UNIVERSITY OF MISSOURI-COLUMBIA
PHYSICS DEPARTMENT

PART I Qualifying Examination

January 17, 2008, 9:00 a.m. to 1:00 p.m.

Instructions: The only material you are allowed in the examination room is a writing instrument and a calculator. You may not store any formulae in your calculator. Paper, mathematical handbooks and question sheets are provided. Each student is assigned a capital English letter; this letter will identify your work on both parts (I and II) of this exam.

In writing out your answers, use only one side of a page, use as many pages as necessary

for each problem, and do not combine work for two different problems on the same page.

Each page should be identified in the upper, right-hand corner according to the following scheme: A 4.3 i.e., student A, problem 4, page 3. Refer all questions to the exam proctor.

In answering the examination questions, the following suggestions should be heeded:

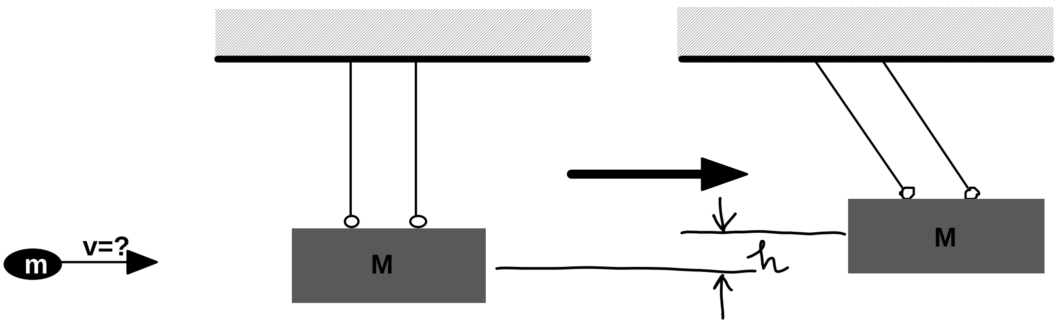
1. Answer the exact question that is asked, not a similar question.

2. Use simple tests of correctness (such as a reasonable value, correct limiting values and dimensional analysis) in carrying out any derivation or calculation.

3. If there is any possibility of the grader being confused as to what your mathematical symbols mean, define them.

You may leave when finished.

1. A block of material of mass M is suspended to form a pendulum as shown in the figure below. A projectile of mass m collides with the block so that the block rises to a height h.



(a) Find the incident speed v of the projectile in terms of m, M, h and g (the acceleration due to gravity) for a perfectly inelastic collision.

(b) Find the incident speed v of the projectile in terms of m, M, h and g for a perfectly elastic collision.

2. A particle of mass m moves on a frictionless horizontal surface under the influence of a central potential given by

$$V(r) = \frac{1}{3}\alpha r^3 \;,$$

where α is a positive constant. At $t = 0$ the particle is at a distance $r(0) = R$ from the center and has a velocity $\vec{v}_0$ oriented perpendicular to the position vector.

(a) Derive and make a sketch of the "effective radial potential", $V_{eff}(r)$, defined by

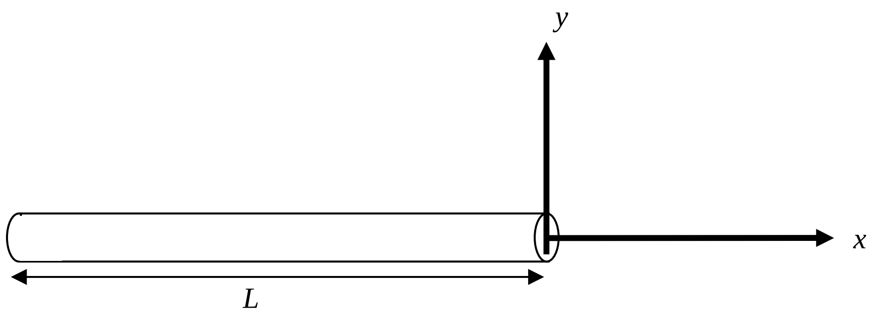
$$E = \frac{m\dot{r}^2}{2} + V_{eff}(r) \quad ,$$

where E is the total energy, and $\dot{r}$ is the radial component of the velocity.

