Smoothed IV quantile regression and quantile Euler equations

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Outline

Standard consumption Euler equation

- Expected utility maximization, $U(C) = C^{1-\gamma}/(1-\gamma)$:

$$0 = \mathbb{E}[\beta(1+R_{t+1})(C_{t+1}/C_t)^{-\gamma} - 1 \mid \Omega_t],$$

  Ω_t: information set at time t
  R_{t+1}: real rate of return of asset
  C_t: real consumption at time t
  β: discount factor (e.g., $\beta = 0.99$)
  $1/\gamma$: elasticity of intertemporal substitution (EIS)
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- Estimation: use variables in Ω_t as instruments (inflation, etc.); run GMM, or IV/2SLS after log-linearization
- Drawback: no separation of EIS ($1/\gamma$) and risk aversion (γ)
- Drawback: approximation error from log-linearization

Quantile Euler equation?

- Standard: $0 = \mathbb{E}[\beta(1 + R_{t+1})(C_{t+1}/C_t)^{-\gamma} - 1 \mid \Omega_t]$
- Replace $\mathbb{E}[\cdot \mid \Omega_t]$ with conditional τ-quantile $Q_\tau[\cdot \mid \Omega_t]$:

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- Advantage: $1/\gamma$ is EIS, but both τ and γ capture risk attitude
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- Advantage: robust to fat tails in consumption
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- Grounded in decision theory? (next slide)
- Practical to estimate? (SIVQR)

Quantile Euler equation: decision theory

- Quantile utility maximization, static setting: Manski (1988), Chambers (2009), and Rostek (2010) (axiomatization)
- Dynamic setting: de Castro and Galvao (2017) show dynamic consistency and derive Euler equation

Quantile Euler equation: estimation

- Can write as $Q_\tau[\epsilon_{t+1} \mid \Omega_t] = 1$, $\epsilon_{t+1} \equiv \beta(1 + R_{t+1})(C_{t+1}/C_t)^{-\gamma}$
- Since $\ln(\cdot)$ is strictly increasing, $Q_\tau[\ln(W)] = \ln(Q_\tau[W])$
- In contrast, $\mathbb{E}[\ln(W)] \leq \ln(\mathbb{E}[W])$ (Jensen's); approx error

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$$\ln(\epsilon_{t+1}) = \ln(\beta) + \ln(1 + R_{t+1}) - \gamma \ln(C_{t+1}/C_t),$$
$$\ln(C_{t+1}/C_t) = \gamma^{-1} \ln(\beta) + \gamma^{-1} \ln(1 + R_{t+1}) - \gamma^{-1} \ln(\epsilon_{t+1})$$

- $\gamma > 0 \implies -\gamma^{-1} \ln(\epsilon)$ strictly ↓ in ϵ:

$$0 = \ln(1) = \ln\big(Q_\tau[\epsilon_{t+1} \mid \Omega_t]\big) = Q_\tau[\ln(\epsilon_{t+1}) \mid \Omega_t]$$
$$= Q_{1-\tau}[-\ln(\epsilon_{t+1}) \mid \Omega_t] = Q_{1-\tau}[-\gamma^{-1} \ln(\epsilon_{t+1}) \mid \Omega_t]$$

- Parameters for τ-quantile maximization correspond to the $1 - \tau$ IV quantile regression of $\ln(C_{t+1}/C_t)$ on a constant and $\ln(1 + R_{t+1})$

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- Very difficult to compute IVQR estimator numerically
- Chernozhukov and Hansen (2006): iid, linear model, only 1 or 2 endogenous regressors (so can't have many interactions, polynomial terms, etc.)
- Other methods (also iid): compliers/LQTE (Abadie, Angrist, and Imbens, 2002), MCMC (Chernozhukov and Hong, 2003; Lancaster and Jun, 2010), triangular system (Lee, 2007, and others), very slow MIQP (Chen and Lee, 2017)

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- de Castro, Galvao, and Kaplan (2017): dependent data, nonlinear model; fast and robust computation, consistency and asymptotic normality
- New results underway from Xin Liu: smoothed two-step GMM, computation (code) and asymptotic theory

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Smoothed IVQR (SIVQR): benefits

- Approach: smooth the moment conditions (estimating equations)
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- Approach: smooth the moment conditions (estimating equations)
- For now: use just-identified system for numerical robustness; if over-identified, just take linear combination of instruments
- Benefit #1: computation is feasible, fast, scalable (many endogenous regressors), and numerically robust
- Benefit #2: often improves MSE (Kaplan and Sun, 2017)
- Benefit #3: important first step toward true IV quantile GMM (in progress with Xin Liu)

Smoothed QR (not IV): literature

- Horowitz (1998): smooths criterion fn instead of moments; Studentized bootstrap refinement
- Whang (2006): same moment smoothing used here, but for empirical likelihood QR; also in Otsu (2008)
- Fernandes, Guerre, and Horta (2017): kernel-smoothed QR criterion; FOC same as smoothed moments above
- MaCurdy and Hong (1999): original IVQR smoothing? (unpub'd notes)

