Customization with Vertically Differentiated Products*

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Abstract

We consider a duopoly market with heterogenous consumers. The firms initially produce vertically differentiated standard products located at the end points of the variety interval. Customization provides ideal varieties for consumers but has no effect on quality. The firms first choose whether to customize their products, then engage in price competition. We show that the low quality firm never customizes alone; customization becomes more likely as the difference between the firms' qualities increases; and less likely as the fixed cost of customization increases.

We extend the base model by relaxing two important assumptions – uniform pricing and exogenous quality. The main conclusions with uniform pricing continue to hold when price customization is allowed. In the second extension the firms' qualities are endogenously determined. We show that the firms choose to be either substantially differentiated in quality or non-differentiated.

Key words: customization, horizontal differentiation, vertical differentiation

JEL codes: D43, L13, C72

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1 Introduction

Customization is a flexible technology designed to produce individually tailored products without significantly compromising cost efficiency. Advances in Internet-based information technologies and improvements in manufacturing flexibility have made customization a reality in many product categories. For example, Dell builds to order notebook and desktop computers; Lands' End offers custom-crafted pants and shirts; Timissimo customizes wrist watches; Rug Rats create rugs and carpets that reflect each customer's personal needs.[1]

Most of the existing theoretical literature on customization adopt one-dimensional horizontal differentiation settings (e.g., Dewan, Jing, and Seidmann 2003, Syam and Kumar 2006, Alexandrov 2008, Mendelson and Parlaktürk 2008, and Xia and Rajagopalan 2009). Customization enables consumers to get their ideal products represented by their locations in the product attribute space. Even though many aspects of customization are captured by these studies, important issues have yet to be examined. Casual empiricism indicates that higher quality/priced firms are more likely to offer customization. For example, National Bicycle Industrial Company custom manufactures high-end bicycles (Kotha 1995); Nike and Adidas (Moser, Muller, and Piller 2006) allow consumers to create their most preferred athletic pair of shoes, whereas lower quality sports shoe producers do not offer customization; Timbuk2 (Cattani, Dahan, Schmidt 2005) customizes its "good-looking, tough-as-Hell" messenger bags and backpacks, while bags of mediocre quality are not customized.

In the operations management literature, a number of authors have pointed out the importance of product quality as a success factor of customization (Pine 1993, Tu, Vonderembse, and Ragu-Nathan 2001, Broekhuizen and Alsem 2002, and Selladurai 2004). However, quality has not been a focus in any of the existing theoretical studies. In the present paper we fill this gap by incorporating quality into product customization competition.

We consider an industry in which products are characterized by variety and quality. Variety is a horizontal attribute and quality is a vertical attribute.[2] Consumer preferences are heterogenous in two dimensions. Each consumer has a most preferred variety and a quality valuation. There are two firms that initially produce standard products located at the end points of the variety interval. The firms are asymmetric due to having different (exogenously given) qualities. Customization provides ideal varieties for consumers but has no effect on quality. The firms play a sequential two-stage game. In the first stage, they simultaneously decide whether to customize their products. In the second stage, they simultaneously choose their prices.

In the base model, we assume that customizing firms are restricted to uniform pricing strategies. Obviously, the idea of setting a different price for each variant of a customized product is very appealing. While some firms do engage in price customization (e.g., Dell, Ford, and Rug Rats), many firms practice uniform pricing. For example, Lands' End charges $74 for a pair of jeans regardless of the options chosen; Timbuk2's price is not linked to color and fabric selections. The theoretical

[1] For more examples see Piller, Moeslin, and Sotko (2004) and the case studies cited in Moser and Piller (2006).

[2] Our model is based on the literature that combines horizontal and vertical differentiation, e.g., Economides (1989) and Neven and Thisse (1990).

literature on customization (reviewed below) is divided between models with uniform pricing and models with price customization. In an extension of our base model we allow customizing firms to customize prices.

Our equilibrium analysis shows that either both firms customize, only the high quality firm customizes, or no firm customizes. The appearance of this sequence of outcomes is monotone in the fixed cost of customization. The key to this result is that the high quality firm always gains more from customization than the low quality firm. As a result, both firms customize when the cost of customization is small, only the high quality firm customizes for intermediate levels of customization cost, and neither firm customizes when the cost is high.

We show that even if customization is costless, each of the three equilibria mentioned above is possible. In particular, both firms choose not to customize when the difference between their qualities is small, only the high quality firm customizes for moderate quality differences, and both firms customize when the quality difference is large. The intuition behind this result is as follows. Customization by one or both firms makes the rivals "closer" to each other, thus intensifying price competition. The smaller is the quality difference, the tougher is price competition. In the extreme case in which the quality difference is zero and both firms customize, price competition results in the Bertrand outcome. Therefore, when the quality difference is small, the firms do not customize their products in order to avoid a price war. When the quality difference is large, the firms customize to take advantage of consumers' desires for ideal varieties. The intermediate case involves customization by the high quality firm only.

We extend our base model by relaxing two central assumptions – uniform pricing and exogenous qualities. In the first extension, we allow customizing firms to set a different price for each variety. With price customization, the main conclusions of the base model continue to hold. Namely, customization becomes more likely as the fixed cost of customization decreases and/or the quality difference increases, and the low quality firm never customizes alone. Price customization offers an additional benefit to the customizing firm by enhancing its ability to compete. This benefit is key to the two differences between this extension and the base model. While in the base model the firms do not customize when the quality difference is small, in the extension both firms customize provided the fixed cost of customization is low. Secondly, the high quality firm is less likely to be the only firm customizing in the extension compared to the base model.

Our base model points to a tradeoff – firms want to minimize quality difference to avoid customization and hence intensified price competition; meanwhile, a firm wants to excel in quality to benefit disproportionately from customization. It is therefore interesting to investigate what qualities will result if they are determined endogenously. In the second extension, we assume that the firms simultaneously choose their qualities prior to the customization stage. There is a quality-dependent cost that the firms incur in this new stage. We show that in equilibrium the firms are either substantially differentiated in quality or non-differentiated. The former results in customization by one or both firms, the latter leads to no customization.

Among the theoretical studies on customization, the paper that is closest to ours is Syam, Ruan, and Hess (2005). In both papers the consumer space is two-dimensional. Syam et al. (2005) endow products with two horizontal attributes, for which consumers have heterogeneous preferences. The firms are initially maximally differentiated with respect to both attributes. They first simultaneously choose whether to customize both, one, or none of the attributes, then compete in prices. The key difference between Syam et al. (2005) and our study is that they work with ex ante symmetric firms and examine how the possibility of customizing multiple attributes affect customization choices, whereas we work with asymmetric firms and focus on the role of quality in customization competition.

Another closely related paper is Bernhardt, Liu, and Serfes (2007), in which ex ante symmetric firms first acquire information about consumers and then customize their products as best as they can to match consumer needs. Similar to our paper, consumer preferences are two-dimensional, corresponding to two attributes of the product, and the second attribute – brand loyalty – cannot be customized. There are two main differences between Bernhardt et al. (2007) and our study. First, Bernhardt et al. (2007) emphasize the cost side of customization, whereas our focus is shifted towards the strategic effects of customization. Second, brand loyalty is a horizontal attribute, not vertical as quality in our paper is.

A number of papers have studied customization using a one-dimensional consumer space. Dewan, et al. (2003) provide a deliberate treatment of customization technology, and assume that customizing firms customize prices, like in our first extension. Syam and Kumar (2006) examine the role of standard products in customization competition. Alexandrov (2008) extends Salop's (1979) model in which firms can offer interval-long adjustable "fat" products. In a dynamic setting Chen (2006) studies two marketing innovations, one of which is essentially a form of product customization. While these studies assume symmetric firms, in Mendelson and Parlaktürk (2008) one firm has a margin advantage (higher difference between reservation price and unit cost) over the other. As in our paper, Mendelson and Parlaktürk (2008) consider both uniform pricing and price customization.

Our paper as well as Syam et al. (2005) model customization as zero-one decisions, so that all consumers get their most preferred varieties when they purchase a customized product. In contrast, all the other papers mentioned above treat customization as continuous choices. Both approaches match aspects of reality and have their advantages. With zero-one decisions, more attention can be devoted to the strategic effects of customization. With continuous customization choices, one can focus on how efficiency considerations determine the range of customization.

The rest of the paper is organized as follows. In the next section we introduce the base model. In Section 3 the pricing stage is analyzed. In Section 4 we study the firms' customization choices. The two extensions, price customization and endogenous quality, are studied in Section 5. Concluding remarks are provided in Section 6. Proofs of all lemmas and propositions, as well as derivations for some expressions and claims, are relegated to the appendix.

2 The Base Model

Consider a market in which each product i is characterized by its variety $x_i \in [0,1]$ and its quality $q_i \geq 0$. The first characteristic corresponds to horizontal differentiation and the second to vertical differentiation. Consumers are heterogenous in two dimensions. Each consumer has a most preferred variety $x \in [0,1]$ and a quality valuation $y \in [0,1]$. A consumer of type (x, y) derives the following utility from buying one unit of product i:

$$v + q_i y - t|x - x_i| - p_i,$$

where v is a positive constant, t is a preference parameter, and p_i is the price of product i. Consumers as represented by (x, y) are uniformly distributed over the unit square $[0,1] \times [0,1]$ with a total mass of 1. We assume that v is large enough for all consumers to find a product that yields positive payoff in equilibrium.

