1. A firm has the following production function \( Q = (K^{1/3})(E^{2/3}) \). It will produce 80 units of output and faces prices for labor and capital as follows: \( w=10 \), \( r=15 \). Find the cost-minimizing bundle of labor and capital, \((E^*, K^*)\).

**DETAILED ANSWER:**

Cost-minimizing condition:

\[
\frac{MP_E}{MP_K} = \frac{w}{r}
\]

Take derivatives of production function to get \( MP_E \) and \( MP_K \):

\[
MP_E = (2/3)(K^{-1/3})(E^{-1/3})
\]

\[
MP_K = (1/3)(K^{-2/3})(E^{2/3})
\]

\[
\frac{MP_E}{MP_K} = \frac{2K}{E}
\]

\[
\frac{2K}{E} = \frac{10}{15} \quad (w=10 \text{ and } r=15 \text{ per above})
\]

\[
10E = 30K
\]

\[
E^* = 3K^*
\]

Now plug in \( 3K \) for \( E \) into production function, and set \( Q = 80 \) as indicated in the problem:

\[
80 = (K^{1/3})(3K^{2/3})
\]

Pull out the term “\(3^{2/3}\)” and add the exponents on the \( K \)-terms:

\[
80 = (3^{2/3})(K)
\]

\[
K^* = 80/(3^{2/3}) = 38.46
\]

\[
E^* = 3K^* = 115.38
\]

Informal check to make sure there are no silly errors:

\[
80 = ((38.46)^{1/3}((115.38)^{2/3}) \quad \text{CHECK}
\]
2. A firm has the following production function $Q = 2(K^{3/4})(E^{1/2})$. It will produce 200 units of output and faces prices for labor and capital as follows: $w=6$, $r=2$. Find the cost-minimizing bundle of labor and capital, $(E^*, K^*)$.

**DETAILED ANSWER:**

Cost-minimizing condition:

$$\frac{MP_E}{MP_K} = \frac{w}{r}$$

Take derivatives of production function to get $MP_E$ and $MP_K$:

$$MP_E = \frac{1}{2}(2)(K^{3/4})(E^{1/2})$$

$$MP_K = \frac{3}{4}(2)(K^{-1/4})(E^{1/2})$$

$$\frac{MP_E}{MP_K} = \frac{2K}{3E}$$

$$\frac{2K}{3E} = \frac{6}{2}$$ \text{(w =6 and r=2 per above)}

$$18E = 4K$$

$$K^* = 4.5E^*$$

Now plug in 4.5E for K into production function, and set Q = 200 as indicated in the problem:

$$200 = 2((4.5E)^{3/4})(E^{1/2})$$

Pull out the term “$4.5^{3/4}$”, multiply it by 2, and add the exponents on the E-terms:

$$200 = (2*4.5^{3/4})(E^{5/4})$$

$$E^{5/4} = 200/(2*4.5^{3/4}) = 200/6.18 = 32.36$$

Raise both sides to the power of 4/5 to get rid of the exponent on E:

$$E^* = (32.36^{4/5}) = 16.14$$

$$K^* = 4.5E^* = 72.65$$

Informal check to make sure there are no silly errors:

$$200 = 2(72.65^{3/4})(16.14^{1/2})$$ \text{CHECK}
Here are a few other problems that you can work on without answers.

3. A firm has the following production function $Q = (K^{1/4})(E^{3/4})$. It will produce 100 units of output and faces prices for labor and capital as follows: $w=4$, $r =2$. Find the cost-minimizing bundle of labor and capital, $(E^*,K^*)$.

4. A firm has the following production function $Q = (K^{1/2})(E^{1/3})$. It will produce 200 units of output and faces prices for labor and capital as follows: $w=20$, $r =4$. Find the cost-minimizing bundle of labor and capital, $(E^*,K^*)$. 