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Menger's Theory of Money: Some Experimental Evidence

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More than a century ago, Carl Menger (1981[1870]: chap. 8; 1892) sought to explain how the social institution of money---a generally accepted medium of exchange---could develop without deliberate design in an economy of self-interested individuals. Rejecting as unhistorical earlier theories treating money as a product of some explicit agreement or edict, Menger portrayed it as a product of spontaneous evolution. Menger's theory ultimately helped to inspire a large modern literature on the spontaneous emergence of exchange media, including contributions by Jones (1976) and Kiyotaki and Wright (1989, 1993).

Despite its originality and enduring value, Menger's theory leaves many questions unanswered. In particular, Menger had little to say about the dynamic process by which one out of many potential commodity monies becomes a universally acceptable medium of exchange. Modern theories of the evolution of money likewise suffer from a relative lack of attention to evolutionary dynamics.

Here we use computer simulations to elaborate upon Menger's theory, examining both the dynamics of monetary evolution and the robustness of Mengerian convergence to a single money.

MENGER'S THEORY

Menger's account begins in a barter economy. A trader wishing to obtain some good A may have difficulty trading endowment good B directly for A because the seller of A doesn't want B. The trader may then try instead to trade indirectly, swapping B for C in order to swap C for A.

Good C may have even less use value to the trader than good B, but is assumed to have a higher degree of marketability (Absatzfaehigkeit) than B.

Under barter all goods have very limited marketability: the absence of what Jevons termed a ‘double coincidence of wants’ makes it difficult for any trader to exchange his or her endowment directly for something having a higher use value to that trader. Consequently, a significant difference exists between each good’s normal buying price and the sale price it will command if it must be disposed of quickly---the bid-ask spread. Still, Menger argues, certain goods are more marketable, and hence exhibit lower bid-ask spreads, than others. Such goods ‘can be disposed of...at any convenient time’ with relatively little loss from their normal purchasing prices (Menger 1892: 244). Any trader intending to trade goods of relatively low marketability will therefore benefit by trading those goods ‘not only for such as he happens to be in need’ but also for a more marketable good that might be utterly useless to the trader except as a medium that can be exchanged more readily for the goods the trader ultimately wishes to acquire.

At first, Menger argues, only a few traders will recognize the advantages of employing especially marketable goods as media of exchange. However, the success of a small number of innovative traders will spur imitation. As traders begin to employ a particular marketable good as an exchange medium, the demand for that good broadens, and its marketability is further enhanced. Eventually, some good that was once only somewhat more widely accepted than other goods becomes money---a universally accepted medium of exchange.

In game-theoretic terms, the adoption of a particular good as money can be understood as the solution to a ‘pure coordination’ game: in principle, traders could employ any good as money. The more agents that adopt a particular good, the greater the good’s marketability, and the more attractive it becomes in the eyes of other agents. Once a good becomes universally accepted, it normally will not be in any trader's interest to abandon that good in favor of any other less marketable indirect exchange medium.

If the monetary coordination game just described has many possible Nash equilibrium solutions---one for each exchangeable commodity---how do agents manage to coordinate around one particular equilibrium? To answer this question, Menger appeals to what Thomas Schelling (1960: 57) later referred to as coordination-problem 'focal points'. In Menger's account, certain goods are more marketable than others before the monetary selection process begins. The greater marketability of these goods makes them more prominent candidates for adoption as indirect exchange media.[1] Monetary equilibria based on these more prominent goods become focal points of the monetary coordination game, while other potential equilibria are ignored. In the simplest case, one good is initially more marketable and hence a more prominent candidate for adoption as money than others, so that traders focus on it to the neglect of all other potential candidates. In this way, the economy selects one particular Nash equilibrium 'solution' to the monetary selection game out of numerous potential alternatives.

SHORTCOMINGS OF MENGER'S ACCOUNT

Although Menger's theory provided a plausible and enduring answer to the question 'How can money emerge?', the theory leaves many questions unanswered. Several of these questions raise doubts concerning the robustness of Menger's main conclusion that barter will inevitably give way to monetary exchange without need for public interference.

First, Menger's explanation of the origins of money lacks an adequate treatment of the dynamics of the monetary selection process. Menger's argument, with its reliance upon a monetary coordination game with preexisting and directly observable 'focal points', suggests that agents initially assumed to be engaged in barter would 'jump' immediately from barter to a monetary equilibrium upon discovering that one good is more marketable than the rest. Agents should realize at once that the most marketable good is bound to become money, and so proceed immediately to accept that good in exchange. Instead of evolving gradually, as Menger would have it, money emerges in a flash.

Menger himself doesn't reach this conclusion, because he assumes that many agents hesitate at first to take advantage of goods' differing degrees of marketability. But this assumption seems patently ad hoc: if goods' varying degrees of marketability are directly observable, why should anyone refrain from immediately putting this knowledge to use? On the other hand, if agents do not directly observe goods' varying degrees of marketability, how can convergence upon a particular medium be guaranteed? Clearly, we need to relax Menger's assumption that marketability is directly observable, both to allow for more interesting evolutionary dynamics and to determine whether Menger's main conclusion holds for an economy with imperfectly informed agents.

Menger's assumption that some goods are initially more marketable than others also begs an important question. Suppose that all goods are at first equally marketable. Would such a starting point preclude convergence upon a particular monetary equilibrium?

Finally, we may ask whether the answers to the above questions depend upon the size of the economy being considered, that is, the number of distinct goods and agents. Does it become more difficult (or perhaps impossible) for an economy to select a monetary equilibrium as the number of distinct goods exchanged in the economy increases?

These questions are only a few raised by Menger's original analysis. Here, we provide preliminary answers to several of them with the help of computer simulations of the monetary selection process.

THE EXPERIMENTAL FRAMEWORK

Imagine an economy in which N agents trade J distinct goods. Each agent is endowed with a single unit of one of the J goods, and wishes to consume one unit of some other good. Agents visit a central marketplace on a daily basis, and each encounters another randomly chosen agent on each visit. An agent can limit himself to direct exchange, trading his endowment good only for the good he ultimately wishes to consume, or he can offer his good in exchange for either the desired consumption good or some other good considered to be an effective medium of exchange. We assume that an agent restricts himself to accepting in exchange only a single good (perhaps because it is the only good whose quality he can expertly judge) apart from the desired consumption good.

In Menger's analysis, some goods are initially more marketable than others, and agents know goods' degrees of marketability. Applying Schelling's 'focal point' theory, Menger's framework leads to trivial 'learning' dynamics in which the initially most saleable good would immediately become universally accepted were it not for some agents' (unexplained) hesitation to engage in indirect exchange.

Here we assume that, although all agents are prepared to take advantage of indirect exchange, they are not equipped with perfect knowledge of goods' marketability. Instead, they have very limited knowledge, which they acquire by sampling the market. Before any visit to the marketplace, a trader communicates with some randomly selected agent (not necessarily the one he will encounter in his next trip to the marketplace), and notes which good that agent is willing to receive in exchange for his endowment. If the sampled agent has not yet chosen an indirect exchange medium, that agent's desired consumption good is noted. Otherwise, the agent's desired medium of exchange is noted. Traders then visit the marketplace, willing to accept as an exchange medium the good that appears most saleable according to their market sample.

In this incomplete-information framework, different agents may initially choose different exchange media based on their particular market sampling results: the economy no longer 'jumps' at once to a particular monetary equilibrium. But does the economy <u>eventually</u> converge upon a particular monetary equilibrium, and, if so, under what circumstances? Does convergence depend on the presence of a starting 'focal point', where one good is initially acceptable to more agents than the rest? How is the likelihood or speed of convergence affected by changes in the number of agents or in the variety of goods?

SIMULATION

To answer the above questions, we programed a computer to simulate trade interactions in the described environment. The program simulates a 'Polya urn' experiment, named after famed Stanford mathematician George Polya (1887--1985). For a physical analogy to our economy with N agents and J distinct types of goods, imagine an urn filled with beads. Each bead represents a unit of demand (marketability) for a particular type of good, and each type of good is represented by a distinct colour. The beads are placed in an urn, where the initial number of beads is at least equal to the initial number of agents. Thus, if initially there are four agents and four types of goods---yellow, blue, green, and red---then, if each trader demands one unit of some good with which he is not endowed, and no two traders demand the same type of good, the urn will initially contain one bead of each colour only, the 'market' for each type of good consisting of one agent only.

