COSMIC NEEDLES VERSUS COSMIC MICROWAVE BACKGROUND RADIATION

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ABSTRACT

It has been suggested by a number of authors that the 2.7 K cosmic microwave background (CMB) radiation might have arisen from the radiation of "Population III" objects thermalized by conducting cosmic graphite/iron needle-shaped dust. Due to a lack of an accurate solution to the absorption properties of exceedingly elongated grains, in existing literature which studies the CMB thermalizing process they are generally modeled as (1) needle-like spheroids in terms of the Rayleigh approximation, (2) infinite cylinders, and (3) antennae. We show here that the Rayleigh approximation is not valid since the Rayleigh criterion is not satisfied for highly conducting needles. We also show that the available intergalactic iron dust, if modeled as infinite cylinders, is not sufficient to supply the required opacity at long wavelengths to obtain the observed isotropy and Planckian nature of the CMB. If appealing to the antenna theory, conducting iron needles with exceedingly large elongations ($>10^4$) appear able to provide sufficient opacity to thermalize the CMB within the iron density limit. But the applicability of the antenna theory to exceedingly thin needles of nanometer/micrometer thickness has not yet been verified.

Subject headings: cosmic microwave background — dust, extinction

1. INTRODUCTION: ABSORPTION PROPERTIES OF COSMIC NEEDLES

The 2.7 K cosmic microwave background (CMB) is generally interpreted as being relic radiation from the early hot universe of a big bang origin. Alternative attempts at explaining the observed CMB as a post–big bang phenomenon have been continuously made in terms of emission from "Population III" objects at high redshift (presumably either a pre-galactic generation of very massive stars or black hole accretion flows) thermalized by hollow spheres (Layzer & Hively 1973) or by long slender conducting cosmic whiskers or "cosmic needles" (Hoyle, Wickramasinghe, & Reddish 1968; Wickramasinghe et al. 1975; Rana 1980; Wright 1982; Narlikar, Wickramasinghe, & Hoyle 1984; Hawkins & Wright 1988; Hoyle & Wickramasinghe 1988; Bond, Carr, & Hogan 1991; Wickramasinghe 1992; Wickramasinghe, Wickramasinghe, & Hoyle 1992; Wickramasinghe & Hoyle 1994; Aguirre 2000).

The reason for invoking "conducting needles" is that neither spherical grains (both dielectric and metallic) nor dielectric needles have high enough opacity in the far-infrared (IR) to be an efficient thermalizing agent unless an unreasonably large amount of dust is invoked. This can be seen from the absorption cross section expressions of spherical or spheroidal grains. Let $\epsilon(\lambda) = \epsilon_1 + i\epsilon_2$ be the dust complex dielectric function at wavelength λ. The absorption cross section $C_{\rm abs}$ per unit volume V for spheres in the Rayleigh regime (Bohren & Huffman 1983) is

$$C_{\rm abs}/V \approx \frac{18\pi}{\lambda}\frac{\epsilon_2}{(\epsilon_1 + 2)^2 + \epsilon_2^2} . \qquad (1)$$

For dielectric spheres, $C_{\rm abs} \propto \lambda^{-1}\epsilon_2 \propto \lambda^{-2}$ approaches zero as $\lambda \to \infty$, since at far-IR, ϵ_1 approaches a constant $\gg \epsilon_2$ while $\epsilon_2 \propto \lambda^{-1}$. For metallic spheres with a conductivity of σ, $C_{\rm abs} \propto \lambda^{-1}\epsilon_2^{-1} \propto \lambda^{-2}$ also approaches zero as $\lambda \to \infty$, since $\epsilon_2 = 2\lambda\sigma/c \propto \lambda$ (c is the speed of light) and $\epsilon_1 \ll \epsilon_2$. Let needle-shaped grains be represented by thin prolate sphe-

roids of semiaxes l along the symmetry axis and a perpendicular to the symmetry axis. In the Rayleigh limit, the absorption cross section per unit volume for needle-like prolate grains ($l \gg a$) is approximately

$$C_{\rm abs}/V \approx \frac{2\pi}{3\lambda}\frac{\epsilon_2}{\left[L_{\parallel}(\epsilon_1 - 1) + 1\right]^2 + \left(L_{\parallel}\epsilon_2\right)^2} , \qquad (2)$$

where $L_{\parallel} \approx (a/l)^2\ln(l/a)$ is the depolarization factor parallel to the symmetry axis. For dielectric needles, $C_{\rm abs} \propto \lambda^{-1}\epsilon_2 \propto \lambda^{-2}$ at far-IR, since we usually have $L_{\parallel}(\epsilon_1 - 1) + 1 \gg L_{\parallel}\epsilon_2$ while ϵ_1 is insensitive to λ at long wavelengths; for metallic needles, it appears at first glance that, for a given value of ϵ_2 (at a given λ)—no matter how large—one can always find a sufficiently long needle with $L_{\parallel}\epsilon_2 \lesssim 1$ and $L_{\parallel}(\epsilon_1 - 1) \ll 1$ (Greenberg 1972) so that $C_{\rm abs} \propto \lambda^{-1}\epsilon_2 \propto \sigma$, which can be very large.[1] Therefore, it is possible for metallic needles with high electrical conductivities to provide a large opacity at long wavelengths to thermalize the cosmic background.

