Branded Websites, Marketplace Selling and Price Competition

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Abstract

I consider a market for differentiated products with an online marketplace (the platform) and two types of firms. Marketplace firms sell through the platform. Branded firms sell to consumers directly and, if they choose so, through the platform. When a branded firm joins the platform, the firm expands its reach beyond its branded website/physical store(s) to consumers who visit the platform for all their purchases. The drawback is that the firm has to pay a referral fee for all sales on the platform, some of which are from its loyal consumers who would otherwise have purchased from the firm directly. I investigate the role of the firm composition in determining the equilibrium outcome. Interestingly, a higher fraction of branded firms translates into more firms on the platform and intense price competition. In the midst of the COVID-19 pandemic consumers who used to shop at physical stores turn to the platform. I show that if they do (do not) look into other products, more (fewer) branded firms will join the platform in equilibrium.

Key words: pricing, competition, online marketplace, platform, brands

JEL codes: C72, D43, L11, L13, M31

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1 Introduction

Online marketplaces, such as those operated by Amazon, eBay, Google, Flipkart (India), Alibaba and Taobao (China), and Ozon (Russia), have become an integral part of many e-commerce businesses. They serve as a two-sided platform in which the intermediary matches buyers with sellers. The sellers pay a commission for this service, but otherwise have control over the prices on the platform. For example, Amazon charges its third-party retailers a referral fee which ranges from 8% to 15% depending on the product’s category. eBay’s fee is 10% for most products, Etsy takes 5%.

Marketplace platforms represent an easy way for new firms to compete in an increasingly competitive landscape of the retail industry. Established firms, too, can take advantage of marketplace selling. When an established firm joins a marketplace platform, the firm expands its reach beyond its branded website/physical store(s) to consumers who visit the platform for all their purchases. The drawback is that the firm has to pay a referral fee for all sales on the platform, some of which are from its loyal consumers who would otherwise have purchased from the firm directly.

In this theoretical study, I consider a setting with an online marketplace (the platform) and two types of firms. Marketplace firms sell through the platform. Branded firms sell to consumers directly; they can join the platform and sell there as well. My first set of research questions is: (i) How are the firms’ pricing choices affected by the presence of the platform? (ii) How many branded firms will join the platform in equilibrium? (iii) What role does the firm composition play in determining the equilibrium outcome?

The COVID-19 pandemic has had a profound impact on how consumers buy anything and everything, from essentials like groceries, to non-essentials like accessories and jewelry. As many businesses and stores had to close or severely limit the number of customers allowed on their premises due to social distancing requirements, consumers were forced to shift their purchases online. McKinsey report that was released in August 2020 noted the overall increase in online shopping by 15% to 30% across most categories, and pointed out several aspects. There has been a massive shock to loyalty: about a third of the U.S. consumers have tried different brands, different stores or different websites, and about a quarter have discovered new private brands. In the midst of the pandemic, consumers appreciated availability and convenience, such as those provided by online marketplace platforms, as supply chains were disrupted, and have been focusing on value, especially for essential categories.1

The proposed framework allows to capture changes in consumer shopping behavior encountered during the COVID-19 pandemic, such as the shift to online shopping and the wavering

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loyalty to brands, in a meaningful way. My second set of research questions is: (i) How do these changes affect the equilibrium outcome? (ii) Will more branded firms join the platform in equilibrium?

1.1 Description of the Model and Results

There is an online marketplace (the platform) and two types of firms, marketplace firms and branded firms. Each firm produces a different variety of a differentiated product. The number of possible varieties is \( N \), the number of marketplace and branded firms are \( n_M \) and \( n_B \) respectively, \( n_M + n_B \leq N \). Marketplace firms sell through the platform. Branded firms sell to consumers directly. They can join the platform and use it in conjunction with their own websites and physical stores. The platform charges its third-party sellers a commission fee.

Some consumers are captive to a particular brand. They purchase only that brand, either directly from the seller’s website/store or from an alternative channel where the seller’s product is available: the platform. Other consumers visit the platform, observe the different product offerings, and choose the one that maximizes their utility. I call them regular consumers and use the spokes model proposed by Chen and Riordan (2007) to describe their preferences.

The spokes model extends the linear city model to arbitrary numbers of product varieties and firms. Resembling the spokes on a bicycle wheel, the preference space consists of \( N \) lines that start from the same central point. Variety \( i \) is located at the extreme end of spoke \( i = 1, 2, \ldots, N \). Consumers are uniformly distributed on the spokes network and incur transportation costs – utility losses due to imperfect matching – as they travel to reach a variety. For a consumer located on spoke \( i \) variety \( i \) is her first preferred variety, and each of the other \( N - 1 \) varieties is equally likely to be her second preferred variety.

Therefore, when a branded firm joins the platform it becomes “visible” to the regular consumers. The drawback is that some (an exogenously fixed fraction \( \rho \)) of the firm’s captive consumers now transact through the platform, which costs the firm the commission fee. Also, the firm must keep in mind that it does not have any brand power over the regular consumers who base their purchasing decisions on the products’ prices and physical characteristics only.

I first find the equilibrium prices for a given number of branded firms on the platform, \( \tilde{n}_B \). The branded firms that are not on the platform set a high price and exploit their captive consumers. The branded firms that are, compete with the marketplace firms for the regular consumers. Their price will be higher than the marketplace firms’ price because of the captive consumers. I then determine how many branded firms will join the platform in equilibrium. Depending on the parameter values, the solution is either boundary, \( \tilde{n}_B = n_B \), or interior, \( \tilde{n}_B < n_B \).

I proceed by analyzing the effect of the firm composition on the equilibrium outcome. Sup-
pose the number of branded firms goes up and the number of marketplace firms goes down by the same amount. At first, one might think a higher fraction of branded firms will soften price competition. I find that in the case of the boundary solution the marketplace firms’ price will stay the same, while the branded firms’ price will actually decrease. This is because an increase in \( n_B \) decreases the number of captive consumers per branded firm, making each branded firm more aggressive when competing for the regular consumers. In the case of the interior solution, the aforementioned change in the firm composition has a snowball effect on the equilibrium outcome: not only the new branded/former marketplace firms, but also some of the branded firms that were not on the platform, will join the platform. There will be more firms on the platform, which will intensify price competition.

I then study how changes in consumer shopping behavior due to the COVID-19 pandemic affect the equilibrium outcome. I consider two scenarios. In the “flight to online” scenario some of the captive consumers who preferred shopping at physical stores now transact through the platform (\( \rho \) goes up). When the boundary solution occurs, the branded firms’ and the marketplace firms’ prices will decrease, and the price difference will shrink. In the case of the interior solution, there will be fewer branded firms on the platform, which will soften price competition. The indirect (through \( \tilde{n}_B \)) effect of the aforementioned change in consumer behavior may reverse the comparative statics results obtained in the case \( \tilde{n}_B = n_B \). In the “shock to consumer loyalty” scenario the branded firms lose their power over some of the captive consumers: they become regular consumers. I show that the branded firms’ and the marketplace firms’ prices will decrease, the price difference will shrink, and in the case of the interior solution more branded firms will join the platform.

