Optimal monetary policy with hidden effort*

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Abstract

In this paper, we compare optimal monetary in two versions of a standard random-relocation overlapping generations economy. In the first version, the Friedman rule is optimal monetary policy. In the second version, we introduce hidden action and show that the planner's allocation has a wedge in the form of unequal consumption; movers consume less than non-movers. In a decentralized setting, the efficient allocation can be achieved by deviating from the Friedman rule. In the second model economy, agents exert costly, but socially beneficial effort that reduces their relocation probability. By deviating from the Friedman rule, greater effort is induced and thus, the decentralized economy replicates the planner's allocation. The key lesson is that deviating from the Friedman rule may be desirable if it provides incentives for agents to take socially beneficial actions. This requires however, that money holders, and *only* they, be the target of the incentive.

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1 Introduction

This paper considers a random-relocation economy in which agents can exert costly effort to reduce the probability of being relocated. If relocation is costly to society, a planner would choose to give less consumption to movers than nonmovers in order to provide incentives for agents to exert effort. We show that the planner's allocation can be implemented by a monetary policy that deviates from the Friedman rule (hereafter FR). To be more concrete, we refer to the FR as the policy implemented by the central bank such that money stock changes in order to equate the return on money with the other competing stores of value.

In this paper, we specify a model economy in which the key frictions are in the form of physical restrictions on the environment. Following an outline described in Kocherlakota (2005), we then derive the planner's allocation and consider what kind of monetary policy can decentralize that allocation. This approach to optimal taxation follows the one developed in Golosov, Kocherlakota and Tsyvinski (2003) (hereafter, GKT). Specifically, GKT study an economy with hidden productivity information. They show that the efficient allocation will consist of wedges as different types will consume different quantities. GKT then show that positive capital income tax rates can obtain the planner's allocation in a decentralized setting. da Costa and Werning (2003) also study an economy with hidden information. They study monetary economiies in which money is valued because it enters directly into the utility function.

We study a random-relocation economy similar to that developed in Champ, Smith, and Williamson (1996). The setting is a two-period lived overlapping generations model where limited communication and stochastic relocation create an endogenous transactions role for fiat money.[1] All agents are *ex ante* identical. Before the end of each period, however, young agents are probabilistically hit with a 'liquidity shock'; that is, a fraction is relocated (the 'movers') to a location different from the one in which they were born.

[1] Economies with spatial separation and limited communication were first studied by Townsend (1980, 1987).

The only asset that can be moved between islands is fiat money. Hence, money is held even if it is dominated in rate of return. (The other asset is a commonly-available linear storage technology with a fixed real return superior to that of money.)

In our analysis, we exclude type-specific taxes/transfers that could be applied *after* the liquidity shock is realized. [2] We justify this restriction by assuming that the government is unable to distinguish between movers and nonmovers at the time the tax is paid.

In this setting, we compare two economies. One in which the probability of moving is exogenous, and one in which there is hidden action. In the latter economy, we specify that the probability of moving can be reduced if agents exert costly and hidden effort. In each case, we assume that the mover has a negative impact on output. We show that in the economy with no hidden effort the planner wants to give to same consumption to movers and nonmovers; that is, there are no wedges in the efficient allocation. In the economy with hidden effort, however, the planner wants to give less consumption to movers. In other words, there is a wedge in the efficient allocation as the planner uses the wedge to provide incentives for agents to exert effort.

We show that movers and nonmovers get the same consumption in the decentralized setting if the central bank (CB) follows the FR. In contrast, movers get less consumption when the money stock grows faster than it would under the FR. Hence, absent other taxes or transfers, the CB should follow the FR in the environment without hidden effort and deviate from the FR in the environment with hidden effort.[3]

Our results add another piece to the optimal monetary policy literature. Phelps' (1973)

[2]That is, we simply prevent the government from taxing/transferring goods between movers and nonmovers once their type has been revealed. Of course, were such taxes allowed and implementable, there would be no demand for money in our economy.

[3]Note that since we choose not to restrict our attention to certain kind of taxes, there are many ways to implement the efficient allocation in the environments we consider. Hence we are not able to prove that deviating from the FR is a necessary condition to implement the planner's allocation. However, we show that if the FR is necessary to implement the efficient allocation in the economy without hidden effort, then deviating from the FR is necessary to implement the efficient allocation in the economy with hidden effort.

criticized Friedman (1969) result, arguing that it only held if lump-sum taxes/transfers were available. Since the Phelps critique, researchers have studied economies in which only distortionary taxes are available. Despite the distortionary-tax-only restriction, Chari, Christiano and Kehoe (1996) and Correia and Teles (1996) derive conditions under which steady-state welfare is maximized by implementing the Friedman rule. There is no such restriction on taxes in our model economies. Lump-sum taxes do not address the incentive to undersupply effort in our economy. So, even though non-distortionary taxes are available, such taxes alone cannot obtain the efficient allocation in a decentralized setting.

