A 3-D viscoelastoplastic model for simulating long-term slip on non-planar faults

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SUMMARY
The geometrical complexity of non-planar faults is known to affect not only fault slip rates but also crustal deformation and seismicity in the surrounding region, but implementing non-planar faults in numerical models has been challenging. We have developed a 3-D viscoelastoplastic finite element model to simulate long-term, steady-state fault slip along conceptual restraining bends and crustal deformation around them. By incorporating plastic yielding in both fault zones and the surrounding crust, this model can predict the quasi-steady state stress field and strain partitioning while avoiding pathological stress build-up encountered in traditional viscoelastic models. Here we detail the formulation of this model and explore major model parameters. The model results show that restraining bends tend to impede fault slip, increase shear stress in the surrounding crust and localize strain in a pair of belts off the fault zone. These effects are enhanced by high viscosity in the lower crust and upper mantle, high aspect ratio of bend width over fault length, fast relative motion of fault blocks and high fault friction. These model results may help explain the diffuse deformation surrounding the Big Bend of the San Andreas Fault and the more confined deformation around the Lebanon Bend of the Dead Sea Fault.

Key words: Crustal deformation; Fault slip; Finite-element methods; Restraining bends; Strain localization; Transform faults.

1 INTRODUCTION
Non-planar faults, often associated with restraining or releasing bends in major strike-slip faults, are known to change fault slip rates and cause deformation in the surrounding crust (e.g. Lisowski et al. 1991; Spotila et al. 1998; Becker et al. 2005; Meade & Hager 2005; Gomez et al. 2007). Unfortunately, non-planar faults are difficult to simulate in numerical models. Thus, previous fault models are often based on planar fault geometry (e.g. Lynch & Richards 2001) or 2-D vertical transections of faults (e.g. Laviet et al. 2000; Chery et al. 2001; Lavier & Buck 2002; Hetzel & Hampel 2005; Schmalzle et al. 2006). In a few models of restraining bends, the bent fault is approximated by segments of infinitely long fault planes, oblique to the relative plate (block) motion. These models are useful for exploring strain partitioning between strike-slip and thrust on the fault plane (Dewey et al. 1998; Fossen & Tikoff 1998; Teyssier & Tikoff 1998) but not for simulating crustal deformation around the bends.

Other models have implemented non-planar faults with various simplifications. These models include boundary element model (Bilham & Bodin 1992; Gomberg & Ellis 1994; Du & Aydin 1996; Simpson et al. 2001; Griffith & Cooke 2005), contact ele-

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We have developed a general purpose, 3-D viscoelastoplastic finite element model for simulating the deformation of lithosphere with faults and applied the model to investigate long-term fault slip and strain partitioning of the San Andreas Fault (SAF) system with its first-order non-planar fault geometry, including the Big Bend in southern California (Li & Liu 2006, 2007). Contrasting to previous models (e.g. Williams & Richardson 1991; Smith & Sandwell 2003) that assign fault slip rates as a priori condition, ours is a fully dynamic model that predicts along-strike variations of fault slip rates as a result of fault geometry and property, rheological structure of the lithosphere and tectonic boundary conditions.

Our preceding studies (Li & Liu 2006, 2007) were focused on specific issues of the SAF: the along-strike variation of fault slip rates and their impact on the stress field and seismicity in California (Li & Liu 2006) and the interaction between the SAF and the San Jacinto Fault (Li & Liu 2007). In this paper, we detail the mathematical formulation, model setting and numerical schemes of this model. To gain insights into the cause of various patterns of deformation around the restraining bends in the world, we have designed a suite of conceptual restraining bends and systematically explored the effects of the controlling factors and tectonic conditions.

2 Finite element model

Our model simulates long-term, quasi-steady state deformation of a 3-D lithosphere, containing faults. Over such timescales, stick-slip faulting can be approximated by continuous creeping in the fault zones. The fault slip and deformation in the surrounding lithosphere are calculated for specified tectonic loading, fault geometry and property (friction) and rheological structure of the model lithosphere. The deformation field is assumed to follow continuum mechanics and is determined using finite element method.

2.1 Governing equations and rheology

For long-term, steady-state lithospheric deformation, the momentum equation is reduced to that of static equilibrium

\[
\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0, \tag{1}
\]

where \(\sigma_{ij}\) is the stress tensor, \(f_i\) is the gravitational body force. In this model, we assume that the relative motion of the fault blocks is the only form of tectonic loading, thus gravitational body force is neglected, \(f_i = 0\). Furthermore, because deformation results mainly from non-lithostatic stress, we removed the lithostatic pressure in most cases. However, the lithostatic pressure can be important for plastic yielding, especially when the fault has a non-zero frictional coefficient. Thus in such cases (cases 6 and 7 in Table 2), we have incorporated the lithostatic pressure in the calculations of the plastic yield (see eq. 9 below).