1.2 Literature Review

My article relates to other theoretical studies of online marketplace platforms. I will start with the closest ones. Ryan, Sun, and Zhao (2012) consider a single firm that currently sells its product through its own website, but may choose to contract with an online marketplace platform to increase its visibility. Key questions for the firm are whether to sell through the platform and at what price. The branded firms in my model face the same questions, but there is more than one branded firm – an arbitrary number of them. Thus my results revolve around the number of branded firms that join the platform in equilibrium. Additionally, Ryan, Sun, and Zhao (2012) are interested in the marketplace owner’s decision whether to sell a competing product and what fee to set.

Galeotti and Moraga-González (2009) study a market where a platform attracts firms selling differentiated products and consumers interested in those products, a setup similar to my model. The platform’s pricing policy consists of an advertising fee firms have to pay to exhibit their
products and prices, and a subscription fee consumers have to pay to access product and price information. The authors show that in equilibrium the platform lures all firms and all consumers to participate. Apart from the platform’s pricing policy, another important difference between Galeotti and Moraga-González (2009) and the present paper is that they assume ex-ante identical firms and consumers, whereas in my model I have two types of firms and and two types of consumers.

I divided the rest of the applicable literature into two groups of papers. The first group is centered around an online retailer’s decision to become a platform. The second group explores dual-format retailing under which the owner of the platform also sells products under its own name. In my model the platform is already present and its owner does not act as a seller; I focus on firms and consumers instead. The fact that transactions may take place outside the platform allows me to capture the effects of changes in consumer shopping behavior such as those associated with the COVID-19 pandemic.

Abhishek, Jerath, and Zhang (2016) analyze online retailers’ incentives to adopt the agency selling format under which they allow firms direct access to their customers for a fee. The authors find that agency selling leads to lower prices and is more efficient than traditional reselling. An e-tailer should use it when the effect the electronic channel has on sales in the brick-and-mortar channel is negative and when there is increased competition between e-tailers. The strategic choice by an intermediary between the agency selling and reselling formats is also studied in Hagiu and Wright (2015). Drivers of the optimal format include the relative importance of the suppliers’ versus the intermediary’s private information, whether the products are long tail or short tail, and the presence of spillovers across products generated by marketing activities. Johnson (2017) examines the agency model and price-parity clauses. The author shows that a move to a system in which retailers set revenue shares and suppliers set retail prices leads to an increase in the profits of retailers, a decrease in the profits of suppliers, and an increase in consumer surplus. Price-parity clauses, when imposed within the agency model, raise prices and therefore harm consumers.

Jiang, Jerath, and Srinivasan (2011) investigate how a platform owner such as Amazon can learn from the third-party retailers’ early sales which of the products it should procure and sell directly. Hagiu, Jullien, and Wright (2020) explore conditions under which a firm can profitably turn itself into a platform by inviting rivals to sell products or services on top of its core product. Among successful examples are Apple’s iPhone and Salesforce’s customer relationship management software. Mantin, Krishnan, and Dhar (2014) is another study of dual-format retailing. The authors develop a bargaining model between a retailer and a manufacturer. There is a competitive fringe of third-party retailers that sell a substitute product of a lower quality. The authors show that by committing to an active online marketplace, the retailer improves its
bargaining position in negotiations with the manufacturer.

2 Model Setup

Consider a setting with an online marketplace (the platform), \( n_M \) marketplace firms and \( n_B \) branded firms. Let \( n = n_M + n_B \) denote the total number of firms. Each firm produces a single variety of a differentiated product. There are \( N \geq n \) possible varieties, indexed by \( i = 1, \ldots, N \). Marketplace firms sell through the platform. Branded firms sell to consumers directly. They can join the platform and use it in conjunction with their own websites/physical stores. I assume that the platform charges its third-party sellers a commission fee \( f \).

There is a mass of consumers of measure \( \Omega + N/2 \) with unit demands. Each branded firm has a mass \( \Omega/n_B \) of captive consumers who purchase from the firm’s website as long as the price does not exceed \( v \). If a branded firm joins the platform, I assume that a fraction \( \rho \) of its captive consumers will transact through the platform. Regular consumers are of mass \( N/2 \). They visit the platform for their purchases. I use the spokes model proposed by Chen and Riordan (2007) to describe their preferences.

Resembling the spokes on a bicycle wheel, the regular consumers’ preference space consists of \( N \) lines (spokes) that start at the same central point. The lines have length \( 1/2 \) and, like varieties, are indexed by \( i = 1, \ldots, N \). Each spoke \( i \) has variety \( i \) at the extreme end of it. Consumers of total mass \( N/2 \) are uniformly distributed over the spokes network. For a consumer located on spoke \( i \) variety \( i \) is her first preferred variety, and each of the other \( N - 1 \) varieties is equally likely to be her second preferred variety. Consumers have value \( v \) for one unit of their first or second preferred variety, and zero value for other varieties. They incur transportation cost \( t \) per unit of length as they travel along the spokes to reach a variety. Note that not all varieties are active (available through the platform).

Figure 1 depicts a radial network with twelve spokes (lines \( l_1, l_2, \ldots, l_{12} \)), four marketplace firms, and three branded firms. The marketplace firms are firms \( M_1, M_5, M_6, \) and \( M_{10} \): \( M_1 \) produces variety 1, \( M_5 \) variety 5, \( M_6 \) variety 6, and \( M_{10} \) variety 10. The branded firms are firms \( B_3, B_8, \) and \( B_{11} \): \( B_3 \) produces variety 3, \( B_8 \) variety 8, and \( B_{11} \) variety 11. Suppose only firms \( B_3 \) and \( B_8 \) have joined the platform. To indicate that variety 11 is not available through the platform, I made the dot that represents it hollow.

Consider a consumer located on \( l_1 \) at distance \( x_1 \) from variety 1. Variety 1 is her first preferred variety; she obtains utility \( v - tx_1 \) from consuming it. The consumer’s second preferred variety is variety \( j \neq 1 \) chosen by nature with probability \( 1/(N - 1) \). Too bad if \( j = 2, 4, 7, 9, \) or 12, as these varieties are not produced by any of the firms. If \( j = 11 \), that is not good either, because variety 11 is not available through the platform. Only if \( j = 3, 5, 6, 8, \) or 10 the
consumer’s second preferred variety is available to her; she obtains utility \( v - t(1 - x_1) \) from consuming it.

Next, consider a consumer located on \( l_2 \) at distance \( x_2 \) from variety 2. Unfortunately for this consumer, variety 2 is not produced by any of the firms. The consumer’s second preferred variety is available to her if \( j = 1, 3, 5, 6, 8, \) or 10; she obtains utility \( v - t(1 - x_2) \) from consuming it. Note that firms \( B_3 \) and \( B_8 \) have no brand power over regular consumers who treat varieties 3 and 8 like any other variety available through the platform.

All firms possess the same constant marginal cost technology with the unit production cost \( c \). I take a reduced form approach and assume that the platform’s fee \( f \) is given. The game unfolds in two stages. In the first stage the branded firms decide whether to join the platform. In the second stage the firms simultaneously set their prices. Consumers observe the prices and make their purchasing decisions.

