Comparing latent inequality with ordinal health data

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Outline

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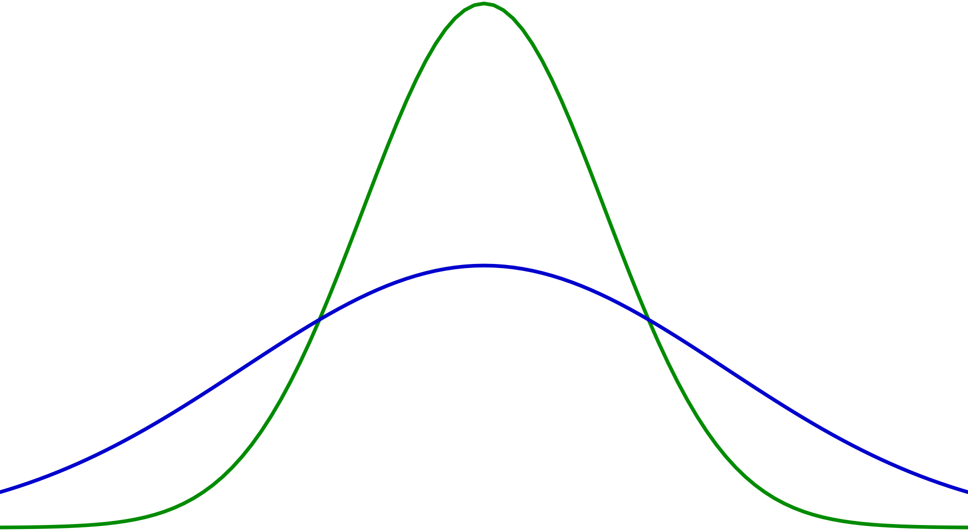
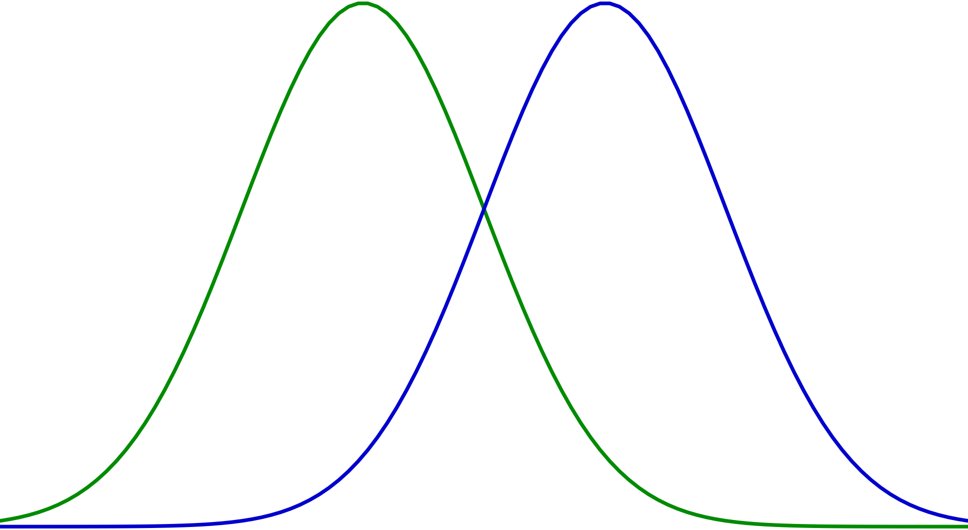
- SRHS: "self-reported health status"
- SRHS scale: excellent, very good, good, fair, poor.
- SRHS benefits:
  1. "Useful over the complete adult life cycle"
  2. Strongly correlated with objective measures
  3. Widely available (PSID, NHIS, etc.)
  4. Synthesizes all dimensions of health

Inequality #1: within-group (dispersion)



Inequality #2: between-group (better/worse)