There are two firms, A and B, operating with zero marginal cost of production. Initially, firm A offers a single (standard) product of quality q_A and variety $x_A = 0$, whereas firm B offers a single product of quality $q_B \geq q_A \geq 0$ and variety $x_B = 1$. That is, firm B is the higher quality firm and the two firms have maximum variety differentiation.

We will normalize t to 1. This amounts to a monotonic transformation of preferences. The utilities of a consumer of type (x, y) from buying firm A's and firm B's standard products are

$$v + q_A y - x - p_A \tag{1}$$

and

$$v + q_B y - (1 - x) - p_B, \tag{2}$$

respectively.

Investing $K \geq 0$ into product-customization technology allows a firm to produce a product that exactly matches a given consumer's preferred variety. The utilities of type (x, y) from buying firm A's and firm B's customized products are

$$v + q_A y - p_A \tag{3}$$

and

$$v + q_B y - p_B. \tag{4}$$

The game involves two stages, a customization stage followed by a pricing stage. In the customization stage the firms simultaneously decide whether to customize their products. These decisions become common knowledge after they are made. In the pricing stage the firms simultaneously choose prices. Consumers subsequently decide which products to purchase, and profits are realized. The equilibrium concept employed is subgame perfect Nash equilibrium. The analysis of consumer choices is straightforward. We, therefore, focus on the firms' choices and proceed using backward

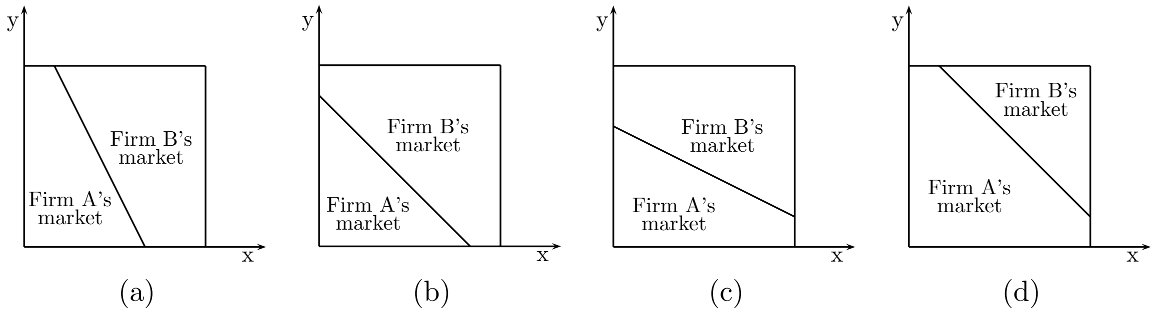

Figure 1: Indifference line and market areas

induction.

3 Analysis of the Pricing Stage

In this section we investigate the firms' pricing decisions, taking as given their customization choices in the first stage of the game. There are four subgames to consider, corresponding to the following scenarios: no firm customizes; firm A (the low quality firm) customizes; firm B (the high quality firm) customizes; and both firms customize. We denote these subgames by "SS," "CS," "SC," and "CC," where "S" and "C" stand for offering standard and customized products, respectively.

3.1 Subgame SS: No Firm Customizes

When no firm customizes, utilities from firm A's and firm B's standard products are given by (1) and (2). Therefore, for a given level of quality valuation y, the marginal consumer type in terms of variety x is

$$\widehat{x}(y) = \frac{1}{2}\left(1 - qy + p_B - p_A\right), \tag{5}$$

where

$$q \equiv q_B - q_A$$

denotes the quality difference between the firms' products. For any $y \in [0, 1]$, consumers in the interval $x \in [0, \widehat{x}(y)]$ will purchase from firm A, whereas those with $x \in (\widehat{x}(y), 1]$ will purchase from firm B. There are four possible positions for the indifference line (5), as illustrated in Figure 1. The slope of the indifference line equals $-2/q$. An increase in q makes the line flatter. An increase in p_B (and/or decrease in p_A) shifts the line to the right, thereby reducing the market size of firm B.

Let $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$ denote the demand functions of firms A and B. The expressions for these functions (detailed in the appendix) depend on the position of the indifference line. The firms choose simultaneously p_A and p_B to maximize their profits,

$$\Pi_A(p_A, p_B) = D_A(p_A, p_B)p_A$$

and

$$\Pi_B(p_A, p_B) = D_B(p_A, p_B) p_B.$$

Lemma 1 (Equilibrium prices and profits when no firm customizes)**.** *Suppose no firm customizes. The equilibrium prices and profits in the pricing stage are as follows.*

(i) *If $q \leq 3/2$,*

$$\begin{cases} p_A^{SS} = 1 - \frac{1}{6}q \\ p_B^{SS} = 1 + \frac{1}{6}q \end{cases} \textit{and} \begin{cases} \Pi_A^{SS} = \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2 \\ \Pi_B^{SS} = \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) *If $q \in (3/2, 3]$,*

$$\begin{cases} p_A^{SS} = \frac{1}{8}\left(1 + \sqrt{1+16q}\right) \\ p_B^{SS} = \frac{1}{8}\left(-5 + 3\sqrt{1+16q}\right) \end{cases} \textit{and} \begin{cases} \Pi_A^{SS} = \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^3 \\ \Pi_B^{SS} = \left(1 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)\frac{-5+3\sqrt{1+16q}}{8} \end{cases}$$

(iii) *If $q > 3$,*

$$\begin{cases} p_A^{SS} = \frac{1}{3}q \\ p_B^{SS} = \frac{2}{3}q \end{cases} \textit{and} \begin{cases} \Pi_A^{SS} = \frac{1}{9}q \\ \Pi_B^{SS} = \frac{4}{9}q \end{cases}$$

It is easy to verify that the corresponding prices and profits in parts (i) and (ii) are equal when evaluated at $q = 3/2$. Similarly, the prices and profits in parts (ii) and (iii) are equal at $q = 3$. Therefore, the equilibrium prices and profits vary continuously when q changes.

Case (i) corresponds to Figure 1(a) in which the quality difference is small. In equilibrium, both firms serve consumers with all quality valuations, and each firm attracts consumers closer to its position on the variety interval. Case (ii) corresponds to Figure 1(b). Firm A attracts only consumers who are close to its variety position and have low quality valuations (i.e., small x's and small y's). Firm B uses its quality advantage to capture all the other consumers. Case (iii) corresponds to Figure 1(c) in which q is large. In this case, the low quality firm A competes aggressively and the high quality firm B does better setting a high price and exploiting consumers with high quality valuations. In equilibrium, firm A serves consumers of all variety preferences, and so does firm B. Firm A attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations.

It is worthwhile to note the intuitive outcome implied by Lemma 1 that in all three cases the high quality firm B sets a higher price, serves a larger market area, and earns a higher profit than firm A. Furthermore, Figure 1(d) does not arise in equilibrium. This is also true for the three subgames studied next.

3.2 Subgame CS: Firm A Customizes

When only firm A customizes, utilities from firm A's customized product and firm B's standard product are given by (3) and (2). Therefore, for a given y, the marginal consumer type in terms of

x is

$$\widehat{x}(y) = 1 - qy + p_B - p_A, \tag{6}$$

from which we can calculate the demand and profit functions for the firms. Lemma 2 presents the equilibrium prices and profits for subgame CS.

Lemma 2 (Equilibrium prices and profits when firm A customizes)**.** *Suppose firm A customizes. The equilibrium prices and profits in the pricing stage are as follows.*

(i) *If* $q \leq 1$*,*

$$\begin{cases} p_A^{CS} = \frac{2}{3} - \frac{1}{6}q \\ p_B^{CS} = \frac{1}{3} + \frac{1}{6}q \end{cases} \quad and \quad \begin{cases} \Pi_A^{CS} = \left(\frac{2}{3} - \frac{1}{6}q\right)^2 \\ \Pi_B^{CS} = \left(\frac{1}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) *If* $q > 1$*,*

$$\begin{cases} p_A^{CS} = \frac{1}{3}q + \frac{1}{6} \\ p_B^{CS} = \frac{2}{3}q - \frac{1}{6} \end{cases} \quad and \quad \begin{cases} \Pi_A^{CS} = \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 \\ \Pi_B^{CS} = \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2 \end{cases}$$

Note that the critical values for the cases in this lemma are different from those in Lemma 1, and that the pricing equilibrium in subgame CS leads to only two possible positions of the indifference line. This is due to the fact that the slope of the indifference line (6) is $-1/q$, which is different from that of (5).

Case (i) of Lemma 2 corresponds to Figure 1(a) in which the quality difference is small. Customization enables firm A to overcome its quality disadvantage. Firm A's equilibrium price, market size, and profit are higher than those of firm B. In equilibrium, both firms serve consumers with all quality valuations; firm A serves consumers with small x's and firm B consumers with large x's. To explain why firm B still attracts consumers with low quality valuations, it suffices to consider consumers with $y = 0$. Such consumers do not gain any extra utility buying from the high quality firm, yet some of them are attracted to firm B because of its low price. Case (ii) corresponds to Figure 1(c) in which q is large. In this case, customization does not overcome the quality disadvantage of firm A. In equilibrium, firm A's price, market size, and profit are lower than those of firm B. Firm A serves consumers of all variety preferences and so does firm B. Firm A attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations.