Note that in the present setup a branded firm that sells through the platform does not have incentives to set a different price on its website. If anything, the price would be higher, but then the firm’s captive consumers would all transact through the platform.
3 Equilibrium Analysis

I start the equilibrium analysis with consumers. I solve the pricing game between the marketplace firms and the branded firms that joined the platform, and then determine how many branded firms will join the platform in equilibrium.

3.1 Consumers

Consider consumers for whom \( i \) and \( j \) are the two preferred varieties. Their total number is

\[
\frac{1}{N-1},
\]

since there are \( 1/2 \) consumers on \( l_i \) (\( l_j \)) and fraction \( 1/(N-1) \) of them desire variety \( j \) (variety \( i \)). The marginal consumer between varieties \( i \) and \( j \) is located at distance

\[
\max \left\{ \min \left\{ \frac{1}{2t} (t - p_i + p_j), 1 \right\}, 0 \right\}
\]

from variety \( i \). The marginal consumer between variety \( i \) and no purchase is located at distance

\[
\max \left\{ \min \left\{ \frac{1}{t} (v - p_i), 1 \right\}, 0 \right\}
\]

from variety \( i \).

We now derive the demand that the firm located at the end of spoke \( i \) faces, be it a marketplace firm or a branded firm. If it is a marketplace firm, it sells to two groups of consumers: (i) regular consumers whose two desired varieties are available through the platform, one of which is variety \( i \), and (ii) regular consumers for whom variety \( i \) is a desired variety and who do not have an alternative available through the platform. Firm \( M_i \)'s demand is, therefore,

\[
\frac{1}{N-1} \sum_{j \text{ varieties available through the platform but variety } i} \left\{ \min \left\{ \frac{1}{2t} (t - p_i + p_j), 1 \right\}, 0 \right\}
\]

\[
\frac{1}{N-1} \sum_{j \text{ varieties not available through the platform}} \left\{ \min \left\{ \frac{1}{t} (v - p_i), 1 \right\}, 0 \right\}.
\]

The demand of a branded firm that joined the platform consists of groups (i), (ii), and the firm’s captive consumers. The number of captive consumers is \( \Omega/n_B \), fraction \( \rho \) of them transact through the platform while the rest purchase directly from the firm.

The demand of a branded firm that did not join the platform consists of its captive con-
sumers. All of them purchase directly from the firm.

### 3.2 Pricing

Suppose \( \tilde{n}_B \) branded firms have joined the platform. The branded firms that are not on the platform, hereinafter referred to as stand-alone branded firms, are essentially monopolists. A stand-alone branded firm optimally sets its price to \( v \) and earns the profit of

\[
\Omega_{nB}(v - c).
\]

The players in the pricing game are \( n_M \) marketplace firms and \( \tilde{n}_B \) branded firms. Let \( \tilde{n} \) denote the total number of firms that sell through the platform, \( \tilde{n} = n_M + \tilde{n}_B \). To keep the model tractable and the results focused, I will restrict my attention to the situations in which all consumers whose preferred varieties are available purchase and enjoy a strictly positive surplus. Chen and Riordan (2007) call such situations normal oligopoly competition.

Provided that \( |p_i - p_j| \leq t \) and \( v - p_i > t \) for all \( i, j \), the profit function of a marketplace firm can be written as

\[
\left( \frac{1}{N-1} \sum_{j: \text{variety available through the platform but variety } i} \frac{1}{2t}(t - p_i + p_j) + \frac{N - \tilde{n}}{N-1} \right) ((1 - f)p_i - c).
\]

The profit function of a branded firm that joined the platform has an additional term

\[
\frac{\Omega}{n_B}((1 - \rho f)p_i - c).
\]

Here \( f \) is multiplied by \( \rho \) because fraction \( \rho \) of the firm’s captive consumers transact through the platform.

**Proposition 1 (Equilibrium prices).** Suppose \( \tilde{n}_B \) branded firms have joined the platform, so that there are \( \tilde{n} = n_M + \tilde{n}_B \) firms on the platform. In a symmetric pure-strategy Nash equilibrium the marketplace firms charge

\[
p_M^* = \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n} - n_M}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) t
\]

and the branded firms charge

\[
p_B^* = \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{2\tilde{n} - n_M - 1}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) t.
\]
In Lemma 1 in the Appendix I show that both $p^*_M$ and $p^*_B$ decrease in the total number of firms on the platform. It is easy to see that

\[ p^*_B - p^*_M = \frac{2(N - 1)\Omega(1 - \rho_f)t}{(2\tilde{n} - 1)n_B(1 - f)}, \]

(3)
too, decreases in $\tilde{n}$. The price difference plays an important role in the welfare analysis, as it determines the marginal consumer between varieties $i$ and $j$ when variety $i$ is offered at price $p^*_B$ and variety $j$ is offered at price $p^*_M$:

\[ \hat{x} = \frac{1}{2} - \frac{p^*_B - p^*_M}{2t}. \]

(4)
Here the distance is measured from variety $i$.

The equilibrium profits can easily be calculated from Proposition 1.

**Corollary 1 (Equilibrium profits).** The equilibrium profits in the pricing game between $n_M$ marketplace firms and $\tilde{n}_B$ branded firms are

\[ \Pi^*_M = \frac{\tilde{n} - 1}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n} - n_M}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho_f)}{(2\tilde{n} - 1)n_B(1 - f)} \right)^2 (1 - f)t \]

and

\[ \Pi^*_B = \frac{\tilde{n} - 1}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{2\tilde{n} - n_M - 1}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho_f)}{(2\tilde{n} - 1)n_B(1 - f)} \right)^2 (1 - f)t \]

\[ + \frac{\Omega(1 - \rho)fc}{n_B(1 - f)} \],

(5)
where $\tilde{n} = n_M + \tilde{n}_B$.

### 3.3 Decision to Join the Platform

Having analyzed the pricing stage of the game, we move to the first stage in which the branded firms decide whether to join the platform. Let us first evaluate $\Pi^*_B$ at $\tilde{n} = n$ and compare it with the profit of a stand-alone branded firm. If

\[ \Pi^*_B(n) \geq \frac{\Omega}{n_B}(v - c), \]

then all branded firms will join the platform in equilibrium. Otherwise, $\tilde{n}_B = \tilde{n} - n_M < n_B$, where $\tilde{n}$ satisfies

\[ \Pi^*_B(\tilde{n}) = \frac{\Omega}{n_B}(v - c). \]

(6)
In Lemma 2 in the Appendix I show that $\Pi^*_B$ is a decreasing function of $\tilde{n}$, so the solution to (6) is unique.

3.4 Welfare

Transactions by the captive consumers contribute $\Omega(v - c)$ to the welfare. Let us look at the regular consumers. The regular consumers whose two desired varieties are offered through the platform by firms of the same type travel $1/4$ on average. Their contribution to the welfare is, therefore,

$$\frac{1}{N-1} \left( \frac{n_M(n_M - 1)}{2} + \tilde{n}_B(\tilde{n}_B - 1) \right) \left( v - c - \frac{t}{4} \right).$$

The regular consumers whose two desired varieties are offered through the platform by different types of firms travel $\hat{x} \times \hat{x} / 2 + (1 - \hat{x}) \times (1 - \hat{x}) / 2$ on average. Substituting (4) for $\hat{x}$ yields

$$\frac{1}{4} + \left( \frac{p^*_B - p^*_M}{2t} \right)^2.$$

Such consumers contribute

$$\frac{1}{N-1} n_M \tilde{n}_B \left( v - c - \frac{t}{4} - \left( \frac{p^*_B - p^*_M}{2t} \right)^2 t \right).$$

to the welfare.

