Advance Selling, Competition and Brand Substitutability

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Abstract

This paper studies the impact of competition on the benefits of advance selling. I construct a two-period price-setting game with two firms that produce different brands serving heterogeneous consumers. Some consumers prefer one brand, others prefer the other brand. Consumers derive common value from their preferred brand, but they differ in how strongly they dislike their less preferred brand. One of the firms can offer consumers the opportunity to pre-order its product in advance of the regular selling season. I calculate the benefits of advance selling when this firm faces competition from the other firm in the regular selling season and when it does not. Competition is shown to enhance the benefits of advance selling when in the advance selling season consumers are uncertain about which brand they will prefer. Comparative statics analysis with respect to brand substitutability reveal some interesting results.

Keywords: advance selling, price competition, strategic consumers, valuation uncertainty, consumer heterogeneity, substitutability of brands.

JEL codes: C72, D42, D43, L12, L13, M31.

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1 Introduction

Advance selling is a standard practice in the travel and leisure industries. Consumers often purchase airline tickets, cruises, vacation packages, tickets for sporting events, concerts and Broadway shows in advance. With recent developments in the Internet and information technology, advance selling has also been adopted in retail, including books, movies and CDs, software, video game consoles and smartphones. In the automobile industry, car manufactures frequently offer bonus cash deals to those who place an order prior to the launch of a new model.

The essence of advance selling is that sales of a product precede the date when the product becomes available for consumption. Because advance orders are pre-committed, advance selling helps the firm partially resolve uncertainty about future demand for its product, thus allowing for better production and distribution capacity planning. By placing an advance order, consumers avoid the risk of facing a stock-out, as it guarantees delivery of the product in the regular selling season. At the same time, consumers have to make their purchasing decisions while still uncertain about their valuations for the yet-to-be-seen product, which may allow the firm to sell to more consumers.

The theoretical literature on advance selling is small but growing. Most of the papers show how a monopolist can take advantage of advance selling (see Xie and Shugan, 2001; Boyaci and Özer, 2010; Nocke, Peitz, and Rosar, 2011; Prasad, Stecke, and Zhao, 2011; Li and Zhang, 2013; Wang and Zeng, 2016; Loginova, Wang, and Zeng, 2017, inter alia). Monopolistic models are well-suited for unique experiential products such as a Justin Bieber concert, a new Harry Potter book or the next Need for Speed game. These products have a well defined customer base of devoted followers who would not consider Tolkien a replacement for Rowling or Mario Cart a replacement for Need for Speed.

Many other products have close substitutes. Both American and Delta offer multiple flights between New York and Chicago. On any given night, there are dozens of Broadway shows that target similar audiences. A consumer who is planning to purchase a new SUV can pre-order next year’s Chevrolet Equinox and enjoy a special introductory price, or he can wait until the launch of the model to compare it directly to a Honda CR-V or a Toyota RAV4.

In this paper I inject competition by considering an alternative brand to the firm’s product in the regular selling season. My main goal is to investigate the impact of competition on the benefits of advance selling. Stated differently, does a firm have stronger or weaker incentives to advance sell when it faces competition from another firm in the regular season compared to when it does not? Our first instinct may be to say that competition increases the benefits of advance selling because of the business-stealing effect. However, the effect of advance selling on price competition in the regular selling season must also be taken into account. The latter depends on which consumer types purchase in advance.

The focus herein is on advance selling of tangible products, e.g., new car models, new versions of game consoles, tablets and smart phones. In such markets: (1) Consumer valuation uncertainty can only be resolved when the product becomes available for consumption. In the aforementioned Chevrolet Equinox example, a test drive is not available at the time of pre-order. (2) Advance selling is a marketing strategy (as opposed to standard practice in service industries such as airlines, car rentals, concerts and hotels). (3) There are no hard capacity constraints unlike, say, in the entertainment industry where the number of seats in the venue is limited. (4)
Retailers often cannot commit to future prices in advance. (As a counterexample, conference organizers commit to a certain price structure, i.e., what the early bird vs. on-site registration fees are.)

With these observations in mind, I build a two-period price-setting game in which two firms, A and B, produce and sell two different brands. Consumers are heterogeneous in two dimensions. First, half of the consumers prefer brand A and the other half prefer brand B (type A and type B consumers). Second, while consumers receive the same level of utility from their preferred brand, they differ in how strongly they dislike their less preferred brand. I consider the situation in which one firm (firm A) can offer consumers the opportunity to pre-order its product in advance of the regular selling season. Under the assumption that firm A cannot credibly commit to a particular regular selling season price in advance, I calculate firm A’s benefits of advance selling when it faces competition from firm B in the regular selling season (the competitive setting) and when it does not (the monopoly setting).\(^1\)

Does competition enhance the benefits of advance selling? The answer depends on what consumers know about their valuations for the two brands in the advance selling season. In Model I (Section 3) a consumer knows her disutility from consuming the alternative, but is uncertain about which brand she will prefer. In other words, consumers differ in their “choosiness.” More choosy consumers experience higher utility losses when consuming their nonpreferred brand; less choosy consumers lose less. Cachon and Feldman (2017), Möller and Watanabe (2016), and Guo (2009), which will be discussed in detail later, use similar models of consumer valuation uncertainty under competition. I find that competition enhances the benefits of advance selling. This is because less choosy consumers are more likely to pre-order, which relaxes price competition in the regular selling season.

In Model II (Section 4) a consumer knows which brand she will prefer, but is uncertain about her disutility from consuming the alternative. This is a reasonable assumption in settings where the products are derivatives of the existing products (e.g., new versions of Apple iPhone and Samsung Galaxy). I show that in both competitive and monopoly settings firm A chooses not to implement advance selling. The intuition behind this result is as follows. Type B consumers have stronger incentives to wait until the regular selling season than type A consumers, which reduces firm A’s price in the regular selling season. To encourage type A consumers to buy in the advance selling season, firm A must set its pre-order price below the (already low) regular selling season price. Generating positive demand in the advance selling season turns out to be too costly.

In Model III (Section 5) a consumer does not know which brand she will prefer, nor does she know her disutility from consuming the alternative. In contrast to the previous two models, there is no heterogeneity among consumers in the advance selling season, implying all consumers either pre-order or all wait until the regular selling season. As in Model I, I find that competition enhances the benefits of advance selling.

In the course of the equilibrium analysis, I uncover the interplay between consumer valuation uncertainty (Models I, II and III), competition (whether firm A faces competition in the regular selling season) and the level of brand substitutability.\(^2\) The most interesting results are:

\(^1\)In the monopoly setting with only brand A available for consumption, type A consumers are fans of firm A’s product, while type B consumers care less about it.

\(^2\)I assume that consumers draw their disutilities from the interval \([0, d]\), so \(d\) is used as a measure of brand...
• In the competitive setting of Model I, firm A has stronger incentives to advance sell when the level of brand substitutability is high. The reverse holds in the monopoly setting.

• For some parameter values, firm A is better off in the competitive setting of Model I than in the monopoly setting. This counterintuitive result is driven by the assumption that firm A cannot credibly commit to a particular regular selling season price in advance: In the absence of firm B, firm A sets its regular season price to reach everyone, which puts pressure on the firm’s advance selling price.

• In the competitive setting of Model III, firm A sells in advance to all consumers no matter what the level of brand substitutability is. In the monopoly setting, firm A does not want to implement advance selling when the level of brand substitutability is high.

• Competition enhances the benefits of advance selling in Models I and III; to a greater extent when the level of brand substitutability is high. (In both models consumers are initially uncertain about their preferred brand.)

• There are no benefits to selling in advance when consumers know their preferred brand from the outset (Model II).

In Section 6 I relax the assumption that only one firm can advance sell. The two firms simultaneously and non-cooperatively choose whether to implement advance selling, resulting in four separate scenarios. I show that under the assumptions of Models I and III the game formed by these four scenarios has two asymmetric pure-strategy Nash equilibria: only firm A (firm B) implements advance selling. Thus, all the results obtained in Sections 3 and 5 continue to hold. Next, I find that if consumer valuation uncertainty is as in Model II, the game has two symmetric equilibria. In the first equilibrium neither firm implements advance selling, while in the second one both firms implement advance selling and all consumers pre-order.

• Allowing both firms to advance sell turns out to be unnecessary in Models I and III, as the firms never pursue the option simultaneously. In contrast, advance selling by both firms arises in equilibrium when consumer valuation uncertainty is as in Model II.

That advance selling by both firms arises in equilibrium only when consumers are already fans of certain brands is an important result. It explains many cases where firms with intense competition (such as smartphone markets) advance sell their substitutable products.

1.1 Related Literature

Most of the theoretical papers that examine advance selling are single-firm models. Only six papers study advance selling in a competitive environment. Out of these five, Shugan and Xie (2005), Cachon and Feldman (2017), Möller and Watanabe (2016), and Guo (2009) are most relevant, as they explore the impact of competition on advance selling and take into account strategic behavior of consumers.

substitutability. The lower (higher) is $d$, the higher (lower) is the level of substitutability between the two brands. In the marketing literature, the most commonly used measures of brand substitutability are brand switching probabilities and cross elasticities. See, for example, Bucklin, Russell and Srinivasan (1998).
In Shugan and Xie (2005), two firms simultaneously decide whether to advance sell or spot sell, and at which prices. If a firm advance sells, consumers can only purchase the product in that period. Consumers observe the prices and decide from which firm to purchase according to a market share attraction model (see, for example, Bell, Keeney, and Little, 1975). Provided that the firms’ marginal costs are sufficiently low, the authors show that advance selling by both firms is an equilibrium. Moreover, each firm obtains a higher profit when both firms advance sell than when they both spot sell.

