

## Price experimentation with strategic buyers

Oksana Loginova · Curtis R. Taylor

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**Abstract** A two-period model in which a monopolist endeavors to learn about the permanent demand parameter of a specific repeat buyer is investigated. The buyer may strategically reject the seller's first-period offer for one of two reasons. First, in order to conceal information (i.e., to pool), a high-valuation buyer may reject high prices that would never be accepted by a low-valuation buyer. Second, in order to reveal information (i.e., to signal), a low-valuation buyer may reject low prices that would always be accepted by a high-valuation buyer. Given this, the seller often finds it optimal to post prices that reveal no useful information. Indeed, in the equilibrium where there is no signaling, the seller never charges an informative first-period price. Learning may occur in the equilibrium where there is maximal signaling, but the scope for learning is quite limited even in this case. Indeed, in order to preempt information transmission through signaling, the seller may be induced to set a first-period price strictly below the buyer's lowest possible valuation.

**Keywords** Price experimentation · Learning · Strategic rejections

**JEL Classification** C73 · D81 · D82

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O. Loginova (✉)  
Department of Economics, University of Missouri-Columbia, 118 Professional Building,  
Columbia, MO 65211-6040, USA  
e-mail: loginovao@missouri.edu

C. R. Taylor  
Department of Economics, Duke University, 213 Social Sciences Building,  
Durham, NC 27708-0097, USA  
e-mail: crtaylor@econ.duke.edu

## 1 Introduction

In a classic paper, [Rothschild \(1974\)](#) showed that the pricing problem facing a monopolist with unknown demand is often analogous to a two-armed bandit problem.<sup>1</sup> The optimal policy for such a firm is, therefore, to experiment with prices in order to learn about its unknown demand parameters. It is, however, well-known that the optimal policy may not result in complete learning because of the opportunity cost of experimentation. In addition, the learning process may be severely hampered unless the firm possesses significant prior knowledge about the type of uncertainty confronting it. For instance, even when demand is deterministic, [Aghion et al. \(1991\)](#) show that strong conditions such as continuity and quasi-concavity of the profit function are required to guarantee that a monopolist will eventually learn all the relevant information about its demand. In this paper, a different caveat is added to the list of reasons that a monopolist may have difficulty in learning demand. It may serve customers who do not want their demands to be known!

In the prior literature on price experimentation, the possibility of strategic buyers has been largely ignored. Specifically, it has typically been assumed either that the monopolist faces a sequence of identical customers who exist in the market for only one period or that market demand is composed of a *large* number of *small* customers.<sup>2</sup> There are, however, many real-world situations in which buyers have a significant stake in what a firm learns about their demands. Any time that price discrimination is possible on an individual basis and repeat purchases are likely, buyers possess incentives for strategic manipulation of demand information. In any long-term supply relationship, the buyer wants the supplier to think that he has very elastic demand for the product, and the buyer may strategically reject some offers in order to influence the supplier's beliefs to this end.

In this paper, a two-period experimental pricing and learning environment is analyzed. There is assumed to be a single buyer whose underlying *permanent* demand parameter,  $\lambda$ , and current *transitory* valuation,  $v_t$ , are private information. (Parameter  $\lambda$  is the probability the buyer's valuation  $v_t$  is high.) The seller makes a take-it-or-leave-it offer in the first period and updates her belief about the value of  $\lambda$  based on the buyer's acceptance decision. In particular, acceptance (rejection) of a high price implies that the buyer's first-period valuation for the product,  $v_1$ , was high (low). This leads the seller to update her beliefs about  $\lambda$  and, therefore, to infer that the buyer's second-period valuation,  $v_2$ , is also likely to be high (low). It is shown that if the buyer is not strategic, then the informational value of a high first-period price can lead the seller to charge one when it would otherwise not be optimal to do so (i.e., to experiment).

<sup>1</sup> Many authors have subsequently refined and extended this observation. See, for example, [Aghion et al. \(1991\)](#), [Mirman et al. \(1993\)](#), [Rustichini and Wolinsky \(1995\)](#), and [Keller and Rady \(1999\)](#).

<sup>2</sup> In a related paper, [Segal \(2003\)](#) considers a setting in which there is a finite number of buyers in the market from the outset. He shows that if the common distribution of buyers' valuations is unknown, then learning through price experimentation is dominated by a multi-unit auction which sets a price to each buyer on the basis of the demand distribution inferred statistically from other buyers' bids. Segal's analysis does not apply to the setting considered here where consumer types are assumed to be drawn from different distributions.