Finally, there are consumers with only one desired variety available through the platform. They travel $1/2$ on average and contribute

$$\frac{1}{N-1} \tilde{n}(N - \tilde{n}) \left( v - c - \frac{t}{2} \right)$$

to the welfare. We add up the aforementioned contributions to obtain the welfare. Simplifying, then rearranging and factoring out bring us to

$$W^* = \Omega(v - c) + \frac{1}{N-1} \frac{\tilde{n}(\tilde{n} - 1)}{2} \left( v - c - \frac{t}{4} \right) + \frac{1}{N-1} \tilde{n}(N - \tilde{n}) \left( v - c - \frac{t}{2} \right)$$

$$- \frac{1}{N-1} n_M \tilde{n}_B \left( \frac{p^*_B - p^*_M}{2t} \right)^2 t. \quad (7)$$

The equilibrium allocation of products to consumers is not efficient. Take the regular consumers whose two desired varieties are offered through the platform by firms $B_i$ and $M_j$. Since $p^*_B > p^*_M$, some consumers on $l_i$ purchase their second preferred variety $j$. Hence the last negative term in (7).
Another source of inefficiency is \( \tilde{n} < n \). When firm \( B_i \) does not sell its product through the platform, then the regular consumers on \( l_i \), as well as those who lack their first preferred variety but for whom variety \( i \) is their second preferred variety, cannot obtain variety \( i \).

The social planner can achieve efficiency by requiring that all branded firms join the platform and that all firms charge the same price:

\[
W^o = \Omega(v - c) + \frac{1}{N - 1} \frac{n(n - 1)}{2} \left( v - c - \frac{t}{4} \right) + \frac{1}{N - 1} n(N - n) \left( v - c - \frac{t}{2} \right). \tag{8}
\]

4 Comparative Statics Analysis

In Section 4.1 I investigate the role the firm composition plays in determining the equilibrium outcome. The COVID-19 crisis has forced whole consumer segments to shop differently. The shift to online shopping and the wavering loyalty to brands are among the most prominent changes. I study their effects on the equilibrium outcome in Sections 4.2 and 4.3.

4.1 Firm Composition

We have \( n = n_M + n_B \) firms total. What happens to the equilibrium outcome if the number of branded firms goes up and the number of marketplace firms goes down by the same amount?

Let us start with the boundary case of \( \tilde{n}_B = n_B (\tilde{n} = n) \). Substituting \( \tilde{n} = n \) and \( n_M = n - n_B \) into (1), (2), and (3) yields

\[
p^*_M = \frac{c}{1 - \rho} + \left( \frac{2N - n - 1}{n - 1} + \frac{n_B}{n - 1} \right) \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2n - 1)n_B(1 - f)} t, \tag{9}
\]

\[
p^*_B = \frac{c}{1 - \rho} + \left( \frac{2N - n - 1}{n - 1} + \frac{n + n_B - 1}{n - 1} \right) \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2n - 1)n_B(1 - f)} t, \tag{10}
\]

and

\[
p^*_B - p^*_M = \frac{2(N - 1)\Omega(1 - \rho f)t}{(2n - 1)n_B(1 - f)}. \tag{11}
\]

We see from the above expressions that \( p^*_M \) will stay the same (\( n_B \) cancels out) and \( p^*_B \) will decrease. This is because an increase in \( n_B \) decreases the number of captive consumers per branded firm, \( \Omega/n_B \), making each branded firm more aggressive when competing with the rest of the firms on the platform for the regular consumers.

As to the welfare, only the last term in (7),

\[
- \frac{1}{N - 1} n_M \tilde{n}_B \left( \frac{p^*_B - p^*_M}{2t} \right)^2 t,
\]
will be affected. Substituting \( n_B \) for \( \tilde{n}_B \), (11) for the price difference, and \( n - n_B \) for \( n_M \) yields an increasing function of \( n_B \):

\[
- \frac{1}{N - 1} (n - n_B) n_B \left( \frac{(N - 1) \Omega (1 - \rho f)}{(2n - 1)n_B(1 - \tilde{f})} \right)^2 t.
\]

The welfare will go up due to the decrease in \( p_B^* - p_M^* \). Fewer consumers will end up with the second preferred variety just because it is cheaper than the first preferred variety.

We now consider the case \( \tilde{n}_B < n_B (\tilde{n} < n) \). In part (ii) of Proposition 2 I show that an increase in \( n_B \) has a snowball effect on the equilibrium outcome: not only the new branded/former marketplace firms, but also some of the branded firms that were not on the platform will join the platform. It is the associated decrease in \( \Omega/n_B \) that makes the option of joining the platform more attractive.

Thus, there will be more firms on the platform. Since both \( p_B^* \) and \( p_M^* \) are decreasing functions of \( \tilde{n} \) (Lemma 1), the overall effect of an increase in \( n_B \) on the equilibrium prices will be negative. The welfare will go up due to the decrease in \( p_B^* - p_M^* \) and because more products will be available to the regular consumers.

**Proposition 2** (Firm composition). Suppose the number of branded firms goes up and the number of marketplace firms goes down by the same amount.

(i) If all branded firms join the platform in equilibrium, then \( p_M^* \) will stay the same and \( p_B^* \) will decrease. The welfare will go up.

(ii) If in equilibrium \( \tilde{n}_B < n_B \), then the total number of firms on the platform will go up, both \( p_M^* \) and \( p_B^* \) will decrease. The welfare will go up.

### 4.2 Flight to Online

Picture a consumer who used to purchase his favorite brand from one of the firm’s brick-and-mortar stores. We meet the same consumer again a few months into the COVID-19 pandemic. The consumer no longer shops at that brick-and-mortar store. Instead, he either (a) orders from the firm’s website, (b) finds his favorite brand on the platform and purchases it there, or (c) visits the platform to explore other products.

The goal of Sections 4.2 and 4.3 is to study how changes in consumer shopping behavior due to the COVID-19 pandemic affect the equilibrium outcome. I consider two scenarios. Consistent with responses (a) and (b), in the first scenario I assume that some of the captive consumers who used to purchase from the firm directly now transact through the platform (\( \rho \) goes up). I call this scenario “flight to online.” The second scenario is based on (c), the most dramatic of the three
responses. In it, a fraction $\gamma$ of the captive consumers become regular consumers. I call this scenario “shock to brand loyalty.”

Now on to the flight to online scenario. We see from (9) and (10) that in the case $\tilde{n}_B = n_B$ both $p^*_M$ and $p^*_B$ decrease in $\rho$. Since the price difference (11) also decreases in $\rho$, an increase in $\rho$ will increase the welfare.

