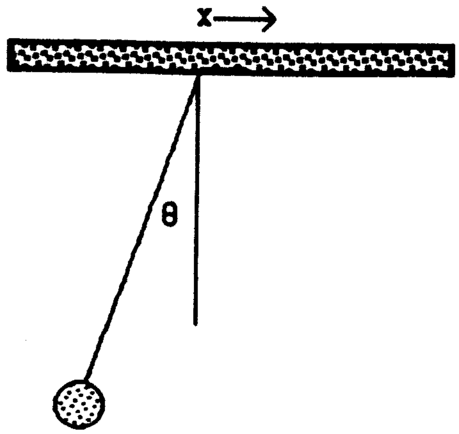
UNIVERSITY OF MISSOURI-COLUMBIA
PHYSICS DEPARTMENT
Qualifying Exam
January 2, 1989

Instructions: The only material you are allowed in the examination room is a writing instrument and a calculator. You may not store any formulae in your calculator. Paper, mathematical handbooks and question sheets are furnished. Each student is assigned a lower-case, Greek letter; this letter will identify your work on both parts (I and II) of this exam. In writing out your answers, use only one side of a page, use as many pages as necessary for each problem, and do not combine work for two different problems on the same page. Each page should be identified in the upper, right hand corner according to the following scheme: β4.3 i.e., student β, problem 4, page 3. Refer all questions to the exam proctor. In answering the examination questions, the following suggestions should be heeded:

1. Answer the exact question that is asked, not a similar question.
2. Use simple tests of correctness (such as a reasonable value, correct limiting values and dimensional analysis) in carrying out any derivation or calcuation.
3. If there is any possibility of the grader being confused as to what your mathematical symbols mean, define them.

You may leave when finished.

1. A simple pendulum is formed from a mass M and a weightless string of length ℓ, and is swinging in a plane. The gravitational field has the unconventional form
$g = g_0(1 + \frac{\delta g}{g_0} sin(\omega t + \phi)$
(This problem i , analogous to a playground swing being "pumped")



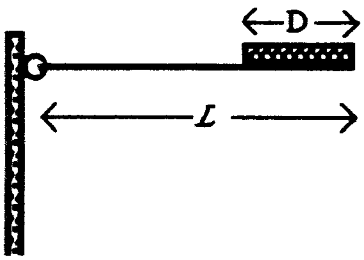
(a) Write the exact equation of motion.

(b) Assume the amplitudes are small, and rewrite the equation as a driven simple harmonic oscillator, moving along the horizontal axis. Identify the driving force.

(c) When δg is small the motion of the mass will be nearly that of the unperturbed SHO, $x = x_0 \ sin \ \omega_p t$. Show that under these conditions the power input from the driving term can be made positive definite for all t with the right choice of ω and ϕ. Find ω and ϕ. (This is a parametric amplifier: feed it at one frequency and get gain at another.)

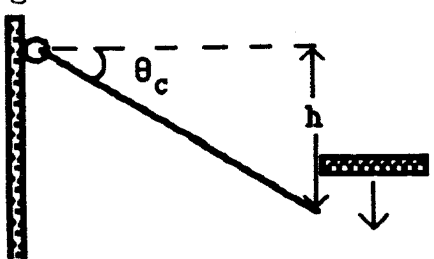
(d) Use energy conservation and the result for ω and ϕ obtained in part c to show that the peak amplitude of oscillation x_0 will grow exponentially with time, and find the growth constant.

2. A coin of diameter D is placed at one end of a horizontal flat ruler of length ℓ, hinged at the other end.



(a) show that if ℓ/D exceeds a minimum value, releasing the ruler wil cause it to rotate away such that the coin is left unsupported and "hanging freely" in mid air. Find ℓ/D minimum.

(b) Assuming ℓ/D has the minimum value, it is not completely clear whether the coin might strike the ruler at some later time as it falls to the floor.