At first, the demand for goods is a consumption demand only. However, each trader will eventually be willing to exchange his endowment either for his desired consumption good or for an indirect exchange medium. Thus, once each trader has chosen an initial medium of exchange, the total number of beads or 'marketability units' in the urn will be equal to N plus the initial number of beads. After each trader makes an initial selection, the original beads representing consumption demands are removed from the urn, because agents, when undertaking further market samples, inquire what medium of exchange a sampled agent is willing to accept, ignoring agents' consumption goods preferences. (We ignore the fact that some agents will not ultimately engage in indirect exchange, because the exchange medium they would select is none other than their preferred consumption good or because they are initially endowed with what turns out to be the generally accepted exchange medium.)

The medium of exchange selection process proceeds as follows: an agent planning to visit the marketplace first draws a bead randomly from the urn (samples for information). That 'colour' of good becomes the agent's exchange medium, and the bead is returned with double replacement. The extra bead represents the extension of the market for the good by one unit. Eventually every agent chooses an exchange medium, and all the beads initially in the urn, representing the consumption demands, are removed. Thereafter, the selection process is repeated, with agents taking new market samples, and revising their medium of exchange choices accordingly. As a new exchange medium is selected, and a corresponding bead is added to the urn, a bead representing the previous period's selection is removed. Although the total number of beads in the urn remains equal to the number of traders, the relative marketability of different goods may continue to change. The economy is said to have converged upon a particular monetary steady-state when all the beads in the urn are of one colour.

Our primary goal is to see how changes in initial conditions---the number of agents, the number of goods, the initial prominence of particular goods---affect the time needed for convergence to a monetary steady-state. We measure time in 'market days', defined as completed rounds of trading. For example, in a model with 20 agents, the first 20 trades (each agent's initial visit to the market) constitute the first market day, the next 20 trades (each agent's second visit to the market) the second market day, and so on. The simulation continues until the economy has converged on a single medium of exchange.[2] Because two simulations with the same initial conditions may produce very different results, we ran a simulation for each set of initial conditions 30 times, then computed the average time to convergence for those 30 simulations.

RESULTS

Our base simulation represents an economy with 10 types of goods and 10 agents, with each agent wishing to consume one unit of one type of good (so that all goods are equally prominent). In this and all other simulations we perform, agents' exchange medium choices are based on a single market sample only.

Results of simulating this base model are given in the first column of Table 1. The economy converged to a monetary steady-state in every run, although the time to convergence varied widely from run to run, ranging from 2.3 to 12.4 market days. The average time to convergence over 30 runs was 4.5 market days (or 45 total trades), with a standard deviation of 2.1 market days. Thus even in an extremely simple framework with limited information and no focal point, a simulated economy converges on a universally accepted medium of exchange in a relatively short period.

[Table 1 about here]

To learn more about the convergence pattern, let us consider in detail the run with the median time to convergence. Figure 1 shows the convergence pattern by plotting the percentages of beads representing each good for each market day until the economy converges. For ease of exposition, we number the goods ex post according to their finish in the simulation: the good that is eventually chosen as the medium of exchange is labeled Good 1; the last good to be eliminated is labeled Good 2; the next-to-last to be eliminated is labeled Good 3; and so on.

[Figure 1 about here]

Figure 1 reveals a striking feature of the convergence process: there is no monotonic increase in the percentage of beads representing the good that eventually becomes the medium of exchange. Indeed, the relative shares of the different beads change unpredictably. For example, Good 2 has 90 percent of the 'money market' after 27 trades (2.7 market days), but begins to lose market share and eventually drops out. This suggests that in an economy described by this simple sampling and belief-updating process, a good may be the economy's most widely accepted medium of exchange at one point, yet not be the good that ultimately becomes a universally accepted medium of exchange.

Changes in the Number of Agents, Number of Goods, and Scale

Next we study the effects of changing the number of agents, holding constant the number of distinct goods. The results appear in the second, third, fourth, and fifth columns of Table 1. We first return to our base model, with 10 agents and 10 goods, and then progressively double the number of agents, keeping the number of distinct goods at 10. As seen in Table 1, doubling the number of agents roughly doubles the average time to convergence. (Note that increasing the number of agents also increases the number of trades per market day, so the time to convergence measured in total trades more than doubles as we double the number of agents.) Figure 2 summarizes the average, minimum, and maximum time to convergence as a function of the number of agents.

[Figure 2 about here]

To see how the convergence pattern changes as we change the number of agents, we again plot convergence paths for the runs of each model having median convergence times. Figures 3A through 3C show convergence paths for the models with 20, 40, and 80 agents, each with 10 goods. As seen in the plots, all four models have similar convergence characteristics. In each case, a good can have a majority share of the market in the early rounds yet not end up the chosen medium of exchange.

[Figures 3A to 3C about here]

For the next set of experiments, we hold constant the number of agents and vary the number of distinct goods. Columns (6), (7), and (8) of Table 1 give the results of three sets of simulations, the first for 20 agents and 5 goods, the second for 20 agents and 10 goods, and the third for 20 agents and 20 goods. Surprisingly, variation in the number of goods has little effect on the average time to convergence. With 5 goods, the mean time to convergence is 8.8 market days. With 10 goods, the mean time to convergence increases slightly, to 10.8 market days. With 20 goods, the mean time to convergence falls to 9.3 market days. Figure 4 summarizes the average, minimum, and maximum time to convergence as a function of the number of goods.

[Figure 4 about here]

Figures 5A through 5C plot the convergence paths for the median-time-to-convergence run of each model.[3] Increasing the number of goods has no systematic effect on the convergence path, because goods with low percentages in the early rounds of trading are quickly eliminated from consideration. Thus increasing the initial number of goods has little effect on agents' ability to coordinate on a medium of exchange.

[Figures 5A through 5C about here]

Finally, we vary the overall scale of the economy by proportionately increasing the numbers of both agents and distinct goods. Columns (9) through (12) of Table 1 reveal the results of four sets of simulations, the first with 10 agents and 10 goods, the second with 20 agents and 20 goods, the third with 40 agents and 40 goods, and the fourth with 80 agents and 80 goods. Figure 6 summarizes the average, minimum, and maximum time to convergence as a function of scale, and Figures 7A through 7D plot convergence paths for the median run in each set. As seen in Table 1 and Figures 7A through 7D, a doubling in the scale of the economy leads to an approximate doubling of the average time to convergence. However, as we learned in the previous experiments, this is primarily due to the effect of increasing the number of agents, not the effect of increasing the number of goods.

[Figure 6 about here]

[Figures 7A through 7D about here]

Simulating a Focal Point

In the previous section we showed how, in a very simple model, a universally accepted medium of exchange can emerge even when all goods are initially equally marketable. As we noted previously, Menger's account of the origins of money assumes that some goods are initially more marketable, thus constituting focal points on which the economy can converge. To explore the effects of focal points, we run sets of simulations in which one good is initially more prominent—-that is, has more beads in the urn—-than the other goods. Results of these simulations are presented in Table 2. The first column of Table 2 gives results from 30 simulations of our base model with 10 agents and 10 equally marketable goods, each represented by a single bead in the urn. The average time to convergence is 4.5 market days. Column (2) reports the result of 30 simulations of a model with 10 agents and 9 goods, one of which is represented by two beads in the urn. We thus begin, as before, with the same number of beads as agents; this time, however, one good is twice as likely to be selected by the first trader as the remaining eight goods.

[Table 2 about here]

Surprisingly, making one good initially more marketable than the others has only a fairly small effect on the average time to convergence. For the simulations reported in column (2) of Table 2, in which the focal good is initially twice as saleable as the other goods, the mean time to convergence is 4.6 market days, essentially the same as that of our base simulation. (The standard deviation is larger, 3.4 to 2.1; one simulation converged in only 1.5 days.) However, the good that is initially twice as marketable is about twice as likely to end up as the generally accepted medium of exchange. The last row of Table 2 shows the percentage of simulations in which the good that is initially more marketable becomes the eventual medium of exchange. In our base simulation with 10 equally marketable goods, each good should be chosen, on average, 3 out of 30 times. For the 30 simulations reported in column (2), the good that is initially more marketable is chosen 6 times. Thus although it might be assumed that a small initial advantage would bring a substantial ex-post advantage, in our model, increasing the probability that a particular good will be chosen in the early rounds gives only a roughly proportionate increase in the probability that that good will be chosen as the medium of exchange.

The third, fourth, and fifth columns of Table 2 report sets of simulations in which we progressively increase the initial probability that a particular good will be chosen: first we give one good three times as many initial beads as the other goods, then five times as many beads, then seven times as many beads. As seen in column (3), giving one good three times as much initial marketability has about the same effect as giving it twice the initial marketability. The mean time to convergence falls slightly, to 4.2 market days, and the initially more marketable good is chosen as the medium of exchange seven times. Increasing the focal good's initial advantage to five times the marketability of the other goods causes a slight increase in the mean time to convergence (4.6 market days), but it again leads to a proportionate increase in the effectiveness of the focal point. The initially more marketable good is chosen as the medium of exchange in 14 of the 30 simulations, roughly five times the probability of any good's being chosen in the simulations without a focal good.