Things are shown to be starkly different, however, when the buyer is strategic. In this case, if the buyer's first-period valuation is high, then he will often attempt to conceal information by strategically rejecting high prices. A buyer with a high value of  $\lambda$  stands to gain the most from concealing a high first-period valuation for the good. This gives rise to "reverse screening" in the sense that only buyers with low values of  $\lambda$  and high valuations for the good will accept high first-period prices. When the first-period price is low, two limiting types of continuation equilibria exist, a Good equilibrium (for the seller) in which all buyer types purchase the product, and a Bad equilibrium in which a buyer with a relatively high value of  $\lambda$  but low valuation for the product strategically rejects the offer in order to signal his low first-period valuation. This signaling at low prices is the mirror image of the screening that occurs at high prices in the sense that strategic rejections at low prices reveal information while strategic rejections at high prices conceal it. When facing a strategic buyer, the seller typically finds it optimal to set a price that reveals no information about his demand parameter. In fact, in the good PBE she never charges an informative first-period price, and in the bad PBE she may even set an equilibrium first-period price strictly below the buyer's lowest possible valuation in order to forestall signaling, the antithesis of price experimentation!

The pricing and acceptance behavior exhibited in the model presented here can be viewed as a manifestation of the ratchet effect familiar from the regulation and agency literature.<sup>3</sup> Specifically, the fact that the seller cannot commit not to use the information it learns in the first period harms it. This lack of commitment generates the strategic rejections by the buyer that give rise to a weakly lower effective first-period demand for the good. This reduction in the probability of a first-period sale leaves the seller worse off as compared with a setting in which she could commit to price non-contingently. This paper also contributes to the burgeoning literature on behavior-based price discrimination.<sup>4</sup> In Internet retailing and many other market settings, firms now have the ability to track the purchasing behavior of individual customers and to tailor price offers to them.<sup>5</sup> To the extent that consumers are aware of this, the findings presented here indicate that they possess significant incentives to manipulate the information collected.

The paper closest to this study is Kennan (2001), where it is shown that persistent private information may lead to stochastic cycles in repeated labor negotiations. While several of the results presented here are reminiscent of Kennan's, the two models differ in important ways. First, in the setting studied by Kennan, a monopolist labor union attempts to learn the current valuation (low or high) of a monopsonist firm in full knowledge of the underlying stochastic process (an infinite-horizon Markov chain). In the present model, by contrast, the monopolist does not know the buyer's current valuation (low or high) and which of a continuum of two-period processes ( $\lambda \in [0, 1]$ ) may have generated it.

<sup>3</sup> See, for example, Laffont and Tirole (1988), and especially Hart and Tirole (1988).

<sup>4</sup> See, for example, Acquisti and Varian (2005), Taylor (2003), Shaffer and Zhang (2000), Fudenberg and Tirole (2000), and Villas-Boas (1999).

<sup>5</sup> See Krugman (2000) and Streitfield (2000).

In Kennan's model, the monopolist may use a screening offer in order to learn the buyer's current valuation and hence, the state of the process. In the present model, the seller may make an offer that reveals the buyer's first-period valuation and allows her to update her beliefs about the underlying process but not to learn it with certainty. In Kennan's model, the seller eventually opts to separate types with a screening offer, while the monopolist in the present model seldom makes separating offers. Interestingly, in both Kennan's model and the present one, the monopolist sometimes makes equilibrium price offers strictly below the minimum valuation of the buyer. In both models, this occurs in order to preempt signaling by a low-value buyer who would reject an offer equal to his valuation. Such signaling would provide valuable information to the monopolist, but the value of the information obtained would not offset the reduced probability of making a current sale.

The infinite-horizon setting of persistent private information studied by Kennan allows him to uncover some interesting and important dynamic effects outside the scope of this investigation. The two-dimensional private information with a permanent component analyzed here, however, adheres more closely to the traditional experimental pricing paradigm where the focus is on learning the structural parameters underlying a noisy demand process.<sup>6</sup>

