the amount of asset liquidated after receiving the productivity signal.

The ex-ante expected utility in autarky is the weighted average of the expected utility in period 1. That is,

$$w_0^{aut} = \alpha u(1) + (1-\alpha)\left[(p_0 q + (1-p_0)(1-q))\, \hat{w}_1^{aut}(H) + (p_0(1-q) + (1-p_0)q)\, \hat{w}_1^{aut}(L)\right].$$

If $\hat{w}_0$ given a banking contract is higher than w_0^{aut} but the contract allows for bank runs in the postdeposit game, then the contract will be accepted ex ante by the depositors, who are aware of the possibility of having a bank run. Bank runs are thus tolerated in the equilibrium.

Proposition 1 *In some economies, the optimal banking contract that is contingent on the public withdrawal history is run-admitting.*

Proof. Prove by example.

Example 2 The utility function and other parameters in the economy are: $u(c) = \frac{(c+b)^{\gamma} - b^{\gamma}}{1-\gamma}$, $b = 0.01$, $\gamma = 1.5$. $\alpha = 0.6$. $\overline{R} = 1.2$, $\underline{R} = 0.8$, $p_0 = 0.75$. $q = 0.9$.

Table 1 describes the payment scheme that an optimal contract provides.

Table 1: An example of an optimal contract that is run-admitting

$\hat{w}_0$	$c^1(1)$	$c^1(1,1)$	$c^2\left(1,0,\overline{R}\right)$	$c^2\left(1,0,\underline{R}\right)$
18.0546	1.0064	0.9936	1.1924	0.7949
$c^1(0,1)$	$c^2\left(0,0,\overline{R}\right)$	$c^2\left(0,0,\underline{R}\right)$	$c^2\left(0,1,\overline{R}\right)$	$c^2\left(0,1,\underline{R}\right)$
1.0296	1.2000	0.8000	1.1644	0.7763

Given this contract, there is a unique perfect Bayesian equilibrium in the postdeposit game. In the equilibrium, the strategies of depositors, if patient, are ($\theta_L^1 = 1$, $\theta_H^1 = 0$, $\theta_{1,L}^2 = 1$, $\theta_{0,L}^2 = 0$, $\theta_{1,H}^2 = 0$, $\theta_{0,H}^2 = 0$). Bank runs occur with positive probability. Some bank runs are partial – only one of the depositors withdraws but he does not need to consume immediately; some are full bank runs – both of the depositors withdraw regardless of their consumption types. The probability of having a partial run conducted by depositor 1 in this example is $\Pr(1 \text{ is patient}) \Pr(S_1 = L) \Pr(2 \text{ is impatient or } S_2 = H) = 0.0864$. The partial runs conducted by depositor 2 happen with probability $\Pr(1 \text{ is impatient and } 2 \text{ is patient}) \Pr(S_2 = L) = 0.072$. The probability of having a full bank run is $\Pr(1 \text{ and } 2 \text{ are patient}) \Pr(S_1 = S_2 = L) = 0.0336$.

In autarky, a depositor leaves all assets in production if a high signal is received and liquidates all assets when a low signal is received. The consumptions of a depositor contingent on the realization of the signals and productivity are summarized as follows: $\{c_1 = 1, c_2\left(H, \overline{R}\right) = 1.2, c_2\left(H, \underline{R}\right) = 0.8, c_2\left(L, \overline{R}\right) = 1, c_2\left(L, \underline{R}\right) = 1\}$. The ex-ante expected utility in autarky, w_0^{aut}, is 18.0540, which is lower than $\hat{w}_0$. Hence, the contract will be accepted, although bank runs will take place with positive probability. ■

In example 2, depositor 2 does not rely solely on his private signals to make a withdrawal decision. Depositor 1's decision to wait reveals that a high signal has been received. If depositor 2 gets a low signal, it will be offset by the high signal inferred. He will not withdraw with the belief of p_0. If depositor 1 withdraws, however, the action sends noisy information that a low signal may be received. When depositor 2 gets a low signal, his private belief is lowered even more, so that he prefers to liquidate the asset immediately to mitigate the loss on investment.

A run-admitting contract is optimal in some economies for the following reasons. First, the contract helps smooth the consumption in an economy with aggregate consumption shocks. Second, in an economy with production uncertainty, a bank run is not necessarily bad. In example 2, if the true state of productivity is low, then depositors get payments in the amount of either 1.0064 or 0.9936. But if both depositors wait, each will get 0.8000. A bank run is a means to terminate low-quality production to mitigate future losses. In this sense, information about productivity is valuable, and a run-admitting contract allows information about productivity to be partially revealed.