Figures 8 and 9 summarize the effects of changes in the strength of the focal point. The horizontal axis measures this strength in terms of the initial share of the focal good. A value of 1 corresponds to the base simulation in which all goods are equally prominent. A value of 2 represents the simulation in which one good has twice the initial share as the other goods, and so on through 7, representing a simulation in which one good has seven times the initial share of the other goods. In Figure 8, the vertical axis measures the average, minimum, and maximum time to convergence. In Figure 9, the vertical axis measures the percentage of simulations (the number out of 30) in which the initially more saleable good is chosen as the medium of exchange.

[Figures 8 and 9 about here]

As the strength of the focal point increases, the mean time to convergence tends to fall. When we increase the initial marketability of the focal good to seven times that of the other goods, the mean time to convergence falls to 3.4 market days, with a standard deviation of 3.2. As the fifth column of Table 2 indicates, the fastest time to convergence is 1.1 market days, significantly less than the fastest time without a focal good. Moreover, the initially more marketable good is chosen as the medium of exchange in 19 of the 30 simulations, roughly seven times the probability of any good's being chosen in the simulations with ten equally marketable goods.

CONCLUSION

In his treatment of the spontaneous origins of money, Carl Menger argued that different goods under barter have different degrees of marketability. The most marketable goods become adopted as indirect exchange media by a subset of traders, thus further enhancing those goods' relative marketability until all traders are willing to trade for them. Money---a universally accepted exchange medium---is thus an outcome of human action, but not of human design.

We have shown that money can emerge spontaneously even where traders have only a very dim perception (based on very limited random sampling) of the marketability of distinct goods, and even where all goods are equally marketable at the onset of the evolutionary process. The assumption of limited information makes for a much more interesting evolutionary process than in Menger's own account, where the only factor preventing an economy from 'jumping' all at once to a particular monetary equilibrium is the assumed hesitation of certain traders to become involved in the monetary selection process. In our framework, agents rely on a single market sample to determine which good is most marketable on any market day. Opinions therefore differ at first, but are drawn together as trade continues. The time required for convergence to a monetary steady state is independent of the number of goods in the economy, but is more or less proportional to the number of agents in the economy.

Although no good has to be particularly marketable or 'prominent' at the onset of a monetary selection process for the process to lead to a monetary steady state, a good's initial prominence has, as one would expect, a direct bearing on its likelihood of becoming money: a good that is initially twice as prominent as all other goods is about twice as likely to become money. Still, under limited information, a good may be quite prominent at first and yet may not be chosen as money. Finally (and somewhat counterintuitively) the initial presence of especially prominent goods does not necessarily result in a more rapid convergence to a monetary equilibrium.

In our endeavour to show how money may evolve even under circumstances where agents have very limited knowledge of goods' relative marketability, we have probably exaggerated the extent of agents' ignorance. A more realistic set of simulations might allow agents to rely upon larger market 'samples' to form opinions concerning goods' relative saleability. Also, while we have assumed an extreme version of adaptive expectations in which agents rely on current sample evidence only, ignoring findings from past samples, a more realistic model might allow agents to assign some positive (but diminishing) weight to earlier sample results. These are just two of many possible changes that might contribute to a more realistic depiction of the monetary selection process. We hope to consider the effects of such changes in later work.

NOTES

1. A good's initial nonmonetary marketability, interpreted narrowly as depending on the number of persons wishing to possess the good for its value-in-use, is only one of several factors that may contribute to its degree of prominence in a monetary coordination game. A good's physical properties---its durability, portability, divisibility, etc.---may also render it more prominent than other candidate goods.

2. To economize on computer resources, we place an upper bound on the total number of trades; the simulation ends either when the economy converges, or when the upper bound has been reached. We used an upper bound of 5,000 trades, which was binding in only five of about 400 total simulations.

3. To clarify the pictures, we plot only the ten goods that attain the highest acceptance rates during the process.

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TABLE 1: SIMULATION RESULTS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	BASE MODEL	CHANGES IN NUMBER OF AGENTS				CHANGES IN NUMBER OF GOODS			CHANGES IN SCALE			
Initial parameters:												
Agents	10	10	20	40	80	20	20	20	10	20	40	80
Goods	10	10	10	10	10	5	10	20	10	20	40	80
Time to convergence (market days):												
Mean	4.5	4.5	10.9	19.2	38.5	8.8	10.8	9.3	4.5	9.3	17.3	34.7
Median	3.9	3.9	9.2	13.2	31.2	7.2	9.2	8.4	3.9	8.4	14.7	32.1
Standard deviation	2.1	2.1	6.7	11.5	15.5	5.8	6.7	4.0	2.1	4.0	7.8	14.5
Minimum	2.3	2.3	3.0	4.9	11.9	1.0	3.0	2.9	2.3	2.9	7.4	13.4
Maximum	12.4	12.4	34.2	49.7	62.5	24.2	34.2	18.5	12.4	18.5	41.5	62.5
Change in mean from first model in set		—	2.422	4.267	8.556	—	1.227	1.057	—	2.059	3.845	7.711
Change in median from first model in set		—	2.359	3.385	8.000	—	1.278	1.167	—	2.141	3.769	8.231

TABLE 2: FOCAL POINT SIMULATION RESULTS

	(1) ALL GOODS EQUALLY MARKETABLE	(2) ONE GOOD TWICE AS MARKETABLE	(3) ONE GOOD THREE TIMES AS MARKETABLE	(4) ONE GOOD FIVE TIMES AS MARKETABLE	(5) ONE GOOD SEVEN TIMES AS MARKETABLE
Initial parameters					
Agents	10	10	10	10	10
Goods	10	9	8	6	4
Initial beads	10	10	10	10	10
Time to convergence (market days):					
Mean	4.5	4.6	4.2	4.6	3.4
Median	3.9	4.1	3.9	3.3	3.2
Standard deviation	2.9	3.4	1.7	3.7	1.7
Minimum	2.3	1.5	2.0	1.1	1.1
Maximum	12.4	20.4	10.1	16.4	6.6
Number of runs (out of 30) in which focal good is chosen as medium of exchange		6	7	14	19