3.3 Subgame SC: Firm B Customizes

When only firm B customizes, utilities from firm A's standard product and firm B's customized product are given by (1) and (4). Therefore, for a given y, the marginal consumer type in terms of x is

$$\widehat{x}(y) = -qy + p_B - p_A. \tag{7}$$

The next lemma presents the equilibrium prices and profits for subgame SC.

Lemma 3 (Equilibrium prices and profits when firm B customizes)**.** *Suppose firm B customizes. The equilibrium prices and profits in the pricing stage are as follows.*

(i) *If $q \le 1/2$,*

$$\begin{cases} p_A^{SC} = \frac{1}{3} - \frac{1}{6}q \\ p_B^{SC} = \frac{2}{3} + \frac{1}{6}q \end{cases} \quad \textit{and} \quad \begin{cases} \Pi_A^{SC} = \left(\frac{1}{3} - \frac{1}{6}q\right)^2 \\ \Pi_B^{SC} = \left(\frac{2}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) *If $q \in (1/2, 2]$,*

$$\begin{cases} p_A^{SC} = \frac{1}{4}\sqrt{2q} \\ p_B^{SC} = \frac{3}{4}\sqrt{2q} \end{cases} \quad \textit{and} \quad \begin{cases} \Pi_A^{SC} = \frac{1}{16}\sqrt{2q} \\ \Pi_B^{SC} = \frac{9}{16}\sqrt{2q} \end{cases}$$

(iii) *If $q > 2$,*

$$\begin{cases} p_A^{SC} = \frac{1}{3}q - \frac{1}{6} \\ p_B^{SC} = \frac{2}{3}q + \frac{1}{6} \end{cases} \quad \textit{and} \quad \begin{cases} \Pi_A^{SC} = \frac{1}{q}\left(\frac{1}{3}q - \frac{1}{6}\right)^2 \\ \Pi_B^{SC} = \frac{1}{q}\left(\frac{2}{3}q + \frac{1}{6}\right)^2 \end{cases}$$

Case (i) corresponds to Figure 1(a), case (ii) to 1(b), and case (iii) to 1(c). Firm B's quality advantage is reinforced by customization, pushing the critical values lower compared to those in Lemma 1.

3.4 Subgame CC: Both Firms Customize

When both firms customize, utilities from firm A's and firm B's customized products are given by (3) and (4). Therefore, consumers with $y \le (p_B - p_A)/q$ will purchase from firm A, those with $y > (p_B - p_A)/q$ will purchase from firm B. The firms' profit functions are

$$\Pi_A(p_A, p_B) = \frac{1}{q}(p_B - p_A)\,p_A \quad \text{and} \quad \Pi_B(p_A, p_B) = \frac{1}{q}(q + p_A - p_B)\,p_B.$$

The profit maximizing first-order conditions

$$\begin{cases} p_B - 2p_A = 0 \\ q + p_A - 2p_B = 0 \end{cases}$$

lead immediately to the following lemma.

Lemma 4 (Equilibrium prices and profits when both firms customize)**.** *Suppose both firms customize. The equilibrium prices and profits in the pricing stage are given by:*

$$\begin{cases} p_A^{CC} = \frac{1}{3}q \\ p_B^{CC} = \frac{2}{3}q \end{cases} \quad \textit{and} \quad \begin{cases} \Pi_A^{CC} = \frac{1}{9}q \\ \Pi_B^{CC} = \frac{4}{9}q \end{cases}$$

In equilibrium the indifference line is horizontal at 1/3 from the bottom side of the unit square. That is, firm A serves all consumers with quality valuations less than 1/3, and firm B serves the rest. The result here is the same as in the standard model of vertical differentiation.

Having derived the profit functions for the pricing stage, we move one step back to study the customization stage.

4 Equilibrium Customization Choices

In the customization stage the firms simultaneously decide whether to customize their products. This is represented by the following matrix.

Firm B

Firm A		S	C
	S	Π_A^{SS}, Π_B^{SS}	$\Pi_A^{SC}, \Pi_B^{SC} - K$
	C	$\Pi_A^{CS} - K, \Pi_B^{CS}$	$\Pi_A^{CC} - K, \Pi_B^{CC} - K$

(8)

It follows that

$$\left.\begin{array}{l}\text{(S,S)}\\ \text{(C,S)}\\ \text{(S,C)}\\ \text{(C,C)}\end{array}\right\} \text{ is a Nash equilibrium if } \left\{\begin{array}{l}K \geq \max\{c_1, r_1\}\\ K \in [r_2, c_1]\\ K \in [c_2, r_1]\\ K \leq \min\{c_2, r_2\}\end{array}\right. \tag{9}$$

where

$$c_1 \equiv \Pi_A^{CS} - \Pi_A^{SS} \quad \text{and} \quad c_2 \equiv \Pi_A^{CC} - \Pi_A^{SC}$$

denote firm A's change in gross profit in the two columns, and

$$r_1 \equiv \Pi_B^{SC} - \Pi_B^{SS} \quad \text{and} \quad r_2 \equiv \Pi_B^{CC} - \Pi_B^{CS}$$

denote firm B's change in gross profit in the two rows.

Lemma 5 (Relative gains from customization)**.** *For any value of* q*,* $\max\{c_1, c_2\} < \min\{r_1, r_2\}$.

Detailed expressions for c_1, c_2, r_1, and r_2 as functions of q are provided in the appendix. This lemma stipulates that the low quality firm A always gains less from customization than the high quality firm B.

It follows from Lemma 5 that the customization stage has a unique Nash equilibrium except for $K = c_2$ and $K = r_1$. For ease of presentation without affecting the results, we will select (C,C) as the Nash equilibrium at $K = c_2$ and (S,C) at $K = r_1$. Accordingly,

$$\left.\begin{array}{l}\text{(S,S)}\\ \text{(S,C)}\\ \text{(C,C)}\end{array}\right\} \text{ is the Nash equilibrium if } \left\{\begin{array}{l}K > r_1\\ K \in (c_2, r_1]\\ K \leq c_2\end{array}\right.$$

Because K is non-negative, the above discussion implies that the signs of c_2 and r_1 are crucial for equilibrium analysis. Specifically, when both c_2 and r_1 are negative, (S,S) is the Nash equilibrium for any K. When $c_2 < 0$ and $r_1 > 0$, either (S,C) or (S,S) is the Nash equilibrium, depending on K. When both c_2 and r_1 are positive, the Nash equilibrium can be (C,C), (S,C), or (S,S).

The next proposition summarizes our main results on the firms' customization choices.

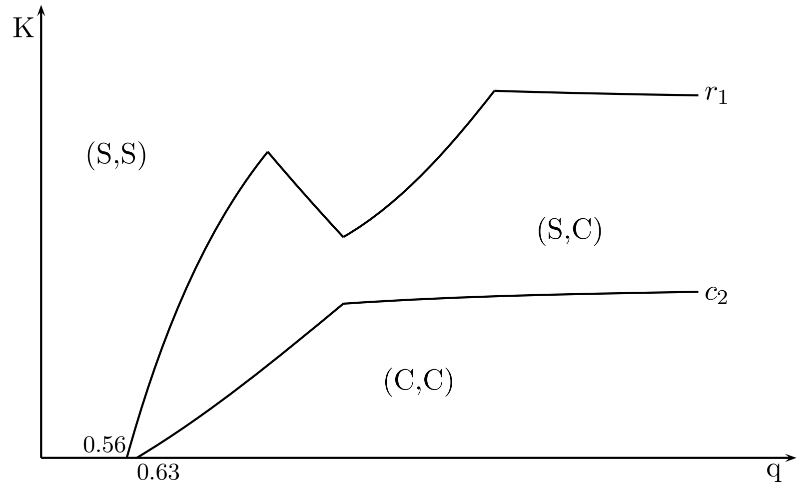

Figure 2: Equilibrium customization choices

Proposition 1 (Equilibrium customization choices)**.** *The following hold for the firms' equilibrium customization choices.*

(i) *If $q \leq 0.56$ then the Nash equilibrium is (S,S) for any value of K.*

(ii) *If $q \in (0.56, 0.63]$ then the Nash equilibrium is (S,C) for $K \leq r_1$ and (S,S) for $K > r_1$.*

(iii) *If $q > 0.63$ then the Nash equilibrium is (C,C) for $K \leq c_2$, (S,C) for $K \in (c_2, r_1]$, and (S,S) for $K > r_1$.*

Figure 2 plots c_2 and r_1 as functions of the quality difference.[3] Depending on the values for the parameters q and K, either both firms customize, only the high quality firm customizes, or no firm customizes. Proposition 1 implies the following:

- The low quality firm never customizes alone.
- Customization is less likely the higher the fixed cost of customization is.
- Customization is more likely the higher the quality difference is.
- Even when customization is costless, the firms will not customize in equilibrium if the quality difference is small.

What drives the first two conclusions is the fact that the high quality firm always benefits more from customization than the low quality firm. Whenever the low quality firm has an incentive to customize, so does the high quality firm. Accordingly, the equilibrium changes from (C,C) to (S,C) to (S,S) as the fixed cost of customization increases.

[3] The number 0.56 is an approximate solution to $81\sqrt{2q} = 2(6+q)^2$. The number 0.63 is an approximate value of 81/126.