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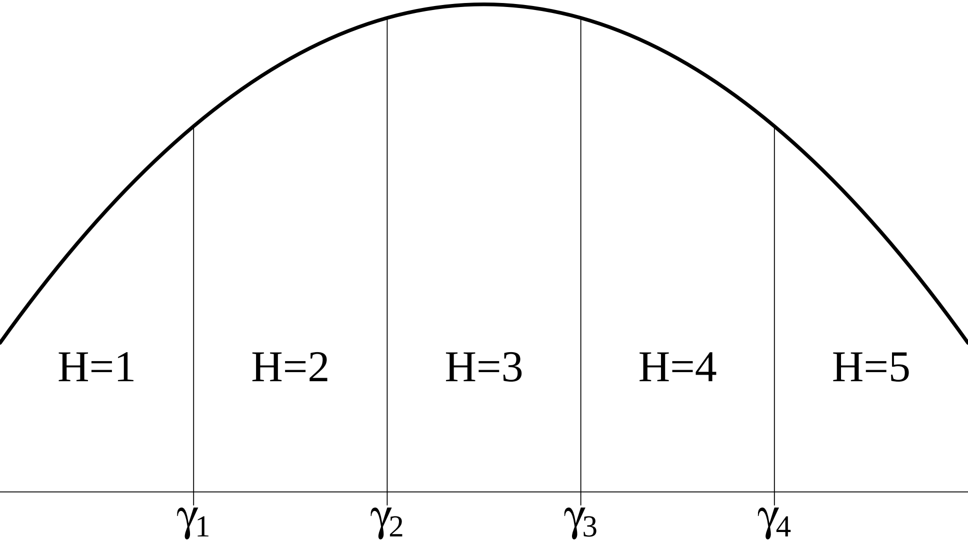
- SRHS H, fixed thresholds γ_j.

Latent model



Literature: SRHS inequality methodology

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- Allison and Foster (2004): "median-preserving spread," called "the breakthrough in analyzing inequality with [SRHS] data" by Madden (2014, p. 206).
- SRHS-based inequality indexes:
  - Good: "complete ordering" of distributions.
  - Bad: many possible indexes/weighting parameters/functions; implicit assumptions.

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- Statistical inference: frequentist and Bayesian. (Code examples.)

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(Partial) Identification

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- One view: latent CDFs are partially identified given thresholds γ_j.
- Related: Stoye (2010).

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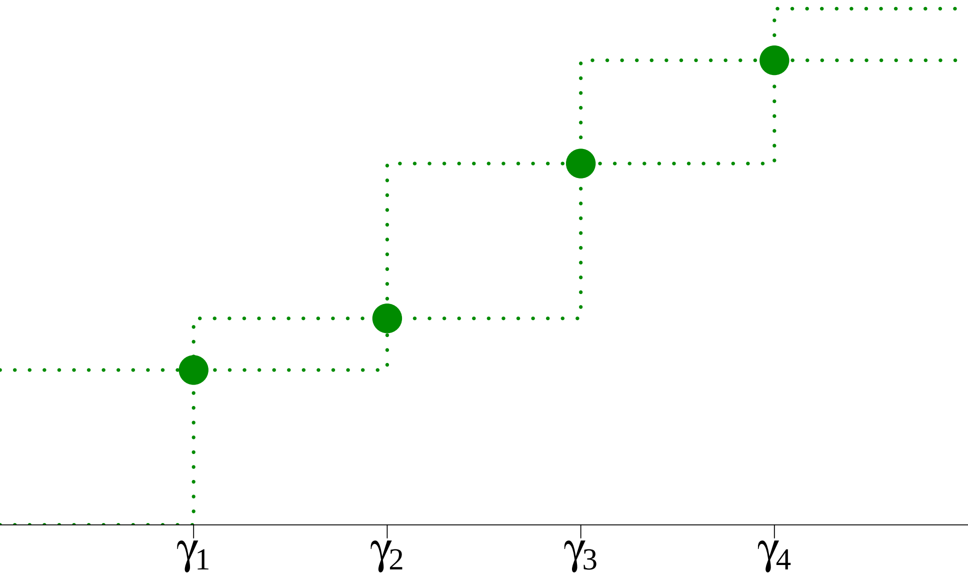

Thresholds: assumptions

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- Evidence of "yes" or "constant shift": Lindeboom and van Doorslaer (2004), Hernández-Quevedo, Jones, and Rice (2005).