(b) Find the value of the initial velocity v_0 for which the total energy is a minimum. What is the trajectory of the particle in this case, and what is the period of the motion?

(c) If the particle has a total energy that is slightly above the minimum that you considered in part (b) while $V_{eff}(r)$ is unchanged, calculate the period of the small amplitude radial oscillations that will occur in the motion of the particle.

3. Electric charge Q is distributed uniformly along an infinitesimally thin rod of length L. Take the electric potential to be defined as zero at infinity.

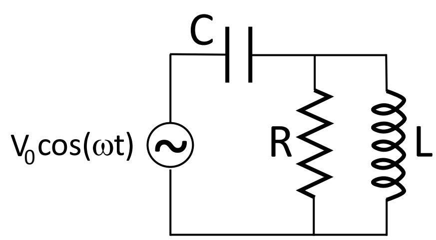


(a) Find the electric potential along the x axis.

(b) As $x >> L$, what does your result in (a) reduce to? What is the physical interpretation of this?

(c) What will be the electric potential along the y-axis when $y>>L$?

4. Consider the AC circuit shown in the figure.



(a) Recognizing that there are both parallel and series connections, find an expression for the amplitude and the phase of the current through the generator.

(b) What is the value of ω at resonance? Will resonance occur for all values of L, R and C?

5.

(a) How does fiber optic cable work?

(b) A specific brand of optical fiber is to be made with a transparent outer coating (called the cladding) that protects the surface of the fiber. The factory has raw materials of two refractive indices: 1.700 and 1.512. Which of the materials should be made into the fiber, and which one the cladding? Explain why!

(c) Calculate the critical angle for the fiber-cladding interface.

(d) The optical fiber is 2 m long and has a diameter of 20 micrometers. A ray of light is incident at an angle α of 40 degrees to the normal of the cross-sectional surface at the end of the fiber. Calculate how many reflections occur before the beam exits the fiber.

(e) For what range of values of the angle α defined in part (d) will the light be transmitted down the fiber?

6. A converging lens is placed on top of a flat surface. When observed in reflection through the lens, using light of wavelength 500 nm, Newton's rings are observed. The fourth bright fringe has a radius of 1.86 mm.

(a) Draw a diagram of the setup, including a sketch of the rays that cause interference.
(b) Will the central fringe be bright or dark? Explain your reasoning.
(c) Calculate the radius of curvature of the lens.
(d) A liquid is introduced between the lens and the flat surface. The radius of the 4^{th} bright fringe changes to 1.43 mm. Calculate the refractive index of the liquid, n_1.

7. A hypothetical speed distribution for a sample of N gas particles is given by

$$P(v) = \begin{cases} (a/v_0)v & \text{for } 0 < v < v_0 \\ a & \text{for } v_0 < v < 2v_0 \\ 0 & \text{for } v > 2v_0 \end{cases} .$$

The distribution function is normalized according to $\int_0^\infty P(v)\,dv = N$.

(a) Express a in terms of N and v_0.

(b) How many of the particles have speeds between $1.5v_0$ and $2v_0$?

(c) Calculate the average and the root mean square speeds.

8. One mole of a monoatomic ideal gas undergoes a slow expansion, from $V_1 = V_0$ to $V_2 = 3V_0$, during which its pressure changes linearly with the volume according to $p = p_0(4 - V/V_0)$. In answering the following questions, express your results in terms of V_0, p_0 and R (the universal gas constant).

(a) Find the volume V_T at which the temperature of the gas is maximum. What is the value of $T_{\max}$?