The deformation (strain and strain rate) is related to stress through the constitutive equation or rheology. It has been shown that plastic rheology is important for simulating long-term deformation of the lithosphere with faults (e.g. Braun & Sambridge 1994; Lavier et al. 2000; Chery et al. 2001; Huismans et al. 2001; Lavier & Buck 2002; Kaus & Podladchikov 2006). In some previous viscoelastoplastic models, plasticity is approximated by adjusting the effective viscosity (Huismans et al. 2001). We implement the viscoelastoplastic rheology in our finite element model, by extending the classic finite element formulation of elastoplastic rheology (Cook et al. 2002) with additional viscous terms. The rheology in the model is a combination of Maxwell viscoelasticity and classic elastoplasticity rheology (Khan & Huang 1995). Plastic deformation occurs when the plastic yield strength is reached. We use the Drucker–Prager yield criterion (Drucker & Prager 1952), which is pressure dependent, similar to the Coulomb failure criterion. The numerical model with the Drucker–Prager criterion is numerically more stable in converging to solutions than with the Coulomb failure criterion.

In the calculations, each strain increment includes viscous, elastic and plastic components,

\[
\{\varepsilon\} = \{\varepsilon^v\} + \{\varepsilon^e\} + \{\varepsilon^p\}, \tag{2}
\]

where superscripts \(v\), \(e\) and \(p\) denote viscous, elastic and plastic strain, respectively. Here \(\{\varepsilon\}\) contains all six components of strain increments.

To simulate the elastoplastic behaviour in the brittle upper crust, we use an artificially high viscosity (\(10^{25}\) Pa s in the model) for the upper crust, making it effectively elastic before reaching the plastic yield strength. For the viscoelastic strain, we use linear Maxwell viscoelastic formulation where viscous strain is related to stress, and elastic strain is related to stress increments:

\[
\{\varepsilon^v\} = [\mathbf{Q}]^{-1}\{\sigma'\}dt, \tag{3}
\]

\[
\{\varepsilon^e\} = [\mathbf{D}]^{-1}\{\varepsilon\}dt, \tag{4}
\]

where \(\{\sigma'\}\) are stresses at time \(t\), \(dt\) is time increment, \(\{\varepsilon\}\) is strain increments, \([\mathbf{Q}]\) is the viscous material property matrix and \([\mathbf{D}]\) is the elastic material property matrix.

\[
[\mathbf{D}] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{pmatrix}
1 & -\nu & 0 & 0 & 0 & 0 \\
-\nu & 1 - \nu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 - \nu \\
0 & 0 & 0 & 0 & 0 & 0.5 - \nu \\
0 & 0 & 0 & 0 & 0 & 0.5 - \nu \\
0 & 0 & 0 & 0 & 0 & 0.5 - \nu
\end{pmatrix}, \tag{5}
\]

where \(\nu\) is Poisson’s ratio, \(E\) is Young’s modulus and \(\nu\) is Poisson’s ratio.

Taking the first-order backward difference scheme in time, stresses at time \(t\) are expressed as follows:

\[
\{\sigma'\} = \{\sigma\} - \{\varepsilon\}dt. \tag{6}
\]

From eqs (2), (3) and (6), we obtain

\[
\{\varepsilon\} = \{\mathbf{D}\}(\{\varepsilon\} - \{\varepsilon^p\}) + \{\varepsilon^p\}, \tag{7}
\]

where

\[
[\mathbf{D}] = ([\mathbf{D}]^{-1} + [\mathbf{Q}]^{-1})^{-1}, \tag{8}
\]

\[
\{\varepsilon^p\} = -[\mathbf{D}]\{\varepsilon\}dt\{\sigma'\}dt. \tag{9}
\]

Plastic deformation occurs when stress reaches the plastic yield criterion (yield envelope), described here by the Drucker–Prager yield function

\[
F = \alpha I_1 + \sqrt{J_2 - k}, \tag{10}
\]

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where \( I_1 \) is the first invariant of the stress tensor and \( J_2 \) is the second invariant of the deviatoric stress tensor. When lithostatic pressure is considered, \( I_1 \) is replaced with the summation of lithostatic pressure and mean normal tectonic stress. Otherwise, \( I_1 \) equals the mean normal tectonic stress. The parameters \( \alpha \) and \( k \) are related to cohesion and effective frictional coefficient, respectively. Plastic strain direction (the flow rule) is specified with a plastic potential function \( G \):

\[
G = \sqrt{J_2}.
\]