The paper is structured as follows. The main analysis involves four distinct stages: (I) the seller sets the first-period price,  $p_1$ ; (II) the buyer observes  $p_1$  and decides whether to purchase the good,  $q_1 = 1$ , or not to purchase it,  $q_1 = 0$ ; (III) the seller observes  $q_1$  and sets the second-period price,  $p_2$ ; (IV) the buyer observes  $p_2$  and decides whether to purchase the good,  $q_2 \in \{0, 1\}$ . The game is solved via backward induction, so the analysis proceeds through these stages in reverse order. Hence, once the model is laid out in the next section, equilibrium second-period purchasing (stage IV) and pricing (stage III) decisions are characterized. Section 3 takes a brief detour to explore the benchmark setting of a non-strategic buyer. The bulk of the analysis appears in Sect. 4 where stage III equilibrium behavior is derived. Specifically, the buyer's equilibrium first-period purchasing decision is characterized for all possible values of  $p_1$  the seller might set. It turns out that there are multiple equilibria of the continuation game if the seller sets a low first-period price. The "bookend" equilibria that involve maximal and minimal signaling by the buyer are characterized. Finally, Sects. 5 and 6 consider stage I of the game in which the seller—anticipating the buyer's purchasing strategy—sets  $p_1$ . The minimal signaling equilibrium is analyzed in Sect. 5 and the maximal signaling equilibrium in Sect. 6. Concluding remarks appear in Sect. 7. Proofs not appearing in the text have been relegated to an Appendix.

<sup>6</sup> In a recent paper, Battaglini (2005) also studies a related setting in which a firm has a long-term relationship with a customer whose demand parameter is determined over time by a Markov process. Battaglini's findings, however, differ markedly from those presented here, primarily because he considers a setting with full-commitment in which the buyer completely reveals his private information at the outset of the relationship.

## 2 The model

There are two risk-neutral players, a seller (S, she) and a buyer (B, he). In each period  $t = 1, 2$ , B demands one unit of a good which S may produce and sell to him. S's production cost is normalized to zero. B's valuation for the good in period  $t$ ,  $v_t$ , equals 1 (a convenient normalization) with probability  $\lambda$  and equals  $v$  with probability  $1 - \lambda$ ,  $v \in (0, 1)$ . The demand parameter  $\lambda$  can be thought of as representing B's income or an underlying preference parameter. It is itself the realization of a random variable which is continuously distributed on  $[0, 1]$  with probability density function  $f(\lambda)$  (cumulative distribution function  $F(\lambda)$ ). Let  $E[\lambda]$  denote the expected value of  $\lambda$  under this prior.

At the beginning of the game, B privately observes  $\lambda$  and  $v_1$ , and he privately observes  $v_2$  at the beginning of the second period. Hence, in any given period, B's "type" has two components, a permanent structural component  $\lambda \in [0, 1]$  and a noisy transitory component  $v_t \in \{v, 1\}$ .<sup>7</sup> In each period  $t = 1, 2$ , B and S play an extensive-form game with the following sequence of events.

- B observes his valuation  $v_t \in \{v, 1\}$ .
- S announces price  $p_t \in \mathfrak{R}_+$  at which she is willing to sell the good to B.
- B either accepts,  $q_t = 1$ , or rejects,  $q_t = 0$ , S's offer.
- B's (contemporaneous) payoff is  $q_t(v_t - p_t)$ , and S's payoff is  $q_t p_t$ .

For ease of exposition, there is assumed to be no discounting.<sup>8</sup> Note that while this game has a recursive structure, it is not a repeated game due to the presence of asymmetric information. Specifically, S updates her prior beliefs about  $\lambda$  from the first period to the second.

Let  $h_S \equiv (p_1, q_1)$  be the history of first-period events observed by S, and let  $h_B \equiv (\lambda, v_1, p_1, q_1)$  be the history of first-period events observed by B at the beginning of period 2. A behavior strategy for S is a pair of probability distributions  $(\Phi_1(p_1), \Phi_2(p_2; h_S))$  over all possible price offers. Similarly, a behavior strategy for B is a pair of functions  $(\gamma_1(\lambda, v_1, p_1), \gamma_2(v_2, p_2; h_B))$ , where  $\gamma_t$  is the probability that B accepts S's offer in period  $t$ .

Let  $f(\lambda|h_S)$  denote S's posterior beliefs about  $\lambda$  at the beginning of period 2. Likewise, let  $E[\lambda|h_S]$  denote her updated expectation. The solution concept employed is efficient perfect Bayesian equilibrium (PBE), i.e., a PBE in which indifference about pricing or purchasing is resolved in favor of efficiency. Because inefficient PBEs occur only for a non-generic set of parameters, the efficiency criterion is fairly innocuous and is suppressed in the discussion below.

Observe that in the second period B optimally accepts any price that does not exceed his valuation regardless of the history. Given this, S will either price at 1 and sell with probability  $E[\lambda|h_S]$ , or she will price at  $v$  and sell with probability one. This is stated formally in the following lemma.

<sup>7</sup> While B's type is multi-dimensional, the focus here is on pricing without commitment rather than on an optimal monopolistic screening mechanism (e.g., [Armstrong 1996](#); [Rochet and Chone 1998](#)).

