On the Determinants of Optimal Border Enforcement[1]

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Abstract

We extend the current immigration-enforcement literature by incorporating both the practice of people smuggling and a role for non-wage income into a two-country, dynamic general equilibrium model. We use the model economy to examine three questions. First, how does technological progress in the smuggling industry affect the level of migration and capital accumulation for a given level of enforcement? Second, do changes in border enforcement affect the level of migration, capital accumulation, and smuggling activity? Third, is the optimal level of enforcement sensitive to technological progress in the smuggling industry? We examine both the global dynamics and the steady state equilibrium with respect to capital accumulation and migration.

JEL: E61, F22, J61, O15

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1 Introduction

1.1 Overview and Main Findings

Illegal immigration into the United States has been steadily rising for decades and reached record levels on the 1990s.[1] One response of the U.S. government to this rise has been to steadily increase border enforcement. Given the concomitant increase in illegal immigration and border enforcement over the past thirty years, the deterrent effect of border enforcement and more generally, the government's policy regarding the desirability and optimality of stemming the flow of migrants deserves greater attention.[2]

The purpose of this paper is to re-examine the issue of optimal immigration policy. We do this by extending the current immigration-enforcement literature to incorporate the practice of people smuggling into a two-country, dynamic general equilibrium model. By developing a dynamic model where migrant's decisions are endogenously determined in response to wage and saving opportunities abroad, we are able to ascertain those conditions under which the optimal government response to illegal immigration is to *not* enforce the border. When enforcement is the optimal policy, we further characterize how the government should effect changes to their policies in response to changes in human smuggling operations. These results are driven by migrants, smugglers, and border enforcement acting optimally in an economy where prices (wages and returns to saving) and quantities (levels of illegal immigration) are determined via market outcomes. As such, our results are not driven by the usual assumptions found in this literature: namely the existence of assumed wage and skill differentials between migrants and natives.

There are three key features of our model that significantly differentiate it from the previous literature. First, we assume migrant workers and host-country natives are identical in terms of their skills.[3] Second, migrant workers can potentially save wage income in the respective country in which it was earned, and thus are allowed to contribute to the host-country capital stock via savings. As a consequence, the capital-labor ratio (and hence wages) are not necessarily diminishing in the number of migrant workers. An increase in the number of migrant workers results in both an increase in the labor supply and also an increase in saving. Therefore, the impact of greater illegal immigration on the capital-labor ratio, and hence wages, depends on the relative changes in labor and savings.[4] Both of these assumptions — the absence of a skill

[1] Over the last decade, net inflows of undocumented immigrants are estimated at 500,000 per year, while the stock of illegals is currently estimated at between 8.7 and 10.9 million (Costanzo *et al.* (2001).)

[2] See Warren (1995).

[3] We use the term host-country to denote the country to which illegal immigrants migrate and home country to denote the country in which migrants were born: i.e., for Mexican – U.S. immigration, Mexico would be the home-country and the United States would be the host-country.

[4] There is evidence that immigrants contribute to the capital stock. Home and business ownership are two examples. See, for instance, Simon (1999), Borjas (1994) and Friedberg and Hunt (1995) for overview of immigrants and saving. More recenty, Benhabib (1996) departs from the conventional view by permitting migrants to bring capital to the host country. More specifically, Benhabib examines a model in which there is capital differential among the set of potential migrants. The migrant's decision depends on the host-country policy and factor income, which depends on the capital-labor ratio. In Benhabib's one-period model, the host-country capital-labor ratio is equal to the host-country endowment plus capital brought by migrants divided by host-country natives plus migrants. For our purposes, it is useful to note that Benhabib also considers an economy

differential and capital accumulation by migrant workers — break from the conventional view. Third, we include a smuggling technology that, *ceteris paribus*, diminishes the efficacy of the government's enforcement technology.[5] Thus, for a given level of border enforcement, improvements in smuggling technology mean that the marginal costs of migrating are lower for migrant workers, and so, potentially, are the marginal benefits of enforcement.

We are particularly interested in addressing three questions. First, how does technological progress in the smuggling industry affect the level of migration and capital accumulation for a given level of enforcement? Second, do changes in border enforcement affect the level of migration, capital accumulation, and smuggling activity? Third, is there an optimal level of border enforcement, and if so, is it sensitive to technological progress in the smuggling industry?

Our key findings can be easily summarized as follows. First, we restrict our analysis to the generic case in which there (possibly) exists two steady state equilibria: one associated with a high capital-labor ratio and a small amount of migration and the other with a low capital-labor ratio and greater migration. In the case of two steady state equilibria, we are able to demonstrate conditions under which the high-capital, low migration steady state is globally stable.

Second, we show that the host country's steady state capital-labor ratio is positively related to technological advances in the smuggling industry in the high-capital (stable) steady state. When smugglers are more efficient, border-crossing frictions are reduced, and migrants spend less time evading enforcement on the border. The impact on migration in this case is ambiguous. Greater capital results in an increase in the wage and this provides an inducement for migrants to spend more time working abroad. However, a greater capital level also reduces the marginal product of capital, thereby decreasing the returns to saving and providing a disincentive to work and save abroad. Thus, equilibrium migration depends on which of these countervailing forces is greater.

Third, we examine the effect of a change in the level of border enforcement. Border enforcement is funded through a lump-sum tax paid by young native, host-country workers. The impact of an increase in border enforcement is to decrease the level of the steady state capital stock while rendering the level of migration ambiguous in the high-capital, stable steady state. The intuition for these results is completely analogous to the case where smuggler's productivity decreases.

Lastly, we explore the case where the government actively chooses the optimal level of border enforcement, conditional on all other individuals' decisions. The optimal level of border enforcement is chosen so as to maximize the welfare of native host citizens. We show that developed countries would never optimally

in which migrants can contribute to the stock of host-country capital.

[5] The United States General Accounting Office (2000) reports on increased alien smuggling. Singer and Massey (1998) provide empirical evidence suggesting that the probability a migrant will employs coyotes is systematically, and positively, related to the quantity of linewatch hours employed by U.S. Border Patrol. See Spener (2001) for a detailed description smugglers and how they operate in south Texas.

choose sufficiently low levels of enforcement relative to no enforcement. In addition, we derive sufficient conditions under which developed countries would choose to never enforce the border. Thus, our results indicate that border enforcement is not a certainty in developed countries. For border enforcement to be welfare maximizing, increases in host-country taxes to fund enforcement must induce a sufficiently large decline in the capital stock to increase the return on savings to make citizens better off. Finally, we examine the impact of technological advances in the smuggling industry on the optimal level of enforcement. We state conditions under which the host country will spend fewer resources on border enforcement as the smugglers become more effective.

1.2 Model Description and Related Literature

We derive these results in the context of a two-country, overlapping generations model economy. The home country is populated by potential migrants and smugglers while the host country has only non-migrant workers. Workers in both countries are identical with respect to labor endowments, preferences, and job skills. Production in the host economy is characterized by a standard Diamond (1965) neoclassical production function while the home country is characterized by self-employment. Capital is not mobile between the two countries although, obviously, labor is.

In the home country, potential migrants choose the fraction of time to spend working at home relative to the fraction spent crossing the border and working in the host country. Because all home-country workers are identical, one could equivalently interpret the equilibrium outcome as the fraction of workers that migrate. Wages earned in the home country are saved via a simple storage technology, while income earned in the host country is saved via capital in the host country. Although capital (savings) is not mobile between countries, the consumption good return from savings is mobile. The decision to migrate rests crucially on the overall return from saving (inclusive of all migration costs) in either country. One key cost of migrating is the productive time lost while crossing the border and evading border enforcement. Here, labor-smuggling services play a useful role. A fraction of the home-country's agents are endowed with a smuggling technology. Smugglers divide their labor endowment between research and development of new border-crossing methods and actually arranging trips across the border. By devoting some of their labor endowment to research, smugglers will endogenously respond to changes in the level of border enforcement.

Host country natives inelastically supply labor to the production process, pay lump-sum taxes, save in the form of capital, and consume only the product produced in the host country. There exists a government in the host country whose sole objective is to provide border enforcement services. Initially we consider the government's enforcement decisions to be exogenously set. Later in the paper we allow for the government to choose taxes (and hence enforcement) so as to maximize the well being of the host-country natives.

The issues we examine with our model have been studied extensively in the literature, although our modelling choices provide a fresh perspective.[6] Ethier (1986) was the first to examine the optimal border enforcement issue. He studies a one-period model and assumes that there is a fixed number of low-skilled potential immigrants. The government chooses immigration policies that determine the number of illegal immigrants employed in the host country. Because production exhibits diminishing marginal product, an influx of low-skilled immigrants lowers national income. Ethier's principle finding is that there exists a mix of enforcement (interdiction) and domestic policies (inspection) that minimize the costs of limiting the quantity of low-skilled, illegal immigrants. His result is essentially an application of the theorem of the second best; the loss of labor market power by host-country natives is smaller when the government chooses a mix of distortionary policies.

Ethier's paper and a subsequent paper by Bond and Chen (1987) characterize the conventional view of the effects of illegal immigration. Ethier argued that skill differential is an essential feature of a model that examines optimal border enforcement. Bond and Chen (1987) extend Ethier's setup to include capital, but assume migrant workers do not accumulate host-country capital.[7] Thus, the conventional view consists of these two tenets – skill differential and the absence of capital accumulation by migrants.

Our model eschews these two tenets, focusing instead on the endogenous response of migrants to their expected total return to migrating. In this sense, the general equilibrium nature of the model is crucially important because prices, particularly wages and returns to capital, are simultaneously determined. Consequently, since both migrants and host-country workers work and save, they base their decisions not only on labor income but also on capital income. Thus, although our model can accommodate skill differentials, it will not be the driving force behind individual's decisions on how to allocate their resources.

We also study a government policy that encompasses both Ethier's interdiction and inspection policies. Our enforcement policy reduces the amount of productive time that a migrant worker spends producing goods in the host country; precisely what the enforcement and domestic policies together accomplish in Ethier's model. In addition, by explicitly modelling the smuggling industry we attempt to further develop the real-world factors affecting the migrant's decisions. It is this commingling of the smuggling industry and the migrants ability to save and add to the host-country's capital stock that drives the key results in our model. Indeed, these features account for why our encompassing border enforcement will not necessarily have the beneficial effects describe in Either. Because we also allow smugglers to endogenously respond to changes in enforcement, we have decidedly tilted the economy's features toward making migrant workers less costly to have in the country and more costly to keep out.

Despite these features that seemingly support an open border, our results show that border enforcement may be desirable. Providing border enforcement consumes resources that could have been invested in capital

[6] See, for instance, the seminal work by Sjaastad (1962), Harris and Todaro (1970) and more recent work by Borjas (1987), Massey and Espinosa (1997), Hanson and Spilimbergo (1999), and Markusen and Zahniser (1999).