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- Simplistic example: linear structural random coefficient model, $Y = \mathbf{X}'\beta(U)$, assume $\mathbf{X}'\beta(U)$ monotonic in unobserved $U \sim \mathrm{Unif}(0,1)$
- If Y is wage, U is "ability": $\mathbf{X}'\beta(0.5)$ traces out potential wage outcomes (given different $\mathbf{X}$) for individual with median ability $(\mathrm{P}(U \leq 0.5) = 0.5)$

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- If instrument vector $\mathbf{Z} \perp\!\!\!\perp U$, then $\mathrm{P}(Y \le \mathbf{X}'\beta(\tau) \mid \mathbf{Z}) = \mathrm{P}(U \le \tau \mid \mathbf{Z}) = \mathrm{P}(U \le \tau) = \tau$: a conditional quantile restriction on the observables Y, $\mathbf{X}$, and $\mathbf{Z}$, and the parameter vector $\beta(\tau)$

Alternative quantile models with endogeneity

- Triangular system: compared in Chernozhukov, Hansen, and Wüthrich (2017, §2.5); like Chesher (2003), Lee (2007), et al.
- LQTE (like LATE): compared in Melly and Wüthrich (2017, §§5–6); like Abadie et al. (2002), Frölich and Melly (2013), et al.

IVQR moment conditions

(linear IV)

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$$\tau = \mathrm{P}(Y \leq h(\mathbf{X},\tau) \mid \mathbf{Z}),\ h(\mathbf{X},\tau) = \mathbf{X}'\boldsymbol{\beta}_{0\tau},\ \mathrm{P}(\cdot) = \mathbb{E}(\mathbb{1}\{\cdot\}) \implies$$

$$\mathbf{0} = \mathbb{E}\Big[\mathbf{Z}\Big(\mathbb{1}\Big\{Y - \mathbf{X}'\boldsymbol{\beta}_{0\tau} \leq 0\Big\} - \tau\Big)\Big]$$ (linear IVQR)

IVQR moment conditions

$$Y = h(\mathbf{X}) + U,\ \mathbb{E}(U \mid \mathbf{Z}) = 0,\ h(\mathbf{X}) = \mathbf{X}'\beta_0 \implies$$

$$\mathbf{0} = \mathbb{E}[\mathbf{Z}(Y - \mathbf{X}'\beta_0)] \qquad \text{(linear IV)}$$

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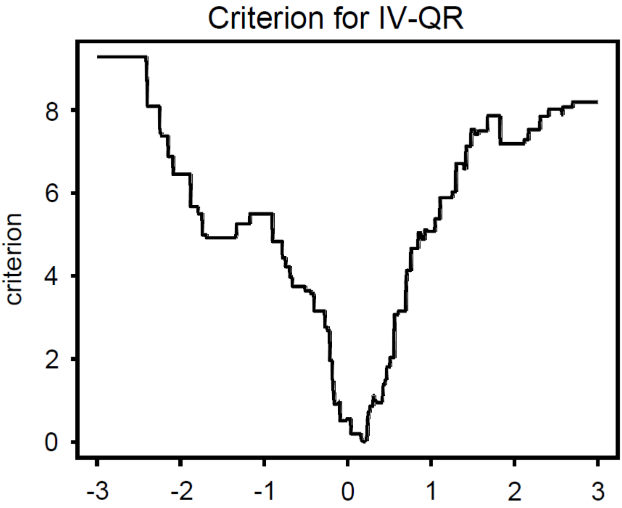
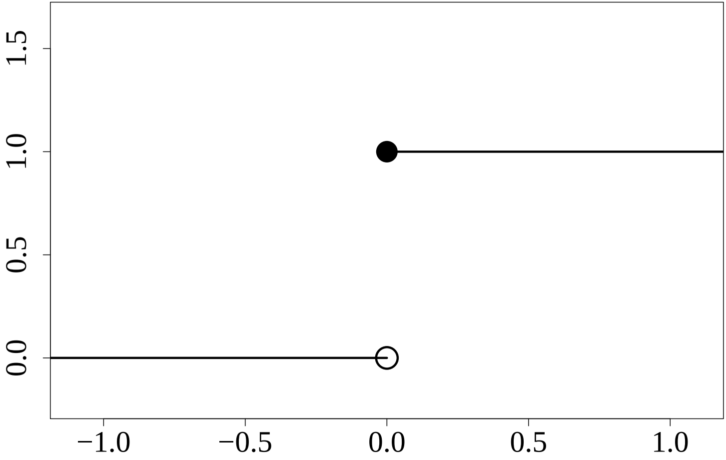
Just run GMM?

Chernozhukov and Hong (2003), Figure 1(a)