An important practical question underlies much of the profession's interest in the FR: Why do central banks around the world choose monetary policies that deviate from Friedman's dictum? Some authors have considered economies with limited enforcement. See, for example, Kocherlakota (2003) or Kiyotaki and Moore (2001). In these environments, the FR would be optimal if it were feasible. However, because of limited enforcement it may not be possible to contract the money supply enough to implement the FR. Others have studied environments in which deviations from the FR provide specific incentives to agents. For example, Head and Kumar (2005) study a search environment in which agents choose their search intensity. They show that the search intensity is suboptimally low at the FR and that deviating from the FR provides incentives for agents to increase their search intensity. Camera (2001) considers an economy in which the use of money promotes illegal activities. In that environment, deviating from the optimal quantity of money provides incentives to reduce illegal activities. Our paper is related to the latter strand of research in that deviations from the FR help provide incentives for agents to exert more socially-beneficial effort than they would under the FR.[4]

The remainder of the paper proceeds as follows. In section 2, we describe a benchmark model with no hidden actions. In this model the planner allocates equal consumption to movers and nonmovers. Following the FR achieves the efficient allocation. In section 3,

[4]The first paper following this approach is probably Levine (1991).

we extend the model to include hidden action. We show that the planner wishes to give more consumption to movers than to nonmovers in order to incentivize agents to exert effort. Deviating from the Friedman rule can achieve the efficient allocation in this model. Section 4 discusses extensions and section 5 concludes.

2 The standard random relocation economy

2.1 The environment

In this section we briefly describe a standard random relocation economy in which the planner's allocation can only be achieved in equilibrium if the CB follows the Friedman rule. [5] Only a succinct description of the model is provided; the interested reader is referred to Schreft and Smith (1997) for more details.

The economy takes place at infinitely many dates indexed by $t = ... - 1, 0, 1, ...$[6] The world is divided into two spatially separated locations. Each location is populated by a continuum of agents of unit mass. Agents live for two periods and receive an endowment of ω units of the single consumption good when young and nothing when old. Let c_t denote old-age consumption of the members of the generation born at date t; their lifetime utility is given by $U(c_t) = \frac{c_t^{1-\rho}}{1-\rho}$, where $\rho \in (0, 1)$.

After depositing their (after-tax) endowment into a bank, agents learn whether they must move to the other location or not. Let π denote the probability that an individual will be relocated. We assume a law of large numbers holds so π is also the measure of agents that is relocated. π is the same on both islands so that moves across locations are symmetric. Movers redeem their bank deposits in the form of money as this is the only way for them to acquire goods in the new location. In contrast, nonmovers redeem their

[5] See also Bhattacharya, Haslag, and Martin (2005).

[6] It is standard in this literature to have an initial period and an initial old generation but to ignore the welfare of that initial generation. To simplify the exposition we choose instead to have no initial period. Bhattacharya, Haslag, and Martin (forthcoming) contrast economies with or without an initial date.

deposits in the form of goods.

Goods deposited in the bank can be used to acquire money from old agents belonging to the previous generation or put into storage. Each unit of the consumption good put into storage at date t yields $x > 1$ units of the consumption good at date $t+1$, where x is a known constant.

The CB chooses the sequence $\{1+z_t\}_{t=-\infty}^{\infty}$, the gross rate of growth of the money supply at each date. It follows that the money supply evolves according to $M_t = (1+z_t)M_{t-1}$. In steady-state, $1+z_t = 1+z$, for all t and $p_t = (1+z)p_{t-1}$. To implement monetary policy, the CB levies lump-sum taxes or subsidies, denoted by τ_t, on the endowment of agents. To remove money from the economy, the CB taxes goods which are later exchanged for cash. A lump-sum subsidy is received in the form of a money injection. τ_t can be either positive or negative and is given by

$$\tau_t = -\frac{(M_t - M_{t-1})}{p_t} = -\frac{z_t}{1+z_t} m_t, \tag{1}$$

where $m_t \equiv M_t/p_t$ denotes real balances.