Customization by one or both firms makes the firms' products less differentiated and intensifies price competition. Competition becomes tougher as the quality difference decreases. In the extreme case in which $q = 0$ and both firms customize, the products become identical in the eyes of consumers, so price competition results in the Bertrand outcome. This intuition is behind the last two conclusions above. Specifically, when q is small, the firms do not customize their products in order to avoid a disastrous price war. When q is large, the firms customize to take advantage of consumers' desires for ideal varieties. The intermediate case involves customization by the high quality firm only.[4]

5 Extensions

In this section we extend the base model by relaxing two central assumptions – uniform pricing and exogenous product qualities.

When a consumer buys a customized product, his ideal variety x is revealed to the firm. In the base model we assumed that firms that customize cannot use this information and, hence, are restricted to uniform pricing strategies. In our first extension we allow customizing firms to customize prices, that is, to link prices to x.

As the quality difference is crucial in determining the firms' equilibrium customization choices, a natural question to ask is: What level of quality difference will emerge if the firms choose their qualities as part of the game? Our second extension endogenizes the firms' qualities. We model this by assuming that the firms simultaneously select their qualities prior to the customization stage.

5.1 Price Customization

Consider a subgame in which one firm customizes and the other does not. With price customization, the customizing firm competes at each location x. The closer is x to the standard product of the rival firm, the fiercer is price competition. Hence, we expect that in equilibrium the customizing firm charges higher prices to consumers located further away from the non-customizing firm. Moreover, price flexibility enables the customizing firm to reach consumers of all variety preferences. This is in contrast to the base model, where the customizing firm does not serve all varieties when q is small (Lemmas 2 and 3).

The above intuition is confirmed below. We need to keep in mind that for price customization to be successful, the price schedule of a customizing firm has to be such that consumers do not have incentives to misrepresent their ideal products.

We start with subgame CS in which firm A customizes and firm B does not. For brevity, the derivations of the equilibrium prices in this subgame are relegated to the appendix.[5] For $q \leq 5/8$,

[4] To be precise, the equilibrium does not always follow the sequence (S,S) to (S,C) to (C,C) as q increases. As can be seen from Figure 2, for some values of K the equilibrium changes from (S,S) to (S,C) to (S,S) to (S,C).

[5] Since the risk of confusion is minimal, in this subsection we employ the same notation as in the base model.

the prices are given by

$$p_A^{CS}(x) = \begin{cases} 1 - \frac{3}{5}q - x, & x \le 1 - \frac{8}{5}q \\ \frac{1}{2} + \frac{1}{5}q - \frac{1}{2}x, & x > 1 - \frac{8}{5}q \end{cases} \quad \text{and} \quad p_B^{CS} = \frac{2}{5}q. \tag{10}$$

For $q > 5/8$, the prices are[6]

$$p_A^{CS}(x) = \frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x \quad \text{and} \quad p_B^{CS} = \frac{2}{3}q - \frac{1}{6}. \tag{11}$$

The case where $q \le 5/8$ corresponds to Figure 1(d) and the case where $q > 5/8$ corresponds to Figure 1(c). Hence, firm A serves consumers of all variety preferences. It is worth noting that $p_A^{CS}(x)$ precludes consumers from misrepresenting their ideal products. Indeed, no consumer would misrepresent his ideal product because

$$p_A^{CS}(x) \le p_A^{CS}(x') + |x' - x|$$

holds for any x and $x' \in [0, 1]$.

Next, consider subgame SC in which only firm B customizes. In the appendix we show that for $q \le 5/4$ the equilibrium prices are given by

$$p_A^{SC} = \frac{1}{5}q \quad \text{and} \quad p_B^{SC}(x) = \begin{cases} \frac{3}{5}q + \frac{1}{2}x, & x \le \frac{4}{5}q \\ \frac{1}{5}q + x, & x > \frac{4}{5}q \end{cases} \tag{12}$$

For $q > 5/4$, the equilibrium prices are[7]

$$p_A^{SC} = \frac{1}{3}q - \frac{1}{6} \quad \text{and} \quad p_B^{SC}(x) = \frac{2}{3}q - \frac{1}{12} + \frac{1}{2}x. \tag{13}$$

The case where $q \le 5/4$ corresponds to Figure 1(b) and the case where $q > 5/4$ corresponds to Figure 1(c). The customizing firm serves consumers of all variety preferences. Note also that $p_B^{SC}(x)$ precludes consumers from misrepresenting their ideal products.

Finally, consider subgames SS and CC. Obviously, subgame SS is the same as in the base model (Lemma 1). In subgame CC the competition at each and every variety x is equivalent to the standard vertical differentiation game, resulting in both firms charging a uniform price. Hence, Lemma 4 applies here.

Having analyzed the pricing stage, we move to the customization stage. Both the game matrix (8) and the general statement (9) regarding Nash equilibrium still apply. However, the firms' equilibrium customization choices may not be the same, because the expressions for c_1, c_2, r_1, and r_2 have changed.

[6] It is easy to verify that for a given value of q, $p_A^{CS}(x)$ is continuous in x, and that (10) coincides with (11) when $q = 5/8$.

[7] It is easy to verify that for a given value of q, $p_B^{SC}(x)$ is continuous in x, and that (12) coincides with (13) when $q = 5/4$.

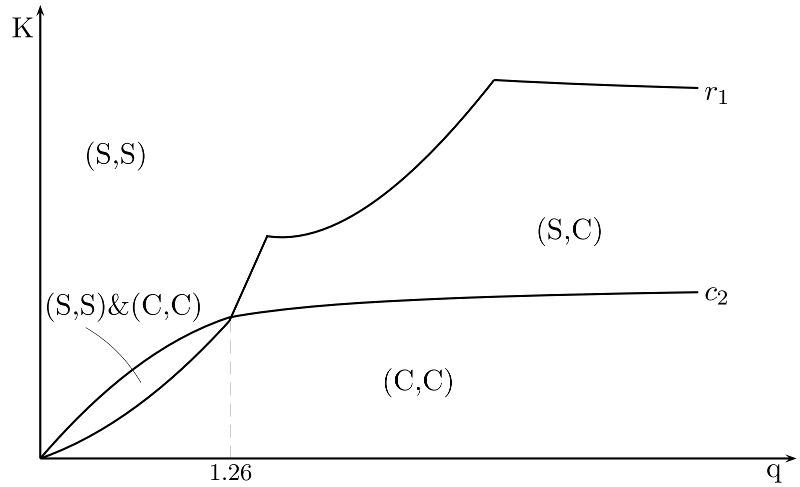

Figure 3: Equilibrium customization choices with price customization

We show in the appendix that $c_1 < \min\{r_1, r_2\}$ and $c_2 < r_2$ hold for any value of q. Figure 3 depicts the firms' customization choices in equilibrium. For $q \leq 1.26$, $c_2 \geq r_1$ and

$$\left.\begin{array}{l}\text{(S,S)}\\ \text{(S,S) and (C,C)}\\ \text{(C,C)}\end{array}\right\}\text{ is (are) Nash equilibrium(a) if }\left\{\begin{array}{l}K > c_2\\ K \in (r_1, c_2]\\ K \leq r_1\end{array}\right.$$

For $q > 1.26$, $c_2 < r_1$ and

$$\left.\begin{array}{l}\text{(S,S)}\\ \text{(S,C)}\\ \text{(C,C)}\end{array}\right\}\text{ is the Nash equilibrium if }\left\{\begin{array}{l}K > r_1\\ K \in (c_2, r_1]\\ K \leq c_2\end{array}\right.$$

The next proposition provides a summary of these results.

Proposition 2 (Equilibrium customization choices with price customization)**.** *The following hold for the equilibrium customization choices when the firms can customize prices.*

(i) *If $q \leq 1.26$ then (C,C) is the Nash equilibrium for $K \leq r_1$; both (C,C) and (S,S) are Nash equilibria for $K \in (r_1, c_2]$; and (S,S) is the Nash equilibrium for $K > c_2$.*

(ii) *If $q > 1.26$ then (C,C) is the Nash equilibrium for $K \leq c_2$; (S,C) is the Nash equilibrium for $K \in (c_2, r_1]$; and (S,S) is the Nash equilibrium for $K > r_1$.*

Proposition 2 implies that with customized prices the main conclusions of the base model (stated following Proposition 1) continue to hold. Namely, the low quality firm never customizes alone; customization becomes less likely as the fixed cost of customization increases; and more likely as the quality difference increases.

Two obvious differences exist between this extension and the base model. With price customization, for any value of q customization by both firms occurs in equilibrium provided that K is sufficiently small, as can be seen from Figure 3. Recall that in the base model no firm customizes when $q < 0.56$ (the last conclusion on page **??**). While decreasing product differentiation, customization takes advantage of consumers' desires for ideal varieties. Price customization offers an additional benefit to the customizing firm by enhancing its ability to compete with the other firm. As a result, in the current extension the firms have incentives to customize even when q is small.

Secondly, the high quality firm is less likely to be the only firm customizing in this extension than in the base model. In particular, the minimum quality difference to support such an outcome is larger in the extension, 1.26 vs. 0.56. (S,C) cannot be an equilibrium when $q < 1.26$ because in this range $c_2 > r_1$. That is, whenever firm B finds it profitable to customize, firm A has an incentive to customize as well. The reason for this is the additional benefit to the customizing firm mentioned in the previous paragraph. This benefit hurts the non-customizing firm, increasingly the lower q is (i.e., less product differentiation). As a result, firm A is compelled to customize in order to soften the loss.