Smoothing the indicator function



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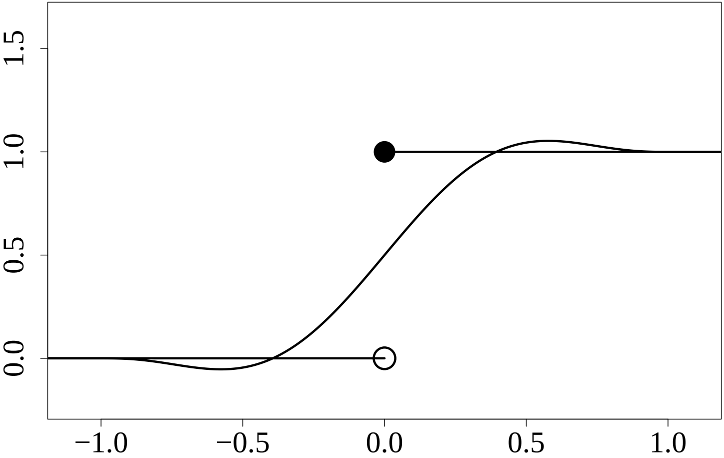
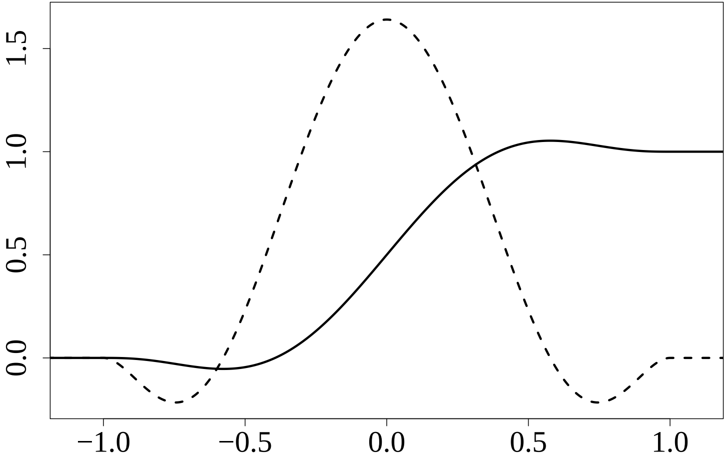
$\tilde{I}(\cdot)$ = solid line



Smoothing the indicator function

$\tilde{I}(\cdot)$ = solid line $\tilde{I}'(\cdot)$ = broken line



Smoothed estimator

- Instead of solving sample moments $\mathbf{0} = \hat{\mathbb{E}}\big[\mathbf{Z}(\mathbb{1}\{\mathbf{X}'\hat{\boldsymbol{\beta}}_\tau - Y \geq 0\} - \tau)\big]$, replace $\mathbb{1}\{\cdot \geq 0\}$ with smoothed $\tilde{I}(\cdot/h_n)$, bandwidth h_n:

$$\mathbf{0} = n^{-1}\sum_{i=1}^{n}\mathbf{Z}_i\left[\tilde{I}\left(\frac{\mathbf{X}_i'\hat{\boldsymbol{\beta}}_\tau - Y_i}{h_n}\right) - \tau\right]$$

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- Can compute Jacobian (wrt $\boldsymbol{\beta}$)
- Easy/fast to compute (unless $h_n \approx 0$), standard solver
- $\tilde{I}'(\cdot)$: rth order kernel; $f_{U|\mathbf{Z},\mathbf{X}}(\cdot)$: $\geq r$ derivatives wrt u

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- $\tilde{I}'(\cdot)$: rth order kernel; $f_{U|\mathbf{Z},\mathbf{X}}(\cdot)$: $\geq r$ derivatives wrt u
- Why exact ID? Robust computation: know when numerical method returns correct $\hat{\boldsymbol{\beta}}$. (If overidentified: can use linear combination of moments, although not efficient.)
- Two-step GMM in progress

Connections: IV, Winsorized mean

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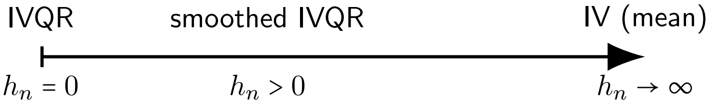

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IVQR — smoothed IVQR — IV (mean)

$h_n = 0$ — $h_n > 0$ — $h_n \to \infty$

- Can try to pick h_n to improve efficiency (like median vs. mean); maybe even better to explicitly average IVQR and IV (and QR) like Hansen (2017)?

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- Special case: $X_i = Z_i = 1$, $\tilde{I}'(u) = \mathbb{1}\{-1 \le u \le 1\}/2$, $\tau = 0.5 \implies$ "Winsorized" mean (Huber, 1964, Ex. iii, p. 79)

Outline

1. Consumption Euler equations
2. Smoothed IV quantile regression (SIVQR)
3. **Results**
4. Conclusion

MSE of SEE ("smoothed estimating equations"), iid/linear

$$Y_i = \mathbf{X}_i' \boldsymbol{\beta}_\tau + U_i, \quad \mathrm{P}(U_i \le 0 \mid \mathbf{Z}_i) = \tau$$

- Ultimately, care more about MSE of $\hat{\boldsymbol{\beta}}_\tau$ than MSE of SEE
- Large statistics literature on optimal EE leading to optimal point estimation for unbiased EE; here: biased
- Connection to MSE of $\hat{\boldsymbol{\beta}}_\tau$ (Kaplan and Sun, 2017)
- MSE of SEE: can compute finite-sample bias/variance; $\hat{\boldsymbol{\beta}}$: asy. approx.
- Also useful for inference; robust to weak IV

MSE of SEE

$$\mathbf{m}_n \equiv n^{-1/2} \sum_{i=1}^{n} \mathbf{Z}_i \left[\tilde{I}\left(\frac{\mathbf{X}_i' \boldsymbol{\beta}_{0\tau} - Y_i}{h_n} \right) - \tau \right] \equiv n^{-1/2} \sum_{i=1}^{n} \mathbf{W}_i$$