The CB is also assumed to operate a discount window. It can make cash loans to banks at date t (after the liquidity shock is revealed) which must be repaid at the start of date $t+1$ at an interest rate of $1+r$. We assume that r is a choice variable of the CB. Let b_t denote the real value of the cash loan made by the DW at date t. Then the bank at date t borrows $p_t b_t$ dollars and is obliged to repay $(1+r)\, p_t b_t$ dollars next period. The CB redistributes the proceeds from the interest rate charged to banks ($r p_t b_t$ dollars) in a lump-sum fashion. In this sense, the DW is self-financing and costless to operate if we assume that printing money is costless. Also note that DW borrowings will not influence the price level since the real value of the DW's dollar holdings is unchanged period to period. Since we focus on steady states, time subscripts are omitted in the remainder of the paper whenever possible.

2.2 Bank behavior

Agents deposit their entire after-tax/transfer endowments with a bank. The bank chooses a gross real return d^m to pay to movers and d^n to pay to nonmovers. In addition, the bank chooses values m and s standing for the real value of money balances bought from old agents, and storage investment, respectively. These choices must satisfy the bank's balance sheet constraint

$$m + s \leq \omega - \tau. \tag{2}$$

Since the discount window opens later, the bank does not choose its real discount window borrowing b at this point. Notice that if b is high, the bank in effect buys very little money from the old movers and therefore stores relatively more.

Banks behave competitively, so they take as given the return on their investments. In particular, the steady state return on real money balances is $p_t/p_{t+1} = 1/(1+z)$. If $x > 1/(1+z)$ and $x > 1+r$, banks want to hold as little liquidity as possible since money is dominated in rate of return. [7]

Banks must possess sufficient liquidity to meet the needs of movers. This is captured by the following expression,

$$\pi d^m(\omega - \tau) \leq \frac{m+b}{1+z}. \tag{3}$$

A similar condition for nonmovers, who consume all the proceeds from the storage technology, is given by

$$(1-\pi)d^n(\omega - \tau) \leq xs - b\frac{1+r}{1+z} + T, \tag{4}$$

[7] If $x = 1/(1+z)$, banks are indifferent between money and storage and there are multiple equilibria. We are not interested in the multiplicity of equilibria because it is not robust in the sense that for $1/1+z$ arbitrarily close to but strictly smaller than x the equilibrium is unique. For this reason, we consider the limiting economy as $1/1+z \to x$.

where $T = rb/(1+z)$ in equilibrium. Substituting equation (2), at equality, to eliminate s, we can write

$$(1-\pi)d^n(\omega-\tau) \leq x(\omega-\tau) - xm - b\frac{1+r}{1+z}. \tag{5}$$

From equations (3) and (5), it is clear that banks choose $m = 0$ if $x > (1+r)/(1+z)$. Indeed, in that case the real interest cost of borrowing cash from the discount window is smaller than the resource cost from reducing investment to sell goods to old movers in exchange for cash. Conversely, if $x < (1+r)/(1+z)$, then banks choose $b = 0$. Whenever $x = (1+r)/(1+z)$, banks are indifferent and we break the tie by assuming that banks borrow.

Banks maximize profits. Because of free entry, they choose their portfolio in a way that maximizes the expected utility of a representative depositor in equilibrium. The bank's problem can thus be written as

$$\max \quad \frac{(\omega-\tau)^{1-\rho}}{1-\rho}\left\{\pi\left(d^m\right)^{1-\rho} + (1-\pi)\left(d^n\right)^{1-\rho}\right\} \tag{6}$$

subject to equations (2), (3), and (4).

2.3 The optimum quantity of money

The CB chooses $1+z \geq 1/x$ in order to maximize agents' utility, taking into account the banks' behavior. The Friedman rule corresponds to the limit as $1+z \to 1/x$. In this case, the rate of return of money is equal to the rate of return of storage. This definition is consistent with Friedman's (1969) dictum.

Haslag and Martin (2005) show that the planner's allocation in this economy is such that $s = \omega$ and $c^m = c^n$.[8] The planner wants to invest as much as possible because the net

[8] We have assumed that the planner can store all goods at date t and pay both movers and non-movers from those stored goods at $t+1$. This corresponds to the unconstrained first best and is tantamount to assuming that the planner does not face the limited communication constraint. Our formulation follows

return on storage is positive. Also, because agents are risk averse, the planner gives them equal consumption.

If the CB can make large enough discount window loans, then the efficient allocation can be replicated (decentralized) by setting $r = 0$, i.e., by offering zero interest cash loans. In that case, $m = 0$ since $x \geq 1/(1+z)$. This implies that $\tau = 0$ and $s = \omega$, implying that the bank only utilizes the DW and simply stores everything just as the planner would. Equations (2), (3), and (4) can be combined to yield

$$\pi d^m + (1-\pi) d^n \leq x.$$

Since depositors are risk averse, banks choose $d^m = d^n$, which implies $c^m = c^n$.