In Proposition 3 I show that, in the case $\tilde{n}_B < n_B$, there will be fewer branded firms on the platform in equilibrium as a result of an increase in $\rho$. Algebraically, an increase in $\rho$ decreases the left-hand side of (6), so $\tilde{n}$ (hence, $\tilde{n}_B$) has to decrease to restore the balance. While the direct effect of $\rho$ on $p^*_M$ and $p^*_B$ is negative, the indirect (through $\tilde{n}_B$) is positive. It is therefore possible for $p^*_M$ and $p^*_B$ to increase.

**Proposition 3 (Flight to online).** Suppose the fraction of captive consumers who prefer the platform to direct purchase increases.

(i) If all branded firms join the platform in equilibrium, then both $p^*_M$ and $p^*_B$ will decrease. The price difference will shrink and the welfare will go up.

(ii) If in equilibrium $\tilde{n}_B < n_B$, there will be fewer branded firms on the platform. This may reverse the comparative statics results obtained in the case $\tilde{n}_B = n_B$.

### 4.3 Shock to Brand Loyalty

In the shock to brand loyalty scenario the branded firms lose power over fraction $\gamma$ of their captive consumers. That is, we now have $(1 - \gamma)\Omega$ captive consumers and $N/2 + \gamma\Omega$ regular consumers. In the proof of Proposition 4 I show that $\Omega$ gets multiplied by the factor

$$\frac{(1 - \gamma)N/2}{N/2 + \gamma\Omega}$$

in the expressions (1), (2), and (3). Since $((1 - \gamma)N/2)/(N/2 + \gamma\Omega) < 1$, it immediately follows that in the case $\tilde{n}_B = n_B$ the new prices and the price difference will be lower.

Next, consider the case $\tilde{n}_B < n_B$. In Proposition 4 I show that more branded firms will join the platform as a result of the aforementioned change in the consumer composition. A higher $\tilde{n}_B$ will reinforce the direct negative effect of the change on the equilibrium prices and the price difference.

**Proposition 4 (Shock to brand loyalty).** Suppose the branded firms lose power over a fraction of their captive consumers. Both $p^*_M$ and $p^*_B$ will decrease, and the price difference will shrink. In the case $\tilde{n}_B < n_B$ there will be more branded firms on the platform.
The parameter changes studied in Sections 4.1 and 4.2 had no effect on $W^\alpha$. Their effects on $W^*$ were, therefore, purely strategic. Here, $W^\alpha$ becomes

$$
W^\alpha = (1 - \gamma)\Omega(v - c) + \frac{N/2 + \gamma\Omega}{N/2} \frac{1}{N - 1} \frac{n(n - 1)}{2} \left( v - c - \frac{t}{4} \right) 
+ \frac{N/2 + \gamma\Omega}{N/2} \frac{1}{N - 1} n(N - n) \left( v - c - \frac{t}{2} \right).
$$

It is lower than (8). When a captive consumer becomes a regular consumer, $v - c$ turns into $v - c - tx$ if at least one of the consumer’s preferred varieties is sold on the platform, 0 if none. Should we then expect that the new $W^*$ will be lower than the old one? Despite the fact that when a captive consumer becomes a regular consumer $v - c$ turns into $v - c - x$ or 0, the positive strategic effect (a higher $\tilde{n}$ and a smaller price difference) may push $W^*$ up!

5 Concluding Remarks

In this paper I considered a market for differentiated products with an online marketplace platform and two types of firms. Marketplace firms sell through the platform. Branded firms sell to consumers directly and, if they choose so, through the platform. I investigated the role the firm composition plays in determining the equilibrium outcome. I found that a higher fraction of branded firms translates into more firms on the platform and intense price competition. The fact that in my model transactions may take place outside the platform allowed me to capture the effects of changes in consumer shopping behavior such as those associated with the COVID-19 pandemic. I found that there will be more branded firms joining the platform when consumer loyalty goes down, although in the case of a lesser impact that is only associated with a general shift to online shopping, fewer firms will join.

I used the spokes model proposed by Chen and Riordan (2017) because it can easily accommodate two or more types of firms, which cannot be said about the circular city model. Indeed, a firm on a circle only competes with its immediate neighbors. With two types of firms, the two neighbors can be of the same type as the given firm, of the other type, or of the mixed types. The equilibrium is sensitive to the configuration of the firms around the circle – an unnecessary complication. Competition in the spokes model is non-localized. For this reason the equilibrium is invariant to the assignment of $n_M$ marketplace firms and $\tilde{n}_B$ branded firms to $N$ spokes. Chen and Riordan’s assumption that each consumer cares about a limited number of product varieties is important for the present study, too. When a branded firm joins the platform, some of the regular consumers who did not purchase because none of their preferred varieties were available through the platform, will now obtain a positive surplus. This allows for a detailed
welfare analysis. It is socially optimal that all branded firms join the platform, which only happens when the boundary solution is obtained, and that all firms on the platform charge the same price (they do not).

References


Appendix

Proof of Proposition 1. Provided that \(|p_i - p_j| \leq t\) and \(v - p_i > t\) for all \(i, j\), a marketplace firm’s demand can be written as

\[
\frac{1}{N-1} \sum_{j: \text{varieties available through the platform but variety } i} \frac{1}{2t}(t - p_i + p_j) + \frac{N - \tilde{n}}{N - 1}.
\]

When \(M_i\) charges \(p_i\) while the rest of the firms adhere to the equilibrium prices, its profit is

\[
\left(\frac{1}{2t}(t - p_i + p_M^*) \frac{n_M - 1}{N - 1} + \frac{1}{2t}(t - p_i + p_B^*) \frac{\tilde{n}_B}{N - 1} + \frac{N - \tilde{n}}{N - 1}\right) (1 - f)p_i - c. \tag{12}
\]

We differentiate it with respect to \(p_i\) and set the derivative to zero:

\[
\left(\frac{1}{2t}(t - p_i + p_M^*) \frac{n_M - 1}{N - 1} + \frac{1}{2t}(t - p_i + p_B^*) \frac{\tilde{n}_B}{N - 1} + \frac{N - \tilde{n}}{N - 1}\right) (1 - f) \\
- \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} (1 - f)p_i - c = 0.
\]

Substituting \(p_i = p_M^*\) yields

\[
\left(\frac{1}{2} \frac{n_M - 1}{N - 1} + \frac{1}{2t}(t - p_M^* + p_B^*) \frac{\tilde{n}_B}{N - 1} + \frac{N - \tilde{n}}{N - 1}\right) (1 - f) \\
- \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} (1 - f)p_M^* - c = 0,
\]

or

\[
\left(\frac{1}{2} \frac{2N - \tilde{n} - 1}{N - 1} + \frac{1}{2t}(p_B^* - p_M^*) \frac{\tilde{n}_B}{N - 1}\right) (1 - f) \\
- \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} (1 - f)p_M^* - c = 0. \tag{13}
\]