<sup>8</sup> Discounting does not change the results qualitatively, but it complicates the analysis by adding a "kink" to expected first-period demand.

**Lemma 1** (Second-period equilibrium behavior) *In any PBE, B's strategy in period 2 is*

$$\gamma_2(v_2, p_2; h_B) = \begin{cases} 1, & \text{if } v_2 \geq p_2 \\ 0, & \text{if } v_2 < p_2. \end{cases}$$

Also, in any PBE,  $\Phi_2(p_2; h_S)$  is a two-point distribution. Specifically, S offers  $p_2 = v$  with probability  $\theta(h_S)$  and  $p_2 = 1$  with probability  $1 - \theta(h_S)$  according to

$$\theta(h_S) = \begin{cases} 0, & \text{if } v < E[\lambda|h_S] \\ \text{some } \theta \in [0, 1], & \text{if } v = E[\lambda|h_S] \\ 1, & \text{if } v > E[\lambda|h_S]. \end{cases}$$

With Lemma 1 in hand, it is possible to derive first-period equilibrium pricing and purchasing behavior, which is the focus of the ensuing sections.

### 3 The classic non-strategic buyer setting

In this section, a benchmark setting is considered. Specifically, the seller's problem when facing a non-strategic buyer originally analyzed by Rothschild (1974) is recast in the simple two-period two-valuation framework of the current paper. In the next section, the straightforward solution to this problem is shown to contrast sharply with the case of a strategic buyer.

Formally, suppose—as in Rothschild (1974)—that S faces two stochastically equivalent (i.e.,  $\lambda$  is the same), but distinct buyers,  $B_1$  and  $B_2$ , who arrive sequentially. In this case, S learns about the demand parameter of  $B_2$  by observing the purchasing behavior of  $B_1$ .

Denote by  $\underline{E}$  the expected value of  $\lambda$  given  $v_1 = v$ , and  $\overline{E}$  the expected value of  $\lambda$  given  $v_1 = 1$ . Straightforward calculations yield

$$\underline{E} \equiv E[\lambda|v_1 = v] = \frac{E[\lambda] - E[\lambda^2]}{1 - E[\lambda]}$$

and

$$\overline{E} \equiv E[\lambda|v_1 = 1] = \frac{E[\lambda^2]}{E[\lambda]}.$$

Observe that  $\underline{E} < E[\lambda] < \overline{E}$ .

**Proposition 1** (Equilibrium with a non-strategic buyer) *In any PBE,  $B_t$  accepts if and only if  $v_t \geq p_t$ . Equilibrium prices are given by:*

- (i)  $p_1 = p_2 = 1$ , if  $v < \underline{E}$ ;
- (ii)  $p_1 = 1$  and  $p_2 = q_1 + (1 - q_1)v$ , if  $v \in [\underline{E}, v']$ ;

(iii)  $p_1 = p_2 = v$ , if  $v \geq v'$ , where

$$v' \equiv \frac{E[\lambda] + E[\lambda^2]}{1 + E[\lambda]} \in (E[\lambda], \bar{E}).$$

Obviously,  $B_t$  will purchase the good if and only if the price does not exceed his valuation. Given this,  $S$  would never charge  $p_t \neq v$  or 1. Note, however, that pricing at  $v$  in the first period provides no information about the demand parameter  $\lambda$  because  $B_1$  always accepts this offer. Pricing at 1 does reveal information because  $B_1$  accepts if and only if  $v_1 = 1$ .

$S$  finds it optimal to charge 1 in both periods if  $v$  is low enough, and she charges  $v$  in both periods if it is sufficiently high. When  $v \in [\underline{E}, v')$ ,  $S$  charges 1 in the first period,  $p_2 = 1$  following acceptance and  $p_2 = v$  following rejection. When  $v \in (E[\lambda], v')$ ,  $S$  experiments by setting  $p_1 = 1$ . That is, her first-period payoff would be maximized at  $p_1 = v$ , but she opts to set  $p_1 = 1$  in order to obtain information about  $B_2$ 's demand parameter  $\lambda$ . It is straightforward to verify that this price experimentation lowers expected social surplus, i.e., the value of the information obtained by  $S$  is less than the concomitant loss in expected consumer surplus. In order to focus on settings where information is potentially valuable to  $S$ , the following necessary condition is assumed to hold throughout the remainder of the paper:

$$\underline{E} < v < \bar{E}.$$

#### 4 The strategic buyer setting

In this section, the case of a single buyer who is interested in purchasing one unit of the good in each period is investigated. In particular,  $B$ 's first-period equilibrium behavior on the continuation game following any first-period price offer  $p_1$  is derived. The analysis begins with two fundamental observations (Lemmas 2, 3), and then moves down the demand curve from consideration of high prices (Corollary 1 and Lemma 4) to consideration of low prices (Corollary 2 and Lemma 5). Gluing these pieces together then yields the first-period equilibrium expected demand facing the seller (Proposition 2).