When $m = 0$, money does not circulate between generations, so it is not clear what the rate of growth of the money supply is. However, we can consider an economy in which the size of the discount window loans made by the CB is constrained so that $m > 0$. Haslag and Martin (2005) show that the planner's allocation corresponds to the limit of a sequence of constrained economies as $m \to 0$ and $1 + z \to 1/x$.[9] This is the sense in which the FR is optimal in this environment.

A few words about the exact working of the DW is in order. The DW lends money to the banks which the banks give to the movers. The movers then relocate to the other island and seek goods to purchase with their money. Young agents in the other island could sell them goods in exchange but so can the bank. The bank needs to obtain cash to repay its loan to the DW. So in the equilibrium with $m = 0$, what the bank does is sell just enough of the goods (deposits) it holds so as to acquire the money to repay the DW. The remaining goods are given to nonmovers. The young agents, deposit all their goods with the bank and do not sell any to the movers. As such, money does not circulate between the generations and yet it is valued in equilibrium because agents believe it has

Smith (2002), Haslag and Martin (2005), and Antinolfi and Keister (forthcoming).

[9]See also Antinolfi and Keister (forthcoming).

value justifying banks' cash borrowings from the DW. [10]

3 An economy with hidden effort

In this section, we modify the above model economy in two ways: First, we assume that depositors can exert costly effort to reduce the probability of being a mover. Second, we assume that moving imposes a cost on the economy.[11] In this economy, as we demonstrate below, the FR is no longer optimal.

Assume agents are endowed with one unit of time when young and nothing when old. This time can be divided between effort and leisure: $e_t + l_t = 1$, for all t, where e denotes the effort applied by young agents and l stands for fraction of time that they enjoy leisure. We posit that the probability of being relocated depends on effort and we let $\pi(e_t)$ denote the probability that an agent will be relocated. The function $\pi(.)$ is assumed to satisfy $\pi' < 0$, $\pi'' > 0$, $\pi(0) = \bar{\pi} \leq 1$, and $\pi(1) = \underline{\pi} \geq 0$. Also, $\lim_{e\to 0} \pi' = -\infty$ and $\lim_{e\to 1} \pi' = 0$. Hence, as effort approaches zero, the marginal effect on the probability of being relocated is large. As effort approaches its maximum level, the effect on that probability vanishes. An individual agent's effort level is private information. We define Π_t as the measure of young agents who relocate at the end of date t.

The disutility from effort is additively separable from the utility derived from consumption. Let $V(1-e_t)$ characterize the function mapping leisure into utility, where $l_t = 1 - e_t$. $V(.)$ is increasing and strictly concave. In other words $V' \geq 0$, $V'' < 0$, $\lim_{l\to 0} V' = \infty$, and $\lim_{l\to 1} V' = 0$. [12]

[10] See Bhattacharya, Haslag, and Russell (2005) for another version of this result where the "simulator" ensures money does not circulate but still has value.

[11] Note that nothing would have changed in the previous section if we had also assumed that moving is costly to the economy in the environment. Hence the key difference between the two models is the fact that there is hidden effort in this section.

[12] Below, we make the parametric assumption that $U(c) = \ln c$. Our result can be extended to utility functions of the form $U(c) = c^{1-\rho}/(1-\rho)$ in a straightforward way.

Movers are assumed to impose a cost to the rest of the economy in the following way. Each depositor can costlessly monitor a project and monitored projects return $x > 1$ for each unit invested. However, movers are unable to monitor the projects to which they are assigned and thus projects assigned to movers have a return of ϕx, where $\phi \in (0, 1)$. It follows that the average return to the storage projects on an island is given by $\Pi\phi x + (1 - \Pi)\, x = x\,[1 - \Pi(1 - \phi)]$, which we assume to be greater than 1. The higher the mass of movers, the lower is the average return. Banks invest in all projects and take the average return as given.

In the remainder of this section, we first determine an agent's optimal effort given his consumption in the moving and non-moving states. Next, we describe the planner's problem for this economy. Finally, show how the planner's allocation can be achieved in a decentralized economy.