Plastic strain increments are given by

\[
\{d\varepsilon^p\} = \left\{ \frac{\partial G}{\partial \varepsilon} \right\} \{d\varepsilon\},
\]

where \( d\varepsilon \) is a ‘plastic multiplier’. With the condition that the stress stays on the yield envelope during plastic yielding, \( dF = 0 \), we obtain from eqs (7), (9), and (11) that

\[
d\varepsilon = \left\{ \frac{\sigma}{J_2} \right\}^T \left( \{D\} \{d\varepsilon\} + \{d\varepsilon\} - \{d\varepsilon_p\} \right) / \left\{ \frac{\sigma}{J_2} \right\}^T \{D\} \left\{ \frac{\sigma}{J_2} \right\}.
\]

From eqs (7), (11) and (12), we obtain the 3-D constitutive equations for the viscoelastoplastic rheology

\[
\{d\sigma\} = \{\tilde{D}\} \{d\varepsilon\} + \{d\varepsilon\} - \{d\varepsilon_p\},
\]

where

\[
\{d\varepsilon_p\} = \left\{ \frac{\sigma}{J_2} \right\}^T \left( \{D\} \{d\varepsilon\} + \{d\varepsilon\} - \{d\varepsilon_p\} \right) / \left\{ \frac{\sigma}{J_2} \right\}^T \{D\} \left\{ \frac{\sigma}{J_2} \right\},
\]

\[
\{d\varepsilon\} = \left\{ \frac{\sigma}{J_2} \right\}^T \left( \{D\} \{d\varepsilon\} + \{d\varepsilon\} - \{d\varepsilon_p\} \right) / \left\{ \frac{\sigma}{J_2} \right\}^T \{D\} \left\{ \frac{\sigma}{J_2} \right\}.
\]

2.2 Finite element formulation

With the Galerkin weighted residual method (Cook et al. 2002), from the equilibrium and constitutive equations we obtain

\[
\int_V \{\delta\varepsilon\}^T \left( \{D\} \{d\varepsilon\} \right) dV = \int_V \{\delta\varepsilon\}^T \{d\varepsilon\} dV - \int_V \{\delta\varepsilon\}^T \{d\varepsilon_p\} dV
\]

\[
+ \int_V \{\delta\varepsilon\}^T \{H'\} d\Gamma - \int_V \{\delta\varepsilon\}^T \{\sigma\} dV,
\]

where \( V \) is the model domain and \( H \) are boundary forces at boundary \( \Gamma \). The equilibrium exists at time \( t \). At the absence of plastic yielding \( (F < 0) \) or transient stress unloading \( (F = 0 \) and \( dF < 0) \), \( \{D\} \) and \( \{d\varepsilon_p\} \) are set to zero. From eq. (15), we obtain element tangent stiffness matrix and load vector

\[
\{K\} = \int_V \{B\}^T \left( \{D\} \{d\varepsilon\} \right) dV,
\]

\[
\{dR\} = \int_V \{B\}^T \{d\varepsilon_p\} dV + \int_V \{N\}^T \{H'\} d\Gamma - \int_V \{B\}^T \{\sigma\} dV,
\]

where \( \{B\} \) is the displacement-strain matrix, \( \{N\} \) is the shape function. By assembling the element matrix, we obtain the system of equations in a matrix form

\[
[K]\{dU\} = \{dR\}
\]

where \( \{dU\} \) is the displacement increment from time \( t - dt \) to \( t \), \( [K] \) is the assembled stiffness matrix, and \( \{dR\} \) is the assembled load vector.

2.3 Computational procedure

This finite element model determines nodal displacements and stresses in a time period with known quasi-static external loads, such as that from continuous relative plate (block) motion. Our computation procedure is similar to that of classic elastoplastic finite element method (Cook et al. 2002). Stress sampling and calculations are performed at each Gaussian point.

We require that deformation and stresses produced at time \( t - dt \) do not violate conditions of equilibrium and compatibility and the plastic yield criterion. The following steps are taken to determine displacements and stresses at time \( t \) (after a time increment \( dt \)): (1) obtain a trial stress with displacement increment from the preceding iteration step; (2) determine the fraction of viscoelastic deformation and the associated change of viscoelastic stress in this time step; (3) calculate the stress change due to plastic deformation with viscoelastoplastic tangent stiffness matrix; (4) adjust the predicted stress to ensure that the new stress state is within plastic yield envelope and (5) update stiffness matrix and load vector with the new stress state and calculate displacement increments with eq. (17). The above steps are iterated until convergence is reached.

Spatial oscillation of the first invariant of stress tensor may occur among Gaussian points because the selective reduced integration method, used in the model, reduces volumetric constraints in each element. The volumetric constraint is applied to element centre instead of Gaussian points. The first stress invariant that is weight-averaged within each element or on element nodes, does not oscillate. So, we use the averaged nodal stresses from the preceding iteration step, rather than the stresses at the Gaussian points, to calculate the first stress invariant in the Drucker–Prager failure criterion (eq. 9).