FIGURE 1: CONVERGENCE PATH FOR BASE MODEL WITH 10 AGENTS, 10 GOODS

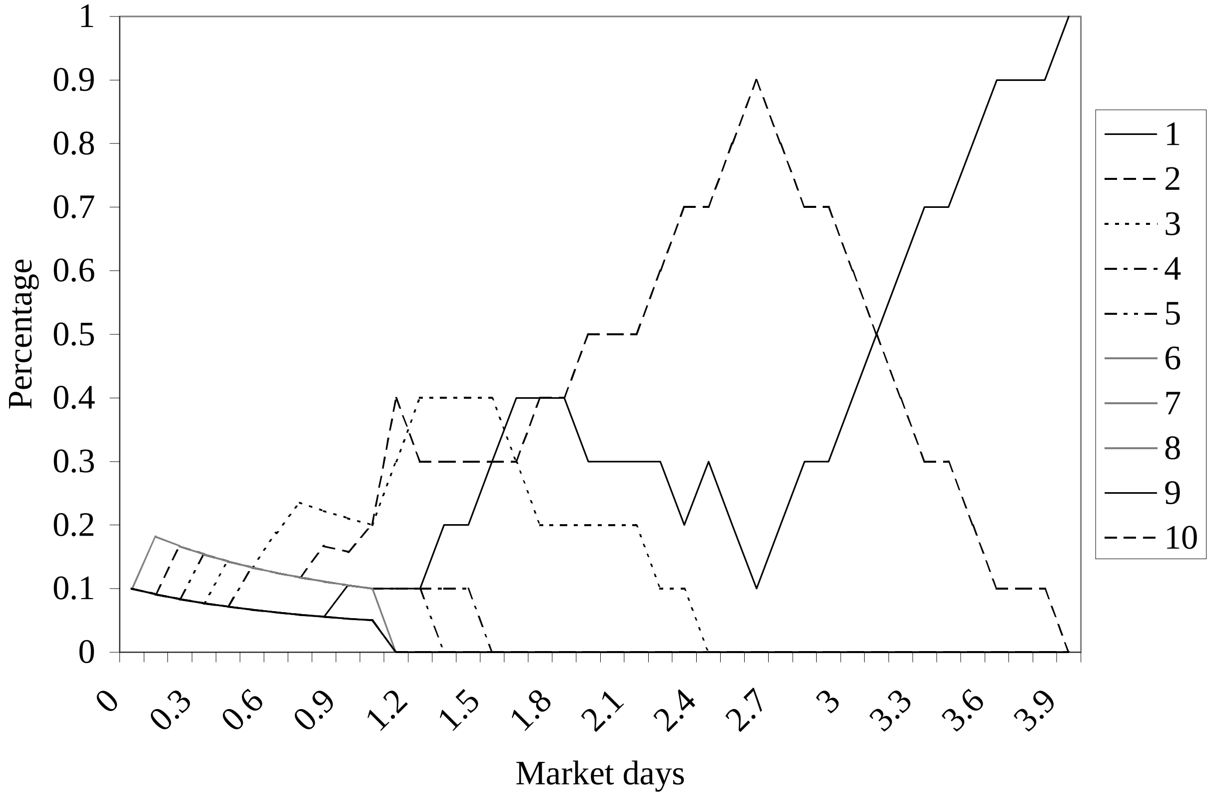
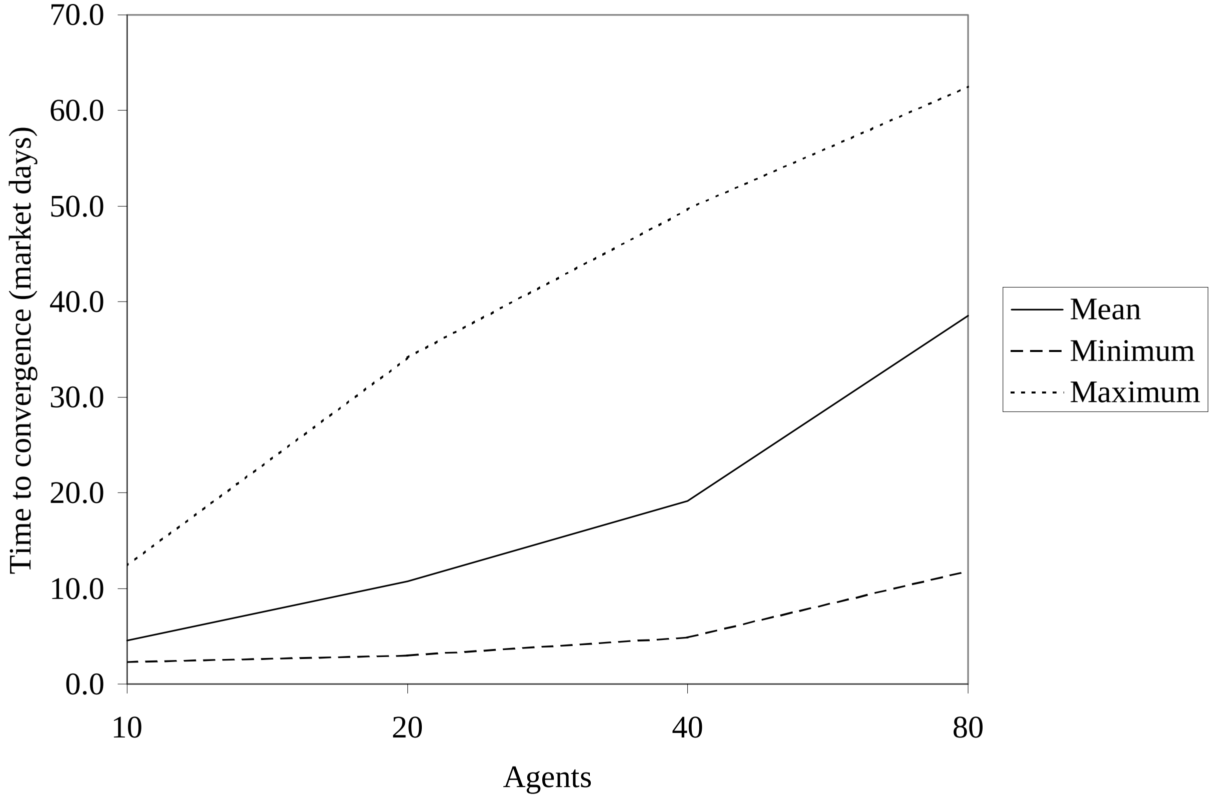
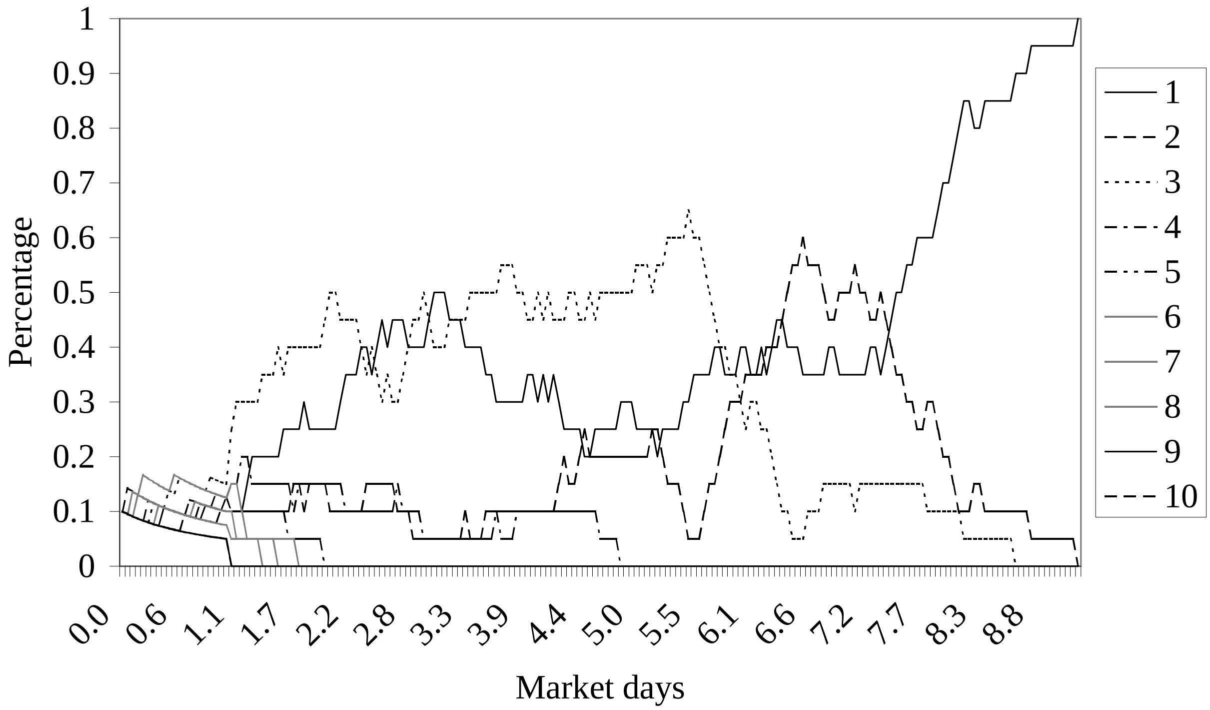