3.1 Effort choice

At the start of a period, agents determine how much effort to exert in reducing their probability of relocation taking as given his consumption in the two states, c^m and c^n. This problem can be written as

$$\max_{e} \quad \pi(e)U(c^m) + [1 - \pi(e)]U(c^n) + V(1 - e). \tag{7}$$

Taking the partial derivative of the objective function with respect to e and setting it to zero yields

$$\pi'\,(.)\,[U(c^m) - U(c^n)] - V'\,(.) = 0. \tag{8}$$

Assuming $V'(1) = 0$, it is apparent from equation (8) that when $c^n = c^m$, the optimal choice of effort is $e = 0$. That is, were the agent offered complete insurance against the risk of relocation, he would exert no effort at reducing this risk. Hence, effort will be positive only if $c^m < c^n$. Intuitively, if movers receive less consumption than non-movers, consumers will be willing to exert some effort to reduce the probability of relocation. Let e^* denote the optimum effort level. This effort level depends on the levels of consumption by movers and non-movers. Thus, it follows from the implicit function theorem that $e^* = e(c^m, c^n)$.

Proposition 1 *Consumer's optimum effort is (i) decreasing in the quantity of consumption good allocated to movers; and (ii) increasing in the quantity of the consumption good allocated to non-movers.*

Proof. After substituting the $e(c^m, c^n)$ into the first-order condition and differentiating, one obtains

$$\pi''[U(c^m) - U(c^n)]de + V''de + \pi'U'dc^m = 0.$$

Collect terms and rearrange, yielding

$$\frac{de}{dc^m} = -\frac{\pi'U'}{\pi''[U(c^m) - U(c^n)] + V''}.$$

As usual, the denominator is the second-order condition. If $\pi'' > 0$ and $V'' < 0$, the sign of the denominator is negative, ensuring that the second-order condition is satisfied. In other words, the value of e^* that satisfies the first-order condition is indeed a maximum.[13] With $\pi' < 0$ and positive marginal utility, the implication is that $\frac{\partial e}{\partial c^m} < 0$. An increase in the quantity of the consumption given to movers will result in less effort by the consumer. As movers get more of the consumption, holding everything else constant, there is less need to use effort to try to indirectly insure themselves against moving.

It is straightforward to show that $\frac{\partial e}{\partial c^n} > 0$ along the same lines as above. ∎

[13]So, $\pi'' > 0$ corresponds to a case in which the impact of effort on the probability of moving is getting algebraically bigger; that is, a smaller negative number.

3.2 The planner's problem

The planner chooses how much to save and how to allocate available goods between movers and nonmovers to maximize the expected utility of a representative generation. Physically, the planner collects endowments from the young on each island. These goods can be consumed by consumers alive in that period or invested in the technology. The planner also collects goods that were invested last period. The planner knows the effect that movers have on the return to the technology. Since the planner can identify movers and non-movers, each type may receive different quantities of goods. Indeed, the planner chooses to give less consumption to movers in order to provide incentive for consumers to spend effort to limit the probability of becoming a mover.

Formally, the planner's problem can be written as

$$\max_{c_t^m, c_t^n} \quad \Pi(e)U(c_t^m) + [1-\Pi(e)]U(c_t^n) + V(1-e_t), \tag{9}$$

subject to

$$\Pi(e_t)c_t^m + [1-\Pi(e_t)]c_t^n = x\left[1-\Pi(e_t)(1-\phi)\right]s_t + \omega_{t+1} - s_{t+1}, \tag{10}$$

$$s_t \leq \omega_t, \tag{11}$$

$$e_t = e(c_t^m, c_t^n). \tag{12}$$

The first constraint states that the planner can provide consumption to consumers born at date t with the goods these agents stored when they were young or with some of the endowment from the next generation. The second constraint stipulates that the quantity of storage cannot exceed the endowment. In a steady state, $s_t = s_{t+1} = s \leq \omega$. Since we assume that $x\left[1-\Pi(e)(1-\phi)\right] > 1$, the planner wants to choose s as big as possible so that $s = \omega$. Hence, constraint (10) becomes

$$\Pi(e)c^m + [1-\Pi(e)]c^n = x\left[1-\Pi(e)(1-\phi)\right]\omega. \tag{13}$$

We can substitute the function for effort, $e_t = e(c_t^m, c_t^n)$, that comes from the consumer's program to get two first-order conditions:

$$e_m\Pi'[U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1-\phi)\omega)] - e_m V' + \Pi(e)\left[U'(c^m) - \lambda\right] = 0$$

and

$$e_n\Pi'[U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1-\phi)\omega)] - e_n V' + (1 - \Pi(e))\left[U'(c^n) - \lambda\right] = 0$$

where λ is the Lagrange multiplier on constraint (13) and $e_n \equiv \frac{\partial e}{\partial c^m}$ and $e_n \equiv \frac{\partial e}{\partial c^n}$.