2.4 Code parallelization

Our codes are parallelized to take advantage of high performance parallel computers. The parallelization is based on the Message Passing Interface (MPI), which is a standard library for passing information among individual computing nodes. The finite element mesh is partitioned into a number of overlapped subdomains with METIS, which is a set of libraries based on multilevel graph partitioning algorithms (Karypis & Kumar 1999).

The finite element computation, including all steps in the procedure described above, is highly parallelized. Each computing node makes element matrices and matrix assemblages for each individual subdomain in parallel. The assembled equations are solved to get incremental nodal displacement with AZTEC, a massively parallel iterative solver library for solving sparse linear systems (Tuminaro et al. 1999), and stress changes are calculated with increments of nodal displacement, in parallel, on multiple processors. Most computation of this work was performed on a 16-node dual-processor PC cluster; we have also run some cases on a 128-node quad-processor PC cluster.
3 NUMERICAL MODEL OF A CONCEPTUAL RESTRAINING BEND

We illustrate the 3-D viscoelastoplastic model by applying it to investigate lithospheric deformation around a conceptual restraining bend. To understand the effects of key model parameters, we first set up a reference model (Fig. 1) and then systematically explore the impacts of each model parameter by changing the parameter and comparing the results with that of the reference model. This reference model covers a region of 400 km × 600 km (Fig. 1). It has a 20-km-thick brittle (elastoplastic) upper crust (the schizosphere) and a 40-km-thick viscoelastic lower crust and mantle (the plastosphere). A strike-slip fault, dipping at 90°, cut through the entire model domain and extends to the top of the plastosphere. The surface trace of the restraining bend is defined by the function of $y = \sin(0.4433x)$. The fault is simulated with a 400-m-thick layer of fault elements, which have a weaker plastic yield strength than the surrounding crust (Table 1).

The model domain is loaded by imposing a 49 mm yr$^{-1}$ velocity in the $y$-direction along its left-hand edge; the right-hand edge is fixed (Fig. 1). The top surface is free; the bottom is free-slip in the horizontal direction and fixed in the vertical direction. All side surfaces are fixed, except that the front and back surfaces are free to move in the $y$-direction.

Model parameters with constant values in all cases are listed in Table 1. These parameter values are within reasonable ranges according to previous measurements and modelling studies (e.g. Lachenbruch & Sass 1980; Kenner & Segall 2000a,b; Pollitz et al. 2001; Pollitz 2003). The major variables explored in this study are given in Table 2.

The numerical simulations start with a zero initial stress. The model region is loaded by the continuous relative block motion. After ~100 000 model yr, the model reaches a quasi-steady state, with nearly constant stresses and strain rates. The results shown below represent the quasi-steady state values. The cumulative fault motion over the entire model period is small (<5 km) comparing

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Table 1. Major parameters in the model.

<table>
<thead>
<tr>
<th>Material block</th>
<th>Young's modulus (Pa)</th>
<th>Poisson's ratio</th>
<th>Viscosity (Pa s)</th>
<th>Frictional coefficient</th>
<th>Cohesion (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper crust</td>
<td>$8.75 \times 10^{10}$</td>
<td>0.25</td>
<td>$10^{33}$</td>
<td>0.4</td>
<td>50</td>
</tr>
<tr>
<td>Lower crust and mantle</td>
<td>$8.75 \times 10^{10}$</td>
<td>0.25</td>
<td>$4 \times 10^{19}$–$10^{21}$</td>
<td>0.4</td>
<td>50</td>
</tr>
<tr>
<td>Fault</td>
<td>$8.75 \times 10^{10}$</td>
<td>0.25</td>
<td>$10^{33}$</td>
<td>0–0.1</td>
<td>10</td>
</tr>
<tr>
<td>Fault downward extension</td>
<td>$8.75 \times 10^{10}$</td>
<td>0.25</td>
<td>$4 \times 10^{19}$–$10^{21}$</td>
<td>0–0.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2. Model variables for numerical experiments.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Viscosity of lower crust and mantle (Pa s)</th>
<th>Fault bend width (km)</th>
<th>Relative block velocity (mm yr$^{-1}$)</th>
<th>Fault friction</th>
<th>Consider lithostatic pressure?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 \times 10^{20}$</td>
<td>100</td>
<td>49</td>
<td>0.0</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>$4 \times 10^{19}$</td>
<td>100</td>
<td>49</td>
<td>0.0</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>$10^{21}$</td>
<td>100</td>
<td>49</td>
<td>0.0</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>$2 \times 10^{20}$</td>
<td>67</td>
<td>49</td>
<td>0.0</td>
<td>no</td>
</tr>
<tr>
<td>5</td>
<td>$2 \times 10^{20}$</td>
<td>67</td>
<td>49</td>
<td>0.0</td>
<td>no</td>
</tr>
<tr>
<td>6</td>
<td>$2 \times 10^{20}$</td>
<td>100</td>
<td>49</td>
<td>0.0</td>
<td>yes</td>
</tr>
<tr>
<td>7</td>
<td>$2 \times 10^{20}$</td>
<td>100</td>
<td>49</td>
<td>0.1</td>
<td>yes</td>
</tr>
</tbody>
</table>
to the dimension of the fault. The resulting changes of the fault geometry, which is important to the model, is insignificant, hence no re-meshing is necessary during time stepping. In this way, distortion of fault element is avoided.