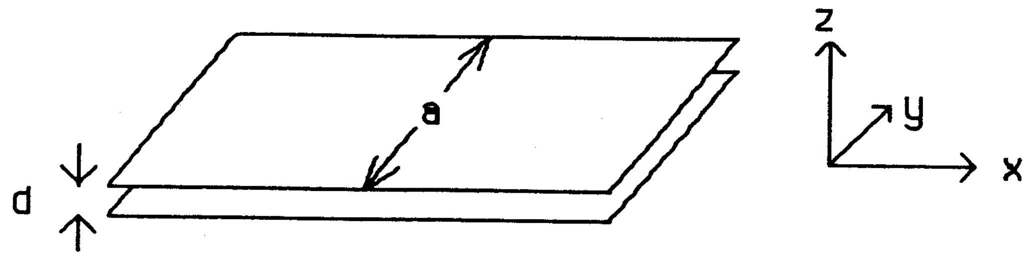
Find the critical angle which swings the ruler entirely out of the way of the falling coin.

(c) Write an exact expression for the time it takes the ruler to rotate through this angle, in terms of an integral.

(d) Show that it takes longer for the coin to fall a distance h than for the ruler to move out of the way.

$$\left[\text{Hint}: \int_0^{\Theta_m} \frac{d\Theta}{(sin\Theta)^{1/2}} < \frac{2\Theta_m^{1/2}}{(1-\frac{\Theta_m^2}{6})} \text{ for } \Theta_m < 1 \text{ is easily proved}\right]$$

3.



An electromagnetic wave propagates in the x-direction between two parallel conducting planes in a transmission line. Assume $a >> d$, so that all end effects are negligible. The electric field is given by:

$$\vec{E} = E_0\hat{z} \quad cos(\omega t - kx) \quad .$$

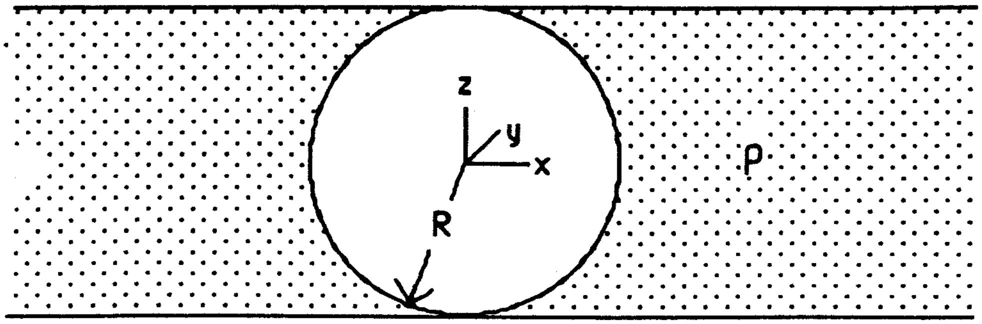
(a) Write down the Maxwell's equations.

(b) Calculate the current $I(x,t)$ flowing along the upper conductor.

(c) Calculate the characteristic impedance, $Z \equiv V/I$, of the transmission line.

$V = \int_0^d E_z \phi z$

4. Consider a spherical void within an infinitely-long cylinder as shown below. The entire



cylinder has a uniform positive charge distribution ρ, except for the void, which is uncharged.

(a) Use the principles of superposition or otherwise compute the electric field at the point $P = (0, 0, x_0), x_0 < R$.

(b) A particle of positive charge q and mass m is placed at the point P at $t = 0$. Describe its motion quantitatively as a function of t.

(c) Is the origin $(0, 0, 0)$ a point of stable equilibrium? Argue why.

5. A radio antenna operating at a wavelength of 400m is located on a high cliff on the edge of a lake. Venus rises above the horizon and is tracked by the antenna. The first minimum in the signal reflected off the surface of Venus is recorded when Venus is 35° above the horizon. Find the height of the cliff. [Remember the reflection from the surface of the lake.]