Solve both equations for λ to get

$$\frac{e_m\Pi'[U(c^m) - U(c^n)] - e_m V' + \Pi(e)U'(c^m)}{e_m\Pi'[c^m - c^n + x(1-\phi)\omega] + \Pi(e)} = \lambda,$$

and

$$\frac{e_n\Pi'[U(c^m) - U(c^n)] - e_n V' + (1-\Pi(e))U'(c^n)}{e_n\Pi'[c^m - c^n + x(1-\phi)\omega] + (1-\Pi(e))} = \lambda.$$

Let $\Delta_1 \equiv e_m\Pi'[U(c^m) - U(c^n)] - e_m V' + \Pi(e)U'(c^m)$, $\Delta_2 \equiv e_m\Pi'[c^m - c^n + x(1-\phi)\omega] + \Pi(e)$, $\Gamma_1 \equiv e_n\Pi'[U(c^m) - U(c^n)] - e_n V' + (1-\Pi(e))U'(c^n)$, and $\Gamma_2 \equiv e_n\Pi'[c^m - c^n + x(1-\phi)\omega] + (1-\Pi(e))$, then it is easily checked that $\frac{\Delta_1}{\Delta_2} = \frac{\Gamma_1}{\Gamma_2}$. Combined with the planner's resource constraint, we have two equations and two unknowns. Once the planner has chosen the allocation – c^{m*}, c^{n*}– then one can solve for the effort level, using $e = e(c^{m*}, c^{n*})$.

Proposition 2 *The planner will choose to allocate different consumption to movers than to non-movers; more formally,* $c^m < c^n$.

Proof. It is straightforward to show that the planner allocates different consumption levels to the different consumer types. Suppose the contradiction: the planner chooses $c^m = c^n = c$. We rearrange the expression $\frac{\Delta_1}{\Delta_2} = \frac{\Gamma_1}{\Gamma_2}$, to get

$$[x(1-\phi)\,\omega]\,U'(c)\Pi' = V' \tag{14}$$

The left hand side of this expression is strictly positive. If $c^m = c^n = c$, consumers exert no effort and $V'(1) = 0$, so the above equation cannot hold. The intuition is that the planner wants agents to exert some effort to reduce their chance of relocation. ■

It is clear from our specification of the planner's problem that both hidden effort and the cost imposed by movers are important features of the model economy. Together, these features can account for why the planner does not want to offer perfect insurance to consumers in this economy. If movers did not impose a cost on the economy, the planner would not be concerned with reducing the mass of movers. It would then be optimal to provide equal consumption to all agents since they are ex-ante identical and risk-averse. Instead, if movers imposed a cost but the probability of moving was exogenous, then the planner would again choose to provide equal consumption to all agents since it would not be possible to mitigate the cost imposed by movers.

If effort were observable, the planner would be able to impose the optimal level of effort, for example by imposing a severe enough punishment. In that case, it would not be necessary to distort the allocation of consumption between movers and nonmovers. Note that the optimal level of effort may be lower than full effort if effort is sufficiently costly.

3.3 The decentralized economy with hidden effort

In this section, we consider the optimal monetary policy in the decentralized version of our modified environment. In doing so, the CB takes into account the deposit contract offered by banks and the effort decision of depositors.

Using the notation described earlier, the bank's problem can now be written as

$$\max_{d^m, d^n} \quad \pi(e)U\left[d^m(\omega-\tau)\right] + [1-\pi(e)]U[d^n(\omega-\tau)] + V(1-e)$$

subject to

$$m + s \leq \omega - \tau, \tag{15}$$

$$\pi(e)d^m(\omega - \tau) \leq \frac{m + b}{1 + z}, \tag{16}$$

$$(1 - \pi(e))d^n(\omega - \tau) \leq x\left[1 - \Pi(1 - \phi)\right]s - (1 + r)\frac{b}{1 + z} + T. \tag{17}$$

We can use equation (15) at equality to eliminate s in equation (17) and get

$$(1 - \pi(e))d^n(\omega - \tau) \leq x\left[1 - \Pi(1 - \phi)\right](\omega - \tau) - x\left[1 - \Pi(1 - \phi)\right]m - (1 + r)\frac{b}{1 + z} + T. \tag{18}$$

As in section 2, the choice of m and s by the bank depends on the relative sizes of $x\left[1 - \Pi(1 - \phi)\right]$ and $(1 + r)/(1 + z)$. Banks choose to borrow as much as possible from the discount window if $x\left[1 - \Pi(1 - \phi)\right] \geq (1 + r)/(1 + z)$, while banks prefer not to borrow form the discount window if $x\left[1 - \Pi(1 - \phi)\right] < (1 + r)/(1 + z)$.