In contrast, Cachon and Feldman (2017) show that even though advance selling by both firms may occur in equilibrium, the firms would in most cases benefit if they could commit to sell on the spot market. In their setup two firms simultaneously announce advance period prices and then spot period prices. Under the first demand specification, consumers initially are unsure about which firm they will prefer as well as how strongly they will value their preferred firm. Specifically, the value from their preferred firm can be high or low, while the value from the other firm is zero. Consequently, the firms optimally set their prices at the high value in the spot period. The authors find that the possibility of advance selling often hurts the firms. This is because consumers are more homogeneous in the advance period, which intensifies price competition and drives down the advance period prices. Under the second demand specification, a fraction of consumers arrive in the advance period and are loyal to one of the two firms, whereas the rest arrive in the spot period. Because consumers buy in advance only if they get a good discount, competition in the spot period forces the firms to further lower their advance period prices, even though they do not directly compete in the advance period. Again, competition lowers the attractiveness of advance selling.

Unlike the current manuscript and the above two papers, Möller and Watanabe (2016) endow firms with the ability to commit to future prices. There are two differentiated products. In the advance purchase period each consumer receives an imperfect signal about the identity of his/her preferred product. More “choosy” consumers derive a higher consumption value from their preferred product and a lower consumption value from the other product. The authors compare the case in which the products are sold by two competing firms with the case in which the products are sold by one firm. They show that advance purchase discounts are larger in the oligopolistic scenario. Competition results in welfare reduction, as it increases the fraction of consumers who buy in the advance purchase period, without full knowledge of their preferences.

Guo (2009) develops a model of refund policy choice for two competing service providers that operate under limited capacity. Low-valuation advance shoppers do not know which service provider they will prefer in the spot period; high-valuation spot shoppers arrive in the spot period already knowing their preferred provider. The interplay between the efficiency-enhancing and the competition-intensifying effects of (partial) refunds leads to a whole range of pure-strategy equilibria.

Tang, Rajaram, Alptekinoğlu, and Ou (2004) and McCardle, Rajaram, and Tang (2004) consider situations in which firms and retailers have to purchase inventories before a short selling season with uncertain demand. Hence, advance selling enables retailers to lock in a portion of the customer demand and use the advance orders to develop more accurate forecasts and supply

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3I assume that if a consumer did not purchase the product from firm A in the advance selling season, she can still purchase it in the regular selling season. Thus, firm A’s regular selling season price influences the pre-order price.
plans. Tang, Rajaram, Alptekinoğlu, and Ou (2004) assume two brands, A and B. A single firm (firm A) launches the advance booking discount (ABD) program by offering a discount to the fixed regular selling season price. The number of consumers who pre-order brand A is an exogenously given increasing function of the discount. The authors evaluate the benefits of the ABD program, characterize the optimal discount price, and analyze how it is affected by the degree of demand uncertainty and the correlation between the demands for brand A and brand B. McCardle, Rajaram, and Tang (2004) extend Tang, Rajaram, Alptekinoğlu, and Ou (2004) by assuming that each of the two retailers simultaneously and independently chooses whether to launch an ABD program. The authors show that if it is optimal for one firm to adopt an ABD program, the unique equilibrium has both firms adopting an ABD program.

Several papers study advance selling in a monopoly setting and, like the present manuscript, involve uncertainty of consumer valuations. Xie and Shugan (2001) consider a variety of situations, including limited capacity, refunds, and exogenous credibility. Their findings suggest that given buyer uncertainty and sufficiently low marginal costs, the monopolist can potentially earn greater profit by either increasing sales to more buyers or by allowing a premium advance price. Fay and Xie (2010) explore two pricing mechanisms employed by a two-product monopolist: advance selling and probabilistic selling. Under advance selling the seller encourages consumers to make decisions before their valuations for the two products are known (thus homogenizing consumers), while under probabilistic selling, the seller motivates consumers with weak product preferences to choose the uncertain option (thus separating heterogeneous consumers). Nocke, Peitz, and Rosar (2011) assume that the seller can credibly commit to future prices. By offering an advance-selling discount, the monopolist induces consumers with high expected valuations to purchase the product in advance. Möller and Watanabe (2010) consider advance purchase discounts and clearance sales. They determine how the comparison of these two pricing strategies depends on price commitment, the availability of temporal capacity limits, the rationing rule, and resale. Chu and Zhang (2011) allow the firm to control the release of information about the product at pre-order. They show that the seller may want to release some information or none, but never all. In Prasad, Stecke, and Zhao (2011) random (possibly correlated) numbers of consumers arrive in the advance and regular selling seasons. The authors find that implementing advance selling is not always optimal. Several papers examine advance selling in an environment where consumers deviate from rational decision making (Zhao and Stecke, 2010; Naisiry and Popescu, 2012; Lim and Tang, 2013). Yu, Ahn, and Kapuscinski (2015) analyze the role of advance selling as a signal of product quality. Yu, Kapuscinski, and Ahn (2015) study the impact of the interdependence of consumers’ valuations on advance selling.

Contribution of this paper to the literature on advance selling is three-fold. First, I factor brand substitutability into the framework, which none of the other theoretical papers have done. Second, I consider two different types of consumer valuation uncertainty. Third, the existing literature arrives at opposing conclusions about the welfare implications of advance selling, or the effect of competition on the firms’ incentives to advance sell. Indeed, while some of the papers reviewed above conclude that firms can extract greater profits when advance selling is possible (Shugan and Xie, 2005), others find that advance selling decreases the firms’ profits.

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4This setup is similar to the present manuscript in the sense that only one firm can advance sell.
This paper discovers that brand substitutability and the nature of consumer valuation uncertainty play an important role in determining the equilibrium outcome, thus shedding some light on the controversy in the literature.

2 Setup

There are two firms: firm A sells brand A and firm B sells brand B. The firms possess identical constant returns to scale technologies with marginal cost $c = 0$.

The market consists of continuum consumers of mass one. Consumers have unit demands and are heterogeneous in two dimensions. First, half of the consumers prefer firm A's product and the rest firm B's product. I will refer to the former group as type A consumers and to the latter as type B consumers. Second, consumers differ in how strongly they dislike their less preferred brand. Specifically, a consumer is willing to pay $v$ for her preferred brand and $v - x$ for the alternative. I assume that $x$ is an i.i.d. draw from a uniform distribution on the interval $[0, d]$, where $d \leq v$. In essence, parameter $d$ measures brand substitutability. The higher (lower) is $d$, the lower (higher) is the level of substitutability between the two brands.

There are two periods. Period 1 is the advance selling season and period 2 is the regular selling season. I analyze three models that differ in the nature of uncertainty that consumers face in the advance selling season. In Model I a consumer knows her $x$, but is uncertain about which brand she will prefer. In Model II a consumer knows her preferred brand, but is uncertain about her $x$. Finally in Model III, a consumer is uncertain about which brand she will prefer as well as her disutility from consuming the alternative. Consumers fully learn their valuations for the two brands in the regular selling season when the brands become available for consumption.

Within each model, I consider two settings: competitive and monopoly. In the competitive setting, one firm (firm A) decides whether to offer consumers the opportunity to pre-order its product in period 1 and at which price, knowing that in period 2 it will compete with the other firm (firm B).

In the monopoly setting, firm A is the only firm serving the market. I maintain the assumption that half of the consumers are willing to pay $v$ for firm A's product, while the other half $v - x$, where $x$ is uniformly distributed on $[0, d]$. Basically, type A consumers are consumers who like brand A, while type B consumers are those who do not like firm A's product that much. This demand is exactly what firm A would face in the competitive setting if firm B set its price so high that no consumers wished to purchase brand B.

In both settings I assume that in the advance selling season firm A cannot commit to a particular regular selling season price. Let $p_1^A$ and $p_2^A$ denote the prices of firm A in periods 1 and 2, and let $p_2^B$ denote the price of firm B in period 2.

2.1 Benchmark: No Advance Selling

Models I, II and III are identical when firm A cannot implement advance selling, because in all three models consumer uncertainty is fully resolved before any purchases are made. I will refer to this model as the no advance selling benchmark. The timeline for the competitive setting is:
1. Consumers learn their valuations for the brands.

2. Firms A and B simultaneously and non-cooperatively choose their regular selling season prices $p_A^2$ and $p_B^2$.

3. Each consumer decides which brand to purchase, if any.

Proposition 1 reports the Nash equilibrium prices. Not surprisingly, as the brands become less substitutable ($d$ increases), price competition subsides.

**Proposition 1 (Benchmark, competitive setting).** *In the competitive setting of the no advance selling benchmark, the Nash equilibrium prices are $p_A^2 = p_B^2 = d$. Each consumer purchases her preferred brand. Equilibrium profits for firm A and firm B, respectively, are*

$$\Pi_A = \Pi_B = \frac{d}{2}.$$

In the monopoly setting of the no advance selling benchmark, firm A chooses $p_A^2$ to maximize its profit.