6. Eyepieces used in microscopes or telescopes are usually made of at least two lenses. To provide good quality images, they are corrected to minimize aberrations, in particular, transverse chromatic aberration (i.e., variation of the combined focal length with refractive index $n(\lambda)$).

(a) Write the lensmaker's equation for a thin convex lens.

(b) Obtain the focal length of two lenses of focal lengths f_1 and f_2 separated by a distance L, made of the same glass.

(c) Minimize the chromatic aberration and obtain a relation between the separation L, f_1 and f_2.

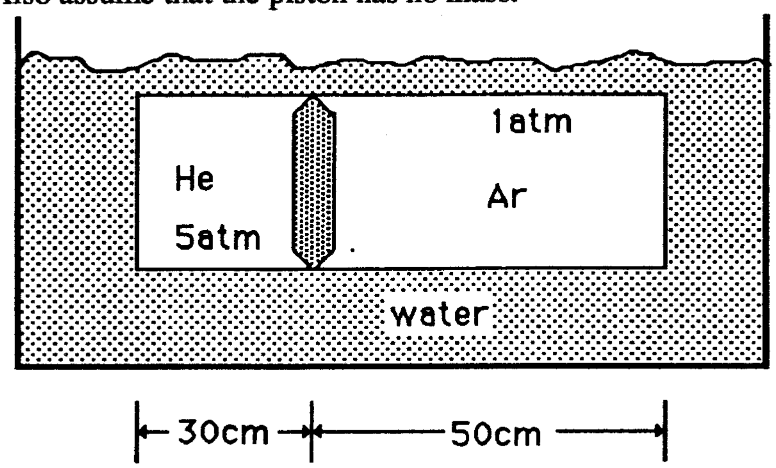
(d) Describe an eyepiece that uses the above criteria.

7. Consider a surface consisting of a square lattice of M adsorption sites per unit area. A 3D ideal gas of pressure P and density ρ is kept in equilibrium with the surface. If each of the N atoms per unit area that sticks has a binding energy of E_a and interactions between adsorbed atoms are zero (accept that only one atom per site is allowed), calculate the coverage, Θ, (Θ=N/M) as a function of temperature (fixed pressure) and the chemical potential, $\mu(T,P)$, of the ideal gas.

$$\mu_{ideal} = -kT \ln\left(\frac{1}{\rho}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right)$$

$$\ln N! \approx N\ln N - N$$

8. A cylindrical container 80cm long is separated into two compartments by a thin piston, originally clamped in a position 30cm from the left end. The left compartment is filled with one mole of helium at 5 atm; the right compartment is filled with argon at 1 atm (the gases are ideal). The cylinder is submerged in 1 liter of water and the whole system is at an equilibrium temperature of 25C. Neglect heat capacities of the cylinder and piston. Also assume that the piston has no mass.



When the piston is unclamped at t=0 it will change position. If the expansion is rapid enough, compared to some equilibrium time τ, we can assume that no heat flows initially and the process is adiabatic at short times. At longer times, $t \gg \tau$, the temperature of the He and Ar must be the same.

a) For $t \ll \tau$, calculate the position of the piston.

b) For $t \gg \tau$ calculate the increase in the water temperature.

c) For $t \gg \tau$ calculate the position of the piston.