When branded firm \(B_i\) charges \(p_i\) while the rest of the firms adhere to the equilibrium prices, its profit is

\[
\left(\frac{1}{2t}(t - p_i + p_M^*) \frac{n_M}{N - 1} + \frac{1}{2t}(t - p_i + p_B^*) \frac{\tilde{n}_B - 1}{N - 1} + \frac{N - \tilde{n}}{N - 1}\right) (1 - f)p_i - c \\
+ \frac{\Omega}{n_B} (1 - \rho f)p_i - c. \tag{14}
\]
We differentiate it with respect to $p_i$ and set the derivative to zero:

$$
\left( \frac{1}{2t} (t - p_i + p_M^*) \frac{n_M}{N-1} + \frac{1}{2t} (t - p_i + p_B^*) \frac{\tilde{n}_B - 1}{N - 1} + \frac{N - \tilde{n}}{N - 1} \right) (1 - f) - \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_i - c) + \frac{\Omega}{n_B} (1 - \rho f) = 0.
$$

Substituting $p_i = p_B^*$ yields

$$
\left( \frac{1}{2t} (t - p_B^* + p_M^*) \frac{n_M}{N-1} + \frac{1}{2} \frac{\tilde{n} - 1}{N - 1} + \frac{N - \tilde{n}}{N - 1} \right) (1 - f) - \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c) + \frac{\Omega}{n_B} (1 - \rho f) = 0,
$$

or

$$
\left( \frac{1}{2t} \frac{2N - \tilde{n} - 1}{N - 1} - \frac{1}{2t} (p_B^* - p_M^*) \frac{n_M}{N-1} \right) (1 - f) - \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c) + \frac{\Omega}{n_B} (1 - \rho f) = 0. \tag{15}
$$

To find the equilibrium prices, we first subtract (15) from (13):

$$
\frac{1}{2t} (p_B^* - p_M^*) \frac{\tilde{n}}{N - 1} (1 - f) + \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} (1 - f)(p_B^* - p_M^*) - \frac{\Omega}{n_B} (1 - \rho f) = 0,
$$

or

$$
\frac{1}{2t} (p_B^* - p_M^*) \frac{2\tilde{n} - 1}{N - 1} (1 - f) = \frac{\Omega}{n_B} (1 - \rho f),
$$

or

$$
p_B^* - p_M^* = \frac{2(N - 1)\Omega(1 - \rho f)t}{(2\tilde{n} - 1)n_B(1 - f)}. \tag{16}
$$

We substitute (16) into (13) to find $p_M^*$:

$$
p_M^* = \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{1}{t} (p_B^* - p_M^*) \frac{\tilde{n}_B}{\tilde{n} - 1} \right) t
$$

$$
= \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n}_B}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) t.
$$

Hence,

$$
p_B^* = p_M^* + \frac{2(N - 1)\Omega(1 - \rho f)t}{(2\tilde{n} - 1)n_B(1 - f)}
$$

$$
= \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n}_B - 1}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) t.
$$
We can use (12) and (13) to calculate $\Pi_M^*$:

$$
\Pi_M^* = \frac{\hat{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_M^* - c)^2 \frac{1}{1 - f}
$$

$$
= \frac{\hat{n} - 1}{N - 1} \frac{1}{2t} \left( \left( \frac{2N - \hat{n} - 1}{\hat{n} - 1} + \frac{\hat{n}_B}{\hat{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\hat{n} - 1)n_B(1 - f)} \right) (1 - f) \right) ^2 \frac{1}{1 - f}
$$

$$
= \frac{\hat{n} - 1}{2(N - 1)} \left( \frac{2N - \hat{n} - 1}{\hat{n} - 1} + \frac{\hat{n}_B}{\hat{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\hat{n} - 1)n_B(1 - f)} \right) (1 - f)t.
$$

We can use (14) and (15) to calculate $\Pi_B^*$:

$$
\Pi_B^* = \left( \frac{\hat{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c) - \frac{\Omega}{n_B} (1 - \rho f) \right) \frac{1}{1 - f} ((1 - f)p_B^* - c)
$$

$$
+ \frac{\Omega}{n_B} (1 - \rho f) p_B^* - c)
$$

$$
= \frac{\hat{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c)^2 \frac{1}{1 - f} + \frac{\Omega(1 - \rho f)c}{n_B(1 - f)}
$$

$$
= \frac{\hat{n} - 1}{2(N - 1)} \left( \frac{2N - \hat{n} - 1}{\hat{n} - 1} + \frac{\hat{n}_B}{\hat{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\hat{n} - 1)n_B(1 - f)} \right) ^2 (1 - f)t
$$

$$
+ \frac{\Omega(1 - \rho f)c}{n_B(1 - f)}.
$$

**Lemma 1.** Calculated in Proposition 1, $p_M^*$ and $p_B^*$ are decreasing functions of $\hat{n}$.

**Proof.** Let

$$
g(\hat{n}) = \frac{2(N - 1)\Omega(1 - \rho f)}{(2\hat{n} - 1)n_B(1 - f)}.
$$

Note that since $(p_B^* - p_M^*)/(2t)$ in (4) cannot exceed $1/2$, $g(\hat{n}) < 1$. We can write (1) as

$$
p_M^*(\hat{n}) = c \frac{1}{1 - f} + \left( \frac{2N - \hat{n} - 1}{\hat{n} - 1} + g(\hat{n}) \frac{\hat{n} - n_M}{\hat{n} - 1} \right) t.
$$

The derivative equals

$$
p_M^*(\hat{n})' = \left( -\frac{2(N - 1)}{(\hat{n} - 1)^2} + g'(\hat{n}) \frac{\hat{n} - n_M}{\hat{n} - 1} + g(\hat{n}) \frac{n_M - 1}{(\hat{n} - 1)^2} \right) t.
$$

Since $g'(\hat{n}) < 0$ and $g(\hat{n}) < 1$,

$$
p_M^*(\hat{n})' < \left( -\frac{2(N - 1)}{(\hat{n} - 1)^2} + \frac{n_M - 1}{(\hat{n} - 1)^2} \right) t = -\frac{2N - n_M - 1}{(\hat{n} - 1)^2} t < 0.
$$
As to $p^*_B$, it decreases in $\tilde{n}$ because it is a sum of two decreasing functions of $\tilde{n}$, $p^*_M$ and $p^*_B - p^*_M$.

**Lemma 2.** Calculated in Corollary 1, $\Pi^*_B$ is a decreasing function of $\tilde{n}$.

**Proof.** We can write (5) as

$$\Pi^*_B(\tilde{n}) = \frac{1}{2(N-1)} \left( \frac{2N - \tilde{n} - 1}{\sqrt{\tilde{n} - 1}} + g(\tilde{n}) \frac{2\tilde{n} - n_M - 1}{\sqrt{\til{n} - 1}} \right)^2 (1 - f) t + \frac{\Omega (1 - \rho f c)}{n_B(1 - f)}.$$

where $g(\tilde{n})$ is from the proof of Lemma 1. We need to show that

$$\frac{\partial}{\partial \tilde{n}} \left( \frac{2N - \tilde{n} - 1}{\sqrt{\tilde{n} - 1}} + g(\tilde{n}) \frac{2\tilde{n} - n_M - 1}{\sqrt{\tilde{n} - 1}} \right) < 0.$$

The derivative equals

$$\frac{1}{\tilde{n} - 1} \left( -\sqrt{\tilde{n} - 1} - \frac{2N - \tilde{n} - 1}{2\sqrt{\tilde{n} - 1}} \right) + g(\tilde{n}) \frac{1}{\tilde{n} - 1} \left( 2\sqrt{\tilde{n} - 1} - \frac{2\tilde{n} - n_M - 1}{2\sqrt{\tilde{n} - 1}} \right) + g'(\tilde{n}) \frac{2\tilde{n} - n_M - 1}{\sqrt{\tilde{n} - 1}}.$$

