UNIVERSITY OF MISSOURI-COLUMBIA
PHYSICS DEPARTMENT

PART I Qualifying Examination

January 3, 2002, 9:00 a.m. to 1:00 p.m.

Instructions: The only material you are allowed in the examination room is a writing instrument and a calculator. You may not store any formulae in your calculator. Paper, mathematical handbooks and question sheets are furnished. Each student is assigned a capital English letter; this letter will identify your work on both parts (I and II) of this exam. In writing out your answers, use only one side of a page, use as many pages as necessary for each problem, and do not combine work for two different problems on the same page. Each page should be identified in the upper, right-hand corner according to the following scheme: A 4.3 i.e., student A, problem 4, page 3. Refer all questions to the exam proctor. In answering the examination questions, the following suggestions should be heeded:

1. Answer the exact question that is asked, not a similar question.

2. Use simple tests of correctness (such as a reasonable value, correct limiting values and dimensional analysis) in carrying out any derivation or calculation.

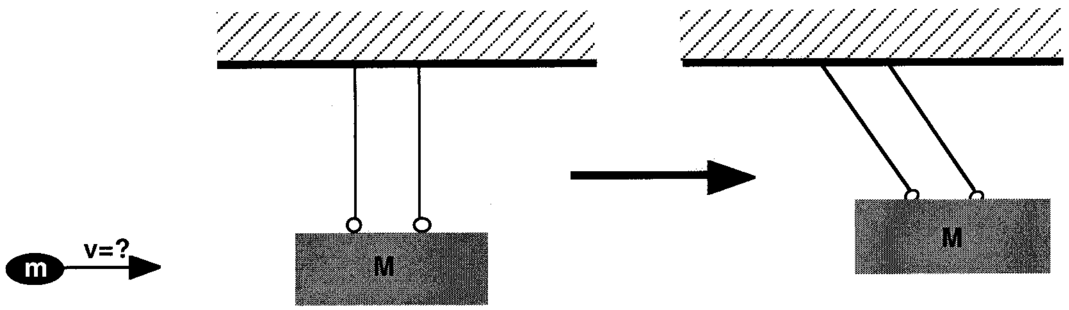
3. If there is any possibility of the grader being confused as to what your mathematical symbols mean, define them.

You may leave when finished.

1. A block of mass M is suspended to form a pendulum as shown in the figure below. A projectile of mass m collides with the block so that the block rises to a height h.

(a) Find the incident speed of the projectile v in terms of m, M, and h for a perfectly inelastic collision.

(b) Find the incident speed of the projectile v in terms of m, M, and h for a perfectly elastic collision.

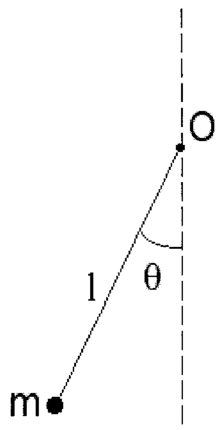


2. A simple pendulum consists of a particle of mass m suspended from a string of length l attached to a fixed point O as shown in the figure below. The mass can freely rotate in a circle of radius l and center O confined to a vertical plane.

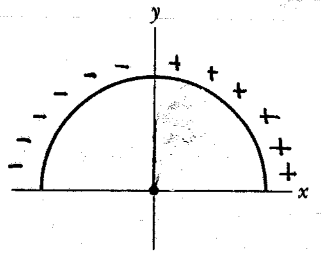
   (a) Calculate the tension T in the string as a function of the particle's mechanical energy E and the angle θ between the string and a downward vertical line through O. (Set the zero of the gravitational potential energy at $\theta = 0$.)

   (b) Assuming that $E = 2mgl$, determine the value of the angle θ at which the string collapses (i.e., the tension vanishes).

   (c) The collapse of the string only occurs in a certain range of energies. What is this range?



3. A thin glass rod is bent into a semicircle of radius R and placed in the upper half of the x-y plane with its center at the origin as shown in the figure. Positive charge $+Q$ is distributed uniformly along the right half of the semicircle and negative charge $-Q$ is distributed uniformly along its left half. Find the electric field at the origin.



4. Consider a nonconducting medium whose permittivity ε and permeability μ are functions of the position $\boldsymbol{r}$ only. There are no free charges or currents present.

   (a) Derive the wave equation for the electric field $\boldsymbol{E}$ (which does not contain the magnetic field $\boldsymbol{B}$).

   (b) For the case in which ε is a function of z only, $\mu = \mu_0$ = constant, and $E_z = 0$, show that the wave equation reduces to that for a uniform medium [except that $\varepsilon = \varepsilon(z)$].

5. (a) Draw a schematic diagram of a Michelson interferometer and explain how it can be used to measure small distances.

   (b) How many fringe shifts will occur if a distance of 100 μm is to be measured? The wavelength of the light used is $\lambda_{\text{source}} = 500$ nm.