[7] See also, Yoshida (1998) for analysis adopting the Ethier setup.

formation. The enforcement reduces the benefits of migrating but the use of resources to provide enforcement results in an increase in the return on capital; an inducement for greater migration. To the extent that the net result provides for a sufficient increase in the return on capital relative to the loss of savings to pay for greater enforcement, enforcement can still be beneficial to native host-country individuals

While our contribution is theoretical, our model also forwards an alternative interpretation of findings presented in the empirical literature. In particular, our sense is that explanations of empirical regularities fail to fully account for the importance of smugglers when considering the linkages between illegal immigration and border enforcement. For example, Hanson *et al.* (2002) have presented evidence on the effects of tighter enforcement on wages along the United States-Mexico border. They find no impact of enforcement on wages in U.S. border states, suggesting enforcement may not be deterring illegal immigrants and hence not "protecting" U.S. workers from the downward wage pressures: one impetus for increased enforcement.[8] Although Hanson *et al.* believe the reason they find no wage impact is because labor markets adjust, by not fully considering the implications of a vibrant smuggling industry, this vein of literature fails to capture changes in migration due to innovations in the smuggling industry, which may have been spurred by the increase in enforcement.

The remainder of the paper is arranged as follows. The basic model is outlined in Section 2 while the conditions that must be satisfied by a competitive equilibrium are stated in section 3. Section 4 solves for existence and number of steady state equilibrium and the global dynamics of the model while section 5 explores the impact of changes in enforcement and smugglers' productivity on the steady state equilibria. Section 6 discusses the optimal level of border enforcement while section 7 concludes.

2 The Model

We consider a world consisting of two countries: a home country, from which individuals may choose to emigrate, and a host country, to which individuals immigrate and from which there is no emigration. The economies of both countries are characterized by a standard two-period lived, overlapping generations model with production. Time is discrete and indexed by $t = 0, 1, 2, ...$ In both countries, each generation is composed of a continuum of individuals having unit mass.[9] All individuals, regardless of their country of origin, are identical with respect to their preferences and endowments; they are endowed with one unit of labor when young and nothing when old, and value only old age consumption.[10] In addition, each country has an initial

[8] One reason why enforcement has had little deterrent effect is that there is typically no penalty for committing 'entry without inspection,' besides being returned to one's country of origin. For Mexicans, this is a brief trip back across the border. For more studies on the efficacy of border enforcement, see Espenshade (1994), Donato *et al.* (1992), and Orrenius (2001). Another vein of the immigration-enforcement literature explores the role of intergenerational conflict in models where agents vote on the desired number of immigrants (level of enforcement); see, Dolmas and Huffman (2001, 2004).

[9] There is no loss in generality by assuming that the population of the two countries are identical.

[10] Ethier (1986) suggests that a good theory of migration should consider the migration of skilled vs. unskilled workers. Since our primary emphasis is on understanding the interrelationship between migrants, smugglers and border enforcement, we

old generation who possess an initial capital endowment.

2.1 Home Country

The home country is characterized by two classes of individuals: migrants and smugglers. Smugglers work only in the smuggling industry while migrants divide their time between home production and host-country production. Migrant production in the home country is characterized by self-employment. It is assumed that migrants produce a single homogenous final good, which is produced and saved in the migrant's first period of life, and then consumed when old. The only input in the production process is labor, and goods are produced according to the decreasing returns to scale production function $F(\mu_t) = A\ln(1+\mu_t)$, where μ_t represents the quantity of labor supplied by migrants in home production.[11]

2.1.1 Migrant's Problem

A fraction γ of individuals within any given generation are potential migrants. Each generation of migrants is endowed with one unit of labor when young and nothing when old. There is no initial old generation of migrants. Since only old-age consumption is valued, this labor is supplied inelastically when young. The migrant must decide what fraction of her labor time, μ_t, to spend working in the home country and what fraction, $1-\mu_t$, to spend crossing the border and working in the foreign country.[12] However, there is a border friction that amounts to reducing the time employed in the host country. Thus, the fraction of time spent emigrating from the home country, $1-\mu$, is further divided into two activities; time spent actually working in the foreign country $M(\cdot)$ and time spent crossing the border, $1-M(\cdot)$.

The amount of time used in crossing the border depends on the level of border enforcement implemented by the host country, e_t, and the amount of services, q_t, a migrant obtains from smugglers. Thus, the amount of time spent working in the host country is a fraction of the time allotment not spent working in the home country; that is, $M(q_t, e_t)(1-\mu_t)$, where $0 \leq M(q_t, e_t) \leq 1$. Conversely, the time lost crossing the border is given by $[1-M(q_t, e_t)](1-\mu_t)$. The level of border enforcement, e_t, is taken as given by the migrant. It is assumed that if $e_t = 0$, that is, there is no border enforcement, then $M(q_t, 0) = 1$ for all $q_t \geq 0$.[13] In addition we assume that $0 > M_e > -\infty$. Thus, an increase in the level of enforcement reduces the amount of time spent working in the host country.

abstract from the issue of skill level in this paper. Similarly, by having individuals value only old age consumption, we abstract from the savings/consumption decision. As with the abstraction from skill levels, this simplification will not qualitatively affect our results.

[11] Although each migrant is self employed, we assume that they produce a homogenous product: an example of this would be agricultural products. The production function utilizing labor as the only input was chosen in an attempt to capture the real world fact that often it is the low skilled workers who are migrating to the foreign country and any self-employment opportunities they have would not be capital intensive.

[12] Bencivenga and Smith (1997) analyze migration from rural to urban areas using an overlapping generations model. In their setup, the risk to migrating was the chance of unemployment in the urban labor market. In our setup, labor time lost during the migration represents the cost to migrating.

[13] Open borders correspond to perfect labor mobility.

Since crossing the border is time consuming, smugglers exist to reduce the crossing time. At date t, migrants can purchase a quantity q_t of smuggling services, taking the price, p_t, as given; where p_t is measured in units of the home-country production good. It is assumed that the greater the quantity of smuggling services obtained, the less time is used to cross the border, that is, $M_q > 0$, and that there are decreasing returns to additional units of smuggling services, $M_{qq} < 0$. In addition, it is assumed that $M_q < \infty$ and $0 < M(0, e_t,) \leq 1$.[14]

Migrants who work in the home country earn income $F(\mu_t)$ from self-employment. This income is saved via a simple storage technology in the home country. For every unit of output saved at time t, the migrant receives x units of consumption good at date $t+1$. Migrants who are successful in crossing the border earn a real wage w_t in the host country and save in the host country via capital accumulation. Thus, income earned at date t, earns a gross real return r_{t+1} at date $t+1$, where r_{t+1} is the rental rate on capital in the host country.[15] Finally, we assume that migrants spend their retirement in the host country.[16] Thus the proceeds from any savings in the home country must be transported across the border. We assume that this is costly and that the fraction δ of home consumption goods is used up in this process.

We can formally write the migrant's problem as

$$\max_{\mu_t, q_t} \; U(c_{t+1}) \tag{MP}$$

subject to

$$\begin{aligned} c_{t+1} &= x(1-\delta)\left[F(\mu_t) - p_t q_t\right] + r_{t+1}\left[M(q_t, e_t)(1-\mu_t) w_t\right], \qquad (1) \\ F(\mu_t) &\geq p_t q_t, \text{ and} \\ 0 &\leq \mu_t \leq 1. \end{aligned}$$

Equation (1) is the sum of home- and host-country income inclusive of all costs while the second equation requires that the cost of smuggler services purchased cannot be greater than the income the migrant has on hand when crossing the border.[17] We assume that $U(c_{t+1})$ satisfies all the standard conditions necessary for an interior solution; namely $U(0) = 0$ and $U'(c_{t+1}) > 0$.

[14] The latter assumption implies that even without the aid of the smuggler, a migrant will eventually cross the border and spend some time working in the foreign country.

[15] We assume that savings (capital) is immobile between countries. Thus, the individual must save in the country in which the income is earned. For a model which allows savings across countries via stock ownership, see Lundborg and Segerstrom (2002).

In addition, as long as there is some positive fraction of saving by migrants in the host-country, our results hold.

[16] Although the prospect of return migration is an important aspect of real world illegal immigration, since our focus is directed toward the effect of enforcement on smugglers and migration and vice versa we ignore the possibility of return migration.

[17] Thus, we are requiring up-front payment for smuggling services and ruling out the possibility of borrowing against future earnings or indentured servitude as means of payment.

The solution to the migrant's maximization problem is characterized by the following two equations:

$$\frac{F'(\mu_t)}{w_t} = \frac{r_{t+1}M(q_t, e_t)}{x(1-\delta)} \tag{2}$$

and

$$p_t = \frac{M_q(q_t, e_t)(1-\mu_t)F'(\mu_t)}{[M(q_t, e_t)]}. \tag{3}$$

Equation (2) indicates the trade-off associated with migrating; the wage ratio in the two countries must equal the ratio of real returns, taking into account time spent crossing the border and the transportation costs associated with moving final goods from home-to-host country. The second condition equates the marginal cost of the smuggling service to the marginal income gain from using smuggling services, where the marginal gain in time working in the host-country labor market is measured by the product $M_q(q_t, e_t)(1-\mu_t)$.

2.1.2 Smuggler's Problem

In each generation, a fraction, $1-\gamma$, of the home-country population are smugglers. Like migrants, smugglers live for two periods. In contrast, smugglers are restricted to only producing smuggling services and may not migrate or work in the home country production sector. When young, smugglers are endowed with one unit of labor that they supply inelastically. As with migrants, smugglers value only old age consumption and are retired when old. Thus they consume the gross return from investing their savings in the same simple storage technology as migrants, which earns a return x in period $t+1$ for every unit of saving invested at time t. Finally, there exists an initial old generation of smugglers who possess smuggling capital a_0.

A smuggler's unit of labor is divided between two activities when young: accumulating smuggling capital (research and development), $a_{t,}$ and selling border crossings. For a smuggler, these operations are ordered sequentially; that is, the young smuggler first accumulates smuggling capital by crossing people, then begins selling services. We think of smuggling capital as the knowledge of methods and means for circumventing the border enforcement of the host country. The smuggler uses the remaining time endowment to arrange border crossings. We let d_t represent the fraction of time which smugglers devote to accumulating smuggling capital and $(1-d_t)$ be the fraction of time devoted to arranging crossings.[18]

When determining the amount of time to devote to accumulating smuggling capital in period t, we assume that the quantity of smuggling capital (knowledge) previously acquired by all past generations, a_{t-1}, is available to the current generations of smugglers; that is, there is no depreciation of smuggling capital.

[18] One can think of the smuggler's first period as divulged into two subperiods. The initial subperiod of his young life is spent as an apprentice to an old smuggler, who has institutional knowledge about crossing and enforcement. In this subperiod, the smuggler undertakes the actual process of crossing migrants over the border. While the apprenticeship provides no income, it does provide the required knowledge to make income-generating arrangements for migrant crossings during the second subperiod.
This is not unlike arrangements smugglers currently make on the U.S.-Mexican border. In practice, apprentice smugglers "run" the migrants across until they have been caught so many times (usually ten) that they risk prosecution if caught again. They then become coordinators and recruiters charged with getting clients for the new generation of runners.