Since $g'(\tilde{n}) < 0$, the derivative is less than

$$\frac{1}{\tilde{n} - 1} \left( -\sqrt{\tilde{n} - 1} - \frac{2N - \tilde{n} - 1}{2\sqrt{\tilde{n} - 1}} \right) + g(\tilde{n}) \frac{1}{\tilde{n} - 1} \left( 2\sqrt{\tilde{n} - 1} - \frac{2\tilde{n} - n_M - 1}{2\sqrt{\tilde{n} - 1}} \right)$$

$$= \frac{1}{2(\tilde{n} - 1)^{2/3}} \left( -2(\tilde{n} - 1) - 2N + \tilde{n} + 1 + g(\tilde{n})(4(\tilde{n} - 1) - 2\tilde{n} + n_M + 1) \right)$$

$$= \frac{1}{2(\tilde{n} - 1)^{2/3}} \left( \tilde{n} + 3 - 2N + g(\tilde{n})(2\tilde{n} - 3 + n_M) \right).$$

Since $g(\tilde{n}) < 1$, the above is less than

$$\frac{1}{2(\tilde{n} - 1)^{2/3}} \left( \tilde{n} + 3 - 2N + 2\tilde{n} - 3 + n_M \right) = -\frac{2N - \tilde{n} - n_M}{2(\tilde{n} - 1)^{2/3}} < 0.$$

**Proof Proposition 2.** Suppose in equilibrium $\tilde{n}_B < n_B (\tilde{n} < n)$. Let us substitute $n - n_B$ for $n_M$ in (6):

$$\tilde{n} - 1 \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{2\tilde{n} + n_B - n - 1}{\tilde{n} - 1} \right) \times \frac{2(N - 1)\Omega (1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)}^2 (1 - f)t$$

$$+ \frac{\Omega(1 - \rho f c)}{n_B(1 - f)} = \frac{\Omega}{n_B}(v - c).$$
Multiplying both sides of the equation by \( n_B \) yields
\[
\frac{(\tilde{n} - 1)n_B}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{2\tilde{n} + n_B - n - 1}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right)^2 (1 - f)t
+ \frac{\Omega(1 - \rho)f c}{1 - f} = \Omega(v - c),
\]
or
\[
\frac{1}{2(N - 1)} F^2(n_B, \tilde{n})(1 - f)t + \frac{\Omega(1 - \rho)f c}{1 - f} = \Omega(v - c),
\]
(17)

Below I show that \( \frac{\partial F(n_B, \tilde{n})}{\partial n_B} > 0 \) and \( \frac{\partial F(n_B, \tilde{n})}{\partial \tilde{n}} < 0 \). Differentiating \( F(n_B, \tilde{n}) \) with respect to \( n_B \) yields
\[
\frac{\partial F(n_B, \tilde{n})}{\partial n_B} = \frac{1}{\sqrt{\tilde{n} - 1}} \frac{\partial}{\partial n_B} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} \sqrt{n_B} \right)
+ \frac{(2\tilde{n} + n_B - n - 1)}{(2\tilde{n} - 1)\sqrt{n_B}(1 - f)} \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)}
\]
\[
= \frac{1}{\sqrt{\tilde{n} - 1}} \left( \frac{2N - \tilde{n} - 1}{2\sqrt{n_B}} + \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \left( \sqrt{n_B} - \frac{2\tilde{n} + n_B - n - 1}{2\sqrt{n_B}} \right) \right)
\]
\[
= \frac{1}{\sqrt{\tilde{n} - 1}} \left( \frac{2N - \tilde{n} - 1}{2\sqrt{n_B}} + g(n_B, \tilde{n}) \left( \sqrt{n_B} - \frac{2\tilde{n} + n_B - n - 1}{2\sqrt{n_B}} \right) \right),
\]
where
\[
g(n_B, \tilde{n}) = \frac{2(N - 1)\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)}
\]
is from the proof of Lemma 1. Thus, we have
\[
\frac{\partial F(n_B, \tilde{n})}{\partial n_B} = \frac{2N - \tilde{n} - 1 + g(n_B, \tilde{n})[n_B - 2\tilde{n} + n + 1]}{2\sqrt{(\tilde{n} - 1)n_B}}.
\]
If the term in square brackets is positive, the derivative is positive. If the term in square brackets is negative, then, since \( g(n_B, \tilde{n}) < 1 \),
\[
\frac{\partial F(n_B, \tilde{n})}{\partial n_B} > \frac{2N - \tilde{n} - 1 + n_B - 2\tilde{n} + n + 1}{2\sqrt{(\tilde{n} - 1)n_B}} = \frac{2N + n + n_B - 3\tilde{n}}{2\sqrt{(\tilde{n} - 1)n_B}} > 0.
\]
Either way, $\partial F(n_B, \tilde{n})/\partial n_B > 0$. Differentiating $F(n_B, \tilde{n})$ with respect to $\tilde{n}$ yields

$$
\frac{\partial F(n_B, \tilde{n})}{\partial \tilde{n}} = \sqrt{n_B} \frac{\partial}{\partial \tilde{n}} \left( \frac{2N - \tilde{n} - 1}{\sqrt{\tilde{n} - 1}} + g(n_B, \tilde{n}) \frac{2\tilde{n} + n_B - n - 1}{\sqrt{\tilde{n} - 1}} \right)
$$

$$
= \frac{\sqrt{n_B}}{\tilde{n} - 1} \left( - \sqrt{\tilde{n} - 1} - \frac{2N - \tilde{n} - 1}{2\sqrt{\tilde{n} - 1}} \right) + \frac{\partial g(n_B, \tilde{n})}{\partial \tilde{n}} \frac{2\tilde{n} + n_B - n - 1}{\sqrt{\tilde{n} - 1}}
$$

$$
+ g(n_B, \tilde{n}) \left( 2\sqrt{\tilde{n} - 1} - \frac{2\tilde{n} + n_B - n - 1}{2\sqrt{\tilde{n} - 1}} \right).
$$

Since $\partial g(n_B, \tilde{n})/\partial \tilde{n} < 0$,

$$
\frac{\partial F(n_B, \tilde{n})}{\partial \tilde{n}} < \frac{\sqrt{n_B}}{\tilde{n} - 1} \left( - \sqrt{\tilde{n} - 1} - \frac{2N - \tilde{n} - 1}{2\sqrt{\tilde{n} - 1}} \right) + g(n_B, \tilde{n}) \left( 2\sqrt{\tilde{n} - 1} - \frac{2\tilde{n} + n_B - n - 1}{2\sqrt{\tilde{n} - 1}} \right)
$$

$$
= \frac{\sqrt{n_B}}{2(\tilde{n} - 1)^{3/2}} \left( - 2N - \tilde{n} + 3 + g(n_B, \tilde{n})(2\tilde{n} - n_B + n - 3) \right).
$$

Since $g(n_B, \tilde{n}) < 1$, the above is less than

$$
\frac{\sqrt{n_B}}{2(\tilde{n} - 1)^{3/2}} \left( - 2N - \tilde{n} + 3 + 2\tilde{n} - n_B + n - 3 \right) = \frac{\sqrt{n_B}}{2(\tilde{n} - 1)^{3/2}}(n + \tilde{n} - 2N - n_B) < 0,
$$

so $\partial F(n_B, \tilde{n})/\tilde{n} < 0$. It follows that (17) describes a positive relationship between $n_B$ and $\tilde{n}$. There will be more firms on the platform as a result of the change in the firm composition.