(b) Find the volume V_S at which the entropy of the gas has a maximum. What is the value of $S_{\max} - S_1$, where S_1 is the entropy of the initial state.

(c) Calculate the total absorbed (Q_{abs}) and released (Q_{rel}) heat during the expansion of the gas.

(d) Show that in the interval $V_T < V < V_S$ of the transformation the specific heat of the gas is negative.

UNIVERSITY OF MISSOURI-COLUMBIA
PHYSICS DEPARTMENT

PART II Qualifying Examination

January 24, 2008, 9:00 a.m. to 1:00 p.m.

Instructions: The only material you are allowed in the examination room is a writing instrument and a calculator. You may not store any formulae in your calculator. Paper, mathematical handbooks and question sheets are provided. Each student is assigned a capital English letter; this letter will identify your work on both parts (I and II) of this exam.
In writing out your answers, use only one side of a page, use as many pages as necessary for each problem, and do not combine work for two different problems on the same page.
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In answering the examination questions, the following suggestions should be heeded:

1. Answer the exact question that is asked, not a similar question.

2. Use simple tests of correctness (such as a reasonable value, correct limiting values and dimensional analysis) in carrying out any derivation or calculation.

3. If there is any possibility of the grader being confused as to what your mathematical symbols mean, define them.

You may leave when finished.

1. A space ship is leaving the Earth and is bound for Pluto. The time of departure is 1:00:00 pm. Two observers of the ship, one on board of the ship, and one on a space station, agree on this time. The average relative speed v of the space ship with respect to the space station is: v= 0.95c. In this problem, we will assume that it only takes an instant to accelerate to this speed, so that we can assume that the speed of the space ship is constant once it is at v = 0.95c.

a) When the ship passes by the space station, the person on the station notes the time as 1:30:00 pm. What is the time noted by the person in the space ship when it passes the space station?

b) If the total rest mass of the space ship (total mass = mass including people) is 10^5 kg, how much energy must have been used up in order to accelerate the space ship from rest to 0.95c?

2. This problem consists of two main parts. If you cannot prove part a), simply take the result of part a) to calculate parts b) and c).

Assume that we have a quantum mechanical particle of mass m in a three dimensional cubic box with side lengths a.

a) Show that the energy levels of this particle are given by: $E_{k,l,n} = (k^2 + l^2 + n^2)\, h^2/(8ma^2)$, with k, l and n integers (1, 2, 3, ...), h Planck's constant (h= 6.62 x 10^{-34} J.s), m the rest mass of the particle. Treat the particle as moving at non-relativistic speeds, that is, ignore any relativistic effects.

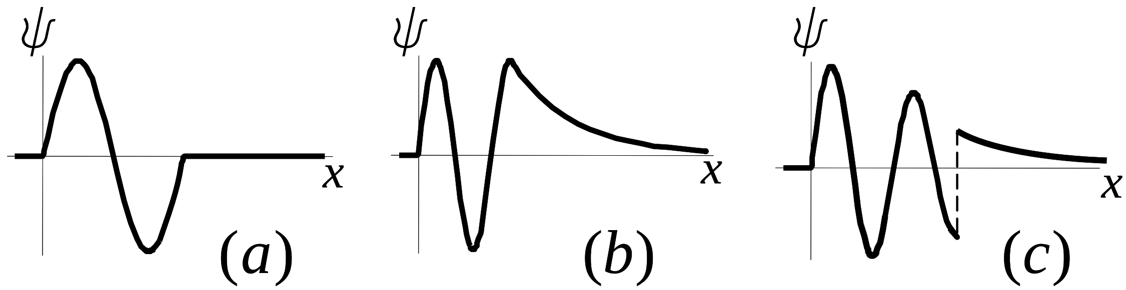
We now apply the particle in a box model to a hypothetical particle called the centurion. The mass of a centurion is m_c=1.7 x 10^{-26} kg, and one centurion consists of three quarks that are confined by the strong nuclear force to be within 1.00 fm (1.00 x 10^{-15}m) from each other. We will assume that a good approximation is to treat the quarks as independent particles that are confined to be in a square well potential of size 1.00 fm.

b) Assume that all quarks are in the lowest energy state. What would be the contribution of the zero-point motion of these quarks to the centurion mass (we do not want numbers here, rather an expression in terms of the rest mass of a quark, m)?