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$$\mathbb{E}(\mathbf{W}_i) = \frac{h_n^r}{r!} \left(\int \tilde{I}'(v) v^r \, dv \right) \mathbb{E}\left[f_{U|\mathbf{Z}}^{(r-1)}(0 \mid \mathbf{Z}_j) \mathbf{Z}_j \right] + o(h_n^r)$$

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$$\mathbb{E}(\mathbf{W}_i \mathbf{W}_i') = \underline{\mathbf{V}} - h_n \left[1 - \int_{-1}^{1} [\tilde{I}(u)]^2 \, du \right] \mathbb{E}\{ f_{U|\mathbf{Z}}(0 \mid \mathbf{Z}_i) \mathbf{Z}_i \mathbf{Z}_i' \} + o(h_n)$$

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- $\uparrow$ bias $\Longrightarrow$ $\downdownarrows$ var

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$$h^* = \left(\frac{(r!)^2 \left[1 - \int_{-1}^{1} \tilde{I}^2(u)\, du \right] f_U(0)}{2r \left(\int \tilde{I}'(v) v^r\, dv \right)^2 \left[f_U^{(r-1)}(0) \right]^2} \frac{d}{n} \right)^{\frac{1}{2r-1}} \quad \text{if } U \perp\!\!\!\perp \mathbf{Z}$$

Brief comments on proofs in de Castro et al. (2017)

- Direct treatment of smoothed estimator (vs. show within $o_p(n^{-1/2})$ of unsmoothed); triangular array (U)LLN/CLT
- Three high-level assumptions, but primitive conditions with dependent data given in each case (using Andrews, 1987, 1988; Kato, 2012; Wooldridge, 1986)
- Smoothing allows the usual mean-value expansion (of the sample moment conditions) to derive asymptotic normality
- For consistency, can just smooth as little as possible

Theoretical results

- Consistency
- Asymptotic normality
- Inference: Wald test based on normality; but bootstrap works better (not proved theoretically); and neither is robust to weak identification like Andrews and Mikusheva (2016), Chernozhukov, Hansen, and Jansson (2009), and others

Setup

- Now: endogenous vector $\mathbf{Y}$, instruments $\mathbf{Z}$ with subset $\mathbf{X}$
- Residual fn $\Lambda(\mathbf{Y}, \mathbf{X}, \beta)$, like $Y_1 - (Y_2, \mathbf{X}')\beta$

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- Now: endogenous vector $\mathbf{Y}$, instruments $\mathbf{Z}$ with subset $\mathbf{X}$
- Residual fn $\Lambda(\mathbf{Y}, \mathbf{X}, \boldsymbol{\beta})$, like $Y_1 - (Y_2, \mathbf{X}')\boldsymbol{\beta}$

$$\mathbf{0} = \mathbb{E}\Big\{\mathbf{Z}_i\Big[\mathbb{1}\{\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta}_{0\tau}) \le 0\} - \tau\Big]\Big\} = \mathbf{M}(\boldsymbol{\beta}_{0\tau}, \tau)$$

Setup

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- Smoothed estimator:

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\begin{aligned} \mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) &\equiv \mathbf{g}_n(\mathbf{Y}_i, \mathbf{X}_i, \mathbf{Z}_i, \boldsymbol{\beta}, \tau) \\ &\equiv \mathbf{Z}_i\big[\tilde{I}\big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\big) - \tau\big]. \end{aligned}$$

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\Big[\tilde{I}\big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\big) - \tau\Big].$$

Assumption A1

Strictly stationary, weakly dependent data.

Assumption A2

$\Lambda(\cdot)$ *known, differentiable in* $\boldsymbol{\beta}$*.*

Assumption A3

Global point identification of $\boldsymbol{\beta}_{0\tau}$*; interior of compact* $\mathcal{B}$*.*

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\Big[\tilde{I}\Big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\Big) - \tau\Big].$$

Assumption A4

ULLN: $\sup_{\boldsymbol{\beta}\in\mathcal{B}}\Big|\hat{\mathbf{M}}_n(\boldsymbol{\beta}, \tau) - \mathbb{E}[\hat{\mathbf{M}}_n(\boldsymbol{\beta}, \tau)]\Big| = o_p(1)$.

Note $\mathbb{E}[\hat{\mathbf{M}}_n(\boldsymbol{\beta}, \tau)] \neq \mathbf{M}(\boldsymbol{\beta}, \tau)$. Paper: example primitive conditions, using Andrews (1987). Use: $\mathbf{g}_{ni} \leq 2|\mathbf{Z}_i|$ and $h_n \to 0$; WLLN from Andrews (1988).

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\big[\tilde{I}\big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\big) - \tau\big].$$

Assumption A5

$\mathbb{E}(\mathbf{Z}_i\mathbf{Z}_i')$ *is positive definite (and finite).*

(No moment restrictions on $\mathbf{Y}_i$.)

Assumption A6

Distribution of $\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})$ *given* $(\boldsymbol{\beta}, \mathbf{Z}_i = \mathbf{z})$ *is continuous at zero.*

E.g., Y_1 cts given $(Y_2, \mathbf{Z})$ for linear IVQR.