However, a bank run can happen when productivity is actually high, due to imperfect signals and the herd effect. In example 2, the partial run conducted by depositor 2 is due to the herd effect. If the probability of having a bank run in a high-productivity state is too large, a run-proof contract will be provided. Example 3 indicates that a run-proof contract is optimal in some economies.

Proposition 2 *In some economies, the optimal banking contract that is contingent on the public withdrawal history is run-proof.*

Proof. Prove by example.

Example 3 The utility function and other parameters in the economy are the same as in the proof of proposition 1 except that $q = 0.5$. In this case, a productivity signal is not informative. Table 2 describes the payment scheme of an optimal contract.

Table 2: An example of an optimal contract that is run-proof

$\hat{w}_0$	$c^1(1)$	$c^1(1,1)$	$c^2\left(1,0,\overline{R}\right)$	$c^2(1,0,\underline{R})$
18.0383	1.0053	0.9947	1.1936	0.7957
$c^1(0,1)$	$c^2\left(0,0,\overline{R}\right)$	$c^2(0,0,\underline{R})$	$c^2\left(0,1,\overline{R}\right)$	$c^2(0,1,\underline{R})$
1.0116	1.2000	0.8000	1.1861	0.7907

Given such a contract, there is a unique equilibrium in the postdeposit game in which depositors withdraw if and only if they are impatient. That is, ($\theta^1_L = 0$, $\theta^1_H = 0$, $\theta^2_{1,L} = 0$, $\theta^2_{0,L} = 0$, $\theta^2_{1,H} = 0$, $\theta^2_{0,H} = 0$).

In autarky, depositors leave all assets in production if they are patient. The ex-ante expected utility in autarky is 18.0383, which is equivalent to that in the banking economy. Depositors weakly prefer to accept the contract and no bank run occurs ex post. ■

To illustrate that the economy is inefficient due to the fact that the productivity information is private, I continue with example 2 but assume that the productivity signals are publicly observable although consumption shocks are still private.

Example 4 In this example, the productivity signals are publicly observable. Depositors have private information about their consumption types. The contract specifies the payment contingent on depositor's arrival time, the productivity signals, and the realization of the signals. The utility function and other parameters are the same as in example 2. The optimal payment scheme is listed in Table 3. The welfare under the optimal payment scheme with public productivity information is $\hat{w}_0 = 18.0567$, which is higher than that in the economy with private productivity signals.

Table 3: An example of an optimal contract when productivity signals are public

$c^1(1,H)$	$c^1(1,L)$	$c^1(1,1,H)$	$c^1(1,1,L)$
1.0084	0.9990	0.9916	1.0010
$c^1(0,1,H,H)$	$c^1(0,1,H,L)$	$c^1(0,1,L,H)$	$c^1(0,1,L,L)$
1.0271	1.0106	1.0106	0.9984
$c^2(1,0,H,H,\overline{R})$	$c^2(1,0,H,H,\underline{R})$	$c^2(1,0,H,L,\overline{R})$	$c^2(1,0,H,L,\underline{R})$
1.1899	0.7933	1.1899	0.7933
$c^2(1,0,L,H,\overline{R})$	$c^2(1,0,L,H,\underline{R})$	$c^2(1,0,L,L,\overline{R})$	$c^2(1,0,L,L,\underline{R})$
1.2012	0.8008	1.0010	1.0010
$c^2(0,0,H,H,\overline{R})$	$c^2(0,0,H,H,\underline{R})$	$c^2(0,0,H,L,\overline{R})$	$c^2(0,0,H,L,\underline{R})$
1.2000	0.8000	1.2000	0.8000
$c^2(0,0,L,H,\overline{R})$	$c^2(0,0,L,H,\underline{R})$	$c^2(0,0,L,L,\overline{R})$	$c^2(0,0,L,L,\underline{R})$
1.2000	0.8000	1	1
$c^2(0,1,H,H,\overline{R})$	$c^2(0,1,H,H,\underline{R})$	$c^2(0,1,H,L,\overline{R})$	$c^2(0,1,H,L,\underline{R})$
1.1675	0.7784	1.1873	0.7915
$c^2(0,1,L,H,\overline{R})$	$c^2(0,1,L,H,\underline{R})$	$c^2(0,1,L,L,\overline{R})$	$c^2(0,1,L,L,\underline{R})$
1.1873	0.7915	1.0016	1.0016

Also note that the optimal payment scheme given public productivity information encourages depositors to truthfully report their consumption types by action. Given any public history, the expected utility of a depositor in the last period is higher than the utility from immediate withdrawal. Depositors withdraw if and only if they are impatient. No bank run occurs under this contract, and the allocation in table 3 is our first-best allocation. This result is similar to Green and Lin's findings that a contract contingent on the withdrawal history can achieve the first-best allocation by eliminating the asymmetric information between the bank and depositors. However, this type of contract can overcome only one-dimensional private information. In my economy with two types of private information, such a contract fails to achieve the first-best allocation.