   (c) Can the device be used with a source of white light? Explain.

   (d) The Michelson interferometer was the key instrument instrument used in the Michelson-Morley experiment. What was the purpose of this experiment? How was the experiment performed?

6. Monochromatic light of wavelength λ passes through a slit of width b and falls on a screen placed at distance $R >> b$ from the slit (Fraunhofer approximation) as shown in the figure below.

   (a) Describe the diffraction pattern and find a general condition for the position of its minima.

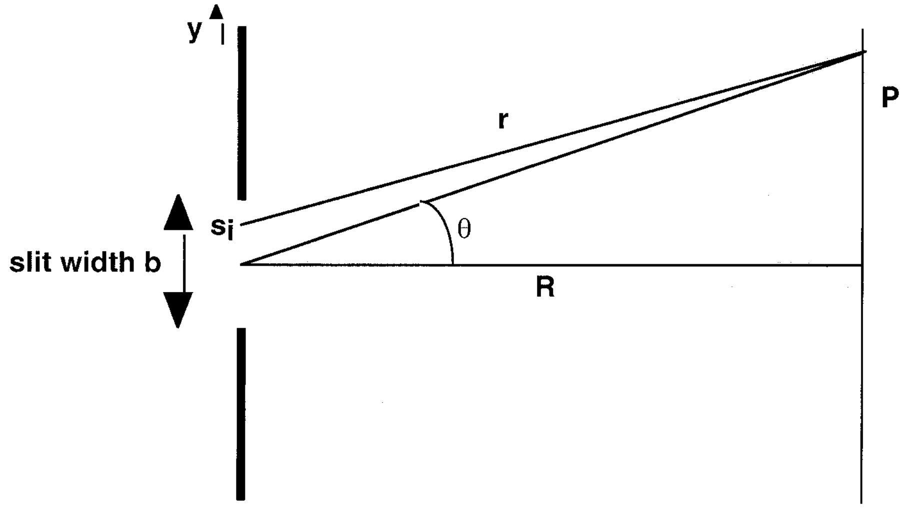
   (b) Show that the intensity of the diffraction pattern on the screen is proportional to

$$\left(\frac{\sin\beta}{\beta}\right)^2$$

   where $\beta = \frac{\pi}{\lambda} b \sin\theta$. Use the following expression for the electric field due to a point source s_i located in the slit:

$$E_i = \frac{E_0}{r} e^{i\left(\omega t - \frac{2\pi r}{\lambda}\right)}$$

   where r is the distance from the source to the screen. [Hint: express r in the phase term as a function of R, y, and θ and use the appropriate approximation for r in the amplitude term. The phase is very sensitive to r, but the amplitude is not.]



7. An object of mass m_1, specific heat c_1, and initial temperature T_1 is placed in thermal contact with a second object of mass m_2, specific heat c_2, and initial temperature $T_2 > T_1$. The two substances are placed in a calorimeter so that no energy is lost to the surroundings. The system consisting of the two objects is allowed to reach thermal equilibrium.

   (a) Find the final equilibrium temperature of the system T_f in terms of m_1, m_2, c_1, c_2, T_1, and T_2.

   (b) Find the total entropy change of the system and determine whether it is positive, negative, or zero.

8. The nuclei of atoms in a certain crystalline solid have spin one. According to quantum theory, each nucleus can therefore be in any one of three quantum states labeled by the quantum number m, where $m = 1, 0$, or -1. This quantum number measures the projection of the nuclear spin along a crystal axis of the solid. Since the electric charge distribution in the nucleus is not spherically symmetrical, but ellipsoidal, the energy of a nucleus depends on its spin orientation with respect to the internal electric field existing at its location. Thus a nucleus has the same energy $E = \varepsilon$ in the state $m = 1$ and the state $m = -1$, compared with an energy $E = 0$ in the state $m = 0$.

   (a) Find an expression, as a function of the absolute temperature T, for the nuclear contribution to the molar internal energy of the solid.

   (b) Find an expression, as a function of T, for the nuclear contribution to the molar entropy of the solid.

   (c) By directly counting the total number of accessible states [i.e., without using your result from part (b)], calculate the nuclear contribution to the molar entropy of the solid at very low temperatures. Calculate it also at very high temperatures. Now show that the expression in part (b) reduces properly to these values as $T \rightarrow 0$ and $T \rightarrow \infty$.

UNIVERSITY OF MISSOURI-COLUMBIA
DEPARTMENT OF PHYSICS & ASTRONOMY

PART II Qualifying Examination

January 10, 2002, 9:00 a.m. to 1:00 p.m.