For conducting needles, equation (2) can be expressed as (Wright 1982)

$$C_{\rm abs} = \frac{C_{\rm abs}^0}{1 + (\lambda/\lambda_0)^2} , \qquad (3)$$

where the long-wavelength cutoff λ_0 is

$$\lambda_0 \equiv \frac{\rho c}{2}\frac{1 + L_{\parallel}(\epsilon_1 - 1)}{L_{\parallel}} \approx \frac{\rho c}{2}\frac{(l/a)^2}{\ln(l/a)} \qquad (4)$$

and

$$C_{\rm abs}^0 \equiv \frac{4\pi V}{3\rho c}\frac{1}{\left[1 + L_{\parallel}(\epsilon_1 - 1)\right]^2} \approx \frac{4\pi V}{3\rho c} , \qquad (5)$$

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[1] This should not be considered inconsistent with the Kramers-Kronig relation, since for a given elongation (l/a) there exists a long-wavelength cutoff (λ_0) for $C_{\rm abs}$: $C_{\rm abs} \propto \lambda^{-2}$ as $\lambda > \lambda_0$ (see eqs. [3], [4], and [5]).

where $\rho = 1/\sigma$ is the dust material resistivity. It is seen that equation (4) establishes a lower bound on the elongation l/a of the needles, which absorb strongly at wavelengths out to λ_0.

Equations (2), (3), (4), and (5) have been widely used in obtaining the dust opacity in the far-IR and microwave range (Rana 1980; Wright 1982; Hawkins & Wright 1988; Hoyle & Wickramasinghe 1988; Bond, Carr, & Hogan 1991; Wickramasinghe et al. 1992; Wickramasinghe & Hoyle 1994). However, none of these has explicitly taken into account the criterion to which the Rayleigh approximation is applicable (Bohren & Huffman 1983):

$$\frac{2\pi l}{\lambda} \ll 1 \; ; \quad |m|\frac{2\pi l}{\lambda} \ll 1 \; , \qquad (6)$$

where $m(\lambda) = m' + im''$ is the complex refractive index ($\epsilon = m^2$). For metals at long wavelengths we have $m' \approx m'' \approx (\sigma\lambda/c)^{1/2}$. Therefore, the Rayleigh approximation (eq. [6]) establishes an upper bound on the needle length:

$$l \ll \frac{1}{2\pi}\left(\frac{\lambda c}{\sigma}\right)^{1/2} = \frac{1}{2\pi}(\lambda\rho c)^{1/2} \; . \qquad (7)$$

The reason for applying this criterion for limiting the needle size is that when it is not satisfied, the cross sections given by equation (2) are overestimates of the true cross sections. It is only when all elements within the particle radiate in phase with each other (i.e., negligible phase shift of light within the particle) that we can achieve the high absorptivities (Greenberg 1980). The implication of equation (7) for cosmic needles is significant. For example, for iron needles of $\rho = 10^{-16}$ s to absorb efficiently out to $\lambda_0 = 5$ mm, equation (4) requires an elongation of $l/a \approx 1600$ (also see Wright 1982). To satisfy the Rayleigh criterion, equation (7) leads to $l \ll 1.9$ μm. A combination of equation (4) and equation (7) requires the needle radius $a \ll 12$ Å. It is unlikely that such tiny iron needles exist in astrophysical environments. After all, for stacks of layers of 2×2 and 3×3 iron atoms the needle radius would already be ≈ 2.8 and 4.2 Å, respectively. To be conservative, we therefore take the minimum radius of iron needles to be $a_{min} = 3.5$ Å. In the following text, we will take the Rayleigh criterion to be

$$|m|\frac{2\pi l_{max}}{\lambda} \approx 0.1 \; ; \quad l_{max} \approx \frac{1}{20\pi}(\lambda\rho c)^{1/2} \; , \qquad (8)$$

where l_{max} is the maximum value of the needle length l for which the Rayleigh approximation is still valid.