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\big[\tilde{I}\big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\big) - \tau\big].$$

Assumption A7

$\tilde{I}'(\cdot)$ is kernel fn (bdd support), like picture.

Assumption A8

$h_n = o(n^{-1/4})$.

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\Big[\tilde{I}\Big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\Big) - \tau\Big].$$

Assumption A9

Let $\Lambda_i \equiv \Lambda\big(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta}_{0\tau}\big)$ *and* $\mathbf{D}_i \equiv \nabla_{\boldsymbol{\beta}}\Lambda\big(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta}_{0\tau}\big)$. *(i)* $f_{\Lambda|\mathbf{Z}}(\cdot \mid \mathbf{z})$ *twice differentiable (also* $f_{\Lambda|\mathbf{Z},\mathbf{D}}$*). (ii) Nonsingular* $\underline{\mathbf{G}} = \nabla_{\beta}\mathbf{M}(\boldsymbol{\beta}_{0\tau}, \tau) = -\mathbb{E}\{\mathbf{Z}_i\mathbf{D}_i' f_{\Lambda|\mathbf{Z},\mathbf{D}}(0 \mid \mathbf{Z}_i, \mathbf{D}_i)\}$.

E.g., $\mathbf{D}_i$ is regressor vector for linear IVQR.

(ii) $\Longrightarrow$ local identification

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\left[\tilde{I}\left(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\right) - \tau\right].$$

Assumption A10

$$-\frac{1}{nh_n}\sum_{i=1}^{n} \tilde{I}'\left(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \hat{\boldsymbol{\beta}}_\tau)/h_n\right)\mathbf{Z}_i \nabla_{\boldsymbol{\beta}}\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \hat{\boldsymbol{\beta}}_\tau)' \xrightarrow{p} \underline{\mathbf{G}}.$$

Closely related to Powell (1984, 1991) kernel estimator for QR covariance. Kato (2012): primitive conditions (w/ weakly dependent data) for linear QR ($\mathbf{Y} = Y$, $\mathbf{Z} = \mathbf{X} = \mathbf{D}$). Readily extended to linear IVQR, but harder if non-constant $\nabla_{\boldsymbol{\beta}}\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \hat{\boldsymbol{\beta}}_\tau)$.

Assumptions

$$\mathbf{0} = \hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_\tau, \tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{g}_{ni}(\hat{\boldsymbol{\beta}}_\tau, \tau),$$

$$\mathbf{g}_{ni}(\boldsymbol{\beta}, \tau) \equiv \mathbf{Z}_i\Big[\tilde{I}\Big(-\Lambda(\mathbf{Y}_i, \mathbf{X}_i, \boldsymbol{\beta})/h_n\Big) - \tau\Big].$$

Assumption A11

Pointwise CLT: $\sqrt{n}\big\{\hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau}, \tau) - \mathbb{E}[\hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau}, \tau)]\big\} \xrightarrow{d} \mathrm{N}(\mathbf{0}, \underline{\boldsymbol{\Sigma}}_\tau)$.

Primitive conditions: moment and dependence restrictions.
Ex: iid, $\mathbb{E}(\|\mathbf{Z}_i\|^2) < \infty$.
Ex: Wooldridge (1986), NED (. . .), $\mathbb{E}(\|\mathbf{Z}_i\|^{2+\epsilon}) < \infty$.

Consistency

Lemma 1

A1–A3 and A5–A8 $\Longrightarrow \sup_{\boldsymbol{\beta}\in\mathcal{B}} \left| \mathbb{E}[\hat{\mathbf{M}}_n(\boldsymbol{\beta},\tau)] - \mathbf{M}(\boldsymbol{\beta},\tau) \right| = o(1)$.

Proof.

Use dominated convergence theorem. Need cts distribution of $\Lambda(\mathbf{Y},\mathbf{X},\boldsymbol{\beta})$ since $\tilde{I}(0) = 0.5 \neq 1 = \mathbb{1}\{0 \geq 0\}$. □

Consistency

Theorem 2

A1–A8 $\implies \hat{\boldsymbol{\beta}}_\tau - \boldsymbol{\beta}_{0\tau} = o_p(1)$.

Proof.

Use Thm 5.9 in van der Vaart (1998), or Thm 2.1 in Newey and McFadden (1994). Combine ULLN (A4) with Lemma 1 (and triangle inequality): $\hat{\mathbf{M}}_n(\cdot) \xrightarrow{p} \mathbf{M}(\cdot)$ uniformly. Maximizer of $-\|\mathbf{M}(\cdot)\|$ is uniquely $\boldsymbol{\beta}_{0\tau}$ (A3), "well-separated" b/c compact $\mathcal{B}$ (A3), cts $\mathbf{M}(\cdot)$ (can show). □

Asymptotic normality

Lemma 3

A1–A3, A5, A7–A9, and A11 $\implies \sqrt{n}\hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau}, \tau) \xrightarrow{d} \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_\tau)$.

Proof.

1) $\mathbb{E}[\hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau}, \tau)] = O(h_n^2)$, like kernel bias.
2) $O(\sqrt{n}h_n^2) = o(1)$ if $h_n = o(n^{-1/4})$ (A8).
3) Apply CLT (A11). □

Asymptotic normality

Theorem 4

A1–A11 $\implies \sqrt{n}(\hat{\boldsymbol{\beta}}_\tau - \boldsymbol{\beta}_{0\tau}) \xrightarrow{d} \mathrm{N}\big(\mathbf{0}, \underline{\mathbf{G}}^{-1}\underline{\boldsymbol{\Sigma}}_\tau[\underline{\mathbf{G}}']^{-1}\big)$.