Instructions: The only material you are allowed in the examination room is a writing instrument and a calculator. You may not store any formulae in your calculator. Paper, mathematical handbooks and question sheets are furnished. Each student is assigned a capital English letter; this letter will identify your work on both parts (I and II) of this exam. In writing out your answers, use only one side of a page, use as many pages as necessary for each problem, and do not combine work for two different problems on the same page. Each page should be identified in the upper, right-hand corner according to the following scheme: A 4.3 i.e., student A, problem 4, page 3. Refer all questions to the exam proctor. In answering the examination questions, the following suggestions should be heeded:

1. Answer the exact question that is asked, not a similar question.

2. Use simple tests of correctness (such as a reasonable value, correct limiting values and dimensional analysis) in carrying out any derivation or calculation.

3. If there is any possibility of the grader being confused as to what your mathematical symbols mean, define them.

You may leave when finished.

1. Find the eigenvalues λ and corresponding normalized eigenvectors $\boldsymbol{r}$ of the matrix

$$A = \begin{bmatrix} 0 & 4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

2. Solve the integral equation

$$Y(t) = 3 \sin t + \frac{1}{2}\int_0^t Y(\tau)\sin(t-\tau)d\tau$$

by the Laplace transform method. Use the convolution theorem

$$\mathrm{L}\left\{\int_0^t F_1(\tau)F_2(t-\tau)d\tau\right\} = \mathrm{L}\{F_1(t)\}\mathrm{L}\{F_2(t)\}$$

where $f(p) = \mathrm{L}\{F(t)\} = \int_0^\infty e^{-pt}F(t)dt$, $\mathrm{Re}(p) > 0$. [Hint: First, evaluate $\mathrm{L}\{\sin at\}$ by integration where a is real.]

3. A proton beam of kinetic energy T (in the laboratory reference frame) enters a hydrogen bubble chamber.

(a) Find the threshold energy T_t for producing antiprotons ($\bar{p}$) in the reaction

$$p^+ + p^+ \rightarrow p^+ + p^+ + p^+ + \bar{p}^-$$

where the target proton is assumed to be at rest. The rest energy of the proton and antiproton is 938 MeV. [Hint: First calculate the velocity of the protons in the reference frame of the center of mass. Then use the relativistic transformation of velocities to calculate the velocity of one proton relative to the other.]

(b) At the threshold energy, what is the speed of the antiproton?

4. A particle in one dimension is in the ground state of an unknown potential $V(x)$. The *probability* density of finding the particle at a position x is found to be

$$\frac{1}{2a\cosh^2\left(\frac{x}{a}\right)}$$

where a is a positive length scale. Assuming that the potential $V(x)$ vanishes at infinity, find:

(a) the ground-state energy;

(b) the potential $V(x)$.

[Hint: Use the Schrödinger equation.]

5. Consider the inelastic scattering of one photon of wavelength λ by a charged particle of mass m which is initially at rest (Compton scattering). Determine the relation between the incident and scattered photon wavelengths λ and λ', respectively, and the scattering angle θ (the angle between the incident and outgoing photon direction). Use relativistic formulæ.

6. Consider a particle in the one-dimensional potential

$$V(x) = \begin{cases} 0, & -a/2 < x < a/2; \\ \infty, & \text{otherwise} \end{cases}$$

a) Write down *normalized* eigenfunctions and eigenvalues for this system.

b) Now a small perturbation $W = \lambda\delta(x)$ is added to the Hamiltonian, where $\delta(x)$ is the Dirac delta function [$\delta(x) = 0$ for all $x \neq 0$, while $\int_{-\infty}^{\infty}\delta(x)dx = 1$]. Determine the first-order correction (proportional to λ) to the energy of the ground state. Repeat for the first excited state.

c) Determine the second-order correction (proportional to λ^2) to the energy of the ground-state. Repeat for the first excited state. Recall that

$$\Delta E_m^{(2)} = \sum_n \frac{|\langle m|W|n\rangle|^2}{E_m^{(0)} - E_n^{(0)}}, \quad n \neq m.$$

Note, you do not need to calculate the summations involved, but you may find the following identity useful: $\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = 1$.

7. When you pass through an airport metal detector, you become part of an alternating current resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) that is part of the circuit.

(a) Calculate the rms (root-mean-square) current versus frequency for a series *RLC* circuit. Sketch your result for three different values of *R*.

(b) To what frequency should the detector be tuned when there is no metal in it? Why?

(c) Define the quality factor of its circuit. If you want the detector to be able to detect a small metal object, should the circuit have a high quality factor or a low one? Why?

8. Answer any four of the following seven questions:

   (a) What is meant by the "protein folding" problem. Why has it been so difficult to solve? What progress has been made recently?

   (b) What is the speed at which gravity is believed to propagate? How might it be measured?

   (c) What is a bilayer lipid membrane?

   (d) What is the cosmic microwave background?

   (e) What is a spin glass?

   (f) Describe a recently synthesized molecule or material which has promise of important technological applications.

   (g) What is meant by "dark matter"?