**Proposition 2 (Benchmark, monopoly setting).** *In the monopoly setting of the no advance selling benchmark:*

(i) *When $d \leq v/3$, firm A charges $p_A^2 = v - d$, all consumers buy, and firm A’s equilibrium profit is*

$$\Pi_A = v - d.$$

(ii) *When $d \in (v/3, v]$, firm A charges $p_A^2 = (v + d)/2$. All type A consumers buy, and type B consumers with $x \leq (v - d)/2$ buy. Firm A’s equilibrium profit is*

$$\Pi_A = \frac{v}{4} + \frac{v^2}{8d} + \frac{d}{8}.$$

All proofs are relegated to the Appendix. In contrast to the competitive setting, in the monopoly setting firm A’s profit decreases in $d$. This is because the differentiation effect of $d$ – that price competition subsides as the brands become less substitutable – is “turned off” in the monopoly setting. Figure 1 depicts the relationships between the firms’ equilibrium profits in the two settings of the benchmark and the brand substitutability parameter $d$.

We will use the results of Propositions 1 and 2 in the next two sections, when calculating the relative benefits of advance selling

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in the competitive and monopoly settings, respectively. When the first ratio is larger (smaller) than the second, we say that competition enhances (diminishes) the benefits of advance selling.

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5Recall that $d \in [0, v]$. In Figure 1 $v$ is normalized to 1, so the range for $d$ becomes $[0, 1]$. 
3 Model I

In Model I consumers are initially uncertain about which brand they will prefer in the regular selling season, and they differ in their “choosiness,” $x$. Less choosy consumers (with $x$’s close to 0) experience small utility losses when consuming their nonpreferred brand; those with $x$’s close to $d$ lose more when consuming their nonpreferred brand.

3.1 Competitive Setting

The timeline for the competitive setting is:

1. At the beginning of period 1, each consumer learns her $x$.
2. Firm A sets a pre-order price for its product, $p^A_1$.
3. Consumers decide whether to pre-order the product from firm A.
4. At the beginning of period 2, each consumer learns her preferred brand.
5. Firms A and B simultaneously and non-cooperatively choose their regular selling season prices $p^A_2$ and $p^B_2$.

6. Those consumers who did not pre-order in period 1 decide which brand to purchase, if any.

Obviously, consumers with high values of $x$ have greater incentives to wait until the regular selling season to learn their preferred brand than those with low values of $x$. Hence, suppose that consumers with $x \in (\hat{x}, d]$ are left on the market in period 2. The next lemma establishes the Nash equilibrium prices in period 2 as functions of the threshold $\hat{x}$.

**Lemma 1** (Model I, prices in period 2). Suppose that consumers with $x \in (\hat{x}, d]$ are on the market in period 2. The Nash equilibrium prices are:

(i) When $d \leq v/2$,

$$ p^A_2(\hat{x}) = p^B_2(\hat{x}) = \begin{cases} d + 2\sqrt{d\hat{x} - \hat{x}^2}, & \text{if } \hat{x} \leq \frac{d}{2} \\ 2d, & \text{if } \hat{x} \in \left(\frac{d}{2}, d\right). \end{cases} $$

(ii) When $d \in (v/2, v]$,

$$ p^A_2(\hat{x}) = p^B_2(\hat{x}) = \begin{cases} d + 2\sqrt{d\hat{x} - \hat{x}^2}, & \text{if } \hat{x} \leq \frac{d}{2} - \sqrt{2vd - v^2} \\ v, & \text{if } \hat{x} \in \left(\frac{d}{2} - \sqrt{2vd - v^2}, d\right). \end{cases} $$

It is easy to verify that the prices in Lemma 1 are (weakly) increasing functions of $\hat{x}$. This is intuitive. The higher is $\hat{x}$, the stronger are the brand preferences of those consumers who remain on the market in period 2, which relaxes price competition.

After the announcement of $p^A_1$, consumers form their expectations about the firms’ regular selling season prices – that $p^A_2$ and $p^B_2$ will be the same price $p^*_2$. If a consumer pre-orders the product from firm A, her payoff will be $v - p^*_2$ if brand A is her preferred brand, and $v - x - p^*_2$ otherwise. Thus, the consumer’s expected payoff is

$$ v - \frac{x}{2} - p^*_2. $$

If the consumer waits until period 2 to purchase her preferred brand (provided that $p_2 \leq v$), she receives payoff

$$ v - p_2. $$

Therefore, the consumer will optimally pre-order the product from firm A if and only if

$$ v - \frac{x}{2} - p^*_2 \geq v - p_2, $$

or

$$ x \leq 2(p_2 - p^*_2). $$
Summarizing the above,
\[
\hat{x} = \begin{cases} 
0, & \text{if } p_1^A > p_2 \\
\min\{d, 2(p_2 - p_1^A)\}, & \text{if } p_1^A \leq p_2.
\end{cases}
\tag{1}
\]

For a given value of \( p_1^A \), a rational expectations equilibrium consists of prices \( p_2^A \) and \( p_2^B \), consumer expectations \( p_2 \), and a threshold \( \hat{x} \) such that: (i) \( p_2^A \) and \( p_2^B \) are the Nash equilibrium prices given \( \hat{x} \) (Lemma 1); (ii) \( \hat{x} \) describes the optimal consumer behavior given \( p_2 \) (equation 1); and (iii) consumer expectations are fulfilled (\( p_2^A = p_2^B = p_2 \)).

The statement of the next lemma, Lemma 2, is relegated to the Appendix because of its length. It gives the equilibrium values of \( \hat{x} \) and \( p_2 \) as functions of \( p_1^A \). As it turns out, both are (weakly) decreasing functions. The intuition is as follows. As \( p_1^A \) increases, the number of consumers who pre-order the product from firm A decreases. Because lower values of \( \hat{x} \) lead to more intense price competition in period 2, \( p_2 \) decreases. That a higher pre-order price leads to lower regular selling season prices is an interesting result driven by the information structure assumed in Model I.

It remains to calculate the optimal pre-order price \( p_1^A \) that maximizes firm A’s expected profit:
\[
\Pi^A(p_1^A) = p_1^A \frac{\hat{x}(p_1^A)}{d} + p_2(p_1^A)\frac{d - \hat{x}(p_1^A)}{2d},
\tag{2}
\]
where the functions \( \hat{x}(p_1^A) \) and \( p_2(p_1^A) \) are from Lemma 2.

**Proposition 3** (Model I, competitive setting). *In the competitive setting of Model I, the equilibrium prices, profits, and consumer behavior are:*

(i) When \( d \leq \frac{v}{2} \), firm A charges \( p_1^A = 3d/2 \) thereby inducing all consumers to pre-order.\(^6\) In period 2 the firms charge \( p_2 = 2d \). The firms’ equilibrium profits are
\[
\Pi^A = \frac{3d}{2} \quad \text{and} \quad \Pi^B = 0.
\]

(ii) When \( d \in (\frac{v}{2}, v] \), firm A charges \( p_1^A = 3v/4 \) thereby inducing consumers with \( x \leq \frac{v}{2} \) to pre-order. In period 2 the firms charge \( p_2 = v \). The remaining consumers purchase their preferred brand. The firms’ equilibrium profits are
\[
\Pi^A = \frac{v}{2} + \frac{v^2}{8d} \quad \text{and} \quad \Pi^B = \frac{v}{2} - \frac{v^2}{4d}.
\]

When the level of brand substitutability is high (\( d \leq \frac{v}{2} \)), price competition in period 2 is intense (part (i) of Lemma 1). Low second-period prices imply firm A’s pre-order price must be low to generate positive demand. When both \( p_1^A \) and \( p_2 \) are low, firm A’s profit is maximized when the total number of consumers who purchase its brand,
\[
\frac{\hat{x}}{d} + \frac{d - \hat{x}}{2d}.
\]

\(^6\)When no consumers are left on the market in period 2, we assume that the firms’ prices are \( p_2^A(d) = p_2^B(d) = \min\{2d, v\} \), obtained by substituting \( \hat{x} = d \) into the expressions for \( p_2^A(\hat{x}) \) and \( p_2^B(\hat{x}) \) in Lemma 1.
is maximized. In equilibrium firm A charges $p_1^A = 3d/2$, which induces all consumers to pre-order: $\hat{x} = d$.

When the level of brand substitutability is low ($d > \nu/2$), price competition in period 2 is less intense (part (ii) of Lemma 1). Maximizing (2) yields an interior solution in the sense that firm A’s equilibrium pre-order price attracts only a fraction of consumers: $\hat{x} < d$.

Recall that in the competitive setting of the no advance selling benchmark (Proposition 1) the firms’ profits are increasing functions of $d$. In contrast, with advance selling in Model I firm A’s profit first increases, then decreases in $d$. Firm B’s profit increases in $d$. Figure 2 depicts the firms’ equilibrium profits as functions of $d$.