For a given wavelength λ and a given dust conductivity σ, which is dependent on dust material and temperature, we can obtain from equation (4) $(l/a)_{min}$—the lower limit on the needle elongation, which displays appreciable opacity at wavelengths up to λ (following Wright 1982, we take $\lambda = 5$ mm for discussion). Likewise, from equation (8) we obtain $(l/a)_{max} = l_{max}/a_{min}$—the upper limit on the needle elongation to which the Rayleigh approximation (eq. [2]) is applicable. In Figure 1 we present $(l/a)_{min}$ and $(l/a)_{max}$ for cosmic iron needles (see § 2) thermalizing the background radiation emitted at redshift z and observed at wavelength $\lambda = 5$ mm. It is seen in Figure 1 that even with $a_{min} = 3.5$ Å, $(l/a)_{max} \ll (l/a)_{min}$ for z up to 200. This clearly indicates that it is not appropriate to adopt the Rayleigh approxima-
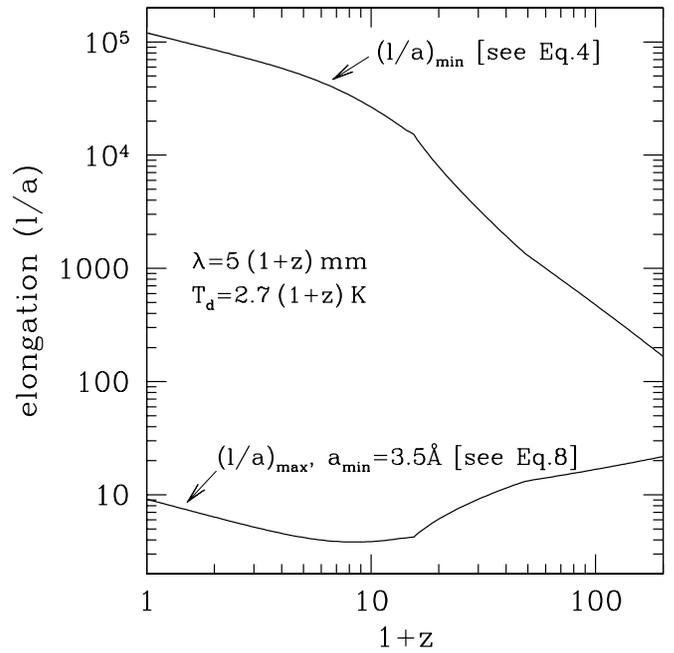

Fig. 1.—Lower $[(l/a)_{min}]$ and upper $[(l/a)_{max}]$ limits on the elongation of iron needles (assumed to thermalize background radiation emitted at z and observed at $\lambda = 5$ mm) as a function of redshift z obtained respectively from the long-wavelength opacity consideration (eq. [4]) and the Rayleigh criterion (eq. [8]). It is apparent that the Rayleigh approximation (eq. [2]) is not valid for studies of the CMB thermalization by iron needles since $(l/a)_{max} \ll (l/a)_{min}$. Note that the dust conductivity σ is a function of temperature T_d (see § 2).

tion (eq. [2]) when studying the CMB thermalization by cosmic iron needles.

The absorption cross sections of needles may be approximated by those of infinite cylinders provided l exceeds a by a factor of ~ 4 (Wickramasinghe 1973) or ~ 9 (Lind & Greenberg 1966). For needles of radius less than ~ 10 Å the classical scattering theory does not apply (Platt 1956; Greenberg 1960). We will adopt the infinite cylinder results but take a cutoff at $\lambda_c \approx 400 \, l$ (Platt 1956). Similarly to Li & Draine (2001), we assume a continuous distribution for the absorption properties of classic infinite cylinders and Platt particles ($a \lesssim 10$ Å):

$$C_{abs}(\lambda) = C_{abs}^{inf}\left[\xi_{QM}\eta_{cut} + \left(1 - \xi_{QM}\right)\right] \; , \qquad (9)$$

$$\xi_{QM}(a) = \min\left[1, (a_\xi/a)^3\right], \quad a_\xi = 10 \text{ Å} \; , \qquad (10)$$

$$\eta_{cut}(\lambda, \lambda_c) = \frac{1}{\pi}\arctan\left[\frac{10^3(y-1)^3}{y}\right] + \frac{1}{2} \; ,$$

$$y = \lambda_c/\lambda, \lambda_c = 400 \, l \; , \qquad (11)$$

where Q_{abs} is the absorption cross section per unit length divided by $2a$, $C_{abs}^{inf}(= 2aQ_{abs})$ is the absorption cross section for infinite cylinders, , and η_{cut} is a cutoff function.