Proof.

Mean value expansion: $\mathbf{0} = \hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau}) + \underline{\dot{\mathbf{M}}}_n(\hat{\boldsymbol{\beta}}_\tau - \boldsymbol{\beta}_{0\tau})$, so
$\sqrt{n}(\hat{\boldsymbol{\beta}}_\tau - \boldsymbol{\beta}_{0\tau}) = -[\underline{\dot{\mathbf{M}}}_n]^{-1}\sqrt{n}\hat{\mathbf{M}}_n(\boldsymbol{\beta}_{0\tau})$.
Apply CMT to Lemma 3 and $\underline{\dot{\mathbf{M}}}_n \xrightarrow{p} \underline{\mathbf{G}}$ (A10). □

Simulation setup

- Compare SIVQR, QR (ignore endogeneity), IV (ignore heterogeneity)
- "JTPA" DGP: iid, binary treatment, randomized offer but self-selection endogeneity
- "TS–IV" DGP: time series regression of y_t on mismeasured x_t, where x_{t-1} is valid IV; normal or Cauchy errors

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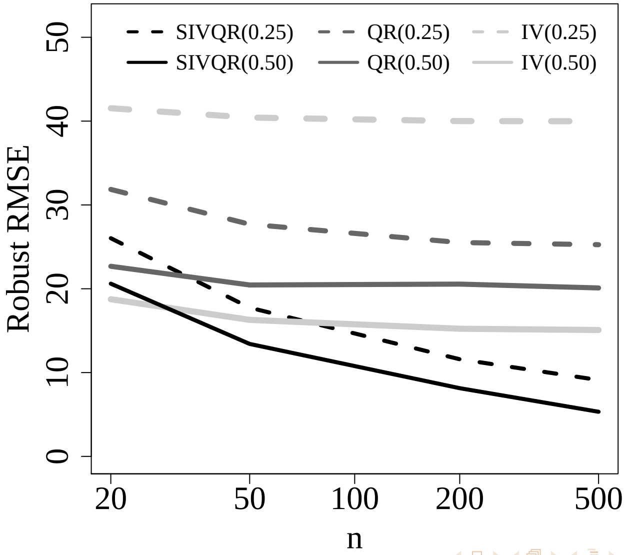
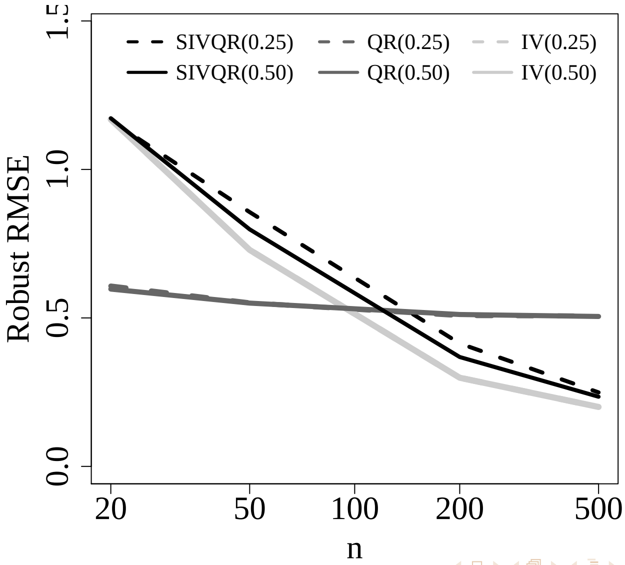
- "Robust RMSE": use median bias, and IQR/1.35, so equals RMSE for normal distribution. (IV has no mean. . .)
- Bandwidth h_n: smallest possible for estimation (only second-order effects over wide range); rule of thumb from Kato (2012) for inference
- LRV est: Bartlett kernel, data-dependent bandwidth from Andrews (1991).
- Stationary bootstrap from Politis and Romano (1994).

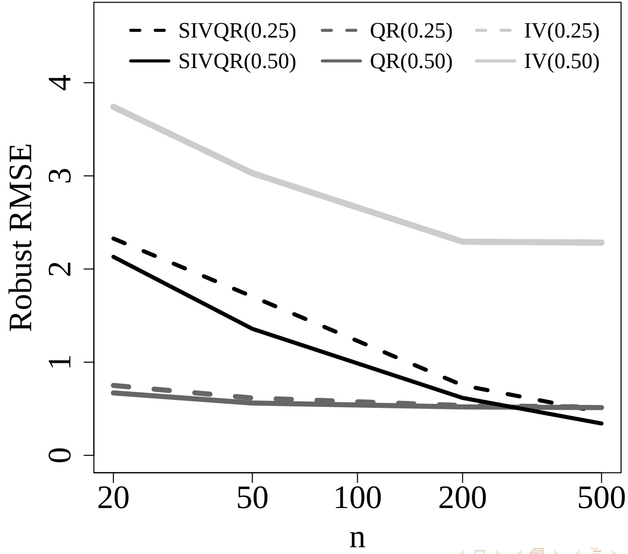
Robust RMSE: JTPA



Robust RMSE: TS–IV, normal



Robust RMSE: TS–IV, Cauchy



Size, 2-sided test of $H_0 : \gamma_\tau = \gamma_0$

DGP	τ	n	α	Wald	BS-t	BS
JTPA	0.25	100	0.10	0.411	0.094	0.196
		1000	0.10	0.415	0.067	0.115
		10,000	0.10	0.226	n/a	n/a
	0.50	100	0.10	0.550	0.074	0.120
		1000	0.10	0.342	0.041	0.101
		10,000	0.10	0.134	n/a	n/a