FIGURE 2: EFFECTS OF CHANGES IN NUMBER OF AGENTS

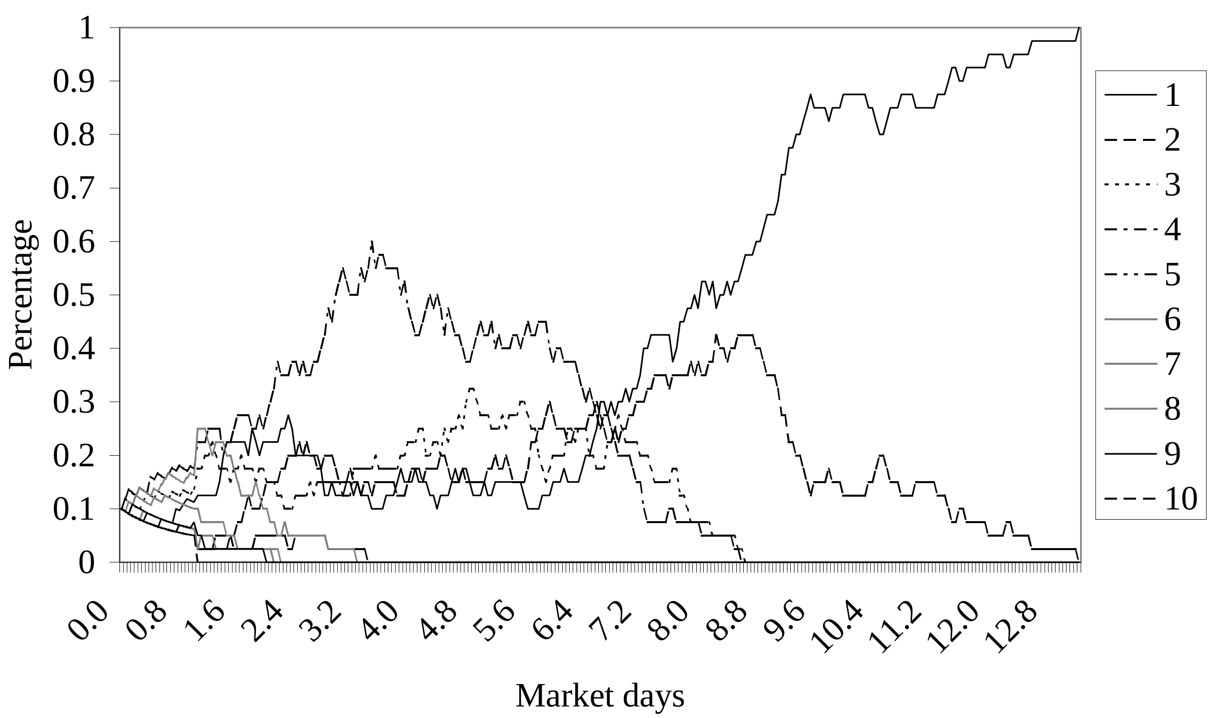


FIGURES 3A–3C: CONVERGENCE PATHS WITH CHANGES IN NUMBER OF AGENTS