4 MODEL RESULTS

4.1 The reference model

To illustrate the effects of key model parameters, we use case 1 (Table 2) as a reference model, with which other cases in Table 2 will be compared.

Fig. 2(a) shows the predicted surface horizontal velocities. The sharp drop of velocities across the fault differ from the GPS velocities across some strike-slip faults (e.g. Meade & Hager 2005; Gomez et al. 2007), because the GPS velocities represent mostly interseismic strain rates, whereas the long-term velocities in Fig. 2(a) represent the time-averaged values of interseismic and coseismic crustal motion. If the fault is planar and parallel to the direction of relative block motion, there would be no motions within the right-hand block, as commonly envisioned in rigid plate motion models (DeMets et al. 1994). However, the restraining bend hampers the relative motion, causing decrease and rotation of velocity vectors near it (Fig. 2a).

The portion of the long-term relative block motion, not accommodated by slip on the fault, must be absorbed by plastic deformation outside the fault (Fig. 2b). The predicted plastic strain rate shown here is the effective strain rate for the upper crust. The predicted plastic strain is concentrated in a pair of belts, branching from the fault where it starts to bend and extending subparallel to the straight segments of the fault. The model also predicts two minor strain belts connecting the major strain belts around the restraining bend. These belts of localized strain may become the sites of future fault zones, as the fault system gradually adjust itself to accommodate the relative crustal motion (Li & Liu 2006, 2007).

The restraining bend may significantly affect regional stress. Fig. 3(a) shows the distribution of deviatoric (shear) stresses, given by the square root of the second invariant of the deviatoric stress tensor. Around the restraining bend, the shear stress increases to ∼50–65 MPa near the straight fault segment, and the high shear stress diffuse broadly along the direction perpendicular to the bend. Fig. 3(b) shows the distribution of the mean normal stress, which is an average of the three principal stresses. Because lithostatic pressure is neglected in this case, the mean normal stress here represents relative stress variation instead of the absolute values. The mean normal stress is compressive around the fault bend as expected, consistent with observed mountain ranges around restraining bends, such as the Transverse Ranges around the Big Bend of the SAF and the Lebanon Mountains around the Lebanon bend of the Dead Sea Fault. As the fault straightens out, the mean normal stress becomes extensional. The extension results from clamping along the bend, which causes the crust blocks across the fault to move apart in the x-direction (Fig. 2a). Parsons (2002a) suggests that long-term interaction of faults may unclamp parts of the faults, causing nearly frictionless faulting. The results here suggest that transpression in restraining bends could also unclamp nearby fault segments.

Fig. 3(c) shows the distribution of the principal stresses in the horizontal plane. Around the fault bend, the model predicts high compressive stresses and low extensional stresses, which are consistent with predominant thrust faulting near restraining bends. The maximum compressive stresses are at high angles (60–80°) to the bent fault trace, similar to that along the Big Bend of the SAF (Townend & Zoback 2004), although stress orientation around the SAF remains debatable (Hardebeck & Michael 2004).

Note that the bands of localized plastic strain (Fig. 2b) do not form in regions of the highest deviatoric stress (Fig. 3a). This is because

Figure 2. Results of the reference model (case 1). The black curve shows the surface trace of the fault. (a) Long-term horizontal surface velocities (arrows); background colour shows velocity magnitude; (b) Plastic strain rate (off-fault regions only).
Figure 3. (a) Deviatoric stress (the square root of the second invariant of the deviatoric stress tensor), (b) mean normal stress (compression is negative) and (c) horizontal principal stresses of the reference model (case 1). Blue bars, compressive stresses; red bars, extensional stresses. The length of the bars is proportional to stress magnitude.