We let the function $g(d_t, a_{t-1}; z_t)$ represent the process by which time devoted to capital accumulation is transformed into smuggler's capital. The variable z_t represents an exogenous technology parameter. Thus we have

$$a_t = g(d_t, a_{t-1}; z_t) \tag{4}$$

where $0 \le d_t \le 1$. We assume that $g(d_t, a_{t-1}; z_t)$ has the following properties: $g_d, g_a, g_z > 0$ and $g_{dd} < 0$. Finally, let $g(0, a_{t-1}; z_t) = 0$, that is, a smuggler must devote some time to actually smuggling people over the border in order to develop knowledge about effective crossing methods and techniques.[19]

The smuggler arranges migration services in a perfectly competitive environment.[20] As such, the representative smuggler takes the price of smuggling services, p_t, as given. In addition, the smuggler also takes as given the level of enforcement, e_t, in period t. Finally, it is only the process of arranging for migrant crossings that generates income. To produce migration services, the smuggler must devote sufficient time to capital accumulation, so that he may overcome the anticipated level of enforcement. Formally, let the quantity of migration services supplied, Q_t, be given by

$$\begin{aligned} Q_t &= B[a_t - e_t](1 - d_t) \text{ for } a_t \ge e_t \\ &= 0 \text{ otherwise and} \end{aligned} \tag{5}$$

where $B > 0$ is a constant scale factor, $a_t - e_t$ is the effectiveness of the smuggling methods relative to enforcement methods, and $1 - d_t$ is the fraction of time devoted to selling migration services.

We can therefore write the smuggler's maximization problem as

$$\max_{d_t} U\left(c^s_{t+1}\right) \tag{CP}$$

subject to the constraints

$$\begin{aligned} s^s_t &= p_t Q_t = p_t B[a_t - e_t](1 - d_t), \\ c^s_{t+1} &= x s^s_t, \text{ and} \\ a_t &= g(d_t, a_{t-1}; z_t) \\ a_t &\ge e_t, \end{aligned}$$

[19] Alternatively, one could think of capital accumulation as simultaneously learning and improving upon existing crossing methods. Thus, it is as costly to copy methods as it is to develop them. In this case, there is no free riding as previous knowledge is not a common good once developed. In contrast, Segerstrom (1991) specifies a model in which research and development costs and copying costs are differentiated.

[20] Perfect competition may not be a completely accurate appraisal of the smuggling industry. For example, in parts of Mexico the smuggling industry is marked by local monopolies. Along the border, however, smuggling operations at popular crossing points are highly competitive..

where c^s (s^s)denotes consumption (saving) by the smuggler. Given the interior solution guaranteed by the properties of the utility function, the efficiency condition for the smuggler is

$$xp_t B \{g_d (d_t, a_{t-1}; z_t) (1 - d_t) - [g (d_t, a_{t-1}; z_t) - e_t]\} = 0 \tag{6}$$

Equation (6) describes the smuggler's trade-off. The first term inside the brackets represents the marginal gain from capital accumulation while the second term inside the brackets represents the marginal cost of time allocated to capital accumulation — time not spent arranging migration services.

2.2 Host Country

All individuals in the host country are identical with respect to endowments and preferences. For simplicity and ease of exposition, individuals born in the host country do not emigrate to the home country. In each period, there is a single consumption good which individuals either consume or invest. The consumption good is produced using capital and labor inputs according to the constant returns to scale production function $F(K_t, L_t)$. Total labor, L_t, represents both host and migrant labor and K_t represents savings from both host and migrant workers from the previous period. Let $f(k_t)$ denote the intensive production function, where $k_t = K_t / L_t$ is the capital-labor ratio inclusive of those who immigrate to the host country. It is assumed that $f(k_t)$ is increasing, strictly concave, satisfies $f(0) = 0$, and that the standard Inada conditions hold. It is also assumed that capital used in the production process each period depreciates completely. Finally, the initial host-country old generation has an aggregate capital endowment of $K_0 > 0$, while subsequent generations have no endowments of either capital or final goods.

2.2.1 Host-Country Native's Problem

Individuals live for two periods and inelastically supply their one unit of labor when young. They take the real wage w_t as given. They also face a lump-sum tax, τ_t, to pay for the level of border enforcement erected by the host country. Since only old-age consumption is valued, they save their entire wage income net of taxes in the form of capital and earn the real rate of return, r_{t+1}, on their savings when old. Thus we can write the individual's problem as

$$\max_{s_t^n} U\left(c_{t+1}^n\right) \tag{FP}$$

subject to

$$s_t^n = w_t - \tau_t, \text{ and}$$
$$c_{t+1}^n = r_{t+1} s_t^n,$$

where c^n (s^n) denotes the consumption (savings) of a native host-country citizen. Obviously, given that the wage rate and gross real return to saving is taken as given, the host individual's maximization problem is trivial.

2.2.2 Host-Country Government

The host-country government is assumed to engage in a single activity — border enforcement. To this end, all revenues generated through the lump-sum tax on host-country native workers is directed towards enforcement efforts.[21] We also assume that the government runs a balanced budget on a period by period basis. Therefore, total government expenditures will exactly equal tax revenue: $g_t = \tau_t$. For simplicity, we assume that the government sets the lump-sum tax in period zero and makes no subsequent changes to it.[22]

Let the function $h(\tau_t)$ denote the technology that transforms government taxes into border enforcement. More specifically, $h(\tau_t)$ transforms the units of the (host country) consumption good collected as taxes into a measure of the enforcement level,

$$h(\tau_t) = e_t.$$

The function $h(\tau_t)$ is assumed to have the following two properties: $h' > 0$ and $h(0) = 0$. In other words, greater tax revenue allows for greater border enforcement and that without any tax revenue, there is no enforcement at the border.

3 Equilibrium

Equilibrium requires that individuals and governments make optimal choices and that the following markets clear: the factor markets in both the home and host country, the goods markets (final goods and capital) in both countries, and the market for smuggler's services in the home country.

3.1 Factor Markets

We begin by characterizing equilibrium in the factor markets. At each date t, we assume that both labor and capital are traded in competitive markets in the host country. Thus, at time t the real wage, w_t, and

[21] Only host country natives pay the income tax in an effort to capture the fact that often times illegal immigrants fall below the radar when it comes to paying federal and state income taxes. Our results would not qualitatively change if both natives and migrants were taxed.

[22] The use of a lump-sum tax, as opposed to an income tax, is driven primarily by real world considerations. In the United States, for example, the INS's annual budget is only 0.14% of total federal expenditures. (Expenditures on border patrols in fiscal year 2001 were $2.9 billion, including all detention and removal expenditures.) Consequently, we believe it is reasonable to treat marginal tax rates as independent of border patrol expenditures.

In addition, we assume that the government takes a passive role in determining the level of border enforcement. This is intended to be a proxy for the real world where the "optimal" level of immigration is usually not the motivating factor underlying policy changes. However, we allow for a more active (utility maximizing) government role in determining the level of border enforcement in section 6.

the rental rate of capital, r_t, are both equal to their marginal products:

$$r_t = f'(k_t)$$

and

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t).$$

3.2 Goods Markets

We now characterize equilibrium in the product markets — consumption goods, capital goods, and migrant and smuggler services. As with the factor markets, all markets are assumed to be perfectly competitive.

In order for the market for migrant services to clear, the efficiency conditions (equations (2) and (3)) most hold. The market for smuggler's capital requires that equation (6) holds. In order for the smuggler's services market to be in equilibrium, it must be the case that total quantity demanded by all migrants equals the total quantity of services supplied by all smugglers,

$$\gamma q_t = (1-\gamma) Q_t.$$

Substitute the market-clearing condition for smuggling services into equation (5), yielding the following

$$q_t^* = \frac{(1-\gamma) B}{\gamma} [a_t - e_t] (1 - d_t) \tag{7}$$

Finally, the goods market in the host country must also clear. However, the goods market clearing is equivalent to the market for investment and savings clearing. Thus it must be the case that

$$\begin{aligned} K_{t+1} &= w_t + \gamma M(q_t, e_t)(1-\mu_t) w_t - \tau_t \\ &= w_t [1 + \gamma M(q_t, e_t)(1-\mu_t)] - \tau_t \end{aligned} \tag{8}$$

where the first term inside the brackets on the right-hand side represents the savings from host-country natives and the second term represents the savings from migrants.

3.3 Equilibrium Law of Motion

We can now describe the evolution of the capital/labor ratio and migration patterns over time. However, it will be useful to first describe the relationship between the level of enforcement and the smuggler's decision on how to allocate his time.

It follows from equation (6), that a smuggler will chose d_t such that

$$g_d(d_t, a_{t-1}; z_t)(1 - d_t) - g(d_t, a_{t-1}; z_t) = -e_t. \tag{9}$$

Using the implicit function theorem we can write $d_t = d(e_t, a_{t-1}, z_t)$. The following lemma describes the properties of $d(e_t, a_{t-1}, z_t)$.

Lemma 1 *a)* $d_e > 0$, *b) for* $g_a > g_{da}$, *then* $d_{a_{t-1}} < 0$ *and c) for* $g_z > g_{dz}$, *then* $d_z < 0$

The results of Lemma 1 follow directly from differentiating equation (9). Parts (b) and (c) hold if $g(d_t, a_{t-1}; z_t)$ is separable in research time, previous smuggler's capital, and the technology shock or if these cross-partials are sufficiently small: an assumption we henceforth make.[23] The intuition for this lemma is straightforward. Part (a) implies that an increase in border enforcement results in the smuggler allocating greater time to research and development in order to overcome the greater level of enforcement. For parts (b) and (c), an increase in the amount of prior smuggler capital or technology will lead to less research and development since for part (b) the effort necessary to overcome the level of border enforcement has been undertaken by previous generations of smugglers and for part (c) each unit of time spent in research and development yields a greater level of capital. Thus less time, d_t, is required to overcome the level of enforcement and greater time $(1 - d_t)$ can be spent arranging crossings (the source of second period consumption).

Using the results of Lemma 1, we can also characterize the effect that changes in enforcement and technology on the equilibrium quantity of smuggling services, q_t^*. Rewriting equation (7) we obtain

$$q_t^* = \frac{(1-\gamma)B}{\gamma}\left\{[g(d(e_t, a_{t-1}, z_t)) - e_t][1 - d(e_t, a_{t-1}, z_t)]\right\}. \tag{10}$$

Again applying the implicit function theorem yields $q_t^* = q^*(e_t, a_{t-1}, z_t)$. The properties of $q^*(e_t, a_{t-1}, z_t)$ are described in the following lemma.

Lemma 2 *a)* $q_e^* < 0$, *b)* $q_a^* > 0$, *and c)* $q_z^* > 0$.

The results of Lemma 2 follow directly from differentiating equation (10) and using equation (9). The intuition is as follows. For part (a), greater enforcement leads to less smuggling activity, as smugglers devote greater time to learning about these new enforcement levels and thus less time actually arranging for crossings (part (a) of Lemma 1). For a given level of enforcement, smugglers with a higher level of accumulated knowledge (smuggler's capital)–part (b)–or better technology–part (c)– will choose to arrange for a greater number of illegal border crossings.