A. 20 Agents, 10 Goods



B. 40 Agents, 10 Goods



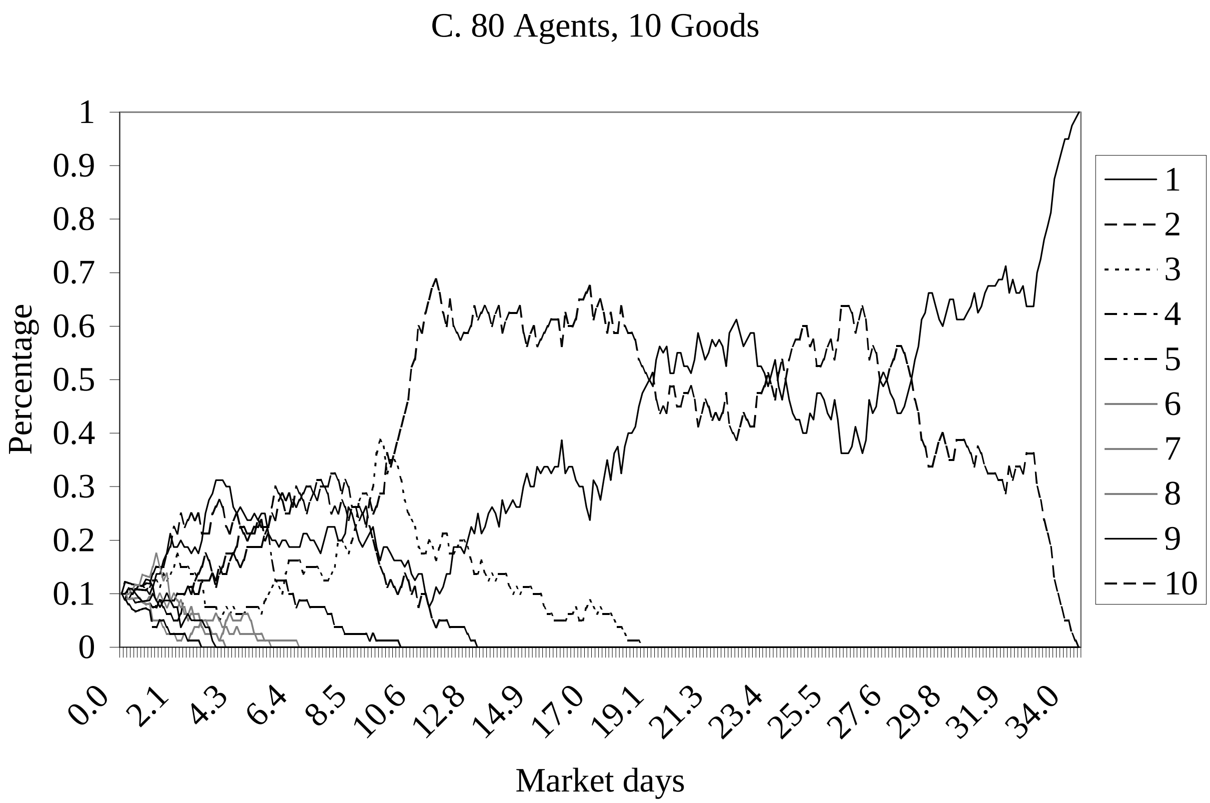
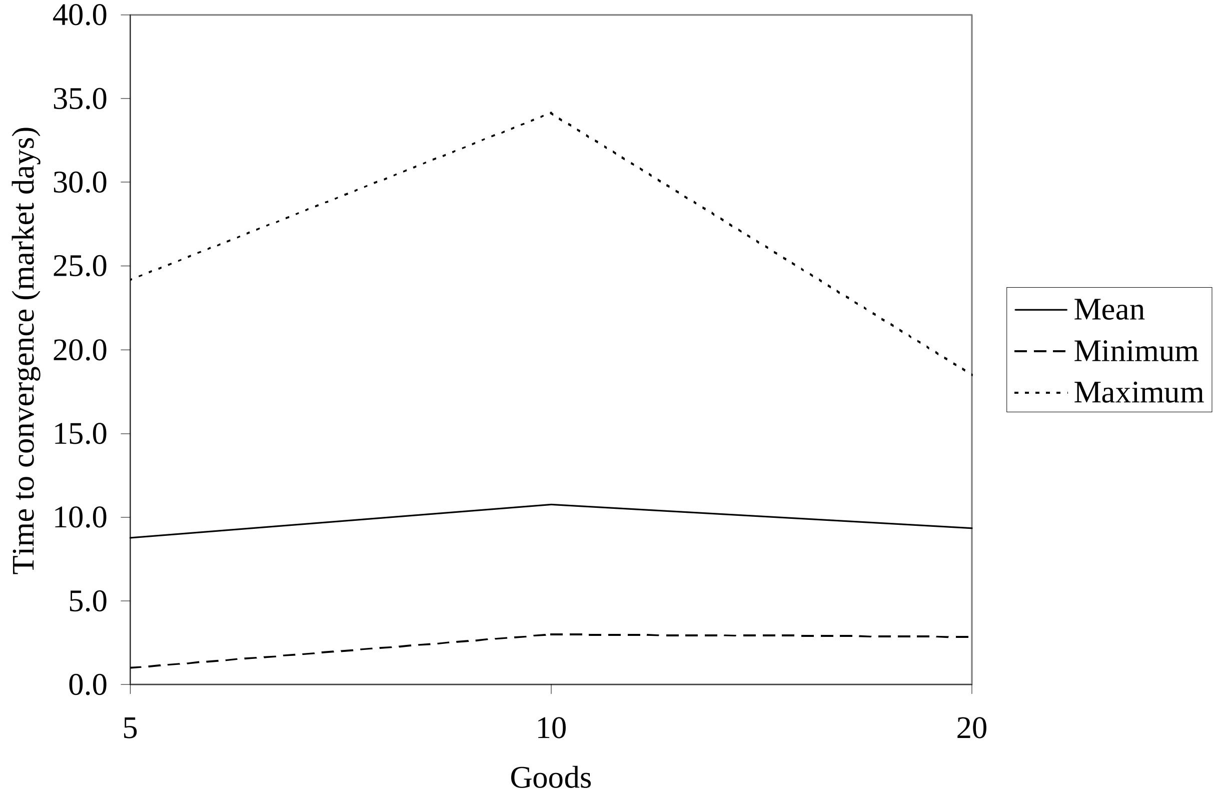
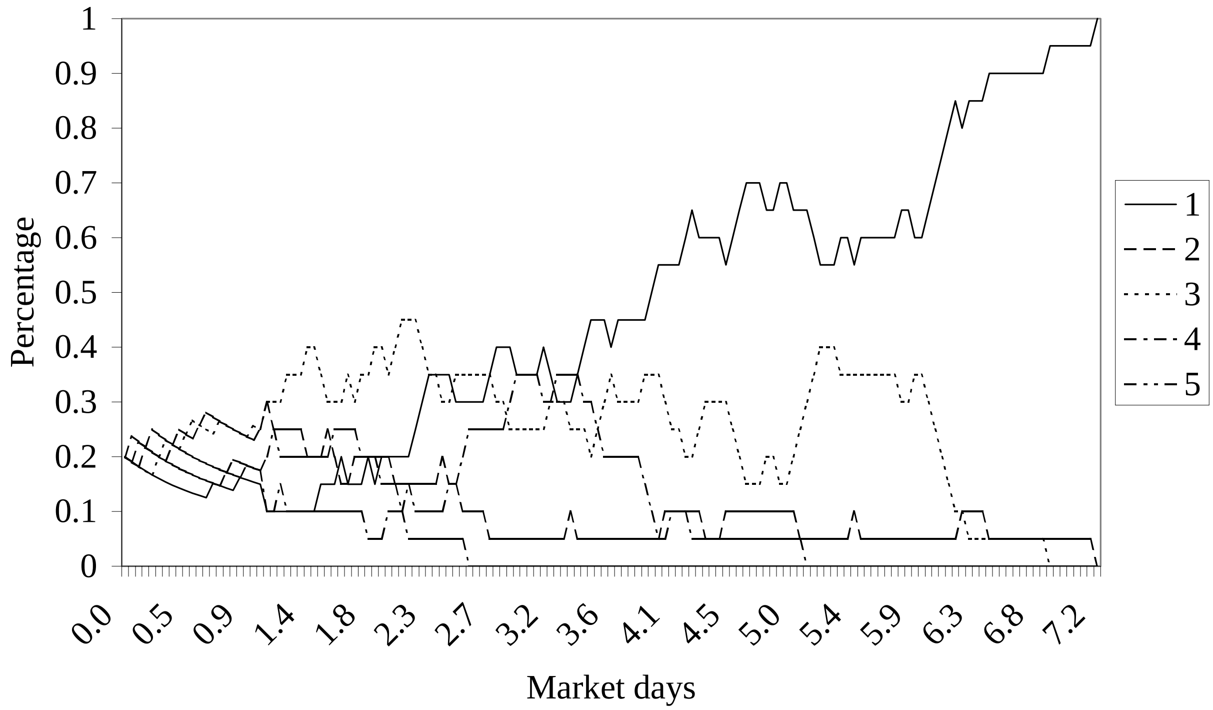