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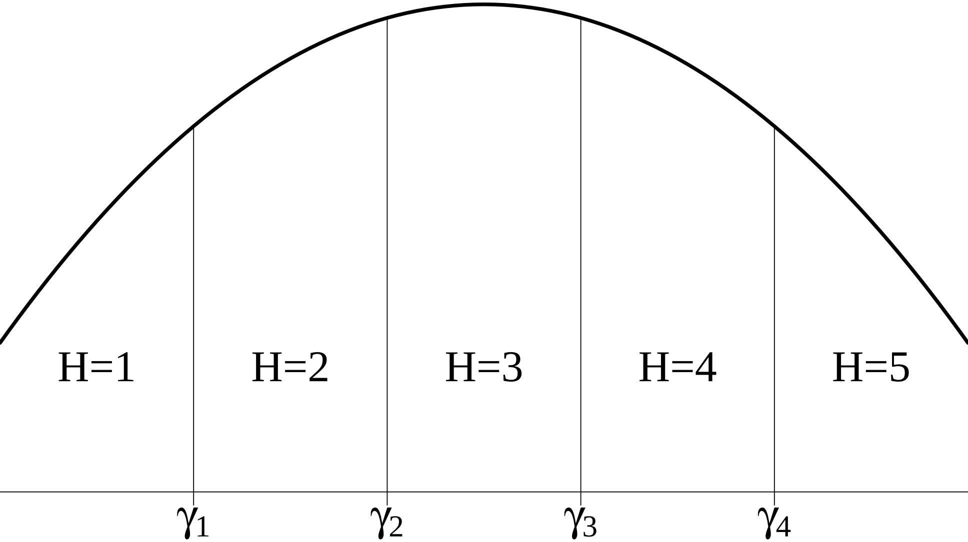
- Proposition 2(i–vi) in paper.

Latent model (again)



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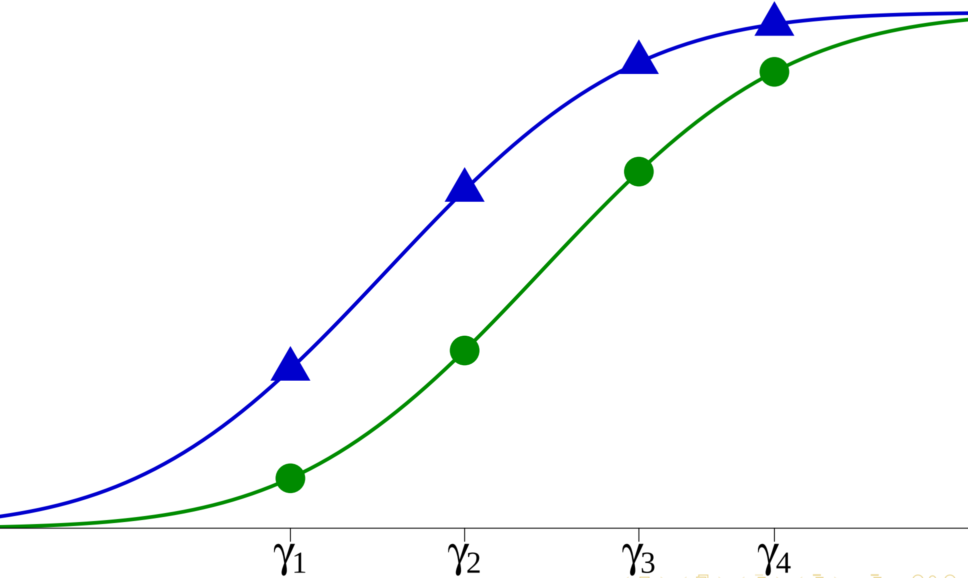
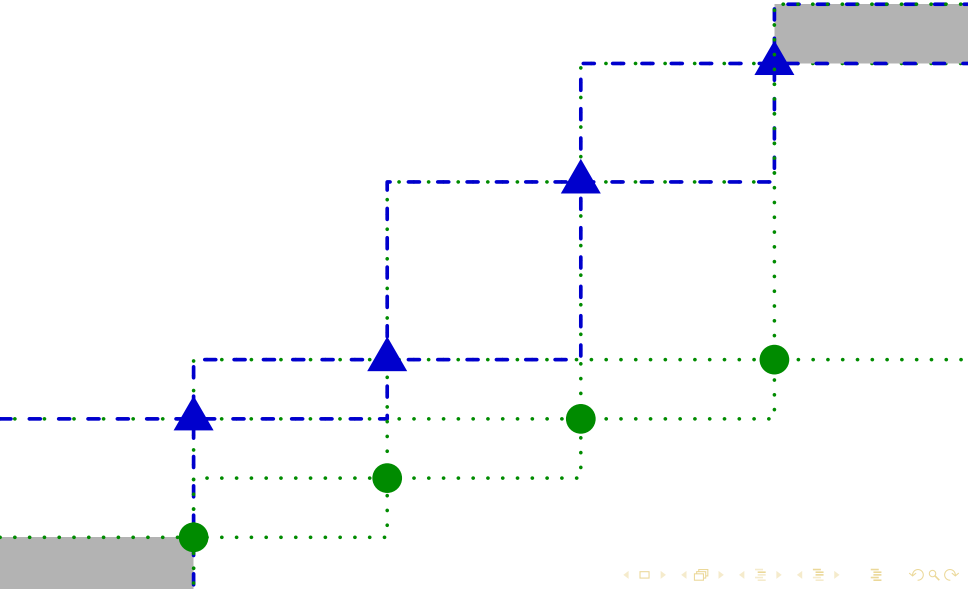
- Prop 2(v,vi): ordinal SD1 $\implies$ latent "restricted SD1" (Atkinson, 1987).