5.2 Endogenous Quality

We next extend the base model by allowing the firms to choose simultaneously their qualities prior to the customization stage. As is usually assumed in the literature, the firms incur quality-dependent fixed costs, and the variable costs of production are not affected.

As either firm may become the high quality firm, we label the two firms as firm 1 and firm 2, and their qualities q_1 and q_2 , which take values in $[0, \infty)$. For any given pair (q_1, q_2), the continuation game is our base model with the quality difference $q = |q_1 - q_2|$.

To keep our analysis more focused, we will present our detailed results for the case $K = 0$, and then briefly for a positive K. Based on Lemmas 1 through 4 and Proposition 1, firm i's profit as a function of qualities is[8]

$$\pi_i(q_i, q_j) = \begin{cases} \frac{1}{9}(q_j - q_i), & q_i - q_j < -0.63 \\ \frac{1}{16}\sqrt{2(q_j - q_i)}, & q_i - q_j \in [-0.63, -0.56) \\ \frac{1}{2}\left(1 - \frac{1}{6}(q_j - q_i)\right)^2, & q_i - q_j \in [-0.56, 0] \\ \frac{1}{2}\left(1 + \frac{1}{6}(q_i - q_j)\right)^2, & q_i - q_j \in [0, 0.56] \\ \frac{9}{16}\sqrt{2(q_i - q_j)}, & q_i - q_j \in (0.56, 0.63] \\ \frac{4}{9}(q_i - q_j), & q_i - q_j > 0.63 \end{cases} \tag{14}$$

for $i = 1, 2$ and $j \neq i$. Firm i's profit is the same as the low quality firm A's when $q_i \leq q_j$ (the first three lines in (14)) and the high quality firm B's when $q_i > q_j$ (the last three lines). The first and last lines of (14) correspond to both firms customizing ($q > 0.63$) and the profits are as in Lemma 4. The second and fifth lines correspond to customization by the high quality firm ($q \in (0.56, 0.63]$) and

[8] One can easily identify from Figure 2 the two critical values (0.56 and 0.63) used in defining $\pi_i(q_i, q_j)$. Note that the critical values change with K.

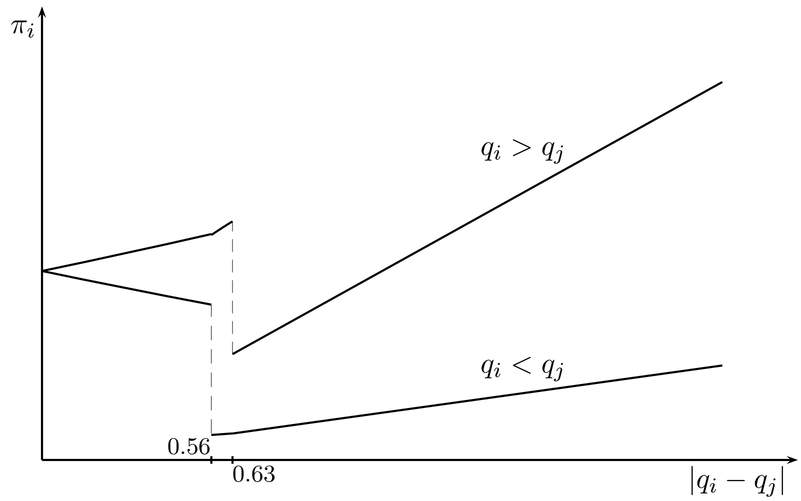

Figure 4: Firm i's profit curves

the profits are as in Lemma 3(ii). The middle two lines correspond to no customization ($q \leq 0.56$) and the profits are as in Lemma 1(i).

Note that firm i's profit $\pi_i(q_i, q_j)$ depends on the quality difference $|q_i - q_j|$ and whether q_i is higher or lower than q_j. The upper curve in Figure 4 depicts firm i's profit function when $q_i > q_j$ and the lower curve when $q_i < q_j$. The two curves start at the same point that corresponds to equal qualities.

Let $c(q_i)$ denote the fixed cost firm i incurs. Firm i's best response function is, therefore, given by

$$q_i(q_j) = \arg\max_{q_i} \pi_i(q_i, q_j) - c(q_i).$$

The firms' quality choices in equilibrium are determined by the intersection point(s) of the two best response curves.

Due to the complexity of the profit function in (14), we are unable to analytically solve the game, even with a specific cost function. In the following we solve the game through numerical simulations. We use the cost function

$$c(q_i) = \alpha q_i^2,$$

where $\alpha > 0$ is a parameter.

Proposition 3 summarizes our simulated results. Figure 5 provides a graphical illustration of the firms' equilibrium quality choices reported in this proposition. The arrows indicate how equilibria change as the cost parameter α increases.

Proposition 3 (Equilibrium qualities)**.** *For $K = 0$, the following hold for the firms' equilibrium quality choices.*

(i) *If $\alpha < 0.066$ then there are two asymmetric Nash equilibria, $(0, q^*)$ and $(q^*, 0)$, where $q^* > 3.37$ and decreases in α.*

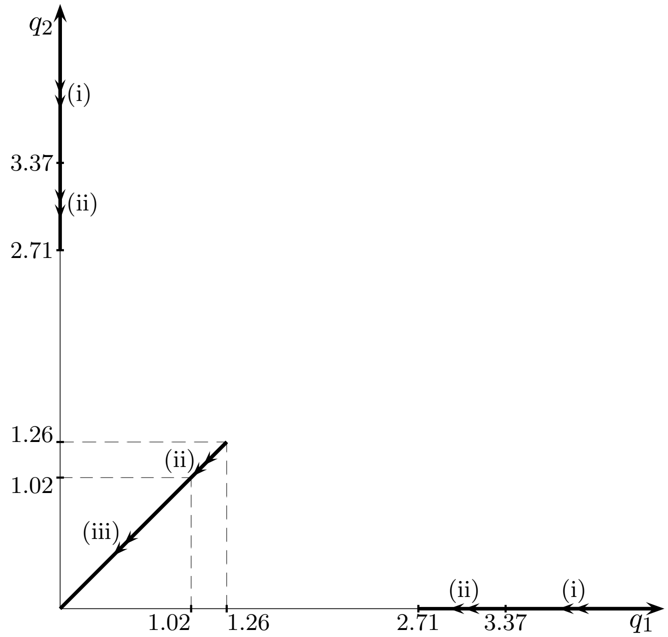

Figure 5: Changes in the equilibrium qualities as α increases within: (i) $\alpha < 0.066$, (ii) $\alpha \in [0.066, 0.082]$, and (iii) $\alpha > 0.082$

(ii) *If* $\alpha \in [0.066, 0.082]$ *then there is one symmetric and two asymmetric Nash equilibria,* $(q^\dagger, q^\dagger)$, $(0, q^*)$, *and* $(q^*, 0)$, *where* $q^\dagger$ *decreases from* 1.26 *to* 1.02 *and* q^* *decreases from* 3.37 *to* 2.71 *as* α *increases from* 0.066 *to* 0.082.

(iii) *If* $\alpha > 0.082$ *then there is one symmetric Nash equilibrium* $(q^\dagger, q^\dagger)$, *where* $q^\dagger < 1.02$ *and decreases in* α.

When α is small (< 0.066), one of the firms sets its quality to zero, while the other chooses positive quality q^*. The equilibrium quality difference (equal to q^*) is high, leading to customization by both firms. Figure 4 helps in providing intuition for this outcome. The low quality firm does not want to raise its quality to decrease the quality gap.[9] This is because doing so is costly and also decreases its gross profit (see the rising portion of the lower curve). For the high quality firm, the portion of the upper profit curve where $q > 1.42$ becomes more attractive than the portion where $q \leq 0.63$, justifying q^* being high (> 3.37).[10]

When $\alpha > 0.082$, the firms choose the same qualities $q^\dagger$ in equilibrium, leading to no customization by either firm. Intuitively, with large α, high qualities are not profitable choices. It follows that high quality differences cannot occur in equilibrium. In fact, the equilibrium is symmetric (zero quality difference). Equilibrium value $q^\dagger$ is such that no firm wants to make a small deviation up or

[9]Here q^* is high so that leapfrogging by the low quality firm to become the high quality firm is too costly relative to the gain in profit.

[10]It is obvious from Figure 4 that any quality gap $|q_i - q_j| \in (0.63, 1.42]$ cannot occur in equilibrium. Indeed, if $|q_i - q_j| \in (0.63, 1.42]$ then the high quality firm benefits from lowering its quality to reduce the gap to 0.63.

down.[11] The profit gain from an upward deviation is less than the increase in cost. The cost saving from a downward deviation is less than the decrease in profit. Moreover, high α prevents either firm from making a large deviation. In the intermediate case ($\alpha \in [0.066, 0.082]$) all forces discussed above are at work. As a result, symmetric and asymmetric equilibria co-exist.

To summarize, Proposition 3 implies that, with endogenous quality determination and zero fixed cost of customization, the firms are substantially differentiated in quality when α is small and non-differentiated when α is large. The former results in customization by both firms, while the latter leads to no customization. Note that (S,C) does not arise in equilibrium.