Remark: Lemmas 1 and 2 provide some insight into how smuggler's behave in equilibrium and the resulting impact their decisions have on the equilibrium quantity of services provided. First, it should be

[23] The conditions $g_a > g_{da}$ and $g_z > g_{dz}$ are sufficient conditions to guarantee that $d_{a_{t-1}}$ and d_z are both less than zero.

noted from equation (9) that how a smuggler decides to spend his time, acquiring capital or arranging crossing, depends only on the level of enforcement and past capital. It is not influenced by any of the state variables: the level of migration, μ_t, or the capital stock, k_t. Lemma 1 highlights the fact that an individual smuggler invests greater time in research and development when confronted with greater border enforcement. We view this result as consistent with the anecdotal evidence that smugglers endogenously respond to more intense border patrols by putting efforts into finding new ways to avoid detection. Given that smuggler's time is limited, this necessarily leads to a reduction in smuggling services provided by an individual smuggler. Since the number of smugglers in the home country, $(1-\gamma)$, is fixed and we do not allow for migrants to become smugglers (or vice versa), the total quantity of all smuggler services in equilibrium must also decline. In addition, since smuggler's time, d_t, is independent of state variables, then so too will be the equilibrium quantity of services. Any changes in state variables are reflected in the price of smuggler's services, p_t, which adjusts to clear the market given the vertical supply curve for smuggler services.

As the reader will see below, we use Lemmas (1) and (2) to characterize the equilibrium laws of motion in this economy. We can now condense the dynamics of the economy down to two equations. Substituting for the value of q_t^*, we can rewrite equation (8) as

$$K_{t+1} = w(k_t)\left\{1 + \gamma M\left[q^*(e_t, z_t), e_t\right](1-\mu_t)\right\} - \tau_t. \tag{11}$$

Writing $M\left[q^*(e_t, z_t), e_t\right]$ as $M(e_t, z_t)$, recall that the capital-labor ratio is defined as

$$\begin{aligned} K_{t+1} &= L_{t+1}k_{t+1} \\ &= \left[1 + \gamma M(e_{t+1}, z_{t+1})\left(1-\mu_{t+1}\right)\right]k_{t+1} \end{aligned}$$

Thus, substituting this last equation into equation (11) yields the first equilibrium law of motion:

$$k_{t+1} = \frac{w(k_t)\left\{1 + \gamma M(e_t, z_t)(1-\mu_t)\right\} - \tau_t}{\left\{1 + \gamma M(e_{t+1}, z_{t+1})\left(1-\mu_{t+1}\right)\right\}}. \tag{12}$$

The second equation necessary to describe the economy is equation (2), which can be rewritten as

$$\frac{Ax(1-\delta)}{1+\mu_t} = f'(k_{t+1})\, w(k_t)\, M(e_t, z_t) \tag{13}$$

Equations (12) and (13) completely describe the economy in terms of the migration decision in the home country and capital accumulation in the host country.

4 Dynamic Equilibria

To ascertain the existence, number, and dynamical properties of any steady state equilibria, it will necessary to know certain properties about the above two equations. For this section, we assume that host-country production is represented by Cobb-Douglas form. Formally, $f(k_t) = k_t^{\alpha}$, where $\alpha \in \left(0, \frac{1}{2}\right]$.[24] Equation (13) can be rewritten as

$$\frac{Ax(1-\delta)}{1+\mu_t} = \alpha k_{t+1}^{\alpha-1}(1-\alpha)k_t^{\alpha}M,$$

where the arguments for the function M, which is independent of movements in k and μ. From this equation, we can derive the following equation

$$k_{t+1} - k_t = 0 = \left[\frac{M\alpha(1-\alpha)}{Ax(1-\delta)}\right]^{\frac{1}{1-\alpha}}(1+\mu_t)^{\frac{1}{1-\alpha}}k_t^{\frac{\alpha}{1-\alpha}} - k_t,$$

which represents the locus of points where the capital stock, k_t, is unchanging over time. For simplicity we define

$$C = \left[\frac{M\alpha(1-\alpha)}{Ax(1-\delta)}\right]^{\frac{1}{1-\alpha}},$$

and thus we have

$$k_{t+1} - k_t = 0 = C(1+\mu_t)^{\frac{1}{1-\alpha}}k_t^{\frac{\alpha}{1-\alpha}} - k_t. \tag{14}$$

One solution to this equation is $k_t = 0$ and $\mu_t \in [0,1]$, i.e., the μ_t-axis as depicted in Figure 1**.** To see if there exists a non-trivial solution to this equation, it will be useful to know the shape of equation (14) in(k_t, μ_t) space. Differentiating equation (14) yields

$$0 = C\left\{\frac{1}{1-\alpha}(1+\mu_t)^{\frac{\alpha}{1-\alpha}}k_t^{\frac{\alpha}{1-\alpha}}\frac{d\mu}{dk_t} + \frac{\alpha}{1-\alpha}(1+\mu_t)^{\frac{1}{1-\alpha}}k_t^{\frac{2\alpha-1}{1-\alpha}}\right\} - 1.$$

After rearranging and simplifying, we obtain

$$\frac{d\mu}{dk_t}\left[\frac{(1+\mu_t)^{\frac{\alpha}{1-\alpha}}k_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\right] = \frac{1}{C} - \frac{\alpha(1+\mu_t)^{\frac{1}{1-\alpha}}k_t^{\frac{2\alpha-1}{1-\alpha}}}{1-\alpha}.$$

Notice that the coefficient on $d\mu_t/dk_t$ is positive and that the sign of $d\mu_t/dk_t$ will thus be the same as the right-hand side of the above equation. The following lemma describes the range over which $d\mu_t/dk_t$ is positive and negative.

[24] This restriction on α is consistent with the US real business cycle literature in which the calibrations have the capital-income ratio between 0.3 and 0.4.

Lemma 3 *Let $\Gamma(\mu_t, k_t)$ be defined as*

$$\Gamma(\mu_t, k_t) = \frac{1}{C} - \frac{\alpha (1+\mu_t)^{\frac{1}{1-\alpha}} k_t^{\frac{2\alpha-1}{1-\alpha}}}{1-\alpha}.$$

For any $\mu_t \in [0,1]$ and $k_t \in [0,\infty)$, $\Gamma(\mu_t, k_t) > 0$ and thus $d\mu_t/dk_t > 0$.

The proof of this lemma can be found in Appendix A.1 and the values of (k_t, μ_t) for which the capital stock is in steady state is depicted in Figure 1. This phase diagram also depicts the directions in which k_t migrates when off the steady state paths.

Turning now to equation (12), we can rewrite this as

$$k_{t+1} = \frac{(1-\alpha) k_t^{\alpha} \{1 + \gamma M (1-\mu_t)\} - \tau_t}{1 + \gamma M \left(1 - \mu_{t+1}\right)}.$$

Substituting equation (13) into the above equation and rearranging terms, one obtains the following equation describing the locus of points where the level of migration, μ_t, is constant over time; that is,

$$\mu_{t+1} - \mu_t = 0 = 1 + \frac{1}{\gamma M} - \frac{(1-\alpha) k_t^{\alpha} [1 + \gamma M (1-\mu_t)] - \tau}{\gamma M C (1+\mu_t)^{\frac{1}{1-\alpha}} k_t^{\frac{\alpha}{1-\alpha}}} - \mu_t. \quad (15)$$

As with equation (14), to ascertain whether there exist any solutions to this equation in the positive orthant we begin by differentiating equation (15). Differentiating and simplifying, one obtains

$$\frac{d\mu_t}{dk_t} \left\{ 1 + \frac{(1+\mu_t)^{\frac{\alpha-2}{1-\alpha}} k_t^{\frac{-\alpha}{1-\alpha}}}{(1-\alpha)\gamma M C} \left\{ \tau - (1-\alpha) [1 + \gamma M (1-\mu_t)] k_t^{\alpha} - (1-\alpha)^2 \gamma M (1+\mu_t) k_t^{\alpha} \right\} \right\} =$$

$$\frac{(1+\mu_t)^{\frac{-1}{1-\alpha}} k_t^{\frac{-\alpha}{1-\alpha}-1}}{\gamma M C} \left(\frac{-\alpha}{1-\alpha} \right) \{ \tau - \alpha (1-\alpha) [1 + \gamma M (1-\mu_t)] k_t^{\alpha} \}.$$

Recall that $w(k_t)[1 + \gamma M (1-\mu_t)] - \tau > 0$. Thus for a sufficiently small value of α, the right-hand side of the above equation is negative.

Assumption 1 (A1) *Let α be such that*

$$\tau - \alpha (1-\alpha) [1 + \gamma M (1-\mu_t)] k_t^{\alpha} > 0$$

for all $\mu_t \in [0,1]$ and $k_t \in [0,\infty)$.

Given Assumption 1, the sign of $d\mu_t/dk_t$ will be the opposite of that of its coefficient. The following lemma describes the range over which $d\mu_t/dk_t$ is positive and negative.

Lemma 4 *Let $\Upsilon(\mu_t, k_t)$ be defined as*

$$\Upsilon(\mu_t, k_t) = 1 + \frac{(1+\mu_t)^{\frac{\alpha-2}{1-\alpha}} k_t^{\frac{-\alpha}{1-\alpha}}}{(1-\alpha)\gamma MC} \left\{ \tau - (1-\alpha)\left[1+\gamma M(1-\mu_t)\right] k_t^{\alpha} - (1-\alpha)^2 \gamma M (1+\mu_t) k_t^{\alpha} \right\}.$$

and let $(\bar{\mu}, \bar{k})$ be all combinations of $(\mu_t . k_t)$ such that $\Upsilon(\bar{\mu}, \bar{k}) = 0$. Then we know
i) $\Upsilon(\bar{\mu}, \bar{k})$ is downward sloping in (k_t, μ_t) space
ii) for all $\bar{k} \in [0, \infty)$ and all $\mu_t > \bar{\mu}$, then $\Upsilon(\mu_t, \bar{k}) > 0$ and $d\mu_t / dk_t < 0$
iii) for all $\bar{k} \in [0, \infty)$ and all $\mu_t < \bar{\mu}$, then $\Upsilon(\mu_t, \bar{k}) < 0$ and $d\mu_t / dk_t > 0$

The proof of this lemma can be found in Appendix A.2 and the values of (k_t, μ_t) for which the migration flow is in steady state is depicted in Figure 2. This phase diagram also depicts the directions in which μ_t migrates when off the steady state path.

Combining the results of Lemmas 3 and 4, our model allows for the possibility of no steady state equilibria, a unique steady state equilibrium, or two steady state equilibria: a low-capital, high-migration steady state and a high-capital low-migration steady state. Since the results from examining the multiple steady state situation are directly applicable to the case of an unique steady state equilibrium, we henceforth restrict ourselves to the multiple equilibria case. Figure 3 depicts equations (14) and (15) and the resulting phase diagram. As is apparent from this diagram, the low-capital, high-migration steady state is an unstable source. The high-capital, low-migration steady state is either a stable spiral that cycles about the steady state value or spirals in towards the steady state (a sink) or an unstable spiral that is a source. If both steady state equilibria are characterized as unstable spirals, then obviously the only equilibria are the steady state equilibria. Consequently, for the remainder of the paper we restrict ourselves to the case where the high-capital, low-migration steady state is characterized as a stable spiral.

5 Comparative Statics

The two exogenous policy variables of interest in this model are 1) the technology parameter, z, associated with the smuggler's human capital accumulation process, and 2) the level of border enforcement, e. Given the government setup, the level of border enforcement is a montonically related to lump-sum taxes, τ, and thus the variable of interest is the level of taxes in economy. Given the complete characterization of the dynamics in the previous section, it suffices to understand how changes in these policy variables impact the steady state equilibria. Thus we begin by simplifying the equations of the previous section to focus solely on steady states.