QUALIFYING EXAM
PART II
January 9, 1989

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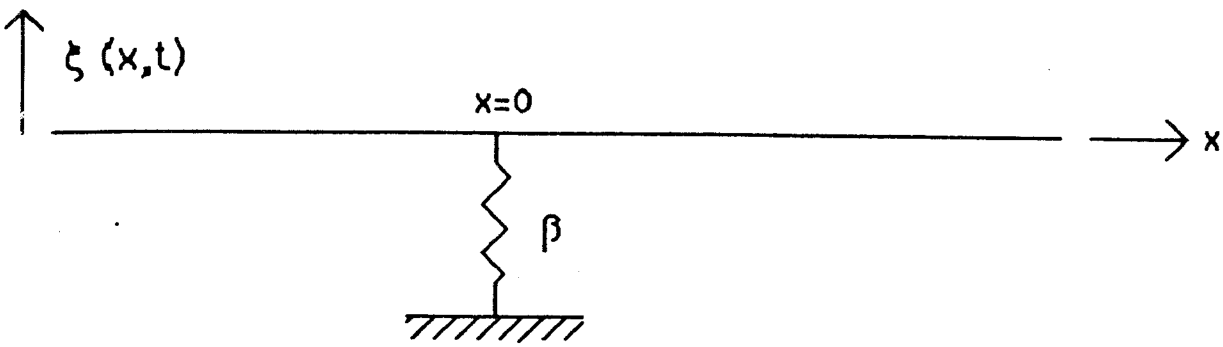
9. Obtain the solution $f(x)$ of the integral equation:

$$f(x) = x + \int_0^1 dy\ xy f(y) \quad .$$

$f(x) = x\left(1 + \int_0^1 dy\, xy\, f(y)\right)$

10. Consider an infinitely long string attached to a massless spring of force constant β as shown below. F is the tension of the string.



(a) If $\xi(x,t)$ represents the displacement of the string, state the origin of the three terms in the equation of motion

$$\rho\ddot{\xi} = F\frac{\partial^2 \xi}{\partial x^2} - \beta\xi\delta(x) \quad . \tag{1}$$

(b) An incident wave described by

$$\xi_i(x,t) = cos(kx - \omega t)$$

$R = \frac{(A-B)^2}{(A+B)^2}$?

$T = \frac{2AB}{(A+B)^2}$

is incident at the junction, $x = 0$, from left. Write a general expression for the reflected wave and for the transmitted wave.

(c) Solve Eq. (1) to get the unknown parameters you introduced in (b).

(d) For what values of β, would there be (i) no transmitted wave (ii) no reflected wave?

11. when μ mesons are stopped by matter, the μ^- can be captured by nuclei in "Bohr orbits", where they emit a characteristic series of x rays as they cascade down to the $n = 1$ level.

(a) Calculate the radius of the $n = 1$ Bohr orbit for a μ^- orbiting a nucleus of charge ze.

$$m_\mu = 106MeV/c^2 = 209me$$

$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = m\frac{v^2}{r}$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e\ e^2} = .529 \times 10^{-10}M$$

(b) At large z the x rays are no longer determined by Bohr theory because the radius of this orbit is now less than the radius of the nucleus. Find the value of z where this effect first appears. $\left(\text{Density of nuclear matter} = 10^{18} kg/m^3 \quad , m_n \simeq m_p = 1.67 \times 10^{-27} kg \right)$

12. (a) Several cosmic ray experiments in the past few years have indicated that we are being bombarded with enormously energetic particles ($E > 10^{15} ev$) from pulsars such as Cygnus X-3, 40,000 light years away. If these particles are protons, they should have been deflected by the galactic magnetic field $B \approx 10^{-10} T$, so we wouldn't be able to identify their origin. If they are neutrons, the 1000 sec $= T_{1/2}$ half life would have caused them to decay on the way.

$$m_p^0 \simeq m_n^0 \approx 10^9 \frac{eV}{c^2}, \quad e = 1.6 \times 10^{-19} C$$

Q: Show both these arguments break down at high enough energy. Estimate the minimum particle energy required for both P^+ and ${}^1_0 n$ to reach us directly from Cygnus X-3. (Hint: The relativistic expression for the path of a changed particle in a magnetic field is the same as the nonrelativistic, except one uses the relativistic mass.)