With Propositions 1 and 3 in hand, we can calculate the relative benefits of advance selling when firm A competes with firm B in the regular selling season:

$$\frac{\text{firm A’s profit in Model I}}{\text{firm A’s profit in the benchmark}} = \begin{cases} 3, & \text{if } d \leq \frac{\nu}{2} \\ \frac{\nu}{d} + \frac{\nu^2}{4d^2}, & \text{if } d \in \left(\frac{\nu}{2}, \nu\right] \end{cases}$$

(3)

The ratio is strictly above one for all values of $d$: firm A always benefits from advance selling in the competitive setting of Model I. Also note that the ratio (weakly) decreases in $d$. That is, the relative benefits of advance selling decrease as the two brands become less substitutable.
3.2 Monopoly Setting

Now we consider the monopoly setting. The results of the equilibrium analysis are summarized in the proposition below.

Proposition 4 (Model I, monopoly setting). In the monopoly setting of Model I, the equilibrium prices, profits and consumer behavior are:

(i) When \( d \leq v/3 \), firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2 firm A charges \( p^A_2 = v - d \). All consumers buy. Firm A's equilibrium profit is

\[ \Pi^A = v - d. \]

(ii) When \( d \in (v/3, v/2] \), firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2 firm A charges \( p^A_2 = (v + d)/2 \). All type A consumers buy, and type B consumers with \( x \leq (v - d)/2 \) buy. Firm A's equilibrium profit is

\[ \Pi^A = \frac{v}{4} + \frac{v^2}{8d} + \frac{d}{8}. \]

(iii) When \( d \in (v/2, v] \), firm A charges \( p^A_1 = 3v/4 \) inducing consumers with \( x \leq v/2 \) to pre-order. In period 2 firm A charges \( p^A_2 = v \). The remaining type A consumers buy. Firm A's equilibrium profit is

\[ \Pi^A = \frac{v}{2} + \frac{v^2}{8d}. \]

When \( d \leq v/2 \), the monopolist sets a high pre-order price so that all consumers wait until the regular selling season. This pricing strategy is payoff equivalent to not implementing advance selling. Inability to set the regular selling season price in advance hurts the monopolist. Indeed, if firm A could commit to \( p^A_2 = v \), then \( p^A_1 = v - d/2 \) would induce all consumers to pre-order. The firm would earn

\[ \Pi^A = v - \frac{d}{2}, \]

a higher profit than that in parts (i) and (ii) of Proposition 4.

When \( d > v/2 \), the monopolist sells in both periods. Observe that firm A sets the same prices and earns the same profit as in the competitive setting of Model I. The only difference between part (ii) of Proposition 3 and part (iii) of Proposition 4 is that in the competitive setting type B consumers with \( x > v/2 \) are served by firm B.

Because firm B focuses on a different set of consumers, its presence actually creates less competitive pressure than the prospective actions by the monopolist. Figure 2 depicts firm A's equilibrium profit as a function of \( d \). Interestingly, firm A's profit may be lower in the monopoly setting than in the competitive setting.\(^7\) Counterintuitive at first, this result is driven by the fact that firm A has to shift gears when \( d \leq v/2 \). While in the competitive setting firm A finds it optimal to pursue the strategy of selling to all consumers in period 1, in the monopoly setting it charges a high pre-order price so that all consumers wait until period 2.

\(^7\)This happens when \( d \in ((1 + \sqrt{12})v/11, v/2] \).
Propositions 2 and 4 imply that the relative benefits of advance selling when firm A is a monopolist equal

\[
\left. \frac{\text{firm A's profit in Model I}}{\text{firm A's profit in the benchmark}} \right|_{\text{monopoly}} = \begin{cases} 
1, & \text{if } d \leq \frac{v}{2} \\
\frac{v^2}{2} + \frac{v^2}{8d} / (8d), & \text{if } d \in \left(\frac{v}{2}, v\right].
\end{cases}
\] (4)

The ratio equals one when \(d \leq \frac{v}{2}\): firm A finds it optimal not to implement advance selling (parts (i) and (ii) of Proposition 4).

We can now answer the main question of the paper, “Does competition enhance the benefits of advance selling?” Figure 3 plots the benefit ratios (3) and (4). We see that in Model I competition always enhances the benefits of advance selling. This is because in Model I advance selling attracts consumers with weak brand preferences (low \(x\)), relaxing price competition in the regular selling season.

4 Model II

In Model II, a consumer knows her preferred brand but is initially uncertain about her disutility from consuming the alternative. We first analyze the competitive setting.
Type B consumers have stronger incentives to wait until period 2 than type A consumers. Lemma 3 (in the Appendix) reports the Nash equilibrium prices when the market in period 2 consists of the fraction \(1 - \beta\) of type B consumers (i.e., all type A consumers and the fraction \(\beta\) of type B consumers have pre-ordered the product from firm A) and when it consists of all type B consumers and the fraction \(1 - \alpha\) of type A consumers (i.e., the fraction \(\alpha\) of type A consumers have pre-ordered the product from firm A).

For a given price \(p^A_1\) that firm A might be charging, Lemma 4 reports the firms’ equilibrium prices in period 2 and the optimal consumer purchasing behavior.

**Lemma 4** (Model II, rational expectations given \(p^A_1\)). Suppose that firm A charges \(p^A_1\) in period 1. In a rational expectations equilibrium:

(a) If \(p^A_1 \leq d/9\), all consumers pre-order. In period 2 the firms charge \(p^A_2 = d/3\) and \(p^B_2 = 2d/3\).\(^8\)

(b) If \(p^A_1 \in (d/9, d/3]\), only type A consumers pre-order. In period 2 the firms charge \(p^A_2 = d/3\) and \(p^B_2 = 2d/3\). Type B consumers with \(x \leq d/3\) purchase brand A, while those with \(x \in (d/3, d]\) purchase brand B.

(c) If \(p^A_1 \in (d/3, d]\), the fraction

\[
\alpha = \frac{3(d - p^A_1)}{2d}
\]

of type A consumers pre-order. In period 2 the firms charge

\[
\begin{align*}
p^A_2 &= \frac{d(3 - 2\alpha)}{3} = p^A_1 \quad \text{and} \\
p^B_2 &= \frac{d(3 - \alpha)}{3} = \frac{d + p^A_1}{2}.
\end{align*}
\]

Type B consumers with \(x \leq \alpha d/3\) and the remaining type A consumers purchase brand A, while type B consumers with \(x \in (\alpha d/3, d]\) purchase brand B.

(d) If \(p^A_1 > d\), no consumer pre-orders. In period 2 the firms charge \(p^A_2 = p^B_2 = d\) and each consumer purchases her preferred brand.

It turns out that firm A maximizes its profit when it sets a high pre-order price so that all consumers wait until period 2 (part (d) of the above lemma).

**Proposition 5** (Model II, competitive setting). In the competitive setting of Model II, firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2 the firms charge \(p^A_2 = p^B_2 = d\). Each consumer purchases her preferred brand. The firms’ equilibrium profits are

\[
\Pi^A = \Pi^B = \frac{d}{2}.
\]

Why is it the case that any \(p^A_1\) that generates positive demand (parts (a), (b) and (c) of Lemma 4) is inferior to not implementing advance selling (charging a high pre-order price so that no consumer pre-orders)? In Model II, type A consumers have stronger incentives to pre-order the

\(^8\)When no consumers are left on the market in period 2, we assume that the firms’ prices are \(p^A_2 = d/3\) and \(p^B_2 = 2d/3\) (as in the first part of Lemma 3).
product than type B consumers, so there will be more type B consumers than type A consumers in period 2. This intensifies price competition. Consider, for example, \( p_A^1 \in (d/9, d/3) \) (part (b) of Lemma 4). Such a price induces type A consumers to pre-order. In period 2 the firms compete for type B consumers. To attract customers, firm A has to lower its price. Firm B responds by lowering its price, too. As a result, \( p_A^2 = d/3 \) and \( p_B^2 = 2d/3 \). (If all type A consumers and all type B consumers were present in period 2, the equilibrium prices would be higher, \( p_A^2 = p_B^2 = d \).

Intense price competition in period 2, in turn, puts pressure on firm A’s pre-order price. Indeed, if firm A pursues a strategy of selling to some consumers in period 1, \( p_A^1 \) cannot be higher than \( p_A^2 \). Thus, it becomes very costly for firm A to generate positive demand in period 1. Firm A is better off charging a high pre-order price so that no consumer pre-orders.

The same result – that firm A finds it optimal not to implement advance selling – also arises in the monopoly setting.

**Proposition 6** (Model II, monopoly setting). In the monopoly setting of Model II, firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2, firm A’s equilibrium price and profit are the same as in the monopoly setting of the no advance selling benchmark (Proposition 2).

The proof of Proposition 6 (in the Appendix) is rather involved despite its simple conclusion. Again, firm A’s inability to commit not to lower its price in period 2 for type B consumers hurts the monopolist. If firm A could credibly commit to \( p_A^2 = v \), then it would charge \( p_A^1 = v - d/2 \), inducing both types of consumers to pre-order. Firm A’s profit of

\[
\Pi^A = v - \frac{d}{2}
\]

would be higher than that in Proposition 2.

To sum up, in both competitive and monopoly settings of Model II firm A finds it optimal to not implement advance selling. This happens because in Model II type B consumers have stronger incentives to wait until the regular selling season than type A consumers, which implies firm A’s price in period 2 cannot be high. To encourage consumers who prefer brand A to pre-order, the firm must set its pre-order price below this low regular selling season price. Generating positive demand in period 1 is too costly.