The conspicuous bands of localized plastic strain in this model may have important implications for long-term evolution of fault systems (Li & Liu 2006, 2007). To investigate the growth of these plastic strain bands and to assess the potential impacts of the boundary conditions in the model, we modified this model by extending the left- and right-hand boundaries further by 100 km (Fig. 4). The continuous loading increases both the shear stress and the mean normal stress along the restraining bend and in the surrounding crust, as shown in Fig. 2. Close to the bend, the stress reaches the failure criterion, that is, the yield function becomes zero (Fig. 4a). Thus, plastic strain initiates at these locations (Fig. 4b). As the loading continues, a larger region reaches the failure criterion (Fig. 4c), hence the broader plastic deformation (Fig. 4d). Note that within the regions that have reached the plastic yield criterion, plastic strain is not uniformly distributed. Rather, the rate of plastic strain depends on the strain rate in the surrounding regions. Further, loading causes more regions to reach the plastic yield criterion, where the stress is capped by the yield strength (Fig. 4e). The plastic deformation gradually concentrates into a belt around the restraining bend and subparallel to the fault (Fig. 4f). The belt is where the velocity gradient is high, and where the shear stress (which promotes plastic deformation) outweighs the mean normal stress (which hinders plastic yielding). Localization of the plastic strain in the belt releases stress in other regions where stress may fall below the yield strength (Fig. 4g). As the model approaches a steady state, the strain pattern becomes stable, featuring two strain belts that are subparallel to the fault, forming a thorough-going pathway that goes around the restraining bend (Fig. 4h). The results in steady state are identical to those in Fig. 2(b), although here the fault is 100 km farther away from the left- and right-hand boundaries. Thus, the predicted strain localization is an intrinsic consequence of the restraining bend.

4.2 Effects of viscosity

One important model parameter is the viscosity of the plastosphere, which includes the lower crust and the uppermost mantle. We run two model cases (Table 2) with a lower (case 2) and higher (case 3) viscosity relative to the reference model (case 1). A higher viscosity of the plastosphere implies stronger mechanical coupling with the brittle upper layer. Consequently plastic deformation outside the fault zone is broader (Fig. 5).

As expected, the restraining bend impedes slip on the fault. Fig. 6 shows the along-strike variation of the predicted long-term slip rates, given by the relative velocity between mesh nodes across fault. The summation of fault slip and the horizontally integrated plastic deformation (along the x-direction) outside the fault is a constant that equals to the relative plate motion. In the reference model case, fault slip accommodates 30–40 mm yr$^{-1}$ of the total 49 mm yr$^{-1}$ relative block motion imposed on the model boundary, with the minimum (∼30 mm yr$^{-1}$) occurring along the restraining bend. For a higher viscosity of the plastosphere, the fault slip rates are lower, and their difference between the bent and straight fault segments are larger. The opposite is true when the viscosity is lower (Fig. 6).

This result differs from that from previous 2-D models (Roy & Royden 2000a,b), which suggest the opposite effect of viscosity.
The difference may arise from the different loading mechanism assumed in these models. Roy & Roydon (2000a, b) uses bottom loading, with specified velocities at the bottom boundary of their model. The viscous lower crust transmits bottom loading to the upper crust. Because the deformation of lithospheric mantle is assumed to be concentrated along plate boundary in the mantle, increasing lower crust viscosity causes more concentrated deformation along the boundary in the upper crust. Our model is driven by the imposed far-field velocity at the side boundary, a condition that is better constrained than the lithospheric mantle deformation. In our model, the viscous lower crust and upper mantle passively flows under the elastoplastic upper crust. Thus, a more viscous lower layer leads to stronger coupling with the upper layer, hence more diffuse deformation.

4.3 Effects of aspect ratio of a restraining bend

Given that restraining bends impede fault slip and diffuse stress, it is reasonable to expect that their sizes, relative to the fault length, would matter. In our conceptual model, the size of the restraining bend is measured by its width, defined here as the orthogonal distance between the two parallel straight fault segments. We
define the aspect ratio of a restraining bend as its width over the fault length. In case 4 (Table 2), we reduced the width of the restraining bend to 67 km from 100 km in case 1; the two models are otherwise identical. The results show less plastic deformation around the restraining bend (Fig. 7), and higher fault slip rates (Fig. 8), for a smaller bend (or more precisely, a smaller aspect ratio). Similar results are obtained by increasing fault length while fixing the bend size.

These results may be explained by the competing effects of the strike-slip faults and the restraining bend: the fault tends to concentrate strain, whereas the restraining bend tends to diffuse deformation into the surrounding regions. The smaller the restraining bend is, the weaker its impact on strain diffusion. Consequently, more strain is accommodated by fault slip.