Example 4 also proves that the direct-revelation rule in Green and Lin's (2003) model is in general infeasible here. Suppose that depositors must announce their private signals before they make decisions at $t = 1$. Depositors have incentives to lie to the bank when they receive low productivity signals. A depositor can instead tell the bank that although he has received a high signal, he needs to consume immediately. If the bank believes him, he could get a larger amount of payment than he should. Table 3 illustrates this point. Assume that depositor 1 always tells the truth about his consumption type and productivity signal. Suppose depositor 1 is a patient type and he receives a low signal. Depositor 2 is also patient, but he receives a low signal. If he tells the true productivity signal and does not withdraw at $t = 1$, he will receive $c^2\left(0, 0, H, L, \overline{R}\right) = c^2\left(0, 0, H, L, \underline{R}\right) = 1$ in the last period, whereas if he claims to be impatient but has received a low signal, he will receive $c^1\left(0, 1, L, H\right) = 1.0106$. Therefore, depositor 2 will not always be truth-telling, and the bank will not trust the reports by the depositors.

5 Conclusion

This paper provides a simple model to study the herd effect in bank runs given a banking contract that is contingent on the withdrawal history. Given such a contract, there exists at least one perfect Bayesian equilibrium in which depositors beliefs and actions are affected by the actions of others. Computed examples indicate that in some economies, the optimal contract tolerates bank runs that bear the herd effect and in other economies the optimal contract is run-proof.

This paper also revisits Green and Lin's mechanism. Green and Lin's mechanism shows that the optimal contract that is contingent on the withdrawal history can eliminate a panic-based run by inducing depositors to truthfully report their consumption type. However, this mechanism fails when there exist more than one uncertainty. A simple zero-one action cannot fully reveal more than one type of private signal. Thus, the asymmetric information between the bank and depositors persists.

A natural extension to this paper is to find a more general banking mechanism that can induce people to report their signals truthfully and achieve a more efficient allocation. More policy implications can be derived from the finding of such a mechanism.

In this paper, the bank has no information on productivity, which is not quite true in reality. In a more complicated model in which the bank receives signals on productivity, there arise problems such as how to eliminate bank's moral hazard incentives due to the information asymmetry between the bank and the depositors and how the bank can reduce the probability of bank runs due to wrong signals. This can be another extension to the paper.

References

[1] Banerjee, Abhijit V., "A Simple Model of Herd Behavior," *Quarterly Journal of Economics,* Vol. 107, No. 3 (August 1992): 797-817.

[2] Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch, "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy,* Vol. 100, No. 5 (October 1992): 992-1026.

[3] Diamond, Douglas, and Philip Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy,* Vol. 91, No. 3 (June 1983): 401-419.

[4] Green, Edward J., and Ping Lin, "Diamond and Dybvig's Classic Theory of Financial Intermediation: What's Missing?" *Federal Reserve Bank Minneapolis Quarterly Review,* 24 (Winter 2000): 3-13.

[5] ——. "Implementing Efficient Allocations in a Model of Financial Intermediation," *Journal of Economic Theory,* Vol. 109, No. 1 (March 2003): 1-23.

[6] Peck, James, and Karl Shell, "Equilibrium Bank Runs," *Journal of Political Economy,* Vol. 111, No. 1 (February 2003): 103-123.

[7] Schumacher, Liliana, "Bank Runs and Currency Run in a System without a Safety Net: Argentina and the 'Tequila' Shock," *Journal of Monetary Economics,* Vol. 46, No. 1 (August 2000): 257-277.

[8] Starr, Martha, and Rasim Yilmaz, "Bank Runs in Emerging-Market Economies: Evidence from Turkey's Special Finance Houses," *Southern Economic Journal,* Vol. 73, No. 4 (April 2007): 1112-1132.

[9] Wallace, Neil, "A Banking Model in Which Partial Suspension is Best," *Federal Reserve Bank Minneapolis Quarterly Review,* 14 (Fall 1990): 3-16.