### 5 Model III

In Model III, a consumer does not know which brand she will prefer, nor her disutility from consuming the alternative. In contrast to the previous two models, there is no heterogeneity among consumers in the advance selling season, implying all consumers either pre-order or all wait until the regular selling season. This makes the analysis of Model III fairly simple.

Let us start with the competitive setting. In equilibrium firm A sets its advance selling price to \( p_A^1 = 3d/4 \), inducing all consumers to pre-order. In period 2, firm A and firm B charge \( p_A^2 = p_B^2 = d \).
Proposition 7 (Model III, competitive setting). *In the competitive setting of Model III, firm A sets its advance selling price to* $p_1^A = 3d/4$ *thereby inducing all consumers to pre-order. In period 2 the firms charge* $p_2^A = p_2^B = d$. *The resulting profits are*

$$\Pi^A = \frac{3d}{4} \quad \text{and} \quad \Pi^B = 0.$$  

I plot the firms’ profits in the competitive setting of Model III, as well as firm A’s profit in the monopoly setting (see Proposition 8 below) in Figure 4.

From Propositions 1 and 7 we calculate the relative benefits of advance selling when firm A competes with firm B in the regular selling season:

$$\frac{\text{firm A’s profit in Model III}}{\text{firm A’s profit in the benchmark}} \bigg|_{\text{competition}} = \frac{3d/4}{d/2} = \frac{3}{2}. \quad (5)$$

Since the ratio is above one for all values of $d$, firm A always benefits from advance selling.

Now consider the monopoly setting. By charging a high pre-order price firm A can achieve the equilibrium profit under the no advance selling benchmark (Proposition 2). The other option is to induce all consumers to pre-order.

Proposition 8 (Model III, monopoly setting). *In the monopoly setting of Model III, the equilibrium prices, profits and consumer behavior are:*

(i) *When* $d \leq v/3$, *firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2 firm A charges* $p_2^A = v - d$. *All consumers buy. Firm A’s equilibrium profit is*

$$\Pi^A = v - d.$$
(ii) When \( d \in (v/3, 2v/(3 + \sqrt{8}) \}, \) firm A sets the pre-order price sufficiently high to foreclose advance sales. In period 2 firm A charges \( p^A_2 = (v + d)/2 \). All type A consumers and type B consumers with \( x \leq (v - d)/2 \) buy. Firm A’s equilibrium profit is

\[
\Pi^A = \frac{v}{4} + \frac{v^2}{8d} + \frac{d}{8}.
\]

(iii) When \( d \in (2v/(3 + \sqrt{8}), v) \}, \) firm A charges \( p^A_1 = v - v^2/(8d) - d/8 \) thereby inducing all consumers to pre-order. In period 2 firm A charges \( p^A_2 = (v + d)/2 \). Firm A’s equilibrium profit is

\[
\Pi^A = v - \frac{v^2}{8d} - \frac{d}{8}.
\]

This proposition implies that when the level of brand substitutability is high \( (d \leq 2v/(3 + \sqrt{8}) \), the monopolist finds it optimal not to implement advance selling. When the level of brand substitutability is low \( (d > 2v/(3 + \sqrt{8}) \), the monopolist induces all consumers to pre-order. Recall that the same result – that firm A’s incentives to advance sell decrease with the level of brand substitutability – was obtained in the monopoly setting of Model I.

Finally, we calculate the relative benefits of advance selling when firm A is a monopolist

\[
\frac{\text{firm A’s profit in Model III}}{\text{firm A’s profit in the benchmark}} \bigg|_{\text{monopoly}} = \begin{cases} 
1, & \text{if } d \leq \frac{2v}{3+\sqrt{8}} \\
\frac{v - v^2/(8d) - d/8}{v/4 + v^2/(8d) + d/8}, & \text{if } d \in \left(\frac{2v}{3+\sqrt{8}}, v\right)
\end{cases}
\]
and compare them with the relative benefits under the competitive setting (5). As we see from Figure 5, competition enhances the benefits of advance selling, to a greater extent when the level of brand substitutability is high ($d$ is low).

6 Both Firms Can Advance Sell

In this section I consider the situation in which both firms have an opportunity to advance sell. Specifically, at the beginning of the game firms A and B simultaneously and non-cooperatively choose whether to implement advance selling. This leads to four different subgames: (N, N), (Y, N), (N, Y) and (Y, Y), where N stands for no advance selling and Y stands for advance selling. I will refer to this new game as the both-firms-can-advance-sell version of Model I (II, III).

Let us start with the both-firms-can-advance-sell version of Model I. Proposition 1 gives us the equilibrium in subgame (N, N) and Proposition 3 covers subgames (Y, N) and (N, Y). It is easy to show that the firms find themselves in a pure Bertrand price war in subgame (Y, Y): $p_A^1 = p_B^1 = c = 0$ and all consumers pre-order. Figures 6(a) and 6(b) depict the payoff matrices for $d \leq v/2$ and $d \in (v/2, v]$, respectively, corresponding to parts (i) and (ii) of Proposition 3.

Comparing firm A’s profit in subgame (Y, N) with that in subgame (N, N), we see that the former is always higher. Indeed, for any $d \leq v/2$

$$\frac{3d}{2} > \frac{d}{2}$$

and for any $d \in (v/2, v]$

$$\frac{v}{2} + \frac{v^2}{8d} > \frac{d}{2}.$$ 

It immediately follows that the game depicted in Figure 6(a) has three pure-strategy Nash equilibria: (Y, N), (N, Y), and (Y, Y); the game in Figure 6(b) has two: (Y, N) and (N, Y). Since (Y, N) and (N, Y) are Nash equilibria under any set of parameter values, the results obtained in Section 3 continue to hold.

Now consider the both-firms-can-advance-sell version of Model II. We already know that in the setting where only one firm can implement advance selling, that firm will optimally choose not to implement advance selling (Proposition 5). What happens in subgame (Y, Y)? Lemma 5
(relegated to the Appendix) shows that the firms set their advance selling prices equal to \( d \) and each consumer pre-orders her preferred brand. The resulting profits are

\[
\Pi^A = \Pi^B = \frac{d}{2}.
\]

Hence, the both-firms-can-advance-sell version of Model II has two symmetric equilibria: one in which none of the firms implement advance selling and the other in which both firms implement advance selling. Thus, allowing both firms to advance sell adds another equilibrium to the one found earlier in Section 4.

Next, consider the both-firms-can-advance-sell version of Model III. Proposition 1 covers subgame (N, N), while Proposition 7 covers subgames (Y, N) and (N, Y). As to subgame (Y, Y), throat-cutting price competition in the advance selling season results in \( p^A_1 = p^B_1 = c = 0 \) and zero profits. Figure 7 depicts the payoff matrix. This matrix makes it clear that the game has three Nash equilibria: (Y, N), (N, Y), and (Y, Y). Again, since (Y, N) and (N, Y) are among the Nash equilibria, the results obtained in Section 5 are valid in the both-firms-can-advance-sell version of Model III.

Finally, suppose there is a fixed cost associated with implementing advance selling. Then – no matter how small this cost is – (Y, Y) will no longer be an equilibrium in the both-firms-advance-sell versions of Models I and III. The earlier result that the equilibrium outcomes obtained in Models I and III are in the subset of the equilibrium outcomes obtained in their both-firms-can-advance-sell versions changes to “the equilibrium outcomes obtained in Models I and III coincide with the ones obtained in their both-firms-can-advance-sell versions.”

After elimination of (Y, Y) from the set of equilibrium outcomes in the both-firms-advance-sell versions of Models I and III, we arrive to an important conclusion. In the situation in which both firms have an opportunity to advance sell, advance selling by both firms occurs in equilibrium only when consumers are already fans of certain brands (Model II).

### 7 Concluding Remarks

This paper examines the effect of competition on the advantages of advance selling driven by consumer valuation uncertainty. I assumed two firms that produce different brands. Some consumers prefer one brand, and others prefer the other brand. Consumers derive the same value from their preferred brand, but they differ in how strongly they dislike their less preferred brand.
One of the firms can offer consumers the opportunity to pre-order its product in advance of the regular selling season. I calculated the benefits of advance selling when this firm faces competition from the other firm in the regular selling season and when it does not, under three demand specifications. In Model I consumers are initially uncertain about the identity of their preferred brand; in Model II consumers are uncertain about their disutility from consuming the alternative; in Model III consumers are uncertain about the identity of their preferred brand as well as their disutility from consuming the alternative.

The first message of the paper is that consumer valuation uncertainty and competition interact to shape the firm’s incentives to advance sell. I showed that the firm does not want to implement advance selling when consumers know their preferred brand from the outset (Model II), and that competition enhances the benefits of advance selling in Models I and III.

The second important message is that the level of brand substitutability plays an important role in determining the equilibrium outcome. Its effect on the firm’s incentives to advance sell can be positive or negative, depending on the model and the setting. For example, I found that in the competitive setting of Model I the firm has stronger incentives to advance sell when the brands are more substitutable, while the reverse holds in the monopoly setting. In the competitive setting of Model III, the firm wants to sell in advance no matter what the level of brand substitutability is. I also showed that as the brands become more substitutable, it strengthens the positive effect of competition on the benefits of advance selling.