From Lemma 1, the expected payoff to  $B$  from accepting  $p_1$  is

$$v_1 - p_1 + \lambda\theta(p_1, 1)(1 - v),$$

and his expected payoff from rejecting is

$$\lambda\theta(p_1, 0)(1 - v).$$

This simple observation serves as proof of the following claim.

**Lemma 2** (Dynamic incentives) *In any PBE,  $B$  accepts  $p_1$  if and only if*

$$v_1 - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - v).$$

Next, note that in any PBE it must be the case that  $E[\lambda|p_1, q_1]$  is derived from B’s first-period behavior given  $\theta(p_1, q_1)$ , and  $\theta(p_1, q_1)$  is optimal for S given  $E[\lambda|p_1, q_1]$ . This interdependence between optimal actions and beliefs gives rise to the following lemma.

**Lemma 3** (Beliefs and actions) *Let  $D_1(p_1)$  denote the probability S assigns to acceptance of her first-period offer. In any PBE, if  $D_1(p_1) \in (0, 1)$ , then*

$$E[\lambda|p_1, 1] \geq E[\lambda|p_1, 0],$$

$$\theta(p_1, 1) \leq \theta(p_1, 0).$$

Intuitively, because B is more likely to accept when  $v_1 = 1$ , S’s beliefs about  $\lambda$  should be higher when she observes  $q_1 = 1$  than when she observes  $q_1 = 0$ . The following result is proven by applying Lemmas 2 and 3.

**Corollary 1** (Honest rejections at high prices) *In any PBE, B with  $v_1 = v$  always rejects  $p_1 \in (v, 1]$ .*

The next step is to determine when B with  $v_1 = 1$  accepts an offer of  $p_1 \in (v, 1]$ . The previous two lemmas indicate that B with high  $\lambda$  has the most to gain from strategically rejecting such an offer (i.e., from pooling with B’s possessing  $v_1 = v$ ). Hence, suppose that B with  $v_1 = 1$  accepts  $p_1 \in (v, 1]$  if and only if  $\lambda \leq \mu$  for some  $\mu \in [0, 1]$ . Then the expected value of  $\lambda$  conditional on acceptance is

$$\alpha(\mu) \equiv E[\lambda|v_1 = 1 \cap \lambda \leq \mu] = \frac{\int_0^\mu \lambda^2 f(\lambda) d\lambda}{\int_0^\mu \lambda f(\lambda) d\lambda},$$

and the expected value of  $\lambda$  conditional on rejection is

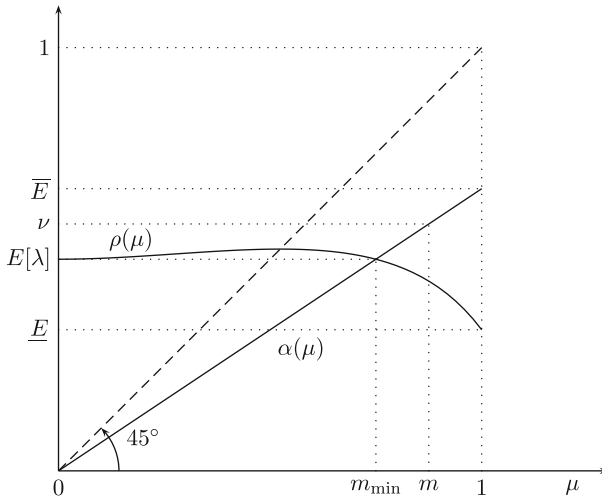
$$\rho(\mu) \equiv E[\lambda|\{v_1 = v\} \cup \{v_1 = 1 \cap \lambda > \mu\}] = \frac{E[\lambda] - \int_0^\mu \lambda^2 f(\lambda) d\lambda}{1 - \int_0^\mu \lambda f(\lambda) d\lambda}.$$

These functions are depicted in Fig. 1.<sup>9</sup> It is straightforward to show that function  $\alpha$  monotonically increases from  $\alpha(0) = 0$  to  $\alpha(1) = \bar{E}$  and is always below the 45-degree line. Function  $\rho$  starts at  $\rho(0) = E[\lambda]$ , increases until it crosses the 45-degree line, and then decreases until it ends at  $\rho(1) = \underline{E}$ . The functions cross once, and at their intersection  $\alpha(m_{\min}) = \rho(m_{\min}) = E[\lambda]$ . It follows that there exists a unique number  $m \in [m_{\min}, 1)$  defined by

$$m \equiv \begin{cases} \rho^{-1}(v), & \text{if } v \in (\underline{E}, E[\lambda]) \\ \alpha^{-1}(v), & \text{if } v \in [E[\lambda], \bar{E}). \end{cases}$$

In Fig. 1,  $m$  is shown for the case  $v > E[\lambda]$ .