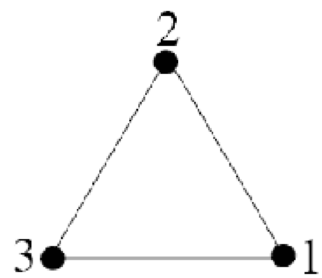
c) If we write the quark mass m as a fraction of the centurion mass m_c as m=fm_c, find the lowest value of f for the numbers given in this problem such that the centurion comes out weighing 1.7 x 10^{-26} kg. Use c = 3 x 10^8 m/s.

3. Sketch the form of the potential energy $V(x)$ for which each of the following wavefunctions $\psi(x)$ are stationary states (if it is possible). In each of the three cases make sure you explain your answer. If you believe that there is no such $V(x)$, explain why this is so. Recall that the Schrodinger's equation for stationary states is

$$-\frac{\hbar^2\partial^2}{2m\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)\,.$$



4. A simple model for a triatomic molecule is shown below. It consists of 3 identical "atoms" at the corners of an equilateral triangle. An electron can occuy any of the three "orbitals" $|a_i\rangle$, $i = 1,2,3$ (which form an orthonormal basis) localized at the atomic sites and having the same energy ε. The electron is able to hop from one orbital to another, thus the eigenfunctions of the system's Hamitonian will be delocalized (i.e. a linear combination of the $|a_i\rangle$). The Hamiltonian of the system in the basis $|a_i\rangle$ is defined by the matrix elements $\langle a_i|H|a_i\rangle = \varepsilon$, $\langle a_i|H|a_j\rangle = -t < 0, \quad i \neq j$.



a) Write down explicitly the Hamiltonian matrix in the basis of the atomic orbitals.
b) Consider an operator R which describes a rotation by 120° in the plane of the molecule with respect to its center: $R|a_1\rangle = |a_2\rangle,\ R|a_2\rangle = |a_3\rangle,\ R|a_3\rangle = |a_1\rangle$ (note that R^3 = 1). Write down the explicit matrix R in the basis of the atomic orbitals.
c) Prove that R commutes with H and explain the physical meaning of this commutator.
d) Prove that the states

$$|R_1\rangle = [e^{-i2\pi/3}|a_1\rangle + e^{+i2\pi/3}|a_2\rangle + |a_3\rangle]/\sqrt{3},$$
$$|R_2\rangle = [e^{+i2\pi/3}|a_1\rangle + e^{-i2\pi/3}|a_2\rangle + |a_3\rangle]/\sqrt{3}, \quad \text{and}$$
$$|R_3\rangle = [|a_1\rangle + |a_2\rangle + |a_3\rangle]/\sqrt{3}$$

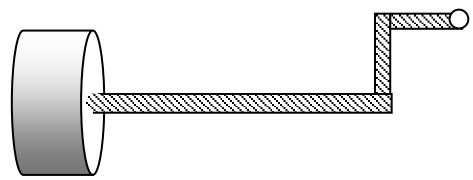
are eigenvectors of R. Calculate the eigenvalues L_i. Checkpoint: remember R^3 = 1.
e) Calculate the energy spectrum of the molecule, i.e., the eigenvalues of the Hamiltonian H together with their degeneracies. Hint: recall $[R,H] = 0$, as stated above.

5. Solve any one (and only one) of the following three problems (labeled **A**, **B** and **C**):

A. This problem concerns semiconducting diodes.
(a) Name three basic materials - elements, molecules or compounds - that can be used to make diodes.
(b) Choosing one of the materials you named in part (a), what changes must be made to these materials in order to use them in diodes? Be specific about the changes to be made.
(c) What techniques can be used to produce the changes to the materials?