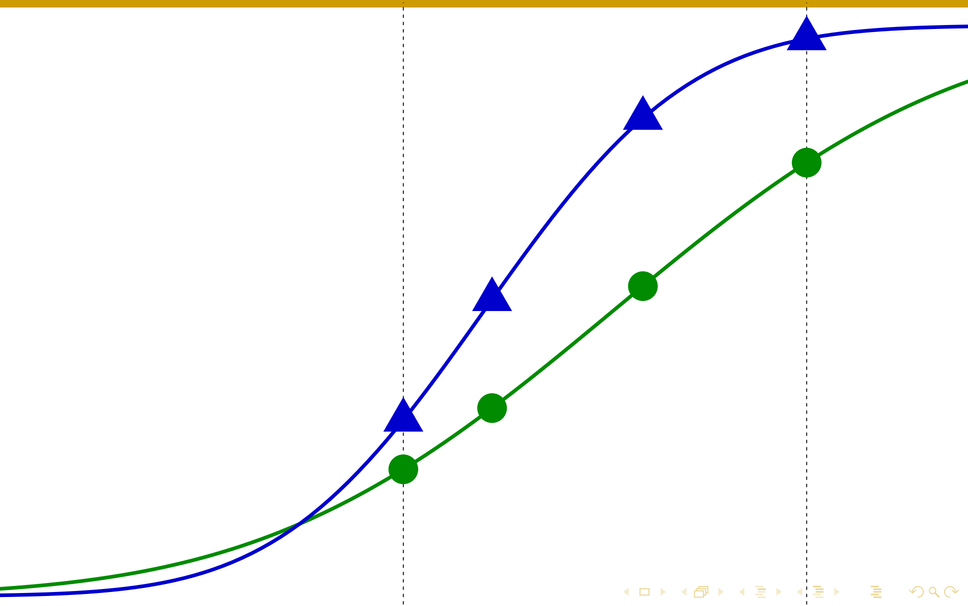
SD1: Prop 2(i)



SD1: Prop 2(v)



SD1: Prop 2(vi)