Finally, I performed a robustness check to demonstrate that in Models I and III even when both firms can advance sell, they never pursue the option simultaneously. In contrast, although the equilibrium obtained in Model II continues to be an equilibrium in its both-firms-can-advance-sell version, another equilibrium obtains in which both firms choose to implement advance selling. This is an important result, as it explains many cases where firms with intense competition advance sell their products.

**Appendix**

**Proof of Proposition 1:** Firm A’s and firm B’s profits are

\[ p^A_2 \left( \frac{1}{2} + \frac{p^B_2 - p^A_2}{2d} \right) \]

and

\[ p^B_2 \left( \frac{1}{2} + \frac{p^A_2 - p^B_2}{2d} \right), \]

respectively. Differentiating the profit of firm A w.r.t. \( p^A_2 \), the profit of firm B w.r.t. \( p^B_2 \) and setting the derivatives to zero yield the system of equations

\[
\begin{cases}
\frac{1}{2} + \frac{p^B_2 - 2p^A_2}{2d} = 0 \\
\frac{1}{2} + \frac{p^A_2 - 2p^B_2}{2d} = 0
\end{cases}
\]

from which the firms’s NE prices can be found: \( p^A_2 = p^B_2 = d \).
Proof of Proposition 2: Any $p^A_2 \in [v - d, v]$ will induce type A consumers and type B consumers with $x \leq v - p^A_2$ buy the product. Firm A faces the following optimization problem:

$$\max_{p^A_2 \in [v - d, v]} p^A_2 \left( \frac{1}{2} + \frac{v - p^A_2}{2d} \right).$$

Differentiating w.r.t. $p^A_2$ and setting the derivative to zero yield

$$\frac{1}{2} + \frac{v - 2p^A_2}{2d} = 0,$$

implying that when $d \leq v/3$ the firm should charge $p^A_2 = v - d$ and sell to all consumers. Otherwise, the firm should charge

$$p^A_2 = \frac{v + d}{2},$$

earning

$$\Pi^A = \frac{v}{4} + \frac{v^2}{8d} + \frac{d}{8}.$$

Proof of Lemma 1: Suppose we found a symmetric Nash equilibrium in which $p^A_2 = p^B_2 = p \geq d$ ($p \geq d$ will be confirmed later), implying that type A consumers buy from firm A and type B consumers from firm B. Since only consumers with $x \in (\hat{x}, [x])$ are on the market in period 2, a firm (say, firm A) have to decrease its price by at least $\hat{x}$ to attract any of type B consumers. Then $p$ is the highest price that makes such a deviation unprofitable. That is,

$$p - \hat{x} \geq p^A_2 \left( \frac{d - \hat{x}}{2d} + \frac{p - \hat{x} - p^A_2}{2d} \right)$$

must hold for any $p^A_2 \in [p - d, p - \hat{x}]$. The right-hand side attains its maximum at $p^A_2 = \min\{p - d, (p + d - 2\hat{x})/2\}$. When $p^A_2 = p - d$, the above inequality becomes

$$p \leq 2d.$$

When $p^A_2 = (p + d - 2\hat{x})/2$, it becomes

$$p \leq d + 2\sqrt{d\hat{x} - \hat{x}^2}.$$

As long as $p \leq v$, we have

$$p = \begin{cases} 
  d + 2\sqrt{d\hat{x} - \hat{x}^2}, & \text{if } \hat{x} \leq \frac{d}{2} \\
  2d, & \text{if } \hat{x} \in \left(\frac{d}{2}, d\right].
\end{cases}$$

When $d > v/2$ and $\hat{x} > d/2 - \sqrt{2vd - v^2}/2$, $p = v$. 

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Lemma 2 (Model I, rational expectations given $p_1^A$). Suppose that firm A charges $p_1^A$ in period 1 and let

$$g(p_1^A) = \frac{1}{17} \left(10d - 2p_1^A + 4\sqrt{2d^2 + 6dp_1^A - 4(p_1^A)^2}\right).$$

Under the rational expectations equilibrium we have:

(i) When $d \leq v/2$,

(a) $p_1^A \leq 3d/2$ induces all consumers to pre-order; in period 2 the firms charge $p_2 = 2d$;

(b) $p_1^A \in (3d/2, 7d/4]$ induces consumers with $x \leq 2(2d - p_1^A)$ to pre-order; in period 2 the firms charge $p_2 = 2d$ and the remaining consumers purchase their preferred brand;

(c) $p_1^A \in (7d/4, (3 + \sqrt{17})d/4]$ induces consumers with $x \leq g(p_1^A)$ to pre-order; in period 2 the firms charge $p_2 = p_1^A + g(p_1^A)/2$ and the remaining consumers purchase their preferred brand;

(d) $p_1^A > (3 + \sqrt{17})d/4$ induces all consumers to wait until period 2; in period 2 the firms charge $p_2 = d$ and each consumer purchases her preferred brand.

(ii) When $d \in (v/2, v]$,

(a) $p_1^A \leq v - d/2$ induces all consumers to pre-order; in period 2 the firms charge $p_2 = v$;

(b) $p_1^A \in (v - d/2, v - (d - \sqrt{2vd - v^2})/4]$ induces consumers with $x \leq 2(v - p_1^A)$ to pre-order; in period 2 the firms charge $p_2 = v$ and the remaining consumers purchase their preferred brand;

(c) $p_1^A \in (v - (d - \sqrt{2vd - v^2})/4, (3 + \sqrt{17})d/4]$ induces consumers with $x \leq g(p_1^A)$ to pre-order; in period 2 the firms charge $p_2 = p_1^A + g(p_1^A)/2$ and the remaining consumers purchase their preferred brand;

(d) $p_1^A > (3 + \sqrt{17})d/4$ induces all consumers to wait until period 2; in period 2 the firms charge $p_2 = d$ and each consumer purchases her preferred brand.

Proof of Lemma 2: We first consider the case $d \leq v/2$ that corresponds to part (i) of the lemma.

(a) Suppose $p_1^A \leq 3d/2$. It follows from Lemma 1 that when $\hat{x} = d$, $p_2 = 2d$. It is left to check that under these prices all consumers will pre-order the product. Indeed, by equation (1)

$$\hat{x} = \min\{d, 2(p_2 - p_1^A)\} = d,$$

as $2(2d - p_1^A) \geq d$ holds for $p_1^A \leq 3d/2$.

(b) Suppose $p_1^A \in (3d/2, 7d/4]$. It follows from Lemma 1 that when $\hat{x} = 2(2d - p_1^A)$, which decreases from $d$ to $d/2$ as $p_1^A$ increases from $3d/2$ to $7d/4$, $p_2 = 2d$. We need to check that it is optimal for consumers with $x \leq 2(2d - p_1^A)$ to pre-order the product. By equation (1)

$$\hat{x} = \min\{d, 2(p_2 - p_1^A)\} = 2(2d - p_1^A),$$

as $2(2d - p_1^A) < d$ holds for $p_1^A \in (3d/2, 7d/4]$. 

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(c) Suppose $p^A_1 \in (7d/4, (3 + \sqrt{17})d/4]$. In this case $\hat{x}$ and $p_2$ are found from the system of equations

$$\begin{cases} p_2 = d + 2\sqrt{d \hat{x} - \hat{x}^2} \\ \hat{x} = 2(p_2 - p^A_1). \end{cases}$$

Substituting $p_2 = p^A_1 + \hat{x}/2$ into the first equation allows us to solve for $\hat{x}$:

$$\hat{x} = \frac{1}{17} \left( 10d - 2p^A_1 + 4\sqrt{2d^2 + 6dp^A_1 - 4(p^A_1)^2} \right).$$

As $p^A_1$ increases from $7d/4$ to $(3 + \sqrt{17})d/4$, $\hat{x}$ decreases from $d/2$ to $(\sqrt{17} - 1)d/(2\sqrt{17})$ and $p_2 = p^A_1 + \hat{x}/2$ decreases from $2d$ to $(4 + \sqrt{17})d/\sqrt{17}$.

(d) Suppose $p^A_1 > (3 + \sqrt{17})d/4$. The above system of equations does not have a solution. In this case $\hat{x} = 0$, implying $p_2 = d$ (Lemma 1). We need to check that under these prices no consumer will pre-order the product. Indeed, by equation (1)

$$\hat{x} = 0,$$

as $p^A_1 > p_2 = d$ holds for $p^A_1 > (3 + \sqrt{17})d/4$.

Proof of part (ii) of the lemma is very similar to that of part (i).

**Proof of Proposition 3:** We first consider the case $d \leq v/2$ that corresponds to part (i) of the proposition.

(a) Suppose $p^A_1 \leq 3d/2$. Then (2) is simply

$$\Pi^A(p^A_1) = p^A_1.$$

The profit is maximized at $p^A_1 = 3d/2$, $\Pi^A(3d/2) = 3d/2$.

(b) Suppose $p^A_1 \in (3d/2, 7d/4]$. Then (2) becomes

$$\Pi^A(p^A_1) = p^A_1 \frac{2(2d - p^A_1)}{d} + 2d - \frac{2(2d - p^A_1)}{2d} = -\frac{2(p^A_1)^2}{d} + 6p^A_1 - 3d.$$

The function attains its maximum at $p^A_1 = 3d/2$, $\Pi^A(3d/2) = 3d/2$.