<sup>9</sup> All figures are drawn for the uniform distribution.



**Fig. 1** Geometric properties of  $\alpha$  and  $\rho$

Given  $m$ , define the constant

$$\bar{p} \equiv 1 - m(1 - \nu).$$

For any  $p_1 \in (\nu, \bar{p}]$ , the next lemma establishes that the marginal type for B that accepts when  $\nu_1 = 1$  is given by the function

$$\mu(p_1) \equiv \frac{1 - p_1}{1 - \nu}.$$

Observe that  $\mu(p_1)$  is monotone decreasing with  $\mu(\nu) = 1$  and  $\mu(\bar{p}) = m$ .

**Lemma 4** (Strategic rejections at high prices) *Suppose  $\nu_1 = 1$ . In any PBE, the following must hold:*

- (i) *If  $p_1 \in (\nu, \bar{p}]$ , then B accepts the price if and only if  $\lambda \leq \mu(p_1)$ . S sets  $p_2 = 1$  following acceptance and  $p_2 = \nu$  following rejection.*
- (ii) *If  $p_1 \in (\bar{p}, 1]$ , then B accepts the price if and only if  $\lambda \leq m$ . When  $\nu < E[\lambda]$ , S sets  $p_2 = 1$  following acceptance and randomizes between  $\nu$  and 1 following rejection according to*

$$\theta(p_1, 0) = \frac{1 - p_1}{m(1 - \nu)}.$$

*When  $\nu > E[\lambda]$ , S sets  $p_2 = \nu$  following rejection and randomizes between  $\nu$  and 1 following acceptance according to*

$$\theta(p_1, 1) = 1 - \frac{1 - p_1}{m(1 - \nu)}.$$

Lemma 4 together with Corollary 1 characterize B’s first-period equilibrium behavior after the announcement of  $p_1 \in (\nu, 1]$ . To understand the result, first consider  $p_1 \in (\nu, \bar{p})$ . Given that B accepts  $p_1$  if and only if  $v_1 = 1$  and  $\lambda \leq \mu(p_1)$ , S updates her beliefs,  $E[\lambda|p_1, 1] = \alpha(\mu(p_1)) > \nu$  and  $E[\lambda|p_1, 0] = \rho(\mu(p_1)) < \nu$ . Hence, S optimally sets  $p_2 = 1$  following acceptance and  $p_2 = \nu$  following rejection. Notice that “reverse screening” occurs over this range in the sense that high prices induce rejection by high long-run types. Indeed, as  $p_1$  increases, the marginal type  $\mu(p_1)$  falls and the set of types willing to accept shrinks. Acceptance and rejection, therefore, become less informative ( $\alpha - \rho$  gets smaller).

Once  $p_1 = \bar{p}$ , further increases in  $p_1$  cannot induce more strategic rejection because acceptance would otherwise indicate a sufficiently low value of  $\lambda$  that S would prefer to set  $p_2 = \nu$ . In particular, for  $p_1 \in (\bar{p}, 1]$ , the marginal type must remain at  $m$ . This requires S to randomize between  $p_2 = \nu$  and  $p_2 = 1$  in order to keep B with  $v_1 = 1$  and  $\lambda = m$  indifferent between accepting and rejecting  $p_1$ .