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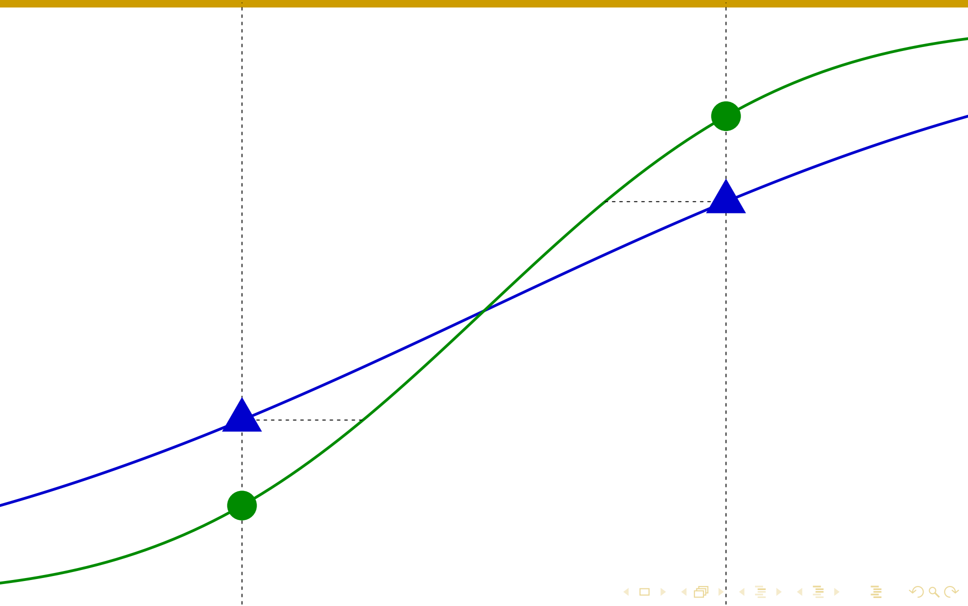
- Prop 3(iii): even stronger assumptions $\implies$ latent SD2.

Dispersion: Prop 3(i,ii)



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- Yes, with stronger assumptions: symmetric, unimodal latent distributions.
- Again, location–scale assumption allows extrapolation.

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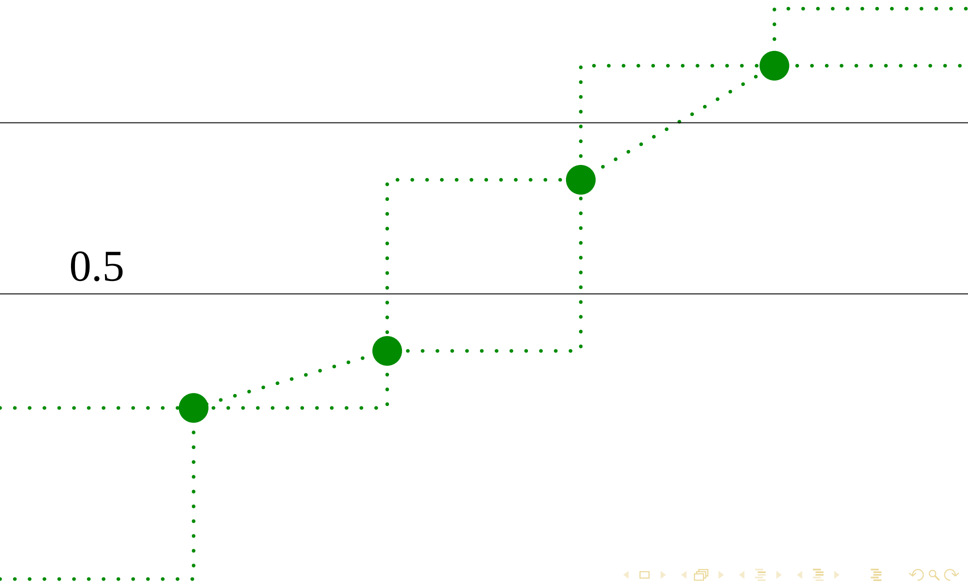

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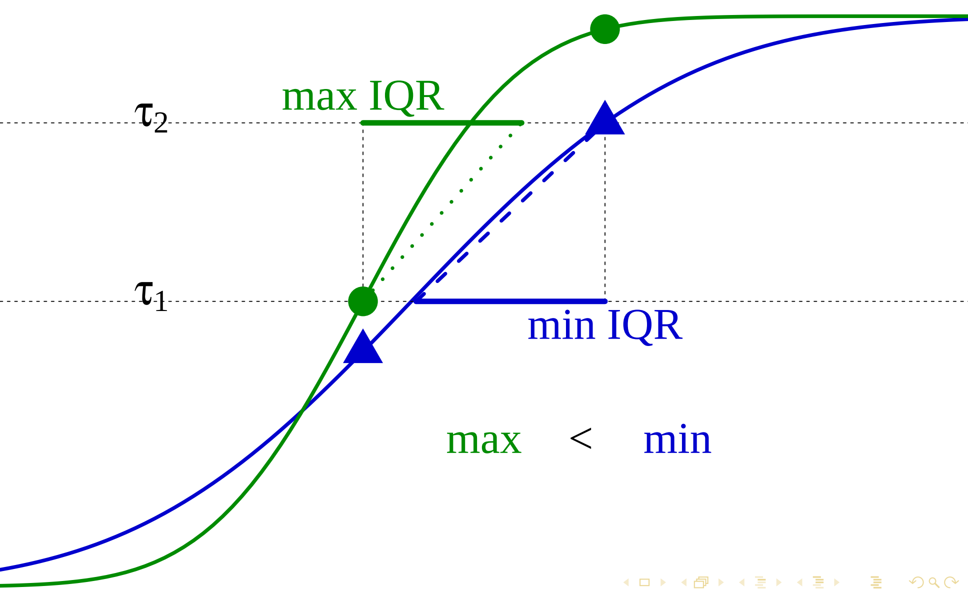