(b.) The supernova SN-1987A in the nearby galaxy GMC emitted neutrinos that were detected on Earth in a burst that lasted roughly 10 seconds, after traveling for 1.6 $\times 10^5$ years. If the rest mass of the neutrino is not zero, that 10 second spread might be due to low energy particles taking longer to arrive. Fortunately, we know the energies. They range from very high (so $v = c$) down to 1 MeV. Show that it is unlikely that the rest mass of neutrinos is much greater than $M_\gamma \approx 10 eV$.

13. Use the Wilson Sommerfeld quantization rule $\oint p_q dq = nh$ to calculate the allowed energy levels of a ball bouncing elastically in the vertical direction.

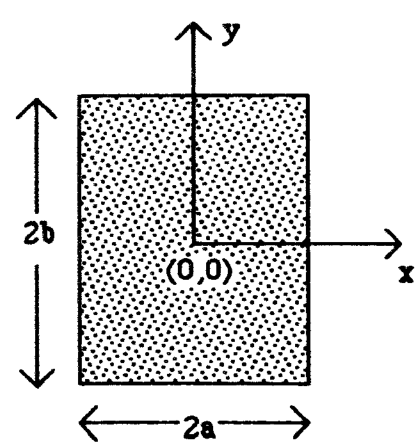
14. A particle of mass M is confined to move in the shaded region of the $x - y$ plane as shown below. Write the Schrödinger equation and find

(a) the unnormalized eigenfunctions for all states and

(b) energy eigenvalues for the four lowest energy states. ($b < 2a$)

(c) Sketch the 4 lowest eigenfunctions (separately along x and y).

(d) Suppose that the potential outside was lowered from ∞ to some finite value, sketch how it would affect the wavefunctions for the lowest state.

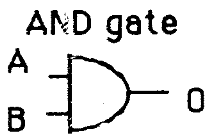


15. Suggest in a phrase or sentence some practical way to reduce or eliminate each of the following kinds of environmental disturbances from an experimental apparatus.
   a) Temperature fluctuations
   b) the earth's electric field
   c) the earth's magnetic field
   d) the AC E&M field in a laboratory
   e) cosmic radiation
   f) building vibrations

16. Choose either part (a) or (b).

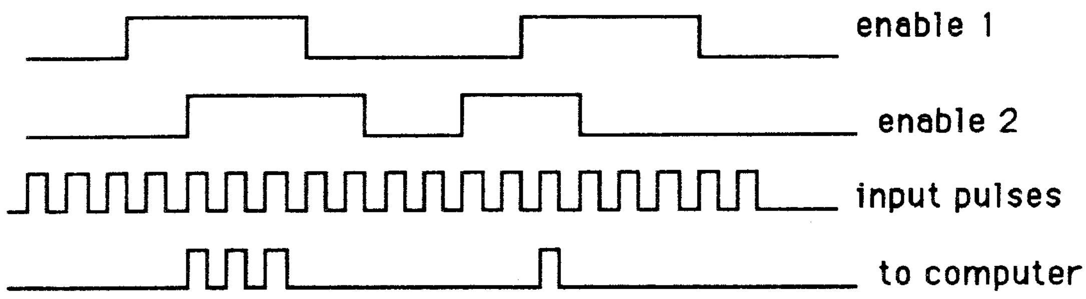
***(a) Complete the truth table below for an AND logic gate.

A	B	O	
1	1	?	1
1	0	?	0
0	1	?	0
0	0	?	0



A detector generates a series of pulses that will be counted by a computer. However, only those pulses which occur when either of the two enables are high are significant. The students research advisor is too broke to buy any new parts so the student must use only the available parts in the lab. Draw a circuit, using only dual input AND gate digital logic chips, that will only send pulses to the computer when the enables are both on.

*** Dave Cowan did not write this question.



(b). Consider the temperature interval 4K- 6,000K. Explain in detail how one would measure temperature in 1) the low temperature range of this interval, 2) the medium temperature range, and 3) in the high temperature range. Explain the physical principle by which each apparatus works, the reasons for the practical temperature limits of the device, and sketch the device.