Let $k_t = k_{t+1} = k$, $\mu_t = \mu_{t+1} = \mu$, and $\tau_t = \tau$, we can rewrite the equilibrium laws of motion from the previous section as

$$k = w(k) - \frac{\tau}{1 + \gamma M(e, z)(1 - \mu)} \tag{16}$$

and

$$\frac{Ax(1 - \delta)}{1 + \mu} = f'(k)\, w(k)\, M(e, z), \tag{17}$$

where we now again explicitly state the arguments affecting the time spent working in the host country, $M(e, z)$. Given the Cobb-Douglas production function assumed in the previous section, it is straight forward to verify that equations (16) and (17) have the general shapes depicted in the steady state diagram in Figure 4.[25] The simple nature of steady state equations (16) and (17) will make it relatively straight forward to analyze the impact of a change in the exogenous policy variables z and τ on the equilibrium steady state values of the capital stock and migration.

5.1 Shock to Smuggler's Technology

To ascertain the effect of a positive shock to the smuggler's technology, it will first be useful to establish how such a shock will impact the crossing times for migrants, $M[q^*(e, z), e]$. From Lemma 2, we have $q_z^* > 0$, and thus it follows that $M_z = M_q q_z^* > 0$. For a fixed value of k, M is increasing in z. Thus, from equation (16), μ must increase to maintain equality. Figure 5 depicts this as an upward shift in equation (16). Using the same logic in equation (17), μ must decline in order to maintain equality, implying a downward shift of that equation in the graph. Throughout our analysis, we focus our attention on the steady state that can be globally stable, examining cases in which it is a sink. As such, the we discuss the impact on the high-capital steady state.[26]

Proposition 1 *An increase in the smuggler's technology, z, results in an increase in the steady state value of capital-labor ratio and an indeterminate effect on migration in the high-capital equilibrium.*

The intuition behind this result is as follows. A positive technology shock makes smugglers more productive. Lemma 2 indicates that the equilibrium level of smuggler services will increase. For migrants, the shock decreases the time spent crossing the border and thus, increases the time available to spend employed. Holding everything else constant, technological progress, for instance, increases the migrant's income, including income in the host country. Income gains translate into increases in the capital stock. We find that capital-labor ratio reflects the fact that the increase in host-country wages translates into additional savings increasing proportionately more than the number of workers. Ultimately, it is ambiguous whether illegal

[25] For the comparative statics which follow, the analysis is also relevant to the case where there exists a unique steady-state equilibria. In section 6 we allow for the government to actively choose the level of border enforcement and discuss more explicitly the conditions under which there exist multiple steady states.

[26] Given the dynamics of the low-capital steady state, under very specific conditions it is possible to transition from the old steady state to the new. Note also that it is possible that the economy moves from the low-capital steady state to the new high-capital steady. We ignore these special cases in the body of this paper.

immigration increases or decreases. The ambiguity stems from the countervailing general equilibrium effects: given that the capital-labor ratio has increased, wages rise but the return to capital falls. For a decline in the product of wages and returns, the migrant's decision may be to spend less time in the host country.[27] Note that the border-crossing time–the $M(e,z)$–is inversely related to changes in the smuggler's technology. The implication is that the migrant worker's *effective* labor supply time increases. Depending on the wage and return to storage in the home country, it is possible that the migrant will maximize lifetime income by spending less time in the host country.[28]

5.2 Increase in Border Enforcement

Next we consider the effect of the government's natural counter-measure to the smugglers' productivity shock — greater enforcement. In our model, an increase in enforcement requires an increase in tax revenues. Thus to analyze the effect of greater enforcement on steady states, we examine the impact of a change in taxes, τ, on the steady state values of the capital-labor ratio and the fraction of migrants' time spent working in the home market.

As with a technology shock, it will first be useful to note the impact that an increase in the tax payment has on migrant's crossing time. Formally, $M_\tau = M_q q_e^* e_\tau + M_e e_\tau$. With $e_\tau > 0$ and from Lemma 2, $q_e^* < 0$, the assumptions on the crossing function imply that $M_\tau < 0$. Thus, from equation (16), for a fixed value of k, μ must decrease to maintain equality. Figure 6 depicts this as a downward shift in equation (16). Using the same logic on equation (17), μ must increase in order to maintain equality, implying an upward shift in this equation. Thus, we have the following proposition.

Proposition 2 *An increase in border enforcement via an increase in taxes, τ, results in a decrease in the capital-labor ratio and an indeterminate effect on migration.*

The intuition is again easily summarized. Indeed, an increase in taxes acts exactly opposite to a positive technological shock in the smuggling industry. Proposition 2 is noteworthy because it bears directly on the questions left open by Hanson, *et al.* They present evidence suggesting that border enforcement may not be much of a deterrent to illegal immigration. Propositions 2 offers an alternative explanation for the positive correlation between border enforcement and the flow of illegal immigrants. We find that illegal immigration may endogenously increase in response to an exogenous increase in border patrols, as described above. Thus, our findings indicate that the market for smuggling services and the general equilibrium effects can account for the empirical regularity reported by Hanson, *et al.* without assuming that the marginal product of border enforcement technology is nonpositive.

[27] Note that $f'(k)w(k)$ is decreasing in k when the technology is Cobb-Douglas.

[28] Although Figure 5 shows an increase in migration at the high-capital steady state, whether migration increases, decrease, or is unchanged depends on the relative magnitudes of the shifts of the curves representing equations (16) and (17).

6 Optimal Border Enforcement Levels

In this section, we analyze the host-country government's problem of choosing the level of border enforcement that maximizes its citizens welfare. We have shown that the model can account for the positive correlation between border enforcement and the flow of illegal immigrants and the growing use of coyotes along the Mexican border. We proceed by turning our attention to finding the level of border enforcement in the host country that maximizes the welfare of the natives.

In our analysis, we solve the problem for a case in which the government takes the actions of migrants and smugglers as given. With one large government solving for the taxes, taking the responses by migrants, smugglers and (trivially) host-country workers as given, one can think of the government's problem as being essentially the same as a Stackelberg leader. Because there is a monotonic relationship between taxes and the intensity of border enforcement, the solution to the government's problem is equivalent to solving for the welfare-maximizing level of lump-sum taxes. Proposition 2 is important in this exercise because the government must take the effect of changes in taxes into account when it solves the maximization problem. In addition, we examine how exogenous changes in the smuggling technology affect the government's choice of taxes. At some point, smuggling technology may build up to the point where open borders are the welfare-maximizing policy.[29]

6.1 Government's Welfare Maximizing Problem

The optimal level of border enforcement (government taxes) is obtained by solving the following Stackelberg leader problem, denoted by $\mathfrak{P}$. The economy consists of four players: the smuggler, the migrant worker, the host-country government, and the home country natives; all players make decisions at the same instant. The host-country natives save all income that is not taxed. The migrant worker and smuggler choose over the set of feasible quantities of smuggling services, Q, taking τ as given; that is, smugglers choose d_t to maximize $U\left(c^s_{t+1}\right)$ and migrant worker chooses μ_t and q_t to maximize $U\left(c_{t+1}\right)$. We denote the optimal decision rules by smugglers and migrant workers as $c^s\left(\tau\right)$ and $c\left(\tau\right)$, respectively. Let $k\left(\tau\right)$ represent the steady state equilibrium level of capital as a function of the tax level, which corresponds to simultaneously solving equation (16) and (17). The government chooses, τ, from the set of feasible, non-negative taxes; $T = \{\tau \in T : 0 \leq \tau \leq s^n_t\}$ so as to maximize its payoff, represented by the welfare of a representative two-period lived agent, taking the reaction function, $k\left(\tau\right)$, as given.

Thus, the equilibrium is the collection, $\{\mathfrak{P}, c\left(\tau\right), c^s\left(\tau\right), T\}$ that solves the problem:

[29] Note that Ethier also finds that in an economy with a single policy instrument–interdiction–the optimal border enforcement may be zero. In Ethier's model, the interdiction policy chiefly redistributes national (wage) income from skilled to unskilled workers. He then claims that interdiction policy, used alone, can probably not redistribute enough income to offset the cost of border enforcement. Our result is a bit stronger. Since our border enforcement encompasses both interdiction and Ethier's inspection policies, we show that it may be optimal to have an open border. Ethier finds that there is an optimal mix of the two policies that will raise national income.

$$\max_{\tau} U\left(c\left(\tau\right)\right) \tag{GP}$$

subject to

$$c\left(\tau, z\right) = r\left(s - \tau\right) = f'\left(k\left(\tau, z\right)\right)\left[w\left(k\left(\tau, z\right)\right) - \tau\right]. \tag{18}$$

For a given z, differentiating equation (18) yields

$$c_{\tau} = k_{\tau}\left(\tau\right)\left\{f''\left(k\left(\tau\right)\right)\left[w\left(k\left(\tau\right)\right) - \tau\right] + f'\left(k\left(\tau\right)\right)w'\left(k\left(\tau\right)\right)\right\} - f'\left(k\left(\tau\right)\right). \tag{19}$$

Equation (19) identifies the basic trade-off facing the benevolent government. Taxes paid when young reduce consumption at the gross real return (the last term of equation (19). The potential benefit from greater enforcement (the first term of equation (19) is a function of the steady state capital-labor ratio. Proposition 2 tells us that higher taxes result in a lower capital-labor ratio. Thus, the marginal benefit of the tax increase requires that the gross real return increases at a fast enough rate to more than offset the decline in pre-tax wages.

It is not analytically possible to show that there exists a $\tau > 0$ which satisfies both equation (19) and $c_{\tau\tau}\left(\tau\right) < 0$. Alternatively, to prove that it is optimal to have some border enforcement, it would be sufficient to show that $c\left(\tau\right) \geq c\left(0\right)$ for some $\tau \in \left(0, w\left(k\left(\tau\right)\right)\right]$. Taking equation (18), assuming a Cobb-Douglas production technology, and simplifying, we obtain the following expression:

$$w\left(k\left(\tau\right)\right) - \tau \geq \left(1 - \alpha\right)^{\frac{a}{1-\alpha}}\left[k\left(\tau\right)\right]^{1-\alpha} \tag{20}$$

Equation (20), which simply states that lump-sum taxes need to be "sufficiently" small relative to the prevailing wage, represents the condition necessary for host-country citizens to have greater consumption with some positive level of border enforcement than with an open border. In general, it is difficult to say much about the optimal level of border enforcement because we have not specified functional forms for smugglers' capital, $g\left(\cdot\right)$, and migrant crossing technology, $M\left(\cdot\right)$. Consequently, in the remainder of this section we focus on deriving conditions under which the host country will decide not to enforce the border and also the impact of a change in smuggler technology on the optimal level of enforcement.

6.2 No Border Enforcement

It will be useful to first define some limiting conditions regarding taxes and equilibrium levels of the capital stock. Consider a case in which taxes *decrease*, equation (17) shifts down while equation (16) shifts up. Consequently, for sufficiently small lump-sum taxes, the low-capital steady state disappears.