C. 80 Agents, 10 Goods
Percentage
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0
2.1
4.3
6.4
8.5
10.6
12.8
14.9
17.0
19.1
21.3
23.4
25.5
27.6
29.8
31.9
34.0
Market days
1
2
3
4
5
6
7
8
9
10

FIGURE 4: EFFECTS OF CHANGES IN NUMBER OF GOODS

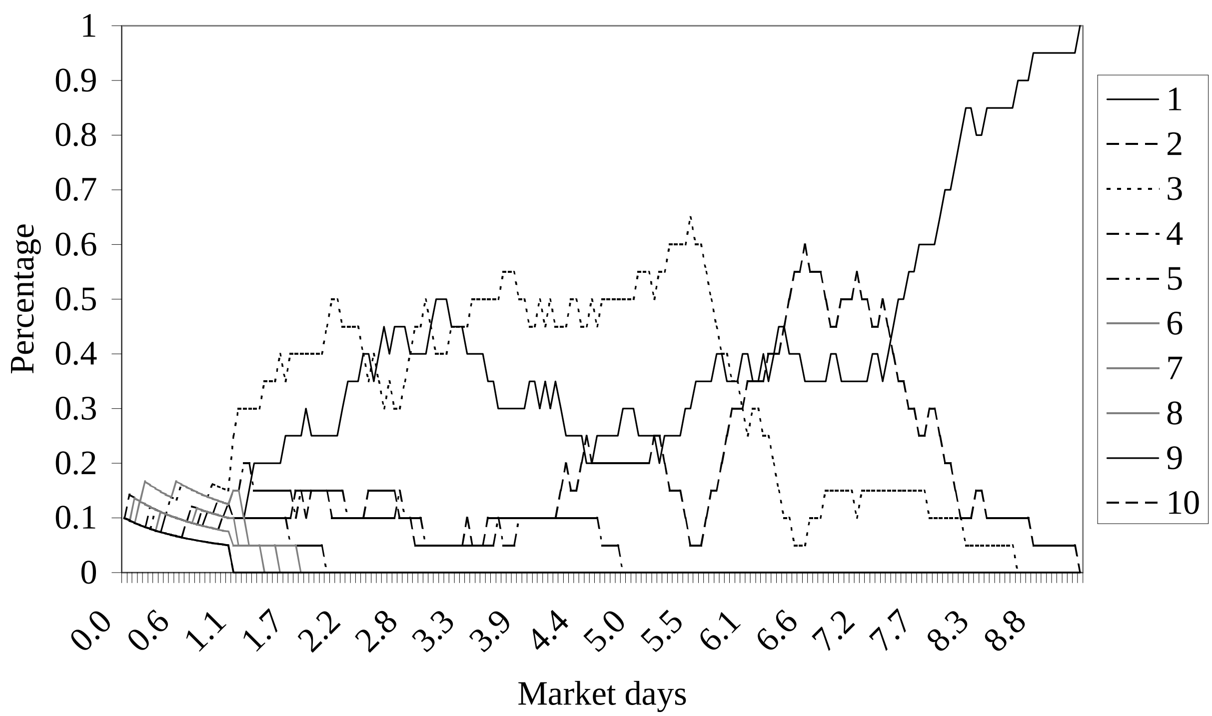


FIGURES 5A–5D: CONVERGENCE PATHS WITH CHANGES IN NUMBER OF GOODS

A. 20 Agents, 5 Goods



B. 20 Agents, 10 Goods



C. 20 Agents, 20 Goods

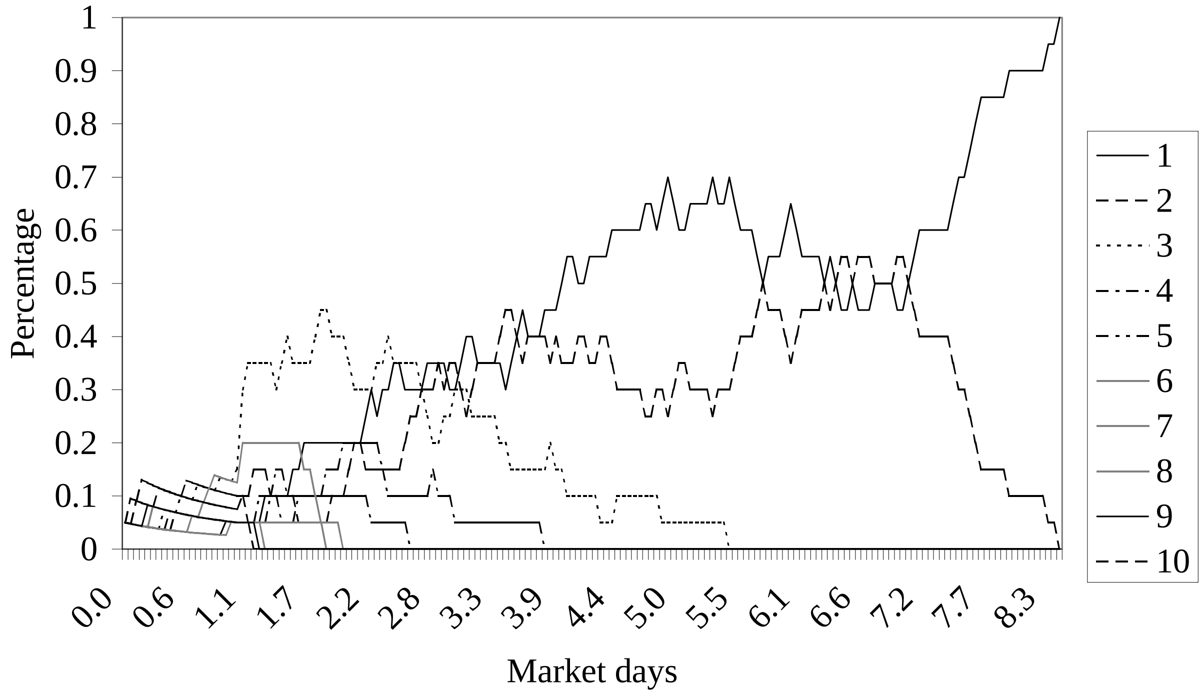
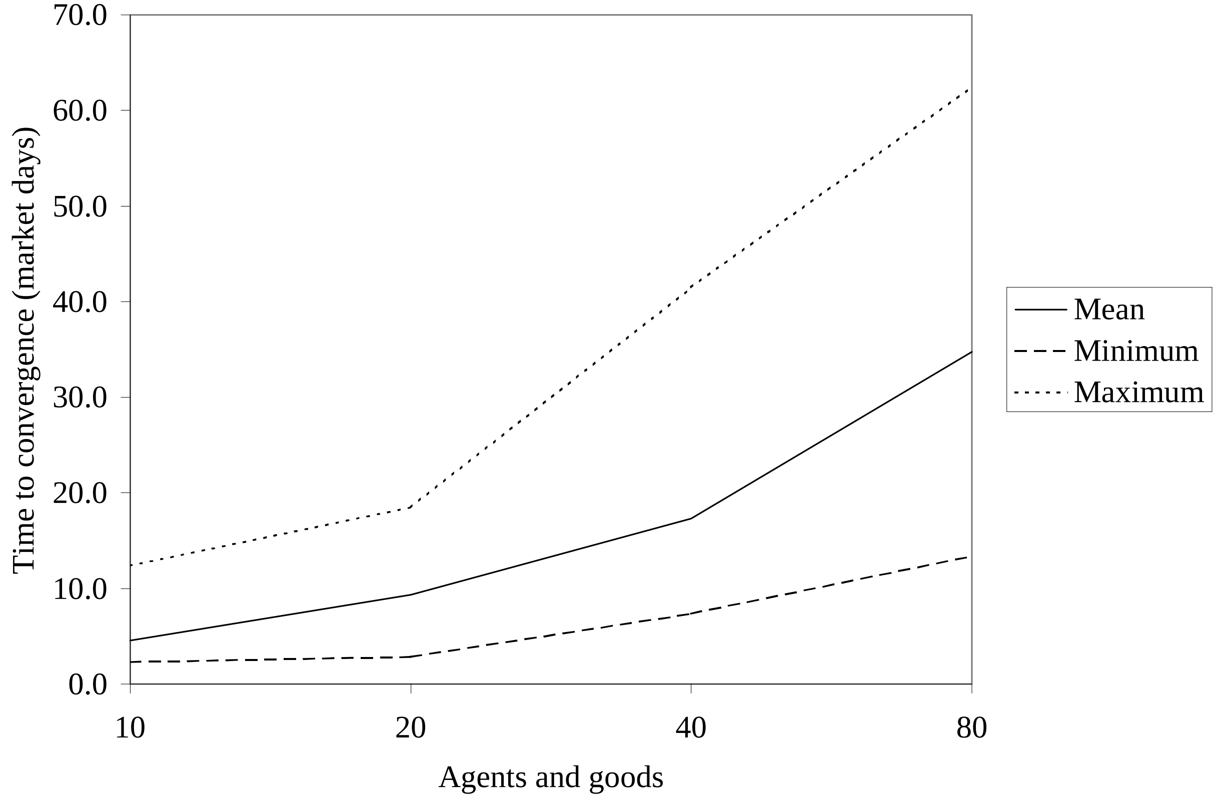
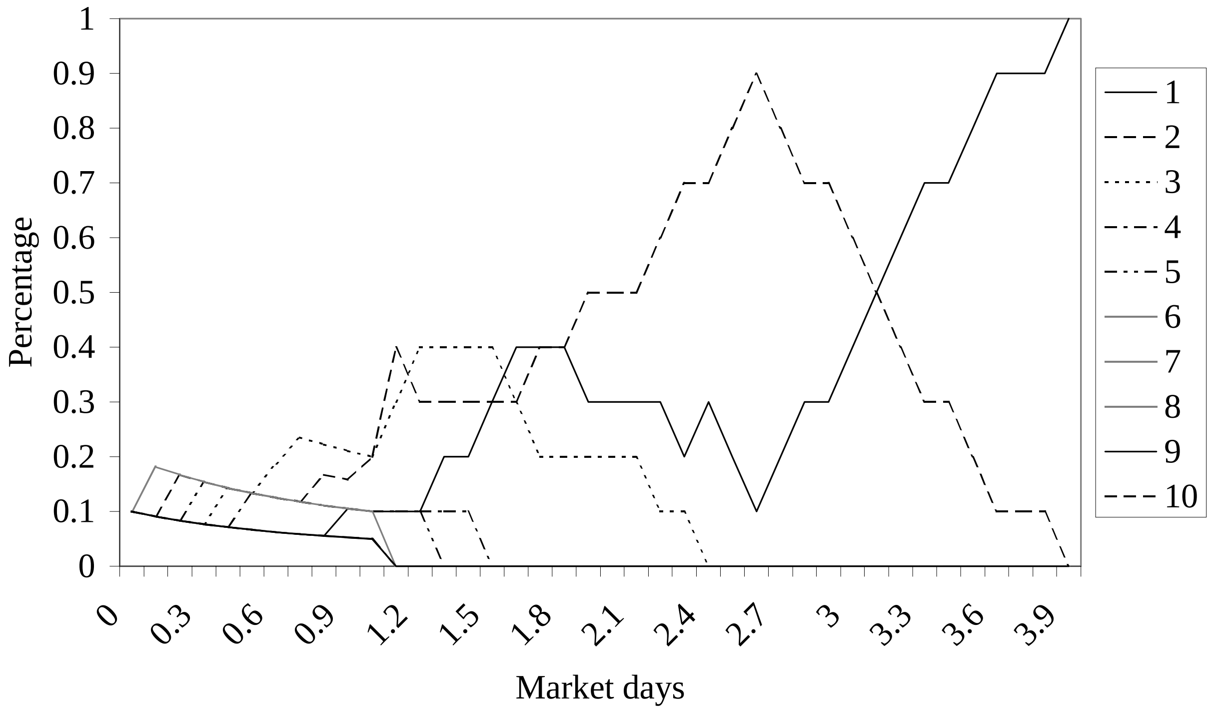