In the far-IR and microwave regions, for metallic needles, both inductance and charge separation effects are ignorable (Hoyle & Wickramasinghe 1988). Therefore, conducting needles can be treated as antennae (Wright 1982): $C_{abs}/V = 4\pi/(3\rho c)$ for $\lambda < \lambda_0$, where λ_0 is the same as in equation (4). Taking into account the quantum mechanical
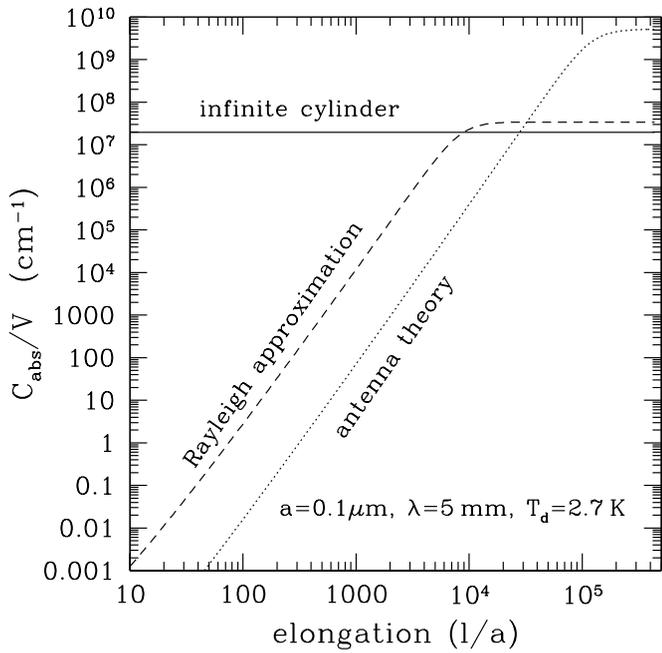
Fig. 2.—Absorption cross sections per unit volume at $\lambda = 5$ mm as a function of elongation for iron needles with of radius $a = 0.1$ μm at $T_d = 2.7$ K calculated from the infinite cylinder representation (*solid line*), Rayleigh approximation (*dashed line*), and antenna theory (*dotted line*). Note that the Rayleigh approximation results should not be considered too seriously since the Rayleigh criterion, which requires $(l/a) < 0.03$, is never satisfied.

effect, the absorption cross section of an antenna can be expressed as

$$C_{\text{abs}}(\lambda)/V = \frac{4\pi}{3\rho c} \left[\xi_{\text{QM}} \eta_{\text{cut}}(\lambda, \lambda_c) + \left(1 - \xi_{\text{QM}}\right) \right] /$$
$$\left[1 + (\lambda/\lambda_0)^2\right] , \qquad (12)$$

where the $1/[1 + (\lambda/\lambda_0)^2]$ term accounts for the cutoff at λ_0, as justified by Wright (1987).

In Figure 2 we compare the absorption cross sections (per unit volume) at $\lambda = 5$ mm calculated from infinite iron cylinders and iron antennae of radius $a = 0.1$ μm and a range of elongations at $T_d = 2.7$ K.[2] Although the Rayleigh approximation is not valid for iron needles capable of efficiently supplying far-IR and microwave opacity, we also present in Figure 2 results obtained from the Rayleigh approximation (eq. [2]) since it is widely used in literature. It is seen in Figure 2 that the infinite cylinder model predicts a much larger 5 mm opacity for $(l/a) < 3 \times 10^4$ and a much smaller one for $(l/a) > 3 \times 10^4$. The antenna model shows a rapid drop at $(l/a) < 1.2 \times 10^5$, which corresponds to a cutoff wavelength $\lambda_0 \approx 5$ mm (see eq. [4]). This is because the long-wavelength cutoff becomes smaller as (l/a) decreases so that the iron opacity at 5 mm decreases rapidly, too. This trend is also seen in the Rayleigh curve.[3] The dramatic dif-

ferences among the absorption cross sections calculated from the three methods will have dramatic effects on the CMB thermalization model (see § 3).

It is the purpose of this work first to show (see above) that the widely adopted Rayleigh approximation is not applicable to studies of the CMB as a result of thermalization by cosmic needles, and then to estimate the quantity of dust required to thermalize the background radiation using the absorption cross sections of either infinite cylinders or antennae. We will only consider iron grains since they absorb more efficiently in the far-IR than graphite and also because the upper bound on the total amount of microwave radiation generated by graphite needles was shown to be considerably smaller than the observed CMB and it was also shown that the optical depth of the graphite needle-containing cloud is not sufficiently large for the cloud to radiate like a blackbody (Sivaram & Shah 1985). Condensed in supernova ejecta, iron grains may likely form as slender whiskers by the "screw dislocation" mechanism which has been verified experimentally[4] and (at least some fraction of them) are then expelled into extragalactic space without significant destruction due to sputtering (see Hoyle & Wickramasinghe 1988 and references therein). It is interesting to note that small iron particles were among the materials initially proposed to be responsible for the interstellar reddening, based on an analogy with small meteors or micrometeorites supposedly fragmented into finer dust (Schalén 1936; Greenstein 1938). In § 2 we calculate the dielectric functions of iron grains based on the Drude theory. In § 3 we calculate the extinction optical depth caused by cosmologically distributed iron needles, estimate the amount of iron needles required to thermalize the CMB, and compare it with the available intergalactic iron density. Concluding remarks are given in § 4.