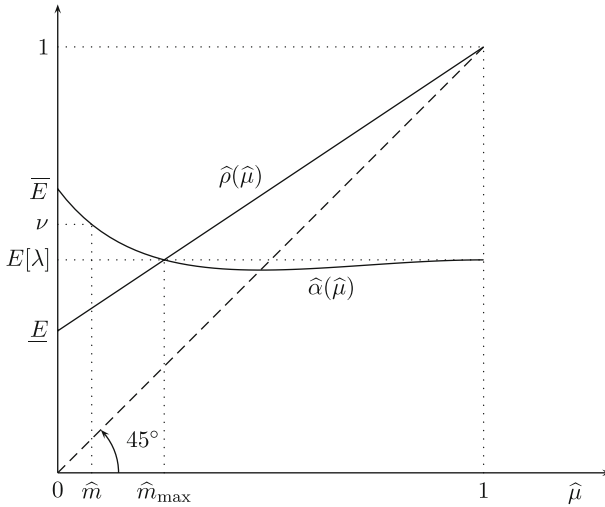
To complete the characterization of equilibrium play for B, it remains to consider  $p_1 \leq \nu$ . The analysis over this range, however, is more complicated because B’s first-period equilibrium behavior is not unique. In particular, there exists a lower bound  $\underline{p}$  such that B accepts in any PBE if  $p_1 \leq \underline{p}$ . For any  $p_1 \in (\underline{p}, \nu]$ , however, either pooling or signaling may occur. To illustrate this, the two extreme equilibria that involve no signaling (i.e., all types accept  $p_1 \leq \nu$ ) and maximal signaling (i.e., strategic rejections by some types for all  $p_1 \in (\underline{p}, \nu]$ ) are derived. For ease of exposition, these equilibria are called (using S’s perspective) respectively *the Good PBE* and *the Bad PBE*, with the understanding that they actually bracket a continuum of intermediate cases. In particular, an Intermediate PBE obtains if the Good PBE holds for some proper subset of prices  $p_1 \in (\underline{p}, \nu]$  and the Bad PBE holds for the complementary subset of prices in this range.

Interestingly, B’s behavior in the Bad PBE for prices  $p_1 \in (\underline{p}, \nu]$  is the mirror image of his behavior for prices in  $(\nu, \bar{p}]$ . Specifically, for  $p_1 \in (\nu, \bar{p}]$ , strategic rejections *conceal* information through pooling. On the other hand, for  $p_1 \in (\underline{p}, \nu]$ , it is shown below that strategic rejections *reveal* information through signaling. The following result is proven by applying Lemmas 2 and 3.

**Corollary 2** (Honest acceptances at low prices) *In any PBE, B with  $v_1 = 1$  always accepts  $p_1 \leq \nu$ .*

The next step is to determine when B with  $v_1 = \nu$  accepts  $p_1 \leq \nu$ . Lemmas 2 and 3 indicate that B with high  $\lambda$  has the most to gain from strategically rejecting such an offer (i.e., from separating from B’s possessing  $v_1 = 1$ ). Hence, suppose that B with  $v_1 = \nu$  accepts  $p_1 \leq \nu$  if and only if  $\lambda \leq \hat{\mu}$  for some  $\hat{\mu} \in [0, 1]$ . Then the expected value of  $\lambda$  conditional on acceptance is

$$\hat{\alpha}(\hat{\mu}) \equiv E[\lambda|\{v_1 = 1\} \cup \{v_1 = \nu \cap \lambda \leq \hat{\mu}\}] = \frac{E[\lambda] - \int_{\hat{\mu}}^1 \lambda(1 - \lambda)f(\lambda) d\lambda}{1 - \int_{\hat{\mu}}^1 (1 - \lambda)f(\lambda) d\lambda},$$



**Fig. 2** Geometric properties of  $\hat{\alpha}$  and  $\hat{\rho}$

and the expected value of  $\lambda$  conditional on rejection is

$$\hat{\rho}(\hat{\mu}) \equiv E[\lambda | v_1 = v \cap \lambda > \hat{\mu}] = \frac{\int_{\hat{\mu}}^1 \lambda(1 - \lambda)f(\lambda) d\lambda}{\int_{\hat{\mu}}^1 (1 - \lambda)f(\lambda) d\lambda}.$$

These functions are depicted in Fig. 2. Function  $\hat{\alpha}$  starts at  $\hat{\alpha}(0) = \bar{E}$ , decreases until it crosses the  $45^\circ$  line, and then increases until it ends at  $\hat{\alpha}(1) = E[\lambda]$ . Function  $\hat{\rho}$  lies above the  $45^\circ$  line, starts at  $\hat{\rho}(0) = \underline{E}$  and increases monotonically until it ends at  $\hat{\rho}(1) = 1$ . The two functions cross once, and at their intersection  $\hat{\alpha}(\hat{m}_{\max}) = \hat{\rho}(\hat{m}_{\max}) = E[\lambda]$ . It follows there exists a unique number  $\hat{m} \in (0, \hat{m}_{\max}]$  defined by

$$\hat{m} \equiv \begin{cases} \hat{\rho}^{-1}(v), & \text{if } v \in (\underline{E}, E[\lambda]) \\ \hat{\alpha}^{-1}(v), & \text{if } v \in [E[\lambda], \bar{E}). \end{cases}$$

In Fig. 2,  $\hat{m}$  is shown for the case  $v > E[\lambda]$ .