Size, 2-sided test of $H_0 : \gamma_\tau = \gamma_0$

DGP	τ	n	α	Wald	BS-t	BS
TS–IV.N	0.25	100	0.10	0.269	0.098	0.040
		1000	0.10	0.154	0.107	0.088
		10,000	0.10	0.116	n/a	n/a
	0.50	100	0.10	0.246	0.103	0.040
		1000	0.10	0.147	0.119	0.090
		10,000	0.10	0.101	n/a	n/a

Size, 2-sided test of $H_0 : \gamma_\tau = \gamma_0$

DGP	τ	n	α	Wald	BS-t	BS
TS–IV.C	0.25	100	0.10	0.389	0.059	0.022
		1000	0.10	0.179	0.075	0.059
		10,000	0.10	0.116	n/a	n/a
	0.50	100	0.10	0.296	0.067	0.019
		1000	0.10	0.159	0.110	0.079
		10,000	0.10	0.092	n/a	n/a

JTPA: context

- Abadie, Angrist, and Imbens (2002), 5102 adult men
- Randomized offer of services to individuals (Z_i), 62% uptake: $\mathrm{P}(D_i = 1 \mid Z_i = 1) = 0.62$. Other regressors: age, race, etc.
- Endogeneity from self-selection into treatment; OLS estimate twice as big as IV est
- Y_i: 30-month earnings (US dollars) in "after" period

JTPA: results

		Quantile index τ				
Regressor	Method	0.15	0.25	0.50	0.75	0.85
Training	AAI	121	702	1544	3131	3378
Training	SEE ($\hat{h}$)	57	381	1080	2630	2744
Training	CH	−125	341	385	2557	3137
Training	tiny h	−129	500	381	2760	3114
Training	huge h	1579	1584	1593	1602	1607
Training	2SLS			1593		
Married	AAI	1564	3190	7683	9509	10,185
Married	SEE ($\hat{h}$)	1132	2357	7163	10,174	10,431
Married	CH	504	2396	7722	10,463	10,484
Married	tiny h	504	2358	7696	10,465	10,439
Married	huge h	6611	6624	6647	6670	6683
Married	2SLS			6647		

JTPA: results

Replace age dummies w/ quartic in age; add continuous baseline measures (wage, weekly hrs worked)

Still computes in one second or less; tiny h takes around 10 seconds

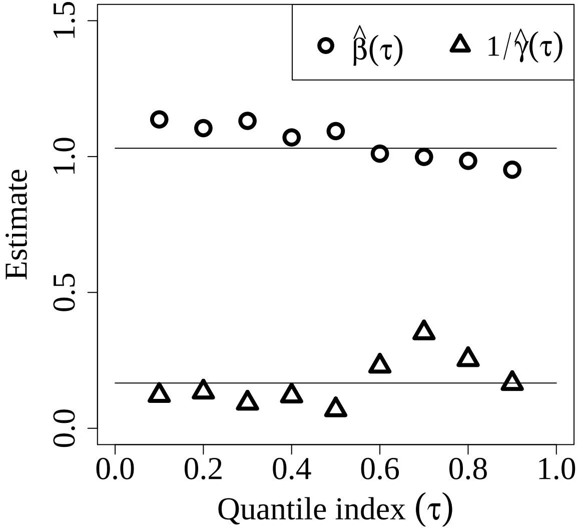
JTPA: results

		Quantile index τ				
Regressor	Method	0.15	0.25	0.50	0.75	0.85
		Original controls				
Training	SEE ($\hat{h}$)	57	381	1080	2630	2744
Training	CH	−125	341	385	2557	3137
Training	tiny h	−129	500	381	2760	3114
Training	2SLS			1593		
		Modified controls				
Training	SEE ($\hat{h}$)	74	398	1045	2748	2974
Training	CH	−20	451	911	2577	3415
Training	tiny h	−50	416	721	2706	3555
Training	huge h	1568	1573	1582	1590	1595
Training	2SLS			1582		