2. OPTICAL PROPERTIES OF IRON GRAINS

We use the Drude free-electron model (Bohren & Huffman 1983) to calculate the iron dielectric functions:

$$\epsilon(\omega) = 1 - \frac{(\omega_p \tau)^2}{(\omega \tau)^2 + i\omega \tau} , \qquad (13)$$

where $\omega = 2\pi c/\lambda$ is the angular frequency; ω_p is the plasma frequency

$$\omega_p^2 = \frac{4\pi e^2 n_e}{m_{\text{eff}}} , \qquad (14)$$

where e is the proton charge; n_e is the density of free electrons; m_{eff} is the effective mass of a free electron; and τ is the collision time

$$\tau^{-1} \approx \frac{\omega_p^2}{4\pi\sigma} + \frac{v_F}{a} , \qquad (15)$$

where v_F is the velocity of electrons at the Fermi surface, which we take to be $v_F = 10^8$ cm s^{-1}. The first term in the right-hand side of equation (15) is for bulk material; the

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[2] Very similar results are obtained for dust at higher temperatures. The only difference lies in the absolute scales of C_{abs}/V since the dust dielectric functions are dependent on temperature T_d (see § 2).

[3] The reason the onset l/a value of the drop in the C_{abs}/V curve for the Rayleigh approximation model differs from that for the antenna model is the second term in the right-hand side of eqs. (4) and (5) does not hold for $l/a < 10^5$ (see Fig. 1).

[4] A large number of elements such as carbon, iron, aluminum, magnesium, potassium, and mercury are observed to grow as whiskers under laboratory conditions (see Wickramasinghe et al. 1975; Hoyle & Wickramasinghe 1988 and references therein). According to Hoyle and his colleagues, the rate of whisker growth is exponential.
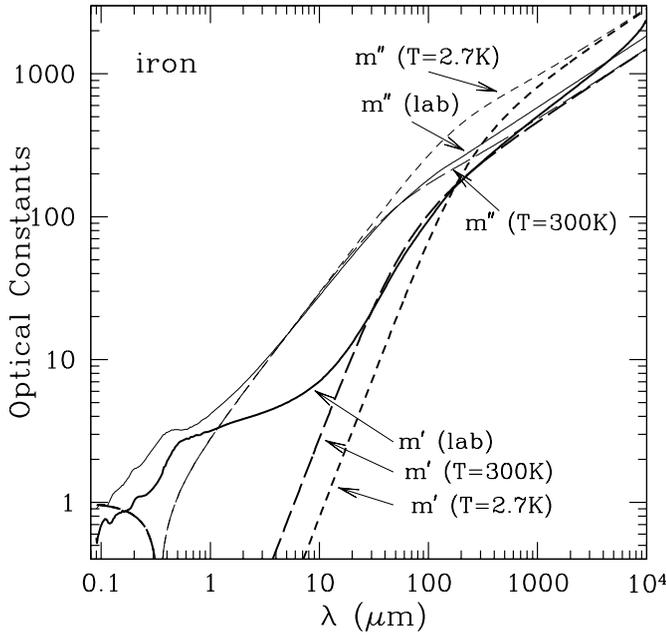
Fig. 3.—Optical constants m' (*thick lines*) and m'' (*thin lines*) of iron calculated for $T = 2.7$ K (*dashed line*) and $T = 300$ K (*long dashed line*) from the Drude free-electron model. Also shown are the experimental data (*solid line*) measured at room temperature.

second term accounts for the small size effect: the inverse of the collision time is increased because of additional collisions with the needle boundary.

Because of lack of knowledge of the temperature dependence of ω_p, we adopt the room temperature value of $\omega_p = 5.56 \times 10^{15}$ s^{-1} (Ordal et al. 1988). We fit the temperature dependent electrical resistance $\rho(T)$ (Lide & Frederikse 1994) by polynomials:

$$
\left[\frac{\rho(T)}{10^{-18} \text{ s}} \right]
$$

$$
= \begin{cases}
0.0286 - 7.57 \times 10^{-4} \, T + 5.48 \times 10^{-5} \, T^2 \, , \\
\qquad\qquad 0 \text{ K} < T < 40 \text{ K} \, , \\
0.0759 - 8.96 \times 10^{-3} \, T + 2.22 \times 10^{-4} \, T^2 \, , \\
\qquad\qquad 40 \text{ K} < T < 130 \text{ K} \, , \\
-0.300 + 0.0120 \, T + 8.38 \times 10^{-5} \, T^2 \, , \\
\qquad\qquad 130 \text{ K} < T < 1000 \text{ K} \, .
\end{cases}
\quad (16)
$$