Lemma 5 *There exists a unique (high-capital) steady state for $\tau \in [0, \bar{\tau})$, where $\bar{\tau}$ satisfies the following equation:*

$$\left[\frac{\alpha(1-\alpha) M(e(\bar{\tau}), z)}{Ax(1-\delta)}\right]^{\frac{1}{1-2\alpha}} = (1-\alpha)\left[\frac{\alpha(1-\alpha) M(e(\bar{\tau}), z)}{Ax(1-\delta)}\right]^{\frac{\alpha}{1-2\alpha}} - \frac{\bar{\tau}}{1+\gamma M(e(\bar{\tau}), z)}$$

The proof of this lemma is left to the appendix. The implication of this lemma is that there exists a bifurcation in the number of steady state equilibria which exist: a result of the migrant's budget constraints and the Cobb-Douglas technology. The total return of working in the host-country is the product of the return on savings and the quantity of savings, $f'(k(\tau))\, w(k(\tau))$. For a Cobb-Douglas production function, this is a decreasing function of the capital stock. Thus, at lower levels of capital, the return to migrating is extremely high. As a result, migrants would optimally want to spend more than their time endowment working in the host-country; clearly violating their budget constraints. Consequently, for $\tau < \bar{\tau}$, the low capital steady state vanishes.

It will be useful when deriving conditions under which border enforcement is not optimal, to consider two cases: (1) there exists only one steady state equilibrium and (2) there exist two steady state equilibria. We begin with the first case. When lump sum taxes are close to zero (that is, very little enforcement) and thus there exists a unique steady state equilibrium, we have the following proposition.

Proposition 3 *For sufficiently small lump sum taxes, $\tau \in [0, \varepsilon)$, $c_\tau(\tau) < 0$. Thus, if the alternative is between small levels of border enforcement and an open border, the welfare maximum is an open border.*

This result follows directly from evaluating equation (19) at $\tau = 0$, and hence the proof is omitted. Thus, for a sufficiently small increase in taxes relative to no taxes, the resulting decline in the steady state level of capital results in the overall decrease in savings, $w(k(\tau)) - \tau$, being greater than the increase in the rate of return on savings, $f'(k(\tau))$.

We interpret Proposition 3 as follows. If the government finances border patrols at a small fraction of output, then welfare is higher in the host country if the border is completely open. With the budget for Immigration and Naturalization Service comprising less than 0.05% of GDP, the conditions for choosing an open border may be satisfied.[30]

More generally, the following proposition states conditions under which no enforcement is preferred when we have two steady state equilibria. As we know from Section 4), only the high-capital steady state equilibrium can be stable. We focus on the conditions for optimal border enforcement for that steady state.

Proposition 4 *For the stable steady state equilibrium $k(\tau)$ and $\tau \geq 0$, if $-k'(\tau) < \frac{\alpha}{1-2\alpha}$, then a benevolent host-country government would choose an open border.*

[30] One may wonder what would happen if immigrants paid taxes on their earnings in the host country. In this model economy, there is less of an incentive to immigrate. This could potentially reduce the capital-labor ratio, harming host-country natives through this mechanism.

The proof can be found in the appendix. The condition in Proposition 4 is a sufficient condition to guarantee that $c'(\tau) < 0$ for the high-capital steady state equilibrium and thus no taxes (enforcement) would be optimal. As taxes increase, the level of capital is decreasing and thus the return to savings is increasing, since $f'(k) < 0$. At the same time the quantity of savings, $w(k) - \tau$, is decreasing. Proposition 4 is merely a sufficient condition which effectively restricts the size of a change in the capital level; consequently, an increase in taxes results in a larger decline in after-tax wages than in rental payments to capital owners.

Together, Propositions 3 and 4 derive sufficient conditions under which no border enforcement would be the optimal policy. This result is reminiscent of Ethier's claims that it is unlikely that interdiction policies – one dimension of border enforcement variable – alone will maximize national income. In both our paper and Ethier's, the goal is to maximize a measure of net earnings. In our setup, the earnings measure is lifetime earnings, while Ethier focused on wage income. More importantly, the mechanisms operating on the alternative income measures are quite different. As mentioned above, Ethier posits a narrow version of the conventional view; that is, as more illegal workers are employed in the host country, wages of native workers decline. What taxes are natives willing to pay to curtail the number of illegal workers? Unlike Ethier, border enforcement affects factor inputs in a way that results in higher pre-tax lifetime income; Proposition 2 tells us that the capital-labor ratio is inversely related to taxes, which means that the product of wages and returns to capital increase. This is only one aspect of the general equilibrium affects operating in our model economy. In addition, the government must take into account how smugglers endogenously react. Overall, we simply introduce a host of different general equilibrium effects that go into the decision to use host-country resources to curtail the flow of illegal immigrants.

6.3 Technological Progress and Enforcement

In sections 3.3 and 5 we analyzed how changes in enforcement affected smugglers decisions and changes in smuggling affected the economy. In this section we consider the effect that technological progress in the smuggling industry would have on the host-country's welfare-maximizing level of border enforcement. We assume that the conditions for some positive level of enforcement are satisfied.

Proposition 5 *Let $\hat{\tau} > 0$ be the optimal tax level derived from solving problem (GP). If there exists a stable steady state, $k(\hat{\tau})$, and $k_{\tau z} \in (-\varepsilon, \infty)$, then $\tau_z < 0$.*[31]

[31] In the case of low-capital, unstable steady state, then if $\hat{\tau} > 0$ is the optimal tax level derived from solving problem (GP) and if there exists a low capital steady state, $k_L(\hat{\tau})$, and $k_\tau > 1/(1-2\alpha)$ and $k_{\tau z} \in (-\infty, \varepsilon)$, then $\tau_z < 0$. It is worth noting that when there exists a high-capital steady state and positive border enforcement, the result of Proposition 5 hinges only on the cross partial of k being "sufficiently" large. Whereas when we have a low-capital steady state equilibrium consistent with border enforcement not only does the cross partial need to be "sufficiently" small, but we also have a second sufficient condition requiring that the capital stock be sufficiently responsive to changes in border policies. This asymmetry in sufficient conditions stems from the fact that changes in the equilibrium level of the capital stock as a result of changes in both smuggler technology and enforcement levels behave exactly opposite in the low and high capital steady states, as discussed in Propositions 1 and 2.

The proof to Proposition 5 can be found in the appendix. This proposition states sufficient conditions under which a benevolent government will reduce border enforcement in response to technological progress in the smuggling industry. In other words, we derive the conditions for the case in which the government would optimally reduce border patrols in the face of a more technologically advanced smuggler. In these cases, the resources (taxes) necessary to maintain the level of immigration and capital stock are greater than the benefits from doing so.

More specifically, our results indicate the government's reaction rests chiefly on the general equilibrium effects on the capital-labor ratio. Generally speaking, after-tax income–$f'(k)\,w(k) - \tau$–is a decreasing function of the capital-labor ratio. Once the direct effects of the smuggler's technology shock are taken into account, the government will respond with higher taxes if doing so results in a higher after-tax income. Alternatively, the government will lower taxes–what we characterize as a version of border "chicken"–if lowering taxes raises after-tax income.

7 Conclusion

Researchers and policymakers have long been interested in understanding the macroeconomic effects of migration. In the absence of definitive answers on this matter, it is not surprising that attention has been directed at a subset of migrants, namely, undocumented workers. We develop a dynamic, general equilibrium model to analyze the effects of illegal immigrants. We include several key features into our model economy. Specifically, we model the decision of the migrant worker, taking into account the wages and returns in both countries, the presence of a coyote, or smuggling, industry and migrant saving in both countries. Capital accumulation in the host country is tied to the level of saving by any worker–migrant or native–that earns wages in the country. We characterize the case in which there are at most two steady state. The steady state with relative high capital and low illegal immigration can be a sink, while the low-capital steady state is a source.

We use the model economy to answer three questions. Does technological progress in the coyote industry affect capital accumulation and the flow of illegal immigrants? We find that the capital-labor ratio is positively related to coyotes exogenously improving their methods. The technological progress means less time is loss in the border crossing. With more time to earn wage income and because capital cannot leave the host country, the capital-labor ratio increases. The flow of migrant workers, however, can either rise or fall in response to this technological shock. Lifetime income falls as the capital-labor ratio increases. As such, fewer migrant workers may work in the host country.

How do changes in border enforcement affect the capital-labor ratio and the number of illegal immigrants? Border enforcement acts in exactly the opposite way to technological progress in the smuggling industry. An increase in border enforcement results in a lower capital-labor ratio. In our model economy, lifetime earnings increase. Accordingly, an increase in border enforcement may actually induce an increase in the equilibrium flow of illegal immigrants. As such, our results offer an alternative interpretation of events documented along the U.S.-Mexico border; namely, the evidence that tighter border enforcement should result in smaller wage differentials along the border. Our results suggest that even if wage differentials shrink, a larger number of forward-looking migrants may still choose to cross the border.

Is the optimal level of border enforcement sensitive to changes in the smuggling industry? We derive conditions in which the host country citizens would prefer the government to expend resources that increase the marginal time lost in crossing the border. With endogenous coyotes, the border crossing issue is analogous to an arms escalation. We derive conditions under which technological progress in the smuggling industry would induce a decline in the optimal level of border enforcement.

We mention two additional topics that we have not considered but would be of interest in future research. Skill differentials are viewed as being an important consideration for an analysis of the illegal immigration issue. It is possible to modify the model economy to incorporate an exogenous level of skill differential. The more general question is how worker heterogeneity, especially if unobservable to some extent, would affect the results presented in this paper. Second, we impose conditions on capital accumulation by migrant workers, restricting the flow of capital out of the host country. It would be interesting to build a model in which the capital friction is eliminated. Our sense is that if all frictions are absent, there could be lump-sum tax and transfer scheme that would actually achieve the first-best solution. It would be interesting to determine the nature of the tax-and-transfer scheme across countries or within the host-country.

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A Appendix

A.1 Lemma 3

To show that $d\mu_t/dk_t > 0$ for all relevant values of μ_t and k_t, i.e. $\mu_t \in [0,1]$ and $k_t \in [0,\infty)$, it suffices to show that $\Gamma(\mu_t, k_t) > 0$ over this range. Recall that $\Gamma(\mu_t, k_t)$ is defined as

$$\Gamma(\mu_t, k_t) = \frac{1}{C} - \frac{\alpha (1+\mu_t)^{\frac{1}{1-\alpha}} k_t^{\frac{2\alpha-1}{1-\alpha}}}{1-\alpha}.$$

Let $\left(\hat{\mu}, \hat{k}\right)$ be the loci of points such that

$$\Gamma\left(\hat{\mu}, \hat{k}\right) = 0 = \frac{1}{C} - \frac{\alpha (1+\hat{\mu})^{\frac{1}{1-\alpha}} \hat{k}^{\frac{2\alpha-1}{1-\alpha}}}{1-\alpha} \tag{A.1.1}$$

Rearranging terms in this equation yields

$$1 + \hat{\mu} = \frac{\hat{k}^{1-2\alpha}}{\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} C^{1-\alpha}}.$$

Thus $\Gamma\left(\hat{\mu}, \hat{k}\right)$ is an upward sloping, concave line that crosses the $k_t - axis$ at the point

$$\left(\left[\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} C^{1-\alpha}\right]^{\frac{1}{1-2\alpha}}, 0\right).$$

It is straightforward to verify that for values of (μ_t, k_t) which lie below the line described by equation (A.1.1), then $\Gamma(\mu_t, k_t) > 0$. In addition note that equation (14) crosses the $k - axis$ at

$$\left(C^{\frac{1-\alpha}{1-2\alpha}}, 0\right).$$

Thus, for all $\mu_t \in [0,1]$ and $k_t \in [0,\infty)$, equation (14) lies below (to the right of) $\Gamma\left(\hat{\mu}, \hat{k}\right)$ and thus $\Gamma(\mu_t, k_t) > 0$.