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Ordinal SD1 inference

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- Bayesian: "nonparametric" posterior for category probabilities $\Longrightarrow$ posterior probabilities for all relationships.

Other relationships

- Other relationships are unions and/or intersections of inequalities.
- Bayes: just compute posteriors.
- Frequentist: intersection–union test (sometimes).

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Treatment effects

- Jones, Molitor, and Reif (2018): large, randomized workplace wellness program.
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- Can compare treated/untreated latent health distributions using SRHS: does treatment improve health (SD1)? increase inequality? etc.
- Can examine selection effects, too.
- Stay tuned. . .

Comparisons of U.S. states

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- Data: 2011 PSID. Around 50–300 observations per state.
- Posteriors: Bayesian bootstrap of Dong, Elliott, and Raghunathan (2014) to incorporate stratification, clustering, weights.

PSID 2011 posterior probabilities (%)

		SD_1		SC		fans out	
X	Y	$\succ$	$\prec$	$\succ$	$\prec$	$\succ$	$\prec$
AZ	MO	0	90	4	2	11	34
NY	UT	0	3	0	94	1	70
IL	NY	20	0	66	1	92	3
MN	NY	24	0	57	0	96	3
IA	MO	0	10	2	16	42	98

PSID 2011 posterior probabilities (%)

	X SD$_1$ Y; Y is:					X SC Y; Y is:				
X	MO	KS	NE	IA	IL	MO	KS	NE	IA	IL
MO	—	0	10	10	0	—	7	67*	16	30
KS	34*	—	20*	10	3	6	—	24	6	16
NE	3	0	—	6	0	2	0	—	3	0
IA	0	0	6	—	0	2	0	66*	—	6
IL	40*	4	18*	43*	—	4	14	48	15	—

Asterisk (*): satisfied in-sample.

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Purpose and setup

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- DGP 1: $\mathrm{P}(X = j) = \mathrm{P}(Y = j) = 1/5$, $j = 1, \ldots, 5$; all SD1 inequalities binding.
- DGP 2: change to $\mathrm{P}(X = j) = 1/10$ for $j = 1, 2, 3$ and $\mathrm{P}(X = 4) = 1/2$; only one binding inequality.

Methods

- KS: Kolmogorov–Smirnov.
- RMS: "refined moment selection" of Andrews and Barwick (2012).
- Bayes: Dirichlet–multinomial, uninformative prior on parameters. Reject if posterior below α.
- adj: adjust prior to $\mathrm{P}(H_0) = 1/2$. (Goutis, Casella, and Wells, 1996)

Results: $\alpha = 0.1$

		$H_0: X \text{ SD}_1\ Y$				$H_0: X \text{ SC } Y$		
DGP	n	KS	RMS	Bayes	adj	RMS	Bayes	adj
1	50	0.038	0.089	0.436	0.204	0.032	0.439	0.175
1	100	0.022	0.084	0.430	0.205	0.029	0.359	0.142
1	500	0.027	0.092	0.447	0.199	0.034	0.428	0.171
1	1000	0.032	0.079	0.454	0.228	0.032	0.408	0.155
2	50	0.004	0.057	0.127	0.032	0.031	0.125	0.034
2	100	0.002	0.068	0.105	0.031	0.085	0.133	0.041
2	500	0.006	0.087	0.098	0.029	0.095	0.114	0.032
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Even with Bayes (adj), cannot treat posterior as p-value, or vice-versa

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- Thank you!
- (And further questions or comments are welcome)

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