Our simulations with other values of K indicate that the result that the firms are either substantially differentiated or non-differentiated is generally true. Sometimes the substantial differentiation leads to customization by the high quality firm alone. In the appendix we report the results for $K = 0.115$.[12] The equilibrium pattern of quality choices is similar to Figure 5, with the critical values for α, 0.066 and 0.082, replaced by 0.051 and 0.091, respectively. In this example, in all asymmetric equilibria only the high quality firm customizes.

As a final note, in all our simulated cases, the high quality firm earns a higher profit (net of all costs) than the low quality firm. This implies that each firm prefers to be the high quality firm in any asymmetric equilibrium.

6 Concluding Remarks

The novelty of our paper is the incorporation of difference in product qualities into customization competition. Customization enables firms to take advantage of consumers' desires for ideal varieties. However, it makes firms less differentiated and, therefore, intensifies price competition. This intuition is behind much of the results in the theoretical literature on customization, and is shown here to be valid when there is vertical differentiation.

The most important finding in our paper is that quality does play an important role in firms' strategic decisions concerning customization. We show that customization can occur in equilibrium only when the quality difference is sufficiently large. Moreover, the high quality firm can always reap a larger benefit from customization than the low quality firm. As a result, the high quality firm may be the only firm customizing in equilibrium, whereas the low quality firm never customizes alone. This result seems to be supported by many real-world observations, several of these were mentioned in the introduction.

These conclusions together with our results in the two extensions have some obvious managerial implications for firms considering customization in competitive situations. First, customization is more likely to be a successful strategy for high quality firms and less so for low quality firms. Low quality firms might consider customization when high quality firms customize and the quality gap is large. Second, to engage in price customization, lower prices should be charged to consumers located closer to the rival's standard product in the attribute space. Third, if quality is part of a firm's

[11] Note that $q^\dagger$ is never zero. This is because at $q_i = q_j = 0$ each firm's marginal profit is 1/6 and marginal cost is 0.
[12] For $K = 0.115$ the equilibrium in the base model is (S,S) when $q < 1$ and (S,C) when $q \geq 1$.

strategic choices, the firm should try to match the competitor's quality when the quality-related costs are high, and differentiate itself from the competitor when the quality-related costs are low.

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Appendix

Proof of Lemma 1. Each case is proven in turn.

(i) Consider $q \leq 3/2$ and suppose the indifference line (5) intersects the unit square as shown in Figure 2(a). Straightforward algebra implies

$$D_A(p_A, p_B) = \frac{1}{2}\left(1 - \frac{1}{2}q + p_B - p_A\right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2}\left(1 + \frac{1}{2}q + p_A - p_B\right)$$

in this case. The profit maximizing first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. It is left to verify that under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\widehat{x}(1) \geq 0$ and $\widehat{x}(0) \leq 1$. Indeed,

$$\widehat{x}(1) = \frac{1}{2}\left(1 - q + p_B^{SS} - p_A^{SS}\right) = \frac{1}{2}\left(1 - \frac{2}{3}q\right) \geq 0,$$

$$\widehat{x}(0) = \frac{1}{2}\left(1 + p_B^{SS} - p_A^{SS}\right) = \frac{1}{2}\left(1 + \frac{1}{3}q\right) < 1$$

hold for $q \leq 3/2$. In fact, $\widehat{x}(1) = 0$ when $q = 3/2$.

(ii) Consider $3/2 < q \leq 3$ and suppose the indifference line (5) intersects the unit square as shown in Figure 2(b). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{4q}\left(1 + p_B - p_A\right)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{4q}\left(1 + p_B - p_A\right)^2.$$

The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, $\widehat{x}(1) \leq 0$ and $\widehat{x}(0) \in [0, 1]$. Indeed,

$$\widehat{x}(1) = \frac{1}{2}\left(1 - q + \frac{1}{4}\left(-3 + \sqrt{1 + 16q}\right)\right) < 0,$$

$$\widehat{x}(0) = \frac{1}{2}\left(1 + \frac{1}{4}\left(-3 + \sqrt{1 + 16q}\right)\right) \in (0, 1]$$

hold for $3/2 < q \leq 3$. Note that $\widehat{x}(1) = 0$ when $q = 3/2$ and $\widehat{x}(0) = 1$ when $q = 3$.

(iii) Consider $q > 3$ and suppose the indifference line (5) intersects the unit square as shown in Figure 2(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q}\left(p_B - p_A\right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q}\left(q + p_A - p_B\right).$$

The first-order conditions yield the equilibrium prices and profits as in part (iii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square.

Algebraically, $\widehat{x}(1) \leq 0$ and $\widehat{x}(0) \geq 1$. Indeed,

$$\widehat{x}(1) = \frac{1}{2}\left(1 - \frac{2}{3}q\right) < 0 \quad \text{and} \quad \widehat{x}(0) = \frac{1}{2}\left(1 + \frac{1}{3}q\right) > 1$$

hold for $q > 3$. Note that $\widehat{x}(0) = 1$ when $q = 3$.

□

Proof of Lemma 2. Each case is proven in turn.

(i) Consider $q \leq 1$ and suppose the indifference line (6) intersects the unit square as shown in Figure 2(a). The firms' demand functions are

$$D_A(p_A, p_B) = 1 - \frac{1}{2}q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2}q + p_A - p_B.$$

The first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. Under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\widehat{x}(1) \geq 0$ and $\widehat{x}(0) \leq 1$. Indeed,

$$\widehat{x}(1) = 1 - q + p_B^{CS} - p_A^{CS} = \frac{2}{3} - \frac{2}{3}q \geq 0 \quad \text{and} \quad \widehat{x}(0) = 1 + p_B^{CS} - p_A^{CS} = \frac{2}{3} + \frac{1}{3}q < 1$$

hold for $q \leq 1$. Note that $\widehat{x}(1) = 0$ and $\widehat{x}(0) = 1$ when $q = 1$.

(ii) Consider $q > 1$ and suppose the indifference line (6) intersects the unit square as shown in Figure 2(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q}\left(\frac{1}{2} + p_B - p_A\right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q}\left(q - \frac{1}{2} + p_A - p_B\right).$$

The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, $\widehat{x}(1) \leq 0$ and $\widehat{x}(0) \geq 1$. Indeed,

$$\widehat{x}(1) = \frac{2}{3} - \frac{2}{3}q < 0 \quad \text{and} \quad \widehat{x}(0) = \frac{2}{3} + \frac{1}{3}q > 1$$

hold for $q > 1$. Note that $\widehat{x}(1) = 0$ and $\widehat{x}(0) = 1$ when $q = 1$.

□

Proof of Lemma 3. Each case is proven in turn.

(i) Consider $q \leq 1/2$ and suppose the indifference line (7) intersects the unit square as shown in Figure 2(a). The firms' demand functions are

$$D_A(p_A, p_B) = -\frac{1}{2}q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = 1 + \frac{1}{2}q + p_A - p_B.$$

The first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. Under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\widehat{x}(1) \geq 0$ and $\widehat{x}(0) \leq 1$. Indeed,

$$\widehat{x}(1) = -q + p_B^{SC} - p_A^{SC} = \frac{1}{3} - \frac{2}{3}q \geq 0 \quad \text{and} \quad \widehat{x}(0) = p_B^{SC} - p_A^{SC} = \frac{1}{3} + \frac{1}{3}q < 1$$

hold for $q \leq 1/2$. Note that $\widehat{x}(1) = 0$ when $q = 1/2$.

(ii) Consider $1/2 < q \leq 2$ and suppose the indifference line (7) intersects the unit square as shown in Figure 2(b). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{2q}\left(p_B - p_A\right)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{2q}\left(p_B - p_A\right)^2.$$

The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, $\widehat{x}(1) \leq 0$ and $\widehat{x}(0) \in [0, 1]$. Indeed,

$$\widehat{x}(1) = -q + \frac{1}{2}\sqrt{2q} < 0 \quad \text{and} \quad \widehat{x}(0) = \frac{1}{2}\sqrt{2q} \in (0, 1]$$

hold for $1/2 < q \leq 2$. Note that $\widehat{x}(1) = 0$ when $q = 1/2$ and $\widehat{x}(0) = 1$ when $q = 2$.

(iii) Consider $q > 2$ and suppose the indifference line (7) intersects the unit square as shown in Figure 2(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q}\left(-\frac{1}{2} + p_B - p_A\right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q}\left(q + \frac{1}{2} + p_A - p_B\right).$$

The first-order conditions yield the equilibrium prices and profits as in part (iii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, $\widehat{x}(1) \leq 0$ and $\widehat{x}(0) \geq 1$. Indeed,

$$\widehat{x}(1) = \frac{1}{3} - \frac{2}{3}q < 0 \quad \text{and} \quad \widehat{x}(0) = \frac{1}{3} + \frac{1}{3}q > 1$$

hold for $q > 2$. Note that $\widehat{x}(0) = 1$ when $q = 2$.