Next, we investigate the effect of an increase in $n_B$ on the equilibrium prices and welfare. Let us substitute $n - n_B$ for $n_M$ in (1) and (2):

$$
p_M^* = \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n} + n_B - n}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho_f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) \times t
$$

and

$$
p_B^* = \frac{c}{1 - f} + \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n} + n_B - n - 1}{\tilde{n} - 1} \times \frac{2(N - 1)\Omega(1 - \rho_f)}{(2\tilde{n} - 1)n_B(1 - f)} \right) \times t.
$$

The direct effect on $n_B$ on the equilibrium prices and the price difference (3) is negative. The indirect (through $\tilde{n}$) is negative, too (Lemma 1). Therefore, both $p_M^*$ and $p_B^*$ will decrease as a result of the change in the firm composition. The welfare will go up due to the decrease $p_B^* - p_M^*$ and the increase in $\tilde{n}$.

**Proof of Proposition 3.** Suppose in equilibrium $\tilde{n}_B < n_B$ ($\tilde{n} < n$). Since an increase in $\rho$ decreases the left-hand side of (6), $\tilde{n}$ has to decrease. A decrease in $\tilde{n}$ increases the prices and
the price difference (Lemma 1). While the direct effect of \( \rho \) on \( p_M^*, p_B^* \), and \( p_B^* - p_M^* \) is negative, the indirect (through \( \tilde{n} \)) is positive. The indirect effect may thus reverse the comparative statics results obtained in the case \( \tilde{n}_B = n_B \).

**Proof of Proposition 4.** Let

\[
\lambda = \frac{N/2}{N/2 + \gamma \Omega}.
\]

Since the number of consumers on each spoke changes from \( 1/2 \) to \( 1/\lambda \), (12) becomes

\[
\frac{1}{\lambda} \left( \frac{1}{2t} (t - p_i + p_M^*) \frac{n_M - 1}{N - 1} + \frac{1}{2t} (t - p_i + p_B^*) \frac{\tilde{n}_B - 1}{N - 1} + \frac{N - \tilde{n}}{N - 1} \right) ((1 - f)p_i - c),
\]

but (13) does not change. Next, (14) becomes

\[
\frac{1}{\lambda} \left( \frac{1}{2t} (t - p_i + p_M^*) \frac{n_M}{N - 1} + \frac{1}{2t} (t - p_i + p_B^*) \frac{\tilde{n}_B - 1}{N - 1} + \frac{N - \tilde{n}}{N - 1} \right) ((1 - f)p_i - c)
\]

\[
+ \frac{(1 - \gamma)\Omega}{n_B} ((1 - \rho f)p_i - c),
\]

which changes (15) to

\[
\frac{1}{\lambda} \left( \frac{1}{2} \frac{2N - \tilde{n} - 1}{N - 1} - \frac{1}{2t} (p_B^* - p_M^*) \frac{n_M}{N - 1} \right) (1 - f)
\]

\[
- \frac{1}{\lambda} \frac{n - \tilde{n}}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c) + \frac{(1 - \gamma)\Omega}{n_B} (1 - \rho f) = 0,
\]

or

\[
\left( \frac{1}{2} \frac{2N - \tilde{n} - 1}{N - 1} - \frac{1}{2t} (p_B^* - p_M^*) \frac{n_M}{N - 1} \right) (1 - f)
\]

\[
- \frac{\tilde{n} - 1}{N - 1} \frac{1}{2t} ((1 - f)p_B^* - c) + \frac{(1 - \gamma)\Omega}{n_B} (1 - \rho f) = 0.
\]

Thus, \( \Omega \) becomes \( (1 - \gamma)\lambda\Omega \) in (1), (2), and (3).
Next, we calculate the equilibrium profit of a branded firm

\[
\Pi_B^* = \left( \frac{1}{\lambda} \frac{\tilde{n} - 1}{2(N - 1)} ((1 - f)p_B^* - c) - \frac{(1 - \gamma)\Omega}{n_B} (1 - \rho f) \right) \frac{1}{1 - f} ((1 - f)p_B^* - c) \\
+ \frac{(1 - \gamma)\Omega}{n_B} ((1 - \rho f)p_B^* - c) \\
= \frac{1}{\lambda} \frac{\tilde{n} - 1}{2(N - 1) 2t} ((1 - f)p_B^* - c)^2 \frac{1}{1 - f} + \frac{(1 - \gamma)\Omega(1 - \rho f)}{n_B(1 - f)} \\
= \frac{1}{\lambda} \frac{\tilde{n} - 1}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + \frac{\tilde{n} + \tilde{n}_B - 1 2(N - 1)(1 - \gamma)\lambda\Omega(1 - \rho f)}{(2\tilde{n} - 1)n_B(1 - f)} \right)^2 (1 - f)t \\
+ \frac{(1 - \gamma)\Omega(1 - \rho f)c}{n_B(1 - f)} \\
\]

and set it equal to the profit of a stand-alone branded firm:

\[
\frac{1}{\lambda} \frac{\tilde{n} - 1}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\tilde{n} - 1} + g(\tilde{n})(2\tilde{n} - n_M - 1) \frac{1 - \gamma}{\lambda} \right)^2 (1 - f)t \\
+ \frac{(1 - \gamma)\Omega(1 - \rho f)c}{n_B(1 - f)} = \frac{(1 - \gamma)\Omega}{n_B}(v - c),
\]

where \( g(\tilde{n}) \) is from Lemma 1. Dividing both sides of the above equation by \( 1 - \gamma \) yields

\[
\frac{\sqrt{\tilde{n} - 1}}{2(N - 1)} \left( \frac{2N - \tilde{n} - 1}{\sqrt{(1 - \gamma)\lambda}} + g(\tilde{n})(2\tilde{n} - n_M - 1) \sqrt{(1 - \gamma)\lambda} \right)^2 (1 - f)t \\
+ \frac{\Omega(1 - \rho f)c}{n_B(1 - f)} = \frac{\Omega}{n_B}(v - c).
\]

More branded firms will join the platform in equilibrium because the introduction of \( \gamma \) shifts the left-hand side of (6) up:

\[
\frac{2N - \tilde{n} - 1}{\sqrt{(1 - \gamma)\lambda}} + g(\tilde{n})(2\tilde{n} - n_M - 1) \sqrt{(1 - \gamma)\lambda} > 2N - \tilde{n} - 1 + g(\tilde{n})(2\tilde{n} - n_M - 1),
\]

\[
2N - \tilde{n} - 1 > g(\tilde{n})(2\tilde{n} - n_M - 1) \sqrt{(1 - \gamma)\lambda}.
\]