We should note that the impact of the aspect ratio discussed here is more accurate near the restraining bend. Away from it, the effects may diminish non-linearly. Furthermore, the effect of the size of the restraining bend may also be affected by the thickness of the elastoplastic upper crust. Nonetheless, the aspect ratio is a useful indicator of how a restraining bend may affect crustal deformation.
4.4 Effects of relative block velocity

Across major strike-slip faults, the relative far-field velocities vary from less than a few to tens of mm yr$^{-1}$ (e.g. Meade & Hager 2005; Gomez et al. 2007). Could the variations of relative block velocity affect deformation around the restraining bends? We explored this issue with case 5 (Table 2), which is identical to case 4, except that the relative block velocity is reduced from 49 to 6 mm yr$^{-1}$. As shown in Fig. 9, when the relative block motion is slower (case 5), plastic deformation outside the fault is much weaker and spatially more confined (note the different scales in Figs 9a and b). Accordingly, a higher fraction of the relative block motion is accommodated by the fault (Fig. 10).

A slower relative block velocity causes lower strain rates, hence lower stress for a given viscosity in the plastosphere. This means weaker mechanical coupling between the schizosphere and the plastosphere, similar to having a less viscous plastosphere (case 2 in Table 2). Thus, in both cases 5 and 2, strain is more concentrated on the fault, and the impact of the restraining bend is reduced.

4.5 Effects of fault friction

Fault friction is important for fault slip but often difficult to constrain (Bird & Kong 1994). Some workers have suggested high frictional coefficient, based on experimental results (Byerlee 1978) and

In all the preceding model cases, the fault frictional coefficient is set to zero, approximating a weak fault (Bird & Kong 1994). Here, we explore the effects of fault friction by comparing two new cases, one with the frictional coefficient set to zero (case 6), the other to 0.1 (case 7). When the fault friction is non-zero, lithostatic pressure needs to be included in calculating the plastic yield strength. For easy comparison, we included the lithostatic pressure in both cases. The lithostatic pressure is taken as the weight of a lithospheric column to a given depth, without considering the dynamic effect of the deformed surface. A comparison of Fig. 11(a) and 11(b) shows that higher friction in the fault causes stronger mechanical coupling across the fault and forces more strain to
be partitioned into the surrounding crust (Fig. 11). Accordingly, the predicted fault slip rates are lower in case 7 than in case 6 (Fig. 12).

Note that case 6 is identical to case 1, except the inclusion of the lithostatic pressure in the calculations of plastic yield. The predicted plastic strain in these two cases (Figs 2b and 11a) is similar but with notable differences. This is because in both cases, a non-zero frictional coefficient (0.4) is used for calculating plastic yield in the off-fault crust. The lithostatic pressure in case 6 hampers plastic deformation in the off-fault regions; thus forces more slip on the fault. Nevertheless, the general similarity between the predicted plastic strain in Figs 2(b) and 11(a) justifies the omission of lithostatic pressure in other cases, when the fault is friction-free.

5 DISCUSSION

The 3-D parallel viscoelastoplastic finite element model presented here provides a useful tool for simulating long-term lithospheric deformation with faults. We applied this technique to investigate deformation associated with restraining bends of major strike-slip faults. To better elucidate crustal deformation with the non-linear
viscoelastoplastic rheology, we modelled a suite of simple conceptual restraining bends and systematically explored key model parameters. The effects of these parameters appear concordant in all model cases, attesting to both the reproducibility and numerical stability of the finite element model.

Although in this study, we have focused on long-term deformation around restraining bends, this 3-D viscoelastoplastic model has been developed as a general-purpose tool for lithospheric geodynamics. Hence, even though long-term crustal deformation may be adequately simulated with viscoplastic rheology (Bird & Kong 1994), we kept elastic strain in the model; so, it can be readily adapted for simulating short-term, transient crustal deformation, associated with seismic cycles (Liu et al. 2007). Some important features have yet to be incorporated into the model; one of them is gravitational body force. Although gravitational body force is critical for deformation at longer timescales (England & Houseman 1989; Sonder & Jones 1999; Flesch 2000), their effects are limited for crustal deformation over timescales of thousands to tens of thousands of years (Liu & Yang 2003), as is the case in this study. The model parameters explored in this study are those, likely responsible for producing various deformation patterns of the restraining bends; thus, not all parameters or boundary conditions are explored. For example, stress boundary conditions or basal velocity field in the ductile lower crust have been used in some studies of transform plate boundary zones (e.g. Bourne et al. 1998). Although one can use these loading mechanisms in our model, far-field velocity boundary conditions used in our model are usually better constrained than stresses at the boundary or flow velocity in the lower crust. Likewise, other reasonable boundary conditions may be applied to the front and back model boundaries. However, the boundary condition (free motion in y-direction), used in this work best fits the assumption that the fault has a finite length and plastic shearing occur not only on the fault zone but also in the crust outside the fault.