In Figure 3 we show the refractive indices calculated for $T = 2.7$ and 300 K for $a = 0.1 \ \mu$m. We have also taken the following "synthetic" approach to obtain the room temperature complex refractive index $m(\lambda) = m' + im''$ for iron: for $0.01 < \lambda < 0.248 \ \mu$m we take m'' from Moravec, Rife, & Dexter (1976); for $0.367 < \lambda < 0.667 \ \mu$m we take m'' from Gray (1972); for $0.667 < \lambda < 285 \ \mu$m we take m'' from Ordal et al. (1988); for $\lambda > 285 \ \mu$m we approximate $m''(\lambda) \approx m''(285\mu\text{m})(\lambda/285 \ \mu\text{m})^{1/2}$ since in the far-IR for metals we have $m'' \propto (\lambda/\rho)^{1/2}$. After smoothly joining the adopted m'' (in so doing, the Gray 1972 data is reduced by a factor of 1.2), we calculate m' from m'' through the Kramers-Kronig relation (Bohren & Huffman 1983). The results are also presented in Figure 3. It is seen in Figure 3 that the Drude free-electron model provides a good approximation for $\lambda > 20 \ \mu$m.

3. ON COSMIC NEEDLES ORIGIN OF CMB

Following Draine & Shapiro (1989), we define $n_d(z) \equiv f_d(z)n_d(0)(1+z)^3$ as the number density of needle-shaped grains at redshift z, where $n_d(0)$ is a constant and f_d is the ratio of the actual number density of grains at z to the number density if the grain number per comoving volume were constant. The extinction optical depth for radiation emitted at redshift z_0 and observed at wavelength λ is

$$
\tau_{\text{ext}}(z_0, \lambda) = \left(\frac{c}{H_0} \right) n_d(0)
$$
$$
\times \int_0^{z_0} \sigma_{\text{abs}} \left[\frac{\lambda}{1+z}, T_0(1+z) \right] \frac{f_d(z)(1+z)dz}{(1+2q_0z)^{1/2}} \, ,
$$
$$
(17)
$$

where H_0 is the Hubble constant, q_0 is the cosmological deceleration parameter, T_0 (≈ 2.7 K) is the current temperature of the microwave background, and σ_{abs} is the absorption cross section[5] integrated over a distribution of needle elongations (l/a)

$$
\sigma_{\text{abs}}(\lambda, T) = \int_{(l/a)_{\text{low}}}^{(l/a)_{\text{upp}}} C_{\text{abs}}(\lambda, T) \frac{dn(l/a)}{d(l/a)} d(l/a) \, , \quad (18)
$$

where the lower cutoff $(l/a)_{\text{low}}$ is taken to be 10; the upper cutoff $(l/a)_{\text{upp}}$ is set at 10^5 since laboratory-grown iron needles display values of (l/a) up to $\sim 10^5$ (see Hoyle & Wickramasinghe 1988 and references therein; Aguirre 2000).

We assume a power-law distribution for the needle elongation for $(l/a)_{\text{low}} \leq (l/a) \leq (l/a)_{\text{upp}}$:

$$
\frac{dn(l/a)}{d(l/a)} = \frac{1}{\ln\left[(l/a)_{\text{upp}}/(l/a)_{\text{low}} \right]} (l/a)^{-1} \, , \quad \beta = 1 \, ; \quad (19)
$$

$$
\frac{dn(l/a)}{d(l/a)} = \frac{(1 - \beta)}{(l/a)_{\text{upp}}^{1-\beta} - (l/a)_{\text{low}}^{1-\beta}} (l/a)^{-\beta} \, , \quad \beta \neq 1 \, , \quad (20)
$$

where β is the power-law index. We will consider two cases: $\beta = 1$ (mass-equipartition distribution) and $\beta = 3.5$; the former may arise from a plausible whisker formation process and can be maintained through an ongoing fragmentation process (Wickramasinghe & Wallis 1996; Aguirre 1999), the latter may result from shattering from grain-grain collisions.

Let ρ_d (≈ 7.86 g cm^{-3}) be the mass density of iron grains and Ω_d be the ratio of the space-averaged comoving mass density of needles to the present critical density: $\Omega_d \equiv f_d(z)n_d(0)V^{\text{tot}}\rho_d(8\pi G/3H_0^2)$ (G is the gravitational constant), where

$$
V^{\text{tot}} = \int_{(l/a)_{\text{low}}}^{(l/a)_{\text{upp}}} \psi \pi a^3 (l/a) \frac{dn(l/a)}{d(l/a)} d(l/a) \, , \quad (21)
$$

where $\psi = 1$ for cylinders and $\psi = 4/3$ for prolates. For simplicity, we take $f_d(z) = 1$ for $0 < z < z_0$ and $f_d(z) = 0$ otherwise (i.e., assuming a uniform comoving number of dust grains back to redshift z_0; see Draine & Shapiro 1989). Therefore, we can estimate the amount of iron needles