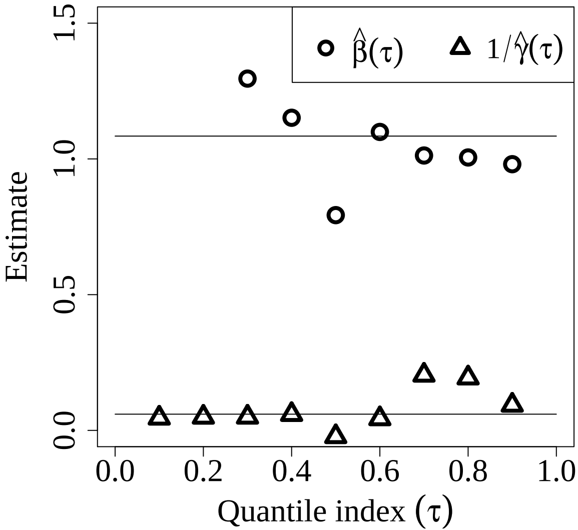
Quantile Euler equation estimates

- Data: from Yogo (2004) (from Campbell, 2003), country-level aggregate time series
- Specification: same as Yogo (2004) Table 2 (but with quantiles): IVQR of $\ln(C_{t+1}/C_t)$ on a constant and $\ln(1 + R_{t+1})$, R =real interest rate
- Excluded instruments are $t-1$ values of: nominal interest rate, inflation, log dividend-price ratio, and $\ln(C_{t-1}/C_{t-2})$; "weak instruments are not a problem" (Yogo, 2004, p. 805)
- Very little smoothing (for estimation)
- β: discount factor ($1 =$ no discount)
- $1/\gamma$: EIS, elasticity of intertemporal substitution

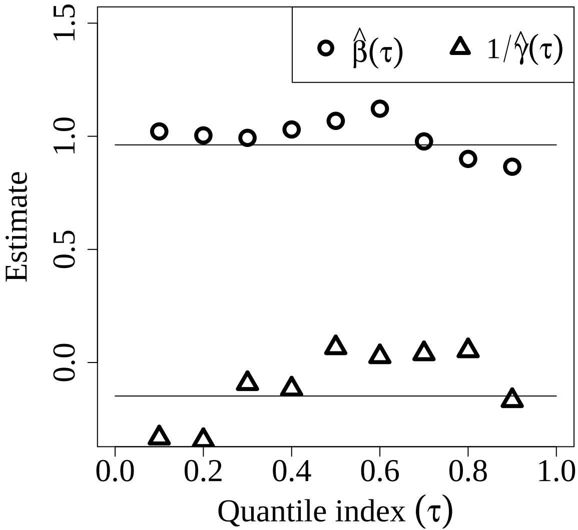
Quantile Euler equation estimates: UK



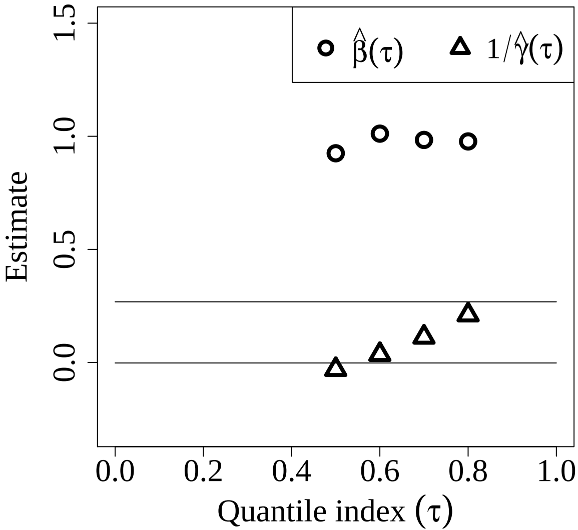
Quantile Euler equation estimates: USA



Quantile Euler equation estimates: Netherlands



Quantile Euler equation estimates: Sweden



Quantile Euler equation estimates

	USA		UK	
τ	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$
0.10	3.18*	22.0*	1.14	8.2*
0.20	1.64	20.5*	1.11	7.5*
0.30	1.30	20.4*	1.13	10.8*
0.40	1.15	17.0*	1.07	8.4*
0.50	0.79	−43.9*	1.09	14.6*
0.60	1.10	23.2*	1.01	4.4*
0.70	1.01	4.9*	1.00	2.8
0.80	1.01	5.2*	0.98	3.9*
0.90	0.98	10.8*	0.95	6.1*
2SLS	1.08	16.7*	1.03	6.0*

*: significantly different from 1 at 10% level (2-sided)

Quantile Euler equation estimates

	US		UK		NTH		SWE	
τ	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$	$\hat{\beta}_\tau$	$\hat{\gamma}_\tau$
0.70	1.01	4.9	1.00	2.8	0.98	25.5	0.98	8.9
0.80	1.01	5.2	0.98	3.9	0.90	19.1	0.98	4.7
2SLS	1.08	16.7	1.03	6.0	0.96	−6.8	0.27	−544.4

Outline

Conclusion

- Smoothed IVQR: fast, scalable, robust computation (and better MSE)
- First theoretical results for feasible IVQR with dependent data (and nonlinear model)
- Quantile Euler equations: decouple EIS and risk attitude, robust to fat tails, no error in log-linearization, more reasonable estimates than 2SLS (in our example)
- Many open questions: quantile GMM (in progress)? averaging estimator like in Hansen (2017) or Cheng, Liao, and Shi (2016) (in progress)? semi/nonparametric? uniformity in τ? determination of τ?

Conclusion

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- Many open questions: quantile GMM (in progress)? averaging estimator like in Hansen (2017) or Cheng, Liao, and Shi (2016) (in progress)? semi/nonparametric? uniformity in τ? determination of τ?
- Thank you!
- (And further questions or comments are welcome)

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