The most conspicuous features in the model results are the localization of plastic strain in bands subparallel to the fault. These belts are symmetric in our simple conceptual model. However, the complexity of natural fault geometry, as well as the existence of other active faults in the fault system, can significantly alter the patterns of strain localization (Li & Liu 2006, 2007). In particular, Li & Liu (2006, 2007) have predicted asymmetric bands of localized strain off the Big Bend, with the major belt of strain localization coinciding with the Eastern California Shear Zone. Our model does not address how such plastic deformation occur in nature, which may include sliding and folding along existing faults or initiating new faults. Doing so would require more sophisticated and specific rheology, such as material degradation and healing (Lyakhovsky et al. 1997, 2001) of the fault zones—a task that is beyond the scope of this study.

The main results of this study are physically intuitive: restraining bends impede fault slip, cause high compressive stress over the bends and high shear stress around the bends and force the surrounding crust to absorb the strain that cannot be accommodated by fault slip. These results are comparable to observations of major restraining bends. For example, along the SAF, the lowest fault slip rates are found over the Big Bend in southern California (Becker et al. 2005). The predicted compressive stress patterns (Fig. 3) are comparable to the patterns and locations of thrust faults and uplifts in the Transverse Ranges over the Big Bend of the SAF (Fig. 13a) and the Lebanon bend of the Dead Sea Fault (Fig. 13b). The high shear stresses around restraining bends are consistent with diffuse seismicity around the Big Bend (Fig. 13a) and the Lebanon Bend (Fig. 13b).

Our model results may also help to explain the major differences between the Big Bend and the Lebanon Bend. The Big Bend is surrounded by broadly distributed seismicity and secondary faults (Fig. 13a). In contrast, around the Lebanon Bend, the seismicity and secondary faults are more confined to be near the bend (Fig. 13b).

Figure 13. (a) Active faults (lines), seismicity and topographic relief in southern California. The labelled faults include the San Andreas Fault (SAF), the San Jacinto Fault (SJF), the Elsinore Fault (EF) and the Imperial Fault (IMF). Circles show epicentres of earthquakes (M > 5.0) from 1800 to present from NEIC catalogue. Red line shows the main trace of the SAF and dashed lines delineate the approximate boundaries of deformation zones outside the SAF, which enclose major active faults and historic earthquakes. (b) Topographic relief map with active faults and seismicity around the Lebanon Bend of the Dead Sea Fault. The labelled faults include the Roum Fault (RF), the Rachaya Fault (RF), the Serghaya Fault (SF), the Yamouneh Fault (YF), the Coastal Flexure (CF), the Offshore Flexure (OF) and the Missyaf Fault (MF). Circles show epicentres of earthquakes (M > 4.0) from 1983 to present and significant historical earthquakes (M > 6.0) since 759 BC from the NEIC catalogue. Red line shows the main trace of the Dead Sea Fault and dashed lines delineate the approximate boundaries of the deformation zone outside the Dead Sea Fault, which enclose major active faults and historic earthquakes. Note the different length scales between (a) and (b). Red arrows show the directions of relative plate motion.
As we have shown, both a higher aspect ratio of bend width relative to the fault length (Fig. 7) and a faster relative motion of the fault blocks (Fig. 9) tend to cause broader crustal deformation. The Big Bend is about ∼125 km wide comparing with the ∼1200 km long SAF, whereas the Lebanon Bend is only ∼60 km wide (from west to east, Fig. 13b) relative to the ∼1000 km Dead Sea Fault, hence the more diffuse deformation around the Big Bend. Furthermore, the far-field relative velocity is ∼49 mm yr⁻¹ for the SAF and only ∼6–7 mm yr⁻¹ for the Lebanon Bend. This also contributes to the more diffuse deformation around the Big Bend.

6 Conclusions

We have developed a 3-D viscoelastoplastic finite element model for simulating lithospheric deformation with faults. The finite element codes are fully parallelized, allowing users to take advantage of the computing power of PC clusters and other parallel computers. The computer codes are available on request from the corresponding author.

In this study, we adapted this model to simulate long-term, quasi-steady state slip on non-planar faults, in restraining bends and deformation in the surrounding crust. Our model results show that restraining bends tend to impede fault slip, causing diffuse deformation in the surrounding crust and localizing strain in bands subparallel to the fault, which may become the future main fault as the fault system evolves. Systematical exploration of model parameters show that the following factors would amplify the impact of restraining bends: (1) high viscosity of the lower crust and upper mantle; (2) high aspect ratio of bend width to fault length of the restraining bend; (3) fast relative motion across the fault blocks and (4) high fault friction. These results may help explain various deformation styles around major restraining bends, such as the broad deformation around the Big Bend of the SAF and the more confined deformation around the Lebanon Bend of the Dead Sea Fault.

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References


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