(d) Describe and draw a diagram that illustrates how the materials must be arranged in order for them to function as a diode.
(e) How does a voltage and its polarity affect the distribution of charges in a diode? Draw a diagram and describe its detailed workings.
(f) Sketch a typical diode's I-V characteristics with sample values for the axes, and describe how different parts of the I-V curve are related to your descriptions above.

B. After surviving a crash of your small airplane on a deserted island, you decide to fabricate a radio transmitter from miscellaneous parts that can be scavenged from the aircraft. You have two 6 V batteries that can deliver up to 20 A of dc current each. You have a 1 nF capacitor, an assortment of inductors and resistors, as well as an ample amount of copper wire. You also have cellophane tape, some small magnets, paper and cardboard. You also have a metallic disk that can be rotated with a hand crank (this was scavenged from the emergency manual landing gear release), pictured below.



(a) You decide to broadcast at a frequency of 12 MHz. Estimate the length of an antenna that would give efficient line-of-sight propagation.

(b) If you assume that the minimum power density detectable by a passing ship radio is 10^{-9} W/m^2 , estimate the maximum distance (assuming line-of-sight propagation) from which you can hope to recruit help.

(c) Because you don't know enough Morse code to convey your location, you decide that you need to fabricate a microphone. How might you build a microphone from the materials given? Draw one or more diagrams that adequately express your design and the essential physical ideas involved.

(d) How would you build the power section of the radio transmitter? Note that you have to overcome the problem that your power source is only dc. Do not worry about how to couple your microphone to the circuit. Just show how you could generate a 12 MHz carrier signal using the given materials. Note any design considerations that would enhance the efficiency of your transmitter.

C. Answer all three of the following (unrelated) astrophysics questions (labeled **C1**, **C2** and **C3**):

C1: Calculate the equilibrium temperature of a planet T_{planet} that has a radius R and that is located at a distance d from the Sun. The planet has an albedo A ($0 \le A \le 1$) which is defined as the fraction of the incident solar energy being reflected by the planet. The Sun's radius is R_Θ

and its temperature is T_Θ ($R_\Theta = 6.96 \times 10^{10}$ cm, $T_\Theta = 5800$ K). We can consider the Sun to be a black-body.

(a) Derive a formula for T_{planet} ;

(b) Calculate T_{planet} for the Earth (R = 6378 km, A = 0.39, $d = 1.496 \times 10^{13}$ cm).

C2. The rotation curve of a galaxy is the plot of the speed *V* of various parts of the galaxy as a function of their distance *R* from the center of the galaxy. Calculate and sketch the rotation curve for the following three different model distributions of matter in the galaxy:
(a) Most of the mass *M* is concentrated at the center of the galaxy in a body whose radius is much smaller than the galactic radius.
(b) $M(R) = k R$, where *k* is a constant of proportionality.
(c) The mass is uniformly distributed within a sphere of radius R_0 (comparable to the galactic radius).

C3. The relative abundances of the major elements in the Sun (by number of atoms with respect to hydrogen) are He/H = 0.1, C/H = 400×10^{-6}, O/H = 500×10^{-6}, Mg/H = 35×10^{-6}, Si/H = 35×10^{-6}, and Fe/H = 35×10^{-6}. The atomic masses, measured in atomic mass unit (amu), are μ_{He}=2, μ_C=12, μ_O=16, μ_{Mg} =24, μ_{Si}=28, μ_{Fe}=56.

(a) Calculate the maximum mass (in amu) of MgFeSiO4 silicate grains that can be formed.
(b) In order to explain the observed interstellar obscuration of visible starlight, which is caused by small solid grains, we would require a silicate mass of 9×10^{-3} (in amu). Is the value you calculated in (a) sufficient? If not, what other grain types may be responsible?