□

Proof of Lemma 4. The results follow immediately from the first-order conditions. □

Proof of Lemma 5. The expressions for c_1, c_2, r_1, and r_2 as functions of q follow immediately from Lemmas 1 through 4,

$$c_1 = \Pi_A^{CS} - \Pi_A^{SS} = \begin{cases} \left(\frac{2}{3} - \frac{1}{6}q\right)^2 - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2, & q \le 1 \\ \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2, & q \in \left(1, \frac{3}{2}\right] \\ \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^3, & q \in \left(\frac{3}{2}, 3\right] \\ \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 - \frac{1}{9}q = \frac{1}{9} + \frac{1}{36q}, & q > 3 \end{cases}$$

$$c_2 = \Pi_A^{CC} - \Pi_A^{SC} = \begin{cases} \frac{1}{9}q - \left(\frac{1}{3} - \frac{1}{6}q\right)^2, & q \le \frac{1}{2} \\ \frac{1}{9}q - \frac{1}{16}\sqrt{2q}, & q \in \left(\frac{1}{2}, 2\right] \\ \frac{1}{9}q - \frac{1}{q}\left(\frac{1}{3}q - \frac{1}{6}\right)^2 = \frac{1}{9} - \frac{1}{36q}, & q > 2 \end{cases}$$

$$r_1 = \Pi_B^{SC} - \Pi_B^{SS} = \begin{cases} \left(\frac{2}{3} + \frac{1}{6}q\right)^2 - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2, & q \le \frac{1}{2} \\ \frac{9}{16}\sqrt{2q} - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2, & q \in \left(\frac{1}{2}, \frac{3}{2}\right] \\ \frac{9}{16}\sqrt{2q} - \left(1 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)\frac{-5+3\sqrt{1+16q}}{8}, & q \in \left(\frac{3}{2}, 2\right] \\ \frac{1}{q}\left(\frac{2}{3}q + \frac{1}{6}\right)^2 - \left(1 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)\frac{-5+3\sqrt{1+16q}}{8}, & q \in (2, 3] \\ \frac{1}{q}\left(\frac{2}{3}q + \frac{1}{6}\right)^2 - \frac{4}{9}q = \frac{2}{9} + \frac{1}{36q}, & q > 3 \end{cases}$$

and

$$r_2 = \Pi_B^{CC} - \Pi_B^{CS} = \begin{cases} \frac{4}{9}q - \left(\frac{1}{3} + \frac{1}{6}q\right)^2, & q \le 1 \\ \frac{4}{9}q - \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2 = \frac{2}{9} - \frac{1}{36q}, & q > 1 \end{cases}$$

Tedious but straightforward numerical calculations confirm that $c_i < r_j$ for $i, j = 1, 2$ and any given value of q. □

Proof of Proposition 1. The results follow immediately from Lemma 5 and the discussion preceding Proposition 1. □

Proofs of (10) and (11). We will start with the easier case of $q > 5/8$, then consider $q \le 5/8$.

(a) Suppose $q > 5/8$. Given firm B's price for its standard product p_B^{CS}, what price should the customizing firm A charge to consumers located at x? Fraction $\widehat{y}(x)$ of these consumers will purchase from firm A:

$$v + q_A y - p_A = v + q_B y - (1 - x) - p_B^{CS} \quad \Longrightarrow \quad \widehat{y}(x) = \frac{1}{q}(1 - x + p_B^{CS} - p_A).$$

The first-order condition for the maximization problem

$$\max_{p_A} \widehat{y}(x) p_A$$

yields

$$p_A^{CS}(x) = \frac{1}{2}\left(1 - x + p_B^{CS}\right).$$

To find firm B's equilibrium price p_B^{CS}, consider a deviation by firm B,

$$v + q_A y - \frac{1}{2}\left(1 - x + p_B^{CS}\right) = v + q_B y - (1-x) - p_B \quad \Longrightarrow \quad \widetilde{y}(x) = \frac{1}{2q}\left(1 - x - p_B^{CS} + 2p_B\right).$$

The first-order condition for the maximization problem

$$\max_{p_B} \int_0^1 (1 - \widetilde{y}(x)) p_B \, \mathrm{d}x$$

is

$$2q - \frac{1}{2} + p_B^{CS} - 4p_B = 0.$$

Substituting $p_B = p_B^{CS}$ into the above equation yields

$$p_B^{CS} = \frac{2}{3}q - \frac{1}{6}.$$

It is worth noting that $q > 5/8$ guarantees

$$\widehat{y}(x) = \frac{1}{q}\left(1 - x + p_B^{CS} - p_A^{CS}(x)\right) = \frac{1}{q}\left(\frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x\right) \in (0,1)$$

for all values of x. In fact, $\widehat{y}(0) = 1$ when $q = 5/8$.

(b) Consider $q \leq 5/8$. In this case

$$p_A^{CS}(x) = \begin{cases} 1 - x - q + p_B^{CS}, & x \leq 1 - 2q + p_B^{CS} \\ \frac{1}{2}\left(1 - x + p_B^{CS}\right), & x > 1 - 2q + p_B^{CS} \end{cases}$$

implying

$$\widehat{y}(x) = \begin{cases} 1, & x \leq 1 - 2q + p_B^{CS} \\ \frac{1}{2q}\left(1 - x + p_B^{SC}\right), & x > 1 - 2q + p_B^{CS} \end{cases}$$

Straightforward but tedious algebra implies that firm B has no incentive to deviate if and only if

$$p_B^{CS} = \frac{2}{5}q.$$

□

Proofs of (12) and (13). We will start with the easier case of $q > 5/4$, then consider $q \leq 5/4$.

(a) Suppose $q > 5/4$. Given firm A's price for its standard product p_A^{SC}, what price should the customizing firm B charge to consumers located at x? Fraction $1 - \widehat{y}(x)$ of these consumers will purchase from firm B:

$$v + q_A y - x - p_A^{SC} = v + q_B y - p_B \quad \Longrightarrow \quad \widehat{y}(x) = \frac{1}{q}(-x - p_A^{SC} + p_B).$$

The first-order condition for the maximization problem

$$\max_{p_B} (1 - \widehat{y}(x))\, p_B$$

yields

$$p_B^{SC}(x) = \frac{1}{2}\left(q + x + p_A^{SC}\right).$$

To find firm A's equilibrium price p_A^{CS}, consider a deviation by firm A,

$$v + q_A y - x - p_A = v + q_B y - \frac{1}{2}\left(q + x + p_A^{SC}\right) \quad \Longrightarrow \quad \widetilde{y}(x) = \frac{1}{2q}\left(q - x + p_A^{SC} - 2p_A\right).$$

The first-order condition for the maximization problem

$$\max_{p_A} \int_0^1 \widetilde{y}(x) p_A \,\mathrm{d}x$$

is

$$q - \frac{1}{2} + p_A^{SC} - 4p_A = 0.$$

Substituting $p_A = p_A^{SC}$ into the above equation yields

$$p_A^{SC} = \frac{1}{3}q - \frac{1}{6}.$$

It is worth noting that $q > 5/4$ guarantees

$$\widehat{y}(x) = \frac{1}{q}\left(-x - p_A^{SC} + p_B^{SC}(x)\right) = \frac{1}{q}\left(\frac{1}{12} + \frac{1}{3}q - \frac{1}{2}x\right) \in (0,1)$$

for all values of x. In fact, $\widehat{y}(1) = 0$ when $q = 5/4$.

(b) Consider $q \leq 5/4$. In this case

$$p_B^{SC}(x) = \begin{cases} \frac{1}{2}\left(q + x + p_A^{SC}\right), & x \leq q - p_A^{SC} \\ x + p_A^{SC}, & x > q - p_A^{SC} \end{cases}$$

implying

$$\widehat{y}(x) = \begin{cases} \frac{1}{2q}\left(q - x - p_A^{SC}\right), & x \leq q - p_A^{SC} \\ 0, & x > q - p_A^{SC} \end{cases}$$

Straightforward but tedious algebra implies that firm A has no incentive to deviate if and only if

$$p_A^{SC} = \frac{1}{5}q.$$

□

Proof of Proposition 2. We will first calculate the firms' equilibrium profits in subgames CS and

SC, then provide expressions for c_1, c_2, r_1, and r_2.

(a) Consider subgame CS. For $q \le 5/8$ the indifference line is

$$\widehat{y}(x) = \begin{cases} 1, & x \le 1 - \frac{8}{5}q \\ \frac{1}{q}\left(\frac{1}{2} + \frac{1}{5}q - \frac{1}{2}x\right), & x > 1 - \frac{8}{5}q \end{cases}$$

Hence,

$$\Pi_A^{CS} = \int_0^1 \widehat{y}(x) p_A^{CS}(x)\,\mathrm{d}x = \int_0^{1-\frac{8}{5}q} \left(1 - \frac{3}{5}q - x\right)\,\mathrm{d}x + \int_{1-\frac{8}{5}q}^1 \frac{1}{q}\left(\frac{1}{2} + \frac{1}{5}q - \frac{1}{2}x\right)^2\,\mathrm{d}x$$
$$= \frac{1}{2}\left(\left(1 - \frac{3}{5}q\right)^2 - q^2\right) + \frac{2}{3q}\left(q^3 - \left(\frac{1}{5}q\right)^3\right)$$

and

$$\Pi_B^{CS} = \int_0^1 (1 - \widehat{y}(x)) p_B^{CS}\,\mathrm{d}x = \int_{1-\frac{8}{5}q}^1 \frac{2}{5}\left(\frac{4}{5}q - \frac{1}{2} + \frac{1}{2}x\right)\,\mathrm{d}x = \frac{32}{125}q^2.$$

For $q > 5/8$ the indifference line is

$$\widehat{y}(x) = \frac{1}{q}\left(\frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x\right).$$