FIGURE 6: EFFECTS OF CHANGES IN SCALE

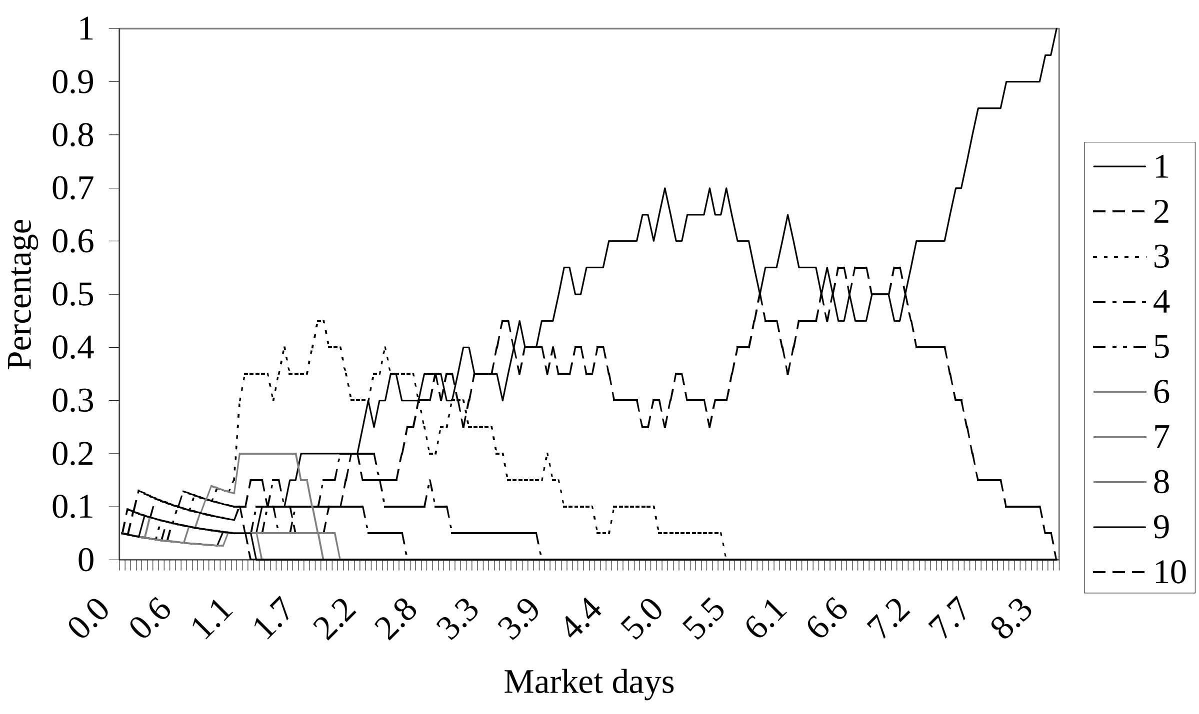


FIGURES 7A–7D: CONVERGENCE PATHS WITH CHANGES IN SCALE

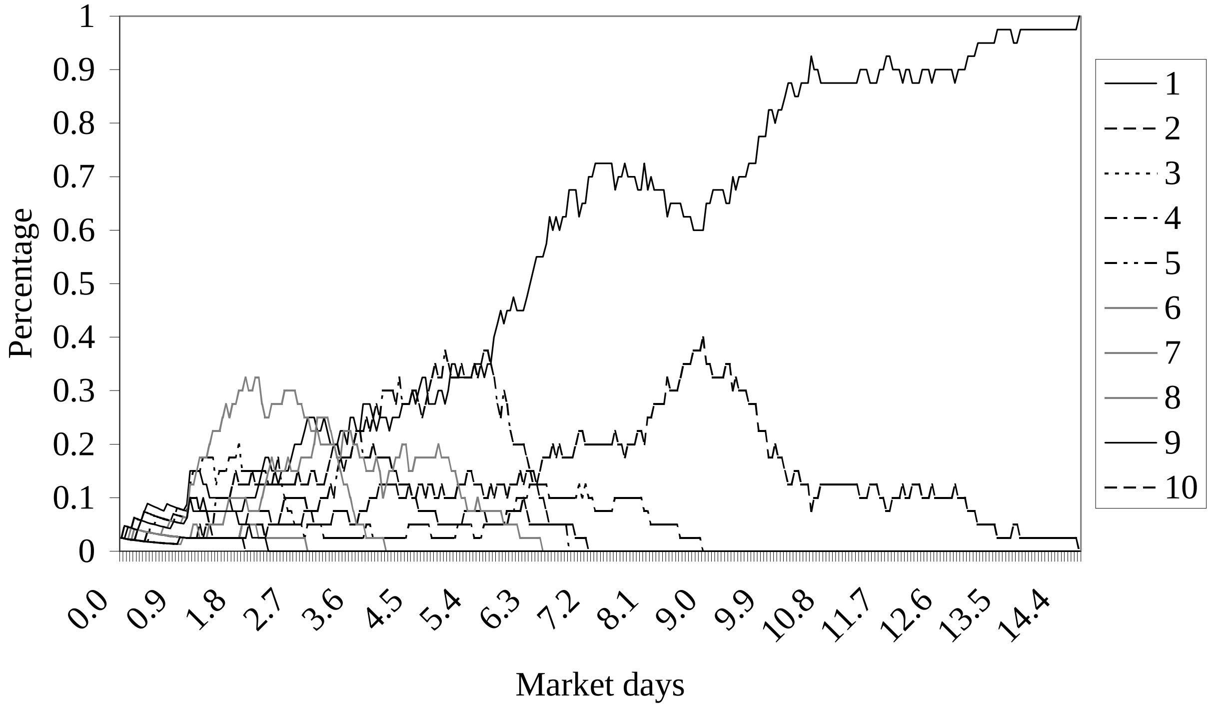
A. 10 Agents, 10 Goods



B. 20 Agents, 20 Goods



C. 40 Agents, 40 Goods



D. 80 Agents, 80 Goods

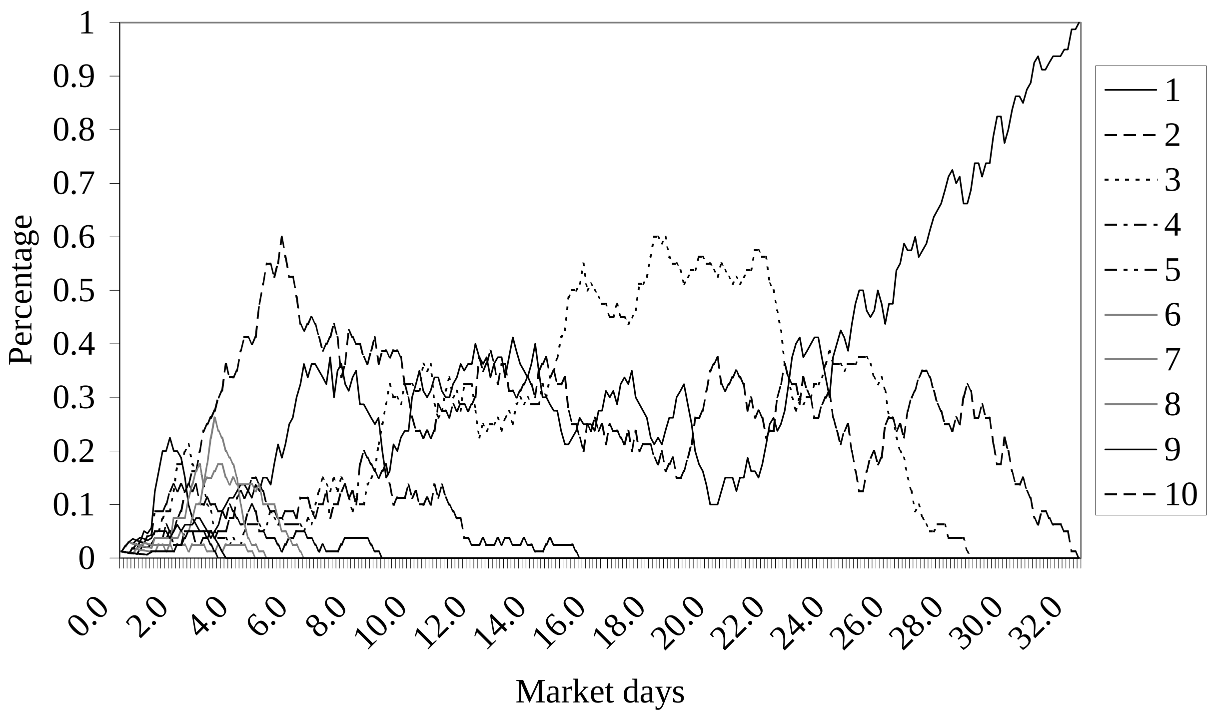


FIGURE 8: EFFECTS OF FOCAL POINT ON TIME TO CONVERGENCE

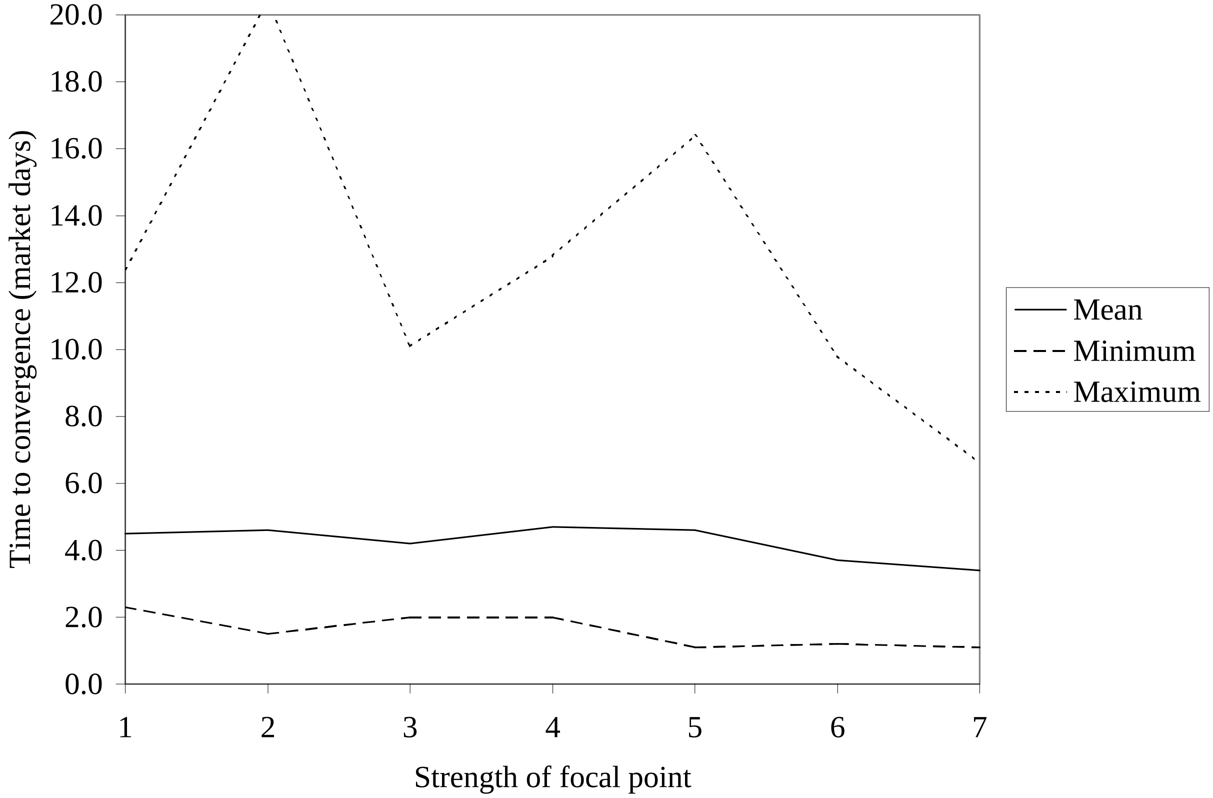


FIGURE 9: EFFECTIVENESS OF FOCAL POINT