A.2 Lemma 4

To show that $d\mu_t/dk_t > (<) 0$ for all relevant values of μ_t and k_t, i.e. $\mu_t \in [0,1]$ and $k_t \in [0,\infty)$, it suffices to show that $\Upsilon(\mu_t, k_t) < (>) 0$ over this range. Recall that $\Upsilon(\mu_t, k_t)$ is defined by

$$\Upsilon(\mu_t, k_t) = 1 + \frac{(1+\mu_t)^{\frac{\alpha-2}{1-\alpha}} k_t^{\frac{-\alpha}{1-\alpha}}}{(1-\alpha)\gamma MC} \left\{\tau - (1-\alpha)\left[1 + \gamma M (1-\mu_t)\right] k_t^{\alpha} - (1-\alpha)^2 \gamma M (1+\mu_t) k_t^{\alpha}\right\}.$$

Let $\left(\bar{\mu},\bar{k}\right)$ be the loci of points such that

$$\begin{aligned}\Upsilon\left(\bar{\mu},\bar{k}\right) &= 0\\ &= 1+\frac{\left(1+\bar{\mu}\right)^{\frac{\alpha-2}{1-\alpha}}\bar{k}^{\frac{-\alpha}{1-\alpha}}}{\left(1-\alpha\right)\gamma MC}\left\{\tau-\left(1-\alpha\right)\left[1+\gamma M\left(1-\bar{\mu}\right)\right]\bar{k}^{\alpha}-\left(1-\alpha\right)^{2}\gamma M\left(1+\bar{\mu}\right)\bar{k}^{\alpha}\right\}.\end{aligned}$$

Rearranging terms in this equation yields

$$\left(1-\alpha\right)\gamma MC\bar{k}^{\frac{\alpha}{1-\alpha}}=\left(1+\bar{\mu}\right)^{\frac{\alpha-2}{1-\alpha}}\left\{\left(1-\alpha\right)\left[1+\gamma M\left(1-\bar{\mu}\right)\right]\bar{k}^{\alpha}-\tau\right\}+\left(1-\alpha\right)^{2}\gamma M\left(1+\bar{\mu}\right)^{\frac{-1}{1-\alpha}}\bar{k}^{\alpha}. \quad \text{(A.2.1)}$$

It is straightforward to verify that as $\bar{k}\longrightarrow 0$, then equation (A.2.1) will hold only if $\bar{\mu}\longrightarrow\infty$. Similarly, as $\bar{\mu}\longrightarrow -1$, then it must be the case that $\bar{k}\longrightarrow\infty$, if equation (A.2.1) is to hold.[32] Thus, in general, $\Upsilon\left(\bar{\mu},\bar{k}\right)$ can be charaterized as a downward trending line (although not necessarily negatively sloped over the entire region $k\in[0,\infty)$) that crosses the $k-axis$ at least once. It is straightforward to verify that for values of (μ_t,k_t) which lie below the line described by equation (A.2.1), then $\Upsilon(\mu_t,k_t)<0$. In other words, for all $\bar{k}\in[0,\infty)$ and all $\mu_t<\bar{\mu}$, then $\Upsilon\left(\mu_t,\bar{k}\right)<0$, since in this case the right-hand side of equation (A.2.1) increases. Converesly, it is also easy to demonstrate that for all $\bar{k}\in[0,\infty)$ and all $\mu_t>\bar{\mu}$, then $\Upsilon\left(\mu_t,\bar{k}\right)>0$.

A.3 Lemma 5

To show that there exists a range of taxes, $\tau\in[0,\bar{\tau})$, over which there is a unique, high-capital steady state, it will suffice to examine where equations (16) and (17) intersect the $k-axis$. Note that on the $k-axis$ we have $\mu=0$ and equation (16) reduces to

$$k\left(\tau\right)=w\left(k\left(\tau\right)\right)-\frac{\tau}{1+\gamma M\left(e\left(\tau\right),z\right)} \quad \text{(A.3.1)}$$

Let $k_1(\tau)$ and $k_2(\tau)$, where $k_1(\tau)<k_2(\tau)$, be the values of the capital stock such that $k_1(\tau)$ and $k_2(\tau)$ are solutions to equation (A.3.1). When there does not exist any enforcement, that is, $\tau=0$, equation (A.3.1) reduces to

$$\begin{aligned}k\left(0\right)&=w\left(k\left(0\right)\right) \quad \text{(A.3.2)}\\ &=\left(1-\alpha\right)k\left(0\right)^{\alpha}\end{aligned}$$

[32] Although it is not possible for $\mu_t<0$ in equilibrium, we are setting $\bar{\mu}=-1$ only for the purposes of determining the shape of the locus defined by $\Upsilon\left(\bar{\mu},\bar{k}\right)$.

in steady state equilibrium for a Cobb-Douglas production function. There are two solutions to equation (A.3.2), the vertical lines $k_1(0)^* = 0$ and $k_2(0)^* = (1-\alpha)^{\frac{1}{1-Ga}}$. In addition, note that

$$\lim_{\tau \to 0} \left(k(\tau) - w(k(\tau)) + \frac{\tau}{1 + \gamma M(e(\tau), z)} \right) = k(0) - w(k(0)) + \frac{0}{1+\gamma}$$
$$= k(0) - w(k(0)),$$

and thus $k_i(\tau)$, for $i = 1, 2$ is continuous at $\tau = 0$. Finally, as state in Proposition 2 (and depicted in Figure 6), $k_1'(\tau) > 0$ and $k_2'(\tau) < 0$.

When $\mu = 0$, the second law of motion, equation (17) reduces to

$$Ax(1-\delta) = f'(k(\tau))\, w(k(\tau))\, M(e(\tau), z). \tag{A.3.3}$$

Let $k_3(\tau)$ be the solution to this equation. When there is no enforcement, that is, $\tau = 0$, this further reduces to

$$\begin{aligned} Ax(1-\delta) &= f'(k(0))\, w(k(0)) \\ &= \alpha(1-\alpha)\, k(0)^{2\alpha-1}. \end{aligned} \tag{A.3.4}$$

The solution to equation(A.3.4) is given by

$$k_3(0) = \left[\frac{\alpha(1-\alpha)}{Ax(1-\delta)} \right]^{\frac{1}{1-2\alpha}} > 0.$$

Finally, note that $k_3'(\tau) < 0$ by Proposition 2.

Thus for

$$\alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}} < Ax(1-\delta),$$

an assumption we henceforth make, we have that

$$0 = k_1(0) < k_3(0) < k_2(0) = (1-\alpha)^{\frac{1}{1-Ga}}$$

and at $\tau = 0$ there exists a unique, high capital steady state equilibrium.[33] Define $\bar{\tau}$ as the level of taxes such that $k_1(\bar{\tau}) = k_3(\vec{\tau})$ and $\bar{\tau} \leq w(k(\bar{\tau}))$. Solving equation (A.3.3) for $k(\bar{\tau})$ and plugging this into equation

[33] It is assumed that equations (16) and (17) have the general shapes as depicted in Figure 6. We refer to equilibria that occur on the downward sloping portion of equation (16) as high capital steady states.

(A.3.1) yields

$$\left[\frac{\alpha(1-\alpha)M(e(\bar{\tau}),z)}{Ax(1-\delta)}\right]^{\frac{1}{1-2\alpha}} = (1-\alpha)\left[\frac{\alpha(1-\alpha)M(e(\bar{\tau}),z)}{Ax(1-\delta)}\right]^{\frac{\alpha}{1-2\alpha}} - \frac{\bar{\tau}}{1+\gamma M(e(\bar{\tau}),z)}$$

which implicitly defines $\bar{\tau}$ as a function of the exogenous parameters. Finally, by the continuity of $k_1(\tau)$ and Proposition 2, we have that for $\tau \in [0, \bar{\tau})$, there exists a unique, high capital steady state equilibrium.

A.4 Proposition 4

To prove this proposition, we need to show conditions under which $U(k_H(\tau)) < U(k_H(0))$ for all $\tau > 0$, i.e., host country individuals would be worse off with any level of enforcement in the high capital steady state. Given the properties of $U(\cdot)$ it is sufficient to state conditions under which $c'(\tau) < 0$ for all $k_H(\tau)$ where $\tau > 0$. Recall that from equation (19) we have

$$c'(\tau) = k_\tau(\tau)\{f''(k(\tau))[w(k(\tau)) - \tau] + f'(k(\tau))w'(k(\tau))\} - f'(k(\tau)).$$

For a Cobb-Douglas production function this can be rewritten as

$$c'(\tau) = \alpha(1-\alpha)k_H(\tau)^{\alpha-2}k_H'(\tau)[(2\alpha-1)k_H(\tau)^\alpha + \tau] - \alpha k_H(\tau)^{\alpha-1}.$$

Thus $c'(\tau) < 0$ if and only if

$$(1-\alpha)k_H'(\tau)[(2\alpha-1)k_H(\tau)^\alpha + \tau] < k_H(\tau)$$

or

$$\tau > \frac{k_H(\tau)}{(1-\alpha)k_H'(\tau)} + (1-2\alpha)k_H(\tau)^\alpha, \tag{A.4.1}$$

where the first term is negative (since $k_H'(\tau) < 0$) and the second term is positive on the right-hand side of the equation. A sufficient condition for equation (A.4.1) to hold is

$$0 > \frac{k_H(\tau)}{(1-\alpha)k_H'(\tau)} + (1-2\alpha)k_H(\tau)^\alpha \tag{A.4.2}$$

since by definition $\tau > 0$. Rearranging terms in equation (A.4.2) yields

$$-k_H'(\tau) < \frac{k_H(\tau)^{1-\alpha}}{(1-\alpha)(1-2\alpha)} \tag{A.4.3}$$

Finally, noting that

$$[\alpha(1-\alpha)]^{\frac{1}{1-\alpha}} \leq k_H(\tau) \leq (1-\alpha)^{\frac{1}{1-\alpha}}$$

and equation (A.4.3) is increasing in $k_H(\tau)$, these conditions together imply that

$$-k_H^{'}(\tau) < \frac{\alpha}{(1-2\alpha)}$$

is sufficient to guarantee that $c'(\tau) < 0$ for all high capital steady state equilibrium.