Our first aim is to show that the CB can achieve the same allocation as the planner would. Clearly, if $d^m(\omega - \tau) = c^m$ and $d^n(\omega - \tau) = c^n$, depositors will make the same effort decision as they would in the planner's economy. In the planner's allocation, all goods are stored, so it must be the case that $m = 0$ in the bank's problem. As noted above, banks choose $m = 0$ if $x\left[1 - \Pi(1 - \phi)\right] \geq (1 + r)/(1 + z)$. Note that if $m = 0$, then $\tau = 0$. In this case, the constraints become

$$\pi(e)d^m\omega \leq \frac{b}{1 + z}, \tag{19}$$

$$(1 - \pi(e))d^n\omega \leq x\left[1 - \Pi(1 - \phi)\right]\omega - (1 + r)\frac{b}{1 + z} + T. \tag{20}$$

We can combine these two equations to get

$$\pi(e)d^m\omega + (1 - \pi(e))d^n\omega \leq x\left[1 - \Pi(1 - \phi)\right]\omega - r\pi(e)d^m\omega + T. \tag{21}$$

If the bank sets $d^m\omega = c^m$, $d^n\omega = c^n$, and $r = x\omega(1-\phi)/c^m$, then constraint (21) becomes identical to (13.) Thus, we state our result in the following proposition.

Proposition 3 *The CB is able to achieve the efficient allocation if it deviates from the FR.*

Indeed, the CB must set the "right" interest rate on borrowed funds. Note that $1+z$ must be big enough so that $x\left[1-\Pi(1-\phi)\right] > (1+r)/(1+z)$. This condition ensures that banks will invest all of their deposits.

Note that banks do not have an incentive to deviate from setting $r = x\omega(1-\phi)/c^m$. Indeed, the "price" of effort in terms of goods, for the planner, is $x\omega(1-\phi)$. This is the term that multiplies $\Pi(e)$ in the planner's budget constraint. The "price" of effort in terms of goods, for the bank, is $x\omega(1-\phi)(d^m\omega/c^m)$. Indeed, all other terms are taken as given by the bank.

If the bank chooses $d^m\omega > c^m$, then it is facing a higher "price" than the planner is for effort. Yet, it is getting less effort, so this cannot be the optimizing choice of d^m. In other words, with $d^m\omega > c^m$ a decrease in effort is more costly, in terms of goods, to the bank than it is to the planner. Since the bank and the planner face the same objective function, the bank should want at least as much effort as the planner. Since increasing d^m will reduce effort, this cannot be the optimal d^m. Conversely, if the bank chooses $d^m\omega < c^m$, then it is facing a lower "price" than the planner is for effort. Yet, it is getting more effort, so this cannot be the optimizing choice of d^m. Again, with $d^m\omega < c^m$ a decrease in effort is less costly, in terms of goods, to the bank than it is to the planner. Since the bank and the planner face the same objective function, the bank should want no more effort as the planner. Since decreasing d^m will increase effort, this cannot be the optimal d^m.

As in section 2, the rate of growth of the money supply is not well defined when money does not circulate between generations. Again, we can consider a sequence of economies that are progressively less constrained such that $m \to 0$. In this economy, the FR is given by $(1+z) = 1/\left(x\left[1-\Pi(1-\phi)\right]\right)$. The CB would like banks to borrow as much as possible from discount window, so it needs to make sure that $x\left[1-\Pi(1-\phi)\right] \geq (1+r)/(1+z)$. The CB must also set $r = x\omega(1-\phi)/c^m > 0$. This implies

$$(1+z) = \frac{1+r}{x\left[1-\Pi(1-\phi)\right]} > \frac{1}{x\left[1-\Pi(1-\phi)\right]}.$$

Thus, deviations ffrom the Friedman rule may be optimal in this economy.

4 Alternative policies

We have seen above that when movers impose a cost on the economy and agents can affect their relocation probability by exerting costly effort, a planner chooses to give less consumption to movers than to nonmovers in order to provide incentives to exert effort. A central bank that has access to a discount window can achieve the planner's allocation if it lends cash at a positive interest rate and deviates from the FR.

There are a number of other ways the efficient allocation could be achieved in the economy we have described. In this section we describe some alternative policies in order to illustrate alternative policies that would achieve the efficient allocation. We also discuss extensions of the model that could make these alternative policies less effective than an inflation tax. This allows us to highlight the specific role played by the inflation tax in our model.