(c) Suppose $p^A_1 \in (7d/4, (3 + \sqrt{17})d/4]$. Then (2) becomes

$$\Pi^A(p^A_1) = p^A_1 \frac{g(p^A_1)}{d} + \left( p^A_1 + \frac{g(p^A_1)}{2} \right) \frac{d - g(p^A_1)}{2d}.$$  

Numerical calculations show that for any value of $p^A_1 \in (7d/4, (3 + \sqrt{17})d/4]$, $\Pi^A(p^A_1) < 3d/2$. 

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(d) Suppose $p_A^1 > (3 + \sqrt{17})d/4$. Then (2) is simply
\[ \Pi^A(p_A^1) = \frac{d}{2}. \]

This is less than $3d/2$.

It follows that when $d \leq v/2$, firm A should set its pre-order price to $3d/2$, which induces all consumers to pre-order. Firm A’s equilibrium profit is $3d/2$ and firm B’s profit is zero. Next, we consider the case $d \in (v/2, v]$ that corresponds to part (ii) of the proposition.

(a) Suppose $p_A^1 \leq v - d/2$. Then (2) is simply
\[ \Pi^A(p_A^1) = p_A^1. \]
The profit is maximized at $p_A^1 = v - d/2$, $\Pi^A(v - d/2) = v - d/2$.

(b) Suppose $p_A^1 \in (v - d/2, v - (d - \sqrt{2vd - v^2})/4]$. Then (2) becomes
\[ \Pi^A(p_A^1) = p_A^1 \frac{2(v - p_A^1)}{d} + v \frac{d - 2(v - p_A^1)}{2d} = - \frac{2(p_A^1)^2}{d} + \frac{3v}{d} p_A^1 + \frac{v(d - 2v)}{2d}. \]
The function attains its maximum at $p_A^1 = 3v/4$, $\Pi^A(3v/4) = v/2 + v^2/(8d)$. This is greater than $v - d/2$.

(c) Suppose $p_A^1 \in (v - (d - \sqrt{2vd - v^2})/4, (3 + \sqrt{17})d/4]$. Then (2) becomes
\[ \Pi^A(p_A^1) = p_A^1 \frac{g(p_A^1)}{d} + \left( p_A^1 + \frac{g(p_A^1)}{2} \right) \frac{d - g(p_A^1)}{2d}. \]
Numerical calculations show that for any value of $p_A^1 \in (v - (d - \sqrt{2vd - v^2})/4, (3 + \sqrt{17})d/4]$, $\Pi^A(p_A^1) < v/2 + v^2/(8d)$.

(d) Suppose $p_A^1 > (3 + \sqrt{17})d/4$. Then (2) is simply
\[ \Pi^A(p_A^1) = \frac{d}{2}. \]

This is less than $v/2 + v^2/(8d)$.

It follows that when $d \in (v/2, v]$, firm A should set its pre-order price to $3v/4$. This price induces consumers with $x \leq v/2$ to pre-order. In period 2 firm A and firm B charge $p_2 = v$. The remaining consumers purchase their preferred brand. The firms’ equilibrium profits are $\Pi^A = v/2 + v^2/(8d)$ and $\Pi^B = v/2 - v^2/(4d)$.

**Proof of Proposition 4:** The proof is completed in four steps.
1. We start with period 2 and assume that consumers with \( x \in (\hat{x}, d] \) are left on the market. Firm A can charge \( p_A^2 = v \) and sell only to type A consumers. In this case it will earn

\[
\frac{v}{2d} \left( d - \hat{x} \right).
\]

If the firm charges \( p_A^2 = v - d \), it will sell to all consumers and earn

\[
(v - d) \frac{d - \hat{x}}{d}.
\]

Finally, the firm can charge \( p_A^2 \in [v - d, v - \hat{x}] \) and sell to all type A consumers and type B consumers with \( x \in (\hat{x}, v - p_A^2) \), earning

\[
p_A^2 \left( \frac{d - \hat{x}}{2d} + \frac{v - p_A^2 - \hat{x}}{2d} \right).
\]

The above function is maximized at

\[
p_A^2 = \begin{cases} 
\frac{v + d - 2\hat{x}}{2}, & \text{if } \hat{x} \leq \frac{3d - v}{2} \\
v - d, & \text{if } \hat{x} \in \left(\frac{3d - v}{2}, d\right].
\end{cases}
\]

It can be shown that \( p_A^2 = v \) maximizes firm A’s profit in period 2 when \( d \in (v/2, v] \) and \( \hat{x} > (d - \sqrt{2vd - v^2})/2 \); \( p_A^2 = v - d \) is optimal when \( d \leq v/3 \) or when \( d \in (v/3, v/2] \) and \( \hat{x} \in ((3d - v)/2, d] \). Otherwise, the firm is better off charging \( p_A^2 = \frac{v + d - 2\hat{x}}{2} \).

2. In period 1, a consumer pre-orders the product if and only if her expected payoff from doing so exceeds her expected payoff from postponing the purchase,

\[
v - \frac{x}{2} - p_A^1 \geq \frac{1}{2} (v - p_A^2) + \frac{1}{2} \max\{0, v - x - p_A^2\}.
\]

3. Let us check that the prices and consumer purchasing behavior postulated in Proposition 4 satisfy the requirements of the rational expectations equilibrium.

(i) Let \( d \leq v/3 \). Suppose firm A charges a high pre-order price so that no consumer purchases in period 1 (\( \hat{x} = 0 \)). Then \( p_A^1 = v - d \) will maximize the firm’s profit in period 2 (see Step 1). Firm A’s profit in this case is as in Proposition 2(i)

\[
\Pi_A^1 = v - d.
\]

(ii) Let \( d \in (v/3, v/2] \). Suppose firm A charges a high pre-order price so that no consumer purchases in period 1 (\( \hat{x} = 0 \)). Then \( p_A^1 = (v + d)/2 \) will maximize the firm’s profit in period 2 (see Step 1). Firm A’s profit in this case is as in Proposition 2(ii)

\[
\Pi_A^1 = \frac{v}{4} + \frac{v^2}{8d} + \frac{d}{8}.
\]
(iii) Let $d \in (v/2, v]$. Suppose firm A charges $p^A_1 = 3v/4$ in period 1 and consumers with $x \leq v/2$ pre-order the product. Then $p^A_2 = v$ will maximize the firm’s profit in period 2 (see Step 1). Pre-ordering the product is optimal if and only if (see Step 2)

$$v - \frac{x}{2} - \frac{3v}{4} \geq 0,$$

or

$$x \leq \frac{v}{2}.$$

Firm A’s profit in this case is

$$\Pi^A = \frac{v^2}{2} + \frac{v^2}{8d}.$$

4. Tedious algebra and numerical calculations reveal that any other combination of prices and consumer purchasing behavior — $p^A_1$, $p^A_2$ and $\hat{x}$ — that satisfies the requirements of the rational expectations equilibrium lead to a lower profit for firm A.

Lemma 3 (Model II, prices in period 2). Suppose the market in period 2 consists of the fraction $1 - \beta$ of type B consumers, then the firms’ Nash equilibrium prices are:

$$p^A_2 = \frac{d}{3} \quad \text{and} \quad p^B_2 = \frac{2d}{3}.$$

Consumers with $x \leq \frac{d}{3}$ will purchase brand A and those with $x \in (d/3, d]$ will purchase brand B. If the market consists of all type B consumers and the fraction $1 - \alpha$ of type A consumers, then

$$p^A_2 = \frac{d(3 - 2\alpha)}{3} \quad \text{and} \quad p^B_2 = \frac{d(3 - \alpha)}{3}.$$

Type B consumers with $x \leq \frac{\alpha d}{3}$ and type A consumers will purchase brand A, while type B consumers with $x \in (\alpha d/3, d]$ will purchase brand B.

Proof of Lemma 3: Suppose the market in period 2 consists of the fraction $1 - \beta$ of type B consumers. Then consumers with $x \leq \hat{x}$ will purchase brand A and the rest brand B, where $\hat{x}$ satisfies

$$v - \hat{x} - p^A_2 = v - p^B_2,$$

$$\hat{x} = p^B_2 - p^A_2.$$

Firm A’s and firm B’s profits in period 2 will be

$$p^A_2 (1 - \beta) \frac{(p^B_2 - p^A_2)}{2d}$$

and

$$p^B_2 (1 - \beta) \frac{(d - p^B_2 + p^A_2)}{2d},$$

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respectively. Differentiating the profit of firm A w.r.t. \( p^A_2 \), the profit of firm B w.r.t. \( p^B_2 \) and setting the derivatives to zero yield the system of equations

\[
\begin{align*}
    p^B_2 - 2p^A_2 &= 0 \\
    d - 2p^B_2 + p^A_2 &= 0
\end{align*}
\]

from which the firms’s NE prices can be found, \( p^A_2 = d/3 \) and \( p^B_2 = 2d/3 \).