Hence,

$$\Pi_A^{CS} = \int_0^1 \frac{1}{q}\left(\frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x\right)^2\,\mathrm{d}x = \frac{2}{3q}\left(\left(\frac{5}{12} + \frac{1}{3}q\right)^3 - \left(\frac{1}{3}q - \frac{1}{12}\right)^3\right)$$

and

$$\Pi_B^{CS} = \int_0^1 \frac{1}{q}\left(\frac{2}{3}q - \frac{5}{12} + \frac{1}{2}x\right)\left(\frac{2}{3}q - \frac{1}{6}\right)\,\mathrm{d}x = \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2.$$

(b) Consider subgame SC. For $q \le 5/4$ the indifference line is

$$\widehat{y}(x) = \begin{cases} \frac{1}{q}\left(\frac{2}{5}q - \frac{1}{2}x\right), & x \le \frac{4}{5}q \\ 0, & x > \frac{4}{5}q \end{cases}$$

Hence,

$$\Pi_A^{SC} = \int_0^1 \widehat{y}(x) p_A^{SC}\,\mathrm{d}x = \int_0^{\frac{4}{5}q} \frac{1}{5}\left(\frac{2}{5}q - \frac{1}{2}x\right)\,\mathrm{d}x = \frac{4}{125}q^2$$

and

$$\Pi_B^{SC} = \int_0^1 (1 - \widehat{y}(x)) p_B^{SC}(x)\,\mathrm{d}x = \int_0^{\frac{4}{5}q} \frac{1}{q}\left(\frac{3}{5}q + \frac{1}{2}x\right)^2\,\mathrm{d}x + \int_{\frac{4}{5}q}^1 \left(\frac{1}{5}q + x\right)\,\mathrm{d}x$$
$$= \frac{2}{3q}\left(q^3 - \left(\frac{3}{5}q\right)^3\right) + \frac{1}{2}\left(\left(\frac{1}{5}q + 1\right)^2 - q^2\right).$$

For $q > 5/4$ the indifference line is

$$\widehat{y}(x) = \frac{1}{q}\left(\frac{1}{12} + \frac{1}{3}q - \frac{1}{2}x\right).$$

Hence,

$$\Pi_A^{SC} = \int_0^1 \frac{1}{q}\left(\frac{1}{12} + \frac{1}{3}q - \frac{1}{2}x\right)\left(\frac{1}{3}q - \frac{1}{6}\right) \mathrm{d}x = \frac{1}{q}\left(\frac{1}{3}q - \frac{1}{6}\right)^2$$

and

$$\Pi_B^{SC} = \int_0^1 \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{12} + \frac{1}{2}x\right)^2 \mathrm{d}x = \frac{2}{3q}\left(\left(\frac{2}{3}q + \frac{5}{12}\right)^3 - \left(\frac{2}{3}q - \frac{1}{12}\right)^3\right).$$

(c) We have

$$c_1 = \begin{cases} \frac{1}{2}\left(\left(1 - \frac{3}{5}q\right)^2 - q^2\right) + \frac{2}{3q}\left(q^3 - \left(\frac{1}{5}q\right)^3\right) - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2, & q \leq \frac{5}{8} \\ \frac{2}{3q}\left(\left(\frac{5}{12} + \frac{1}{3}q\right)^3 - \left(\frac{1}{3}q - \frac{1}{12}\right)^3\right) - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2, & q \in \left(\frac{5}{8}, \frac{3}{2}\right] \\ \frac{2}{3q}\left(\left(\frac{5}{12} + \frac{1}{3}q\right)^3 - \left(\frac{1}{3}q - \frac{1}{12}\right)^3\right) - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^3, & q \in \left(\frac{3}{2}, 3\right] \\ \frac{2}{3q}\left(\left(\frac{5}{12} + \frac{1}{3}q\right)^3 - \left(\frac{1}{3}q - \frac{1}{12}\right)^3\right) - \frac{1}{9}q, & q > 3 \end{cases}$$

$$c_2 = \begin{cases} \frac{1}{9}q - \frac{4}{125}q^2, & q \leq \frac{5}{4} \\ \frac{1}{9}q - \frac{1}{q}\left(\frac{1}{3}q - \frac{1}{6}\right)^2, & q > \frac{5}{4} \end{cases}$$

$$r_1 = \begin{cases} \frac{2}{3q}\left(q^3 - \left(\frac{3}{5}q\right)^3\right) + \frac{1}{2}\left(\left(\frac{1}{5}q + 1\right)^2 - q^2\right) - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2, & q \leq \frac{5}{4} \\ \frac{2}{3q}\left(\left(\frac{2}{3}q + \frac{5}{12}\right)^3 - \left(\frac{2}{3}q - \frac{1}{12}\right)^3\right) - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2, & q \in \left(\frac{5}{4}, \frac{3}{2}\right] \\ \frac{2}{3q}\left(\left(\frac{2}{3}q + \frac{5}{12}\right)^3 - \left(\frac{2}{3}q - \frac{1}{12}\right)^3\right) - \left(1 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)\frac{-5+3\sqrt{1+16q}}{8}, & q \in \left(\frac{3}{2}, 3\right) \\ \frac{2}{3q}\left(\left(\frac{2}{3}q + \frac{5}{12}\right)^3 - \left(\frac{2}{3}q - \frac{1}{12}\right)^3\right) - \frac{4}{9}q, & q > 3 \end{cases}$$

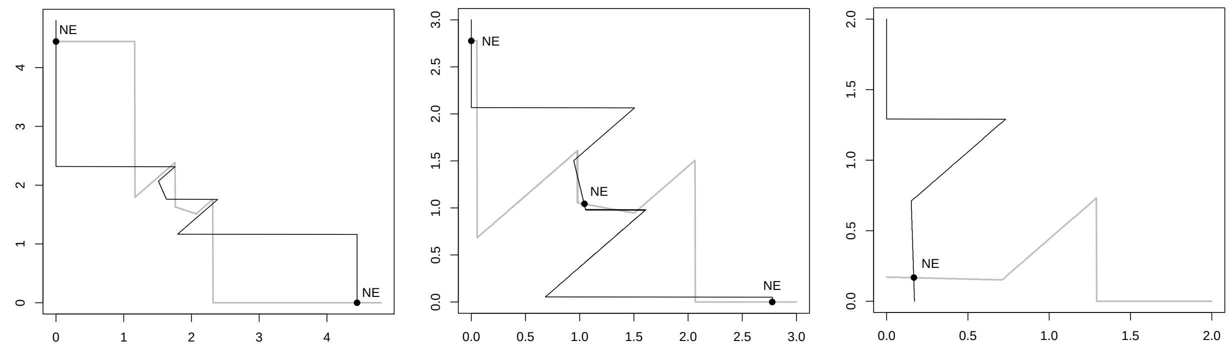
and

$$r_2 = \begin{cases} \frac{4}{9}q - \frac{32}{125}q^2, & q \leq \frac{5}{8} \\ \frac{4}{9}q - \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2, & q > \frac{5}{8} \end{cases}$$

Tedious but straightforward numerical calculations confirm that $c_1 < \min\{r_1, r_2\}$ and $c_2 < r_2$ for any given value of q. The results of Proposition 2 follow immediately.

□

Proof of Proposition 3. The three statements in this proposition are based on the equilibrium qualities for various values of α obtained through simulations. In the accompanying graphs, the black and grey curves are firm 1's and 2's best response functions, respectively. The intersections of these curves yield Nash equilibrium qualities. The graph for $\alpha = 0.05$ is representative of case (i), $\alpha = 0.08$ of case(ii), and $\alpha = 0.5$ of case (iii).



$\alpha = 0.05$ $\alpha = 0.08$ $\alpha = 0.5$

□

Equilibrium qualities for K=0.115. The counterpart of the profit function in (14) is

$$\pi_i(q_i, q_j) = \begin{cases} \frac{1}{q_j - q_i}\left(\frac{1}{3}(q_j - q_i) - \frac{1}{6}\right)^2, & q_i - q_j < -2 \\ \frac{1}{16}\sqrt{2(q_j - q_i)}, & q_i - q_j \in [-2, -1) \\ \frac{1}{2}\left(1 - \frac{1}{6}(q_j - q_i)\right)^2, & q_i - q_j \in [-1, 0] \\ \frac{1}{2}\left(1 + \frac{1}{6}(q_i - q_j)\right)^2, & q_i - q_j \in [0, 1] \\ \frac{9}{16}\sqrt{2(q_i - q_j)} - 0.115, & q_i - q_j \in (1, 2] \\ \frac{1}{q_i - q_j}\left(\frac{2}{3}(q_i - q_j) + \frac{1}{6}\right)^2 - 0.115, & q_i - q_j > 2 \end{cases}$$

Our numerical simulations show that when $K = 0.115$ the following hold for the firms' equilibrium quality choices.

(i) If $\alpha < 0.051$ then there are two asymmetric Nash equilibria, $(0, q^*)$ and $(q^*, 0)$, where $q^* > 5.35$ and decreases in α.

(ii) If $\alpha \in [0.051, 0.091]$ then there is one symmetric and two asymmetric Nash equilibria, $(q^\dagger, q^\dagger)$, $(0, q^*)$, and $(q^*, 0)$, where $q^\dagger$ decreases from 1.64 to 0.92 and q^* decreases from 4.35 to 2.42 as α increases from 0.051 to 0.091.

(iii) If $\alpha > 0.091$ then there is one symmetric Nash equilibrium $(q^\dagger, q^\dagger)$, where $q^\dagger < 0.92$ and decreases in α.

□