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[5] Since the iron dielectric functions are dependent on temperature (see § 2), the absorption cross sections are thus a function of temperature, too. We set $T = T_0(1+z)$ because we want to thermalize the background, not to distort it (see Wright 1982). If iron needles act as a thermalizer to produce the $T_0(1+z)$ background, it is necessary that they are themselves thermalized to $T_0(1+z)$.
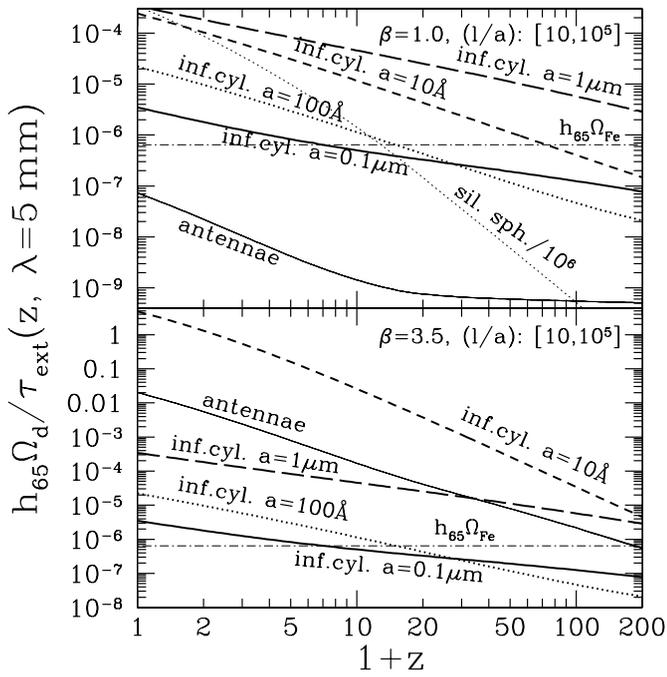
FIG. 4.—Quantity ($h_{65}\Omega_d$) of cosmic iron needles as a function of $1 + z$ required to produce an optical depth τ_{ext} at $\lambda = 5$ mm. The absorption cross sections are calculated from infinite cylinders with a radius of $a = 0.1$ μm (*heavy solid line*), $a = 100$ Å (*heavy dotted line*), $a = 10$ Å (*heavy dashed line*), and $a = 1$ μm (*heavy long-dashed line*), and the antenna theory (*thin solid line*). We assume a power-law distribution $dn/d(l/a) \propto (l/a)^{-\beta}$ for the needle elongation with $\beta = 1$ (*upper panel*) and $\beta = 3.5$ (*lower panel*) with a lower cutoff at $(l/a)_{low} = 10$ and an upper cutoff at $(l/a)_{upp} = 10^5$. For comparison, results for spherical "astronomical silicates" (Draine & Lee 1984) are also shown (reduced by a factor of 10^6; *thin dotted line*). The horizontal dot-dashed line shows the available intergalactic iron density $h_{65}\Omega_{Fe}$. It is seen that the thermalization condition $\tau_{ext} \gg 1$ at microwave wavelengths required by the isotropy and Planckian nature of the CMB is only attainable by the antenna theory for $\beta \lesssim 2.5$. However, we note that it has not been verified whether the antenna theory is valid for extremely thin needles.

required to produce an optical depth of τ_{ext} at redshift z_0 and wavelength λ:

$$\Omega_d = \left(\frac{8\pi G\rho_d}{3cH_0}\right)\tau_{ext}(z_0, \lambda)$$
$$\times \left\{\int_0^{z_0} \sigma_{abs}\left(\frac{\lambda}{1+z}, T_0[1+z]\right) \Big/ V^{tot}\frac{(1+z)dz}{(1+2q_0z)^{1/2}}\right\}^{-1}. \quad (22)$$

Adopting $H_0 = 65$ km s^{-1}Mpc^{-1} and $q_0 = 0.5$, we present in Figure 4 the results on $h_{65}\Omega_d/\tau_{ext}$ given by equation (22) for $\lambda = 5$ mm as a function of z, where $h_{65} \equiv H_0/100$ km s^{-1}Mpc$^{-1} = 0.65$. The absorption cross sections are obtained from infinite cylinders (eq. [9]) and antennae (eq. [12]) of radius $a = 10$, 100, 1000, and 10^4 Å. For comparison, results for spherical "astronomical silicates" (Draine & Lee 1984) are also plotted. It can be seen in Figure 4 that (1) Ω_d decreases with increasing z as expected from equation (22); (2) for a given z, Ω_d decreases with radius a ($a \lesssim 0.1$ μm) for infinite cylinders while it is insensitive to a for antennae, provided $a \gtrsim 10$ Å (as expected from eq. [12]); (3) for infinite cylinders with $a \gtrsim 10$ Å, Ω_d is insensitive to β, as expected from the way we calculate their

absorption cross sections; (4) for antennae Ω_d increases with β since a higher β implies fewer highly elongated needles and thus a lower opacity at $\lambda = 5$ mm since the long-wavelength cutoff shifts to shorter wavelengths for needles with smaller elongations.