A.5 Proposition 5

From equation (19) we have

$$c_\tau = k_\tau(\tau)\{f''(k(\tau))[w(k(\tau))-\tau] + f'(k(\tau))w'(k(\tau))\} - f'(k(\tau)), \tag{A.5.1}$$

which using the Implicit Function Theorem allows us to define

$$\tau = \tau(z).$$

Thus we know that

$$\tau_z = -\frac{c_{\tau z}}{c_{\tau\tau}}.$$

At an interior optimum it must be the case that $c_{\tau\tau} < 0$ and thus the sign of τ_z is equivalent to the sign of $c_{\tau z}$. Differentiating equation (A.5.1) with respect to z yields

$$\begin{aligned} c_{\tau z} = {} & k_{\tau z}\{f''(k(\tau))[w(k(\tau))-\tau] + f'(k(\tau))w'(k(\tau))\} + \\ & k_z\{k_\tau[f'''(k(\tau))(w(k(\tau))-\tau) + 2f''(k(\tau))w'(k(\tau)) + f'(k(\tau))w''(k(\tau))] - f''(k(\tau))\} \end{aligned}$$

Using the fact that $c_\tau = 0$ and a Cobb-Douglas production function gives us

$$c_{\tau z} = k_{\tau z}\frac{\alpha k^{\alpha-1}}{k_\tau} + k_z\left\{\alpha(\alpha-1)k^{\alpha-3}[k_\tau\{(1-\alpha)k^\alpha(4\alpha-2) - (\alpha-2)\tau\} - k]\right\} \tag{A.5.2}$$

The sign of the second term on the right-hand side depends on the sign of the term in the square brackets:

$$k_\tau\{(1-\alpha)k^\alpha(4\alpha-2) - (\alpha-2)\tau\} - k. \tag{A.5.3}$$

Using the fact that at an interior optimum

$$\tau = (1-2\alpha)\,k^{\alpha} + \frac{k}{(1-\alpha)\,k_{\tau}}$$

it is straightforward to show that the expression in (A.5.3) is positive for a high-capital steady state and is negative for a low-capital steady state if

$$k_{\tau} > \frac{1}{1-2\alpha}.$$

Then in a high capital steady state, we have $k_z > 0$, $k_\tau < 0$ and the second term on the right hand side of equation (A.5.2) is negative. Given that $k_\tau < 0$, then if $k_{\tau z} \geq 0$ or if $k_{\tau z} < 0$ but sufficiently close to zero, then $c_{\tau z} < 0$ and $\tau_z < 0$.

When we have a low-capital steady state, then the second term on the right hand side of equation (A.5.2) is negative since $k_z < 0$. Given that $k_\tau > 0$, then if $k_{\tau z} \leq 0$ or if $k_{\tau z} > 0$ but sufficiently close to zero, then $c_{\tau z} < 0$ and $\tau_z < 0$.

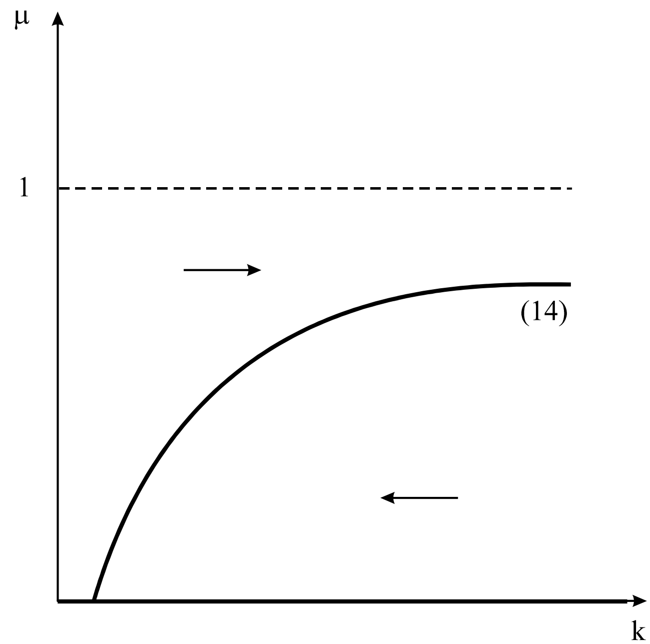

Figure 1: Loci where the capital-labor ratio is in steady state

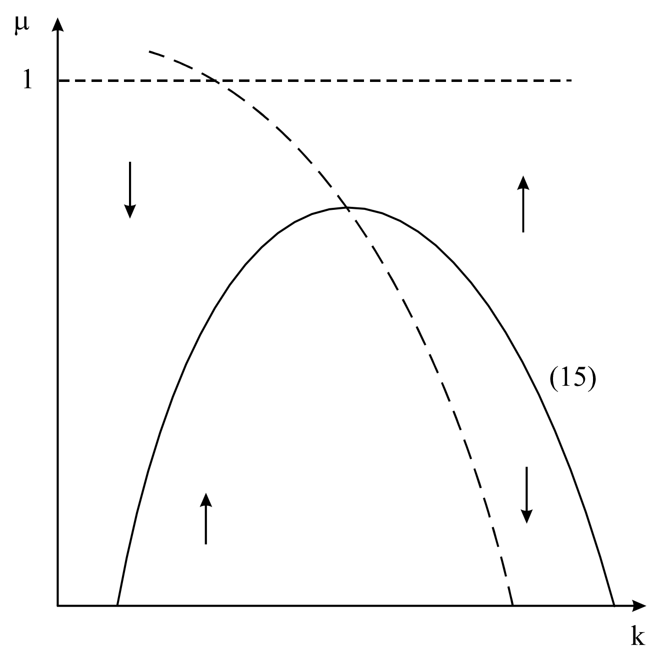

Figure 2: Locus where migration is in steady state

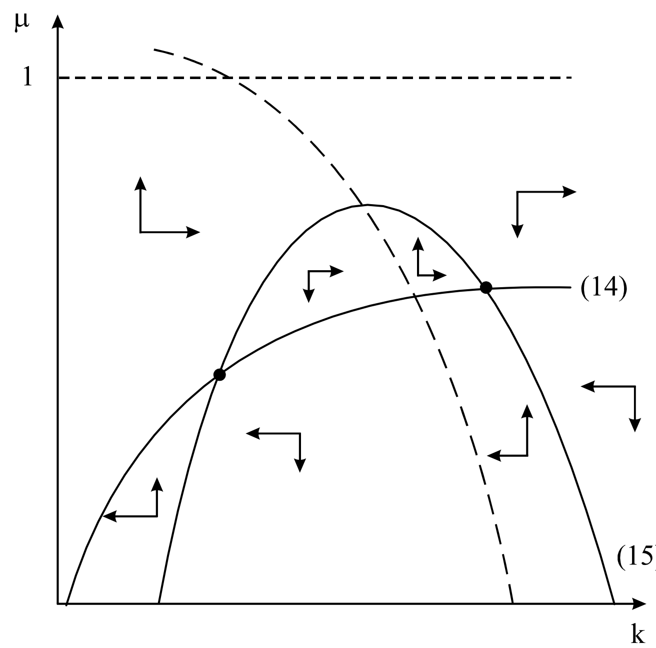

Figure 3: Phase Diagram when there are two steady state equilibria

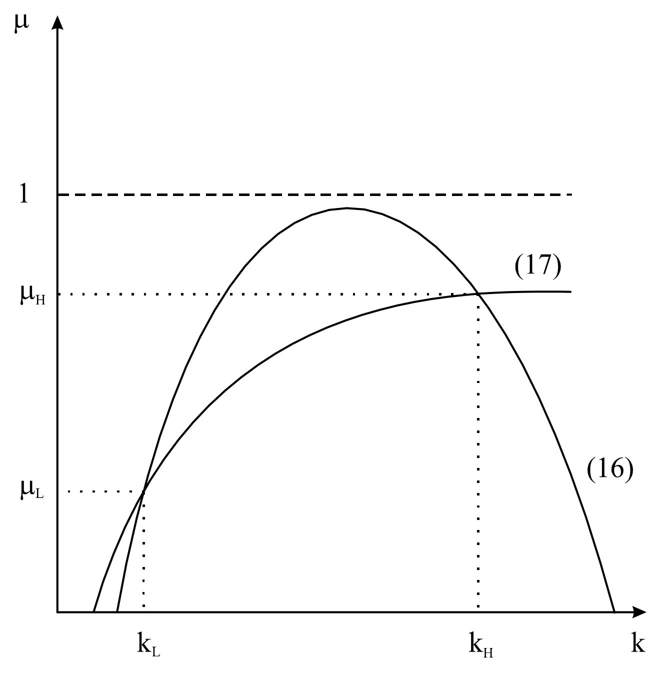

Figure 4: Multiple Steady State Equilibria

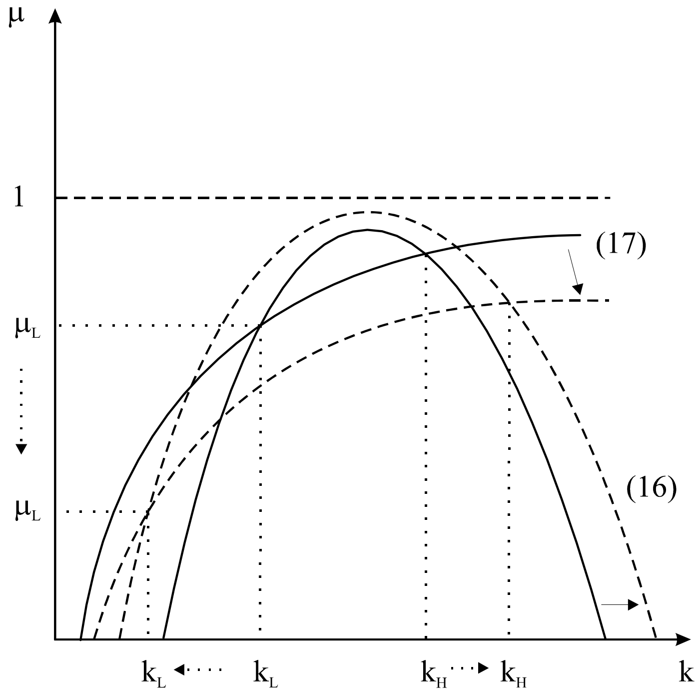

Figure 5: Effect of an Increase in Smuggling Technology

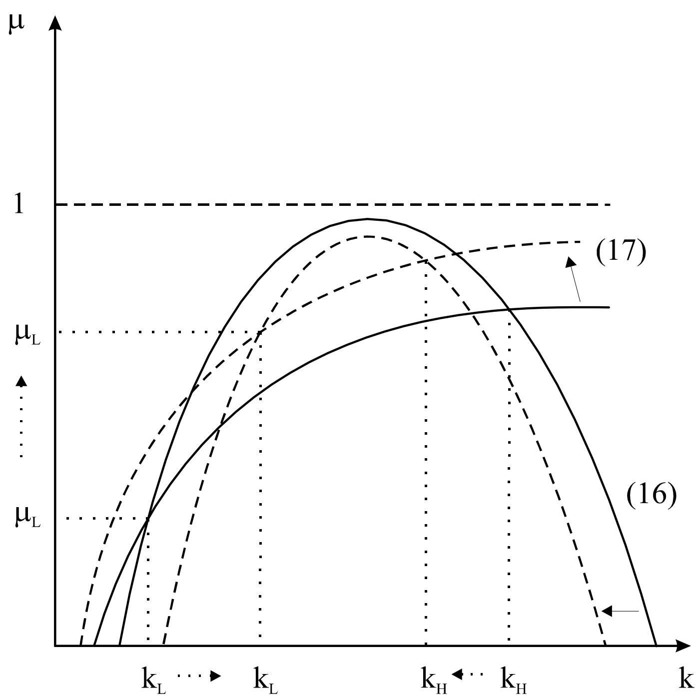

Figure 6: Effect of an Increase in Border Enforcement (Taxes)