In the economy of section 3, the efficient allocation can be achieved by following the FR, taxing movers, and redistributing the proceeds of the tax in lump-sum fashion. However, this policy will no longer be desirable in a slightly different, but related, setting.[14] Assume that a fraction of consumers who did not move when they were young must relocate to the other island when old, before goods are available on their island, but after they have been able to monitor their projects. These agents need money, because they move to the other island and this money can be obtained by banks through discount window loans.

[14] Also, this policy is not feasible if the government is unable to distinguish between movers and nonmovers.

First, observe that the policy described in section 3 is still optimal in this new environment provided the CB charges $r = 0$ for intraperiod loans and $r = x\omega(1-\phi)/c^m > 0$ for interperiod loans. By making intraperiod loans at a rate of zero, the CB does not penalize the old movers. This is appropriate since these agents are able to monitor their projects and thus do not impose a cost on the economy. Interperiod loans carry a positive interest rate in order to provide incentives for consumers to reduce the probability of being a mover. In contrast, a policy of taxing movers, to the extend that it does not distinguish between young and old movers, cannot implement the efficient allocation.[15]

Another policy that would implement the efficient allocation in section 3 is to impose a tax on transactions involving money. Again, this policy would not implement the efficient allocation in an economy with old movers as described in this section.

It is important to note that deviations from the FR do not penalize all agents who move. Moreover, the penalty does not necessarily affect all agents who use money. Rather, such deviations specifically harm agents who carry money from one period to the next. The old movers introduced in this section are similar to young movers in that they relocate and use money, but they are different in that they do not carry money between periods.

The CB does not care about agents carrying money between periods per se. The CB cares about the fact that movers impose a cost on the economy. It turns out that, in the economies described in this paper, only the movers to whom the CB is trying to provide incentives to carry money between periods. Hence, providing incentives through an inflation tax is effective in this setting.

[15]It is also possible to imagine an economy in which some young movers can affect the probability with which they are relocated while others cannot. If the former type consume when they are old while the latter type consume when they are young, then it is possible to implement the efficient allocation as above, by deviating from the FR and charging an interest rate of zero for intraperiod loans and a positive interest rate for inter period loans. In such a case, even a policy that can tax young and old movers differently cannot implement the efficient allocation. What is needed is a policy that can distinguish between movers who consume when young and movers who consume when old.

It is possible to imagine an economy in which the inflation tax would not work so well. For example, there might be two types of movers: Some cannot influence the probability that they will be relocated while some others can. A planner would like to provide incentives to the latter type but would prefer not to distort the allocation of the former type. In such an environment, an inflation tax may not implement the planner's allocation. Taxing movers would not work either. What would be needed is a much more targeted tax.

This leads us to state the following theorem.

Theorem 1 *We can restrict the set of taxes so that (1) in economies without hidden effort, the Friedman rule is necessary for optimality; and (2) in economies with hidden effort, deviating from the Friedman rule is necessary for optimality.*

(A formal proof is being worked out.)

4.1 An application to payment systems

In the economy sketched in this section, there are two kids of movers: Some have an exogenous probability of moving and consume in the same period in which they move. Others can affect the probability with which they must move and consume in the next period. As noted above, in this case the CB can implement the efficient allocation by charging an interest rate of zero for intraperiod loans and a positive interest rate for interperiod loans.

If one interprets intraperiod loans as intraday credit and interperiod loans as overnight credit, then this pattern resembles the practice of many central banks.[16] Hence the framework presented in this paper can help illuminate that issue. The model suggests that intraday credit carries a very low cost because it does not affect the quantity of resources available in the economy. Old movers, in the above economy, do not impose a cost to society because they are able to monitor their projects. In contrast, interperiod credit

[16] See, for example, Martin (2004)

carries a relatively high cost in order to provide incentives to consumers. The availability of interperiod credit can affect the resources available in the economy since it influences the effort made by consumers and thus the mass of projects that will not be monitored.

5 Conclusion

In this paper, we build a model in which following the FR leads agents to exert inefficiently low effort, which is costly to the economy. We show that deviating from the Friedman rule can promote effort and achieve the planner's allocation. We also consider alternative policies that may implement the efficient allocation and discuss extensions of the model that make these alternative policies undesirable.

The key lesson from our paper is that deviating from the FR may be desirable if it provides incentives for agents to take socially beneficial actions. This requires that agents who hold money, and only such agents, be the target of the incentive.

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