Aguirre (2000) argued that the density of intergalactic dust is $\Omega_{dust} \lesssim 10^{-5}$. Assuming a solar-like metallicity ratio pattern, this leads to an iron density of $\Omega_{Fe} \lesssim 10^{-6}$. In order to obtain the observed high degree of spacial isotropy and Planckian nature of the CMB, one requires the optical depth $\tau_{ext} \gg 1$ in the microwave region (Wickramasinghe et al. 1975; Rana 1980; Sivaram & Shah 1985). It is clear from Figure 4 that it is unlikely for the intergalactic iron needles modeled as infinite cylinders to produce $\tau_{ext}(\lambda = 5$ mm$) \gg 1$ even if all available intergalactic iron has been locked up in needles since $\Omega_{Fe}/\Omega_d < 1$ for $z < 10$–20 and Ω_{Fe}/Ω_d is just about 10 for $z \sim 100$–200 given that Population III objects are currently believed to form at $z \sim 20$–30.[6] It is also clear from the spherical silicate model that direct thermalization of pregalactic starlight by intergalactic dielectric dust spherical in shape is not viable since $\Omega_{Si}/\Omega_d \ll 1$.

Wickramasinghe et al. (1992) argued that the infinite cylinder approximation for slender needles is only valid when the inductance and charge separation effects are not important, which requires $\lambda < (\rho c/4)(l/a)^2/\ln(l/a)$. This would strengthen the conclusion drawn above for the infinite cylinder model: there is simply not enough iron dust to produce a large optical depth at microwave range, since with this taken into account, the absorption cross section (per unit dust mass) at this wavelength region would be reduced. Increasing $(l/a)_{low}$ and/or $(l/a)_{upp}$ would have little effect on the infinite cylinder model. Increasing the cylinder thickness ($a > 0.1$ μm) does not help either (see Fig. 4).

However, if appealing to the antenna theory, it appears that $\tau_{ext}(\lambda = 5$ mm$) \gg 1$ is attainable by models with a relatively flat distribution of needle elongation ($\beta \lesssim 2.5$) since the absorption cross sections per unit dust mass for exceedingly elongated grains ($l/a > 3 \times 10^4$) calculated from the antenna theory are much larger than those from the infinite cylinder approximation (see Fig. 2). However, it is not clear whether the antenna theory applies to thin needles with nanometer/micrometer thickness. Unfortunately, the discrete dipole approximation, currently the most powerful tool for solving the light scattering problem of nonspherical grains, is limited to grains of small size parameters and small refractive indices (Draine 1988). Without an accurate solution, we are not at a position to either approve or disprove the antenna theory for exceedingly thin needles.

4. CONCLUSION

A wide variety of works has proposed the cosmic needle model as the CMB thermalizing agent: if cosmic metallic needle-shaped grains absorb strongly at all wavelengths

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[6] All current interstellar dust models assume that the bulk of iron condenses in silicate grains (see Li & Greenberg 2002 for a review) except the model of Chlewicki & Laureijs (1988). Therefore, it is more reasonable to assume $\lesssim 10\%$ of the intergalactic iron is locked up in whiskers (Aguirre 2000) since intergalactic iron needles are of interstellar origin. This leads to $\tau_{ext}(\lambda = 5$ mm$) \lesssim 1$–3 for $z \sim 100$–200, which is not enough to thermalize the CMB even for the most optimized case (i.e., the infinite cylinder model of $a = 100$ Å; see Fig. 4). After all, if the CMB could not be thermalized at $z < 10$–20, the blackbody nature of the CMB could not be attained even if it were thermal at $z > 100$–200.

from IR to microwave wavelengths, it is possible to ascribe the origin of observed background radiation at frequencies greater than 1 cm^{-1} to thermalization, by these slender needles, of the radiation of Population III objects. It is pointed out here that in many of these results insufficient attention was given to the limits of applicability of the small particle (Rayleigh) approximation. It is shown that the widely adopted Rayleigh approximation is not applicable to conducting needles capable of supplying high far-IR and microwave opacities. Due to the lack of an accurate solution to the absorption properties of slender needles, we model them either in terms of infinite cylinders or according to the antenna theory. It is found that the available intergalactic iron dust, if modeled as infinite cylinders, is not sufficient to produce the large optical depth at long wavelengths required by the observed isotropy and Planckian nature of the CMB. In the context of the antenna theory, conducting needles with exceedingly large elongations ($>10^4$) appear to be capable of satisfying the optical depth requirement without violating the iron density limit. However, the applicability of the antenna theory to exceedingly thin needles of nanometer/micrometer thickness needs to be justified.

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