If the market consists of all type B consumers and the fraction \( 1 - \alpha \) of type A consumers, then type B consumers with \( x \leq \hat{x} \) and type A consumers will purchase brand A and the rest brand B, yielding

\[
p^A_2 \left( \frac{p^B_2 - p^A_2}{2d} + \frac{1 - \alpha}{2} \right)
\]

to firm A and

\[
p^B_2 \frac{d - p^B_2 + p^A_2}{2d}
\]

to firm B. Differentiating the profit of firm A w.r.t \( p^A_2 \), the profit of firm B w.r.t to \( p^B_2 \) and setting the derivatives to zero yields the system of equations

\[
\begin{align*}
    p^B_2 - 2p^A_2 + (1 - \alpha)d &= 0 \\
    d - 2p^B_2 + p^A_2 &= 0
\end{align*}
\]

from which the firms’s NE prices can be found, \( p^A_2 = d(3 - 2\alpha)/3 \) and \( p^B_2 = d(3 - \alpha)/3 \).

**Proof of Lemma 4:** In each case we will verify that the firms’ prices and consumer purchasing behavior constitute a rational expectations equilibrium.

**(a)** Suppose \( p^A_1 \leq d/9 \) and all consumers pre-order the product from firm A. Lemma 3 implies \( p^A_2 = d/3 \) and \( p^B_2 = 2d/3 \). Obviously, pre-ordering is optimal for type A consumers. It is left to check that type B consumers are also better off purchasing in period 1:

\[
v - \frac{d}{2} - p^A_1 > v - \frac{1}{3} \left( \frac{p^A_2 + d}{6} \right) - 2 \frac{1}{3} p^B_2 = v - \frac{1}{3} \left( \frac{d}{3} + \frac{d}{6} \right) - \frac{4d}{9},
\]

\[
p^A_1 < \frac{d}{9}.
\]

**(b)** Suppose \( p^A_1 \in (d/9, d/3] \) and only type A consumers pre-order. Lemma 3 implies \( p^A_2 = d/3 \) and \( p^B_2 = 2d/3 \). In period 2, type B consumers with \( x \leq d/3 \) optimally purchase brand A, while those with \( x \in (d/3, d] \) brand B. In period 1, pre-ordering the product is optimal for type A consumers (obviously). Thus, it is left to check that type B consumers are better off purchasing in period 2:

\[
v - \frac{d}{2} - p^A_1 < v - \frac{1}{3} \left( \frac{p^A_2 + d}{6} \right) - 2 \frac{1}{3} p^B_2 = v - \frac{1}{3} \left( \frac{d}{3} + \frac{d}{6} \right) - \frac{4d}{9},
\]

\[
p^A_1 > \frac{d}{9}.
\]
(c) Suppose $p_A^1 \in (d/3, d]$ and fraction $\alpha = 3(d - p_A^1)/(2d)$ of type A consumers pre-order. Lemma 3 implies

$$p_A^2 = \frac{d(3 - 2\alpha)}{3} = p_A^1,$$

and

$$p_B^2 = \frac{d(3 - \alpha)}{3} = \frac{d + p_A^1}{2}.$$  

In period 2, type B consumers with $x \leq \alpha d/3 = (d - p_A^1)/2$ and the remaining type A consumers optimally purchase brand A, while type B consumers with $x \in (\alpha d/3, d]$ brand B. In period 1, type A consumers are indifferent between pre-ordering the product or purchasing brand A in period 2 (obviously), so mixing is optimal. To show that type B consumers are better off purchasing in period 2, it will be sufficient to show that their payoff from pre-ordering the product is below what they can get if they (sub-optimally) purchase from brand B in period 2:

$$v - \frac{d}{2} - p_A^1 < v - p_B^2 = v - \frac{d + p_A^1}{2},$$

$$p_A^1 > \frac{p_A^1}{2}.$$  

(d) Suppose $p_A^1 > d$ and no consumer pre-orders. Lemma 3 implies $p_A^2 = p_B^2 = d$. In period 1, all consumers optimally choose not to pre-order.

**Proof of Proposition 5:** We calculate firm A's profit under each item of Lemma 4. It is easy to show that it is maximized under item (d).

**Proof of Proposition 6:** Suppose firm A sets a high pre-order price so that no consumer purchases in period 1. Then firm A’s optimal price in period 2 and the resulting profit will be as in Proposition 2.

Alternatively, firm A can sell to type A consumers in period 1 and to type B consumers in period 2. In this case $p_A^2$ solves the following maximization problem:

$$\max_{p_A^2 \in [v - d, v]} p_A^2 v - p_A^2.$$

If $d < v/2$, the maximum is achieved at $p_A^2 = v - d$. To induce type A to pre-order, $p_A^1$ cannot exceed $v - d$. The resulting profit is $\Pi^A = v - d$. If $d \in (v/2, v]$, the maximum is achieved at $p_A^2 = v/2$ (type B consumers with $x \leq v/2$ will buy the product). To induce type A to pre-order, $p_A^1$ cannot exceed $v/2$. The resulting profit is

$$\Pi^A = \frac{v}{4} + \frac{v^2}{8d},$$

which is below the one in Proposition 2.
Finally, suppose firm A sells to all consumers in period 1. In this case \( p^A_2 = v - d \) if \( d < v/2 \) and \( p^A_2 = v/2 \) if \( d \in (v/2, v] \) (obtained as a limit as \( \beta \to 1 \)). Consider \( d < v/2 \) first. To induce all consumers to pre-order, \( p^A_1 \) cannot exceed \( v - d \). The resulting profit is \( v - d \). Now consider \( d \in (v/2, v] \). Let us find the value of \( p^A_1 \) that makes type B indifferent between pre-ordering the product and waiting until period 2:

\[
v - \frac{d}{2} - p^A_1 = \frac{v}{2d} \left( v - \frac{v}{4} - \frac{v}{2} \right),
\]

\[
p^A_1 = v - \frac{d}{2} - \frac{v^2}{8d}.
\]

The resulting profit is

\[
\Pi^A = p^A_1 = v - \frac{d}{2} - \frac{v^2}{8d},
\]

which is below the one in Proposition 2.

**Proof of Proposition 7** Suppose the fraction \( \gamma \in [0, 1) \) of consumers have pre-ordered the product. Hence, the market in period 2 consists of the fraction \( 1 - \gamma \) of consumers. The firms’ Nash equilibrium prices are \( p^A_2 = p^B_2 = d \) (as in the competitive setting of the no advance selling benchmark). These prices are independent of \( \gamma \), so we will assume that when \( \gamma = 1 \) (no consumers are left on the market in period 2), \( p^A_2 = p^B_2 = d \). To induce all consumers to pre-order, firm A sets its advance selling price to \( p^A_1 = d - d/4 = 3d/4 \). The resulting profits are

\[
\Pi^A = \frac{3d}{4}
\]

and

\[
\Pi^B = 0.
\]

**Proof of Proposition 8**

Suppose firm A sets a high pre-order price so that no consumer purchases in period 1. Then firm A’s optimal price in period 2 and the resulting profit will be as in Proposition 2.

Alternatively, firm A can set a low pre-order price inducing all consumers to pre-order the product. In this case \( p^A_2 = v - d \) when \( d \leq v/3 \) and \( p^A_2 = (v + d)/2 \) when \( d \in (v/3, v] \) (obtained as a limit as the fraction of consumers left on the market in period 2 goes to zero). The highest pre-order price makes a consumer indifferent between pre-ordering the product and waiting until period 2. First, consider \( d \leq v/3 \). Then \( p^A_1 = v - d \) and the resulting profit is \( \Pi^A = v - d \) (the same as in Proposition 2).

Now consider \( d \in (v/3, v] \):

\[
v - \frac{d}{4} - p^A_1 = \frac{1}{2} \left( v - \frac{v + d}{2} \right) + \frac{v - d}{2d} \left( v - \frac{v - d}{4} - \frac{v + d}{2} \right),
\]

\[
p^A_1 = v - \frac{v^2}{8d} - \frac{d}{8}.
\]
The resulting profit is
\[ \Pi^A = v - \frac{v^2}{8d} - \frac{d}{8}, \]
which is greater than the one in Proposition 2 when \( d \in (2v/(3 + \sqrt{8}), v] \).

**Lemma 5** (Model II, both firms advance sell). Consider subgame \((Y, Y)\) of the both-firms-can-advance-sell version of Model II. In period 1 firm A and firm B charge \( p^A_1 = p^B_1 = d \); each consumer purchases her preferred brand. In period 2 the firms charge \( p^A_2 = p^B_2 = d \). The resulting profits are
\[ \Pi^A = \Pi^B = \frac{d}{2}. \]

**Proof of Lemma 5:** Suppose we found a symmetric Nash equilibrium in which \( p^A_1 = p^B_1 = p \); each consumer buys her preferred brand and the firms’ equilibrium profits are \( \Pi^A = \Pi^B = p/2 \). If firm A deviates and decreases its price by \( d/2 \), it will lose
\[ \frac{d}{4} \]
on its existing customers. At the same time, it will attract all type B consumers, thus increasing its profit by
\[ \frac{1}{2} \left( p - \frac{d}{2} \right). \]
Equilibrium requires
\[ \frac{1}{2} \left( p - \frac{d}{2} \right) = \frac{d}{4}, \]
\[ p = d. \]

**References**