Given  $\hat{m}$ , define the constant

$$\underline{p} \equiv v - \hat{m}(1 - v).$$

For any  $p_1 \in (\underline{p}, v]$  the next result establishes that the marginal type for B that accepts in the Bad PB $\bar{E}$  when  $v_1 = v$  is given by the function

$$\hat{\mu}(p_1) \equiv \frac{v - p_1}{1 - v}.$$

Observe that  $\hat{\mu}(p_1)$  is monotone decreasing with  $\hat{\mu}(\underline{p}) = \hat{m}$  and  $\hat{\mu}(v) = 0$ .

**Lemma 5** (Pooling and signaling at low prices) *Suppose  $v_1 = v$ . In the Good PBE, B always accepts  $p_1 \leq v$ , S sets  $p_2 = 1$  if  $v < E[\lambda]$  and  $p_2 = v$  if  $v > E[\lambda]$ . In the Bad PBE:*

- (i) *If  $p_1 \in (\underline{p}, v]$ , then B accepts if and only if  $\lambda \leq \widehat{\mu}(p_1)$ . S sets  $p_2 = 1$  following acceptance and  $p_2 = v$  following rejection.*
- (ii) *If  $p_1 \leq \underline{p}$ , then B always accepts the price. S sets  $p_2 = 1$  if  $v < E[\lambda]$  and  $p_2 = v$  if  $v > E[\lambda]$ .*

To understand the Bad PBE, first consider  $p_1 \in (\underline{p}, v]$ . Given that B rejects if and only if  $v_1 = v$  and  $\lambda > \widehat{\mu}(p_1)$ , S updates her beliefs:  $E[\lambda|p_1, 0] = \widehat{\rho}(\widehat{\mu}(p_1)) < v$  and  $E[\lambda|p_1, 1] = \widehat{\alpha}(\widehat{\mu}(p_1)) > v$ . Hence, S optimally sets  $p_2 = v$  following rejection and  $p_2 = 1$  following acceptance. Notice that as  $p_1$  falls,  $\widehat{\mu}(p_1)$  increases and the set of long-run types willing to strategically reject shrinks. Acceptance and rejection, therefore, become less informative ( $\widehat{\alpha} - \widehat{\rho}$  gets smaller). For  $p_1 \leq \underline{p}$ , signaling is not possible because rejection would indicate a sufficiently high value of  $\lambda$  that S would prefer to set  $p_2 = 1$ . Lemma 5 shows that any  $p_1 \leq \underline{p}$  must induce acceptance by all types of B in any PBE.

Lemma 4 with Corollary 1 and Lemma 5 with Corollary 2 allow to formulate the expected first-period demand by B and the expected values of  $\lambda$  conditional on acceptance and rejection of the first-period price.

**Proposition 2** (Expected first-period demand and posterior beliefs) *The probability that B accepts  $p_1$  and the expected values of  $\lambda$  conditional on acceptance/rejection of  $p_1$  are as follows:*

- (i) *If  $p_1 > 1$ , then  $D_1(p_1) = 0$  and  $E[\lambda|p_1, 0] = E[\lambda]$ .*
- (ii) *If  $p_1 \in (\overline{p}, 1]$ , then  $D_1(p_1) = \int_0^m \lambda f(\lambda) d\lambda$  and*

$$E[\lambda|p_1, q_1] = \begin{cases} \alpha(m), & \text{if } q_1 = 1 \\ \rho(m), & \text{if } q_1 = 0. \end{cases}$$

- (iii) *If  $p_1 \in (v, \overline{p}]$ , then  $D_1(p_1) = \int_0^{\mu(p_1)} \lambda f(\lambda) d\lambda$  and*

$$E[\lambda|p_1, q_1] = \begin{cases} \alpha(\mu(p_1)), & \text{if } q_1 = 1 \\ \rho(\mu(p_1)), & \text{if } q_1 = 0. \end{cases}$$

- (iv) *If  $p_1 \leq v$  and the Good PBE obtains, then  $D_1(p_1) = 1$  and  $E[\lambda|p_1, 1] = E[\lambda]$ . If the Bad PBE obtains, then*

$$D_1(p_1) = \begin{cases} 1 - \int_{\widehat{\mu}(p_1)}^1 (1 - \lambda) f(\lambda) d\lambda, & \text{if } p_1 \in (\underline{p}, v] \\ 1, & \text{if } p_1 \leq \underline{p} \end{cases}$$

and

$$E[\lambda|p_1, q_1] = \begin{cases} \widehat{\alpha}(\widehat{\mu}(p_1)), & \text{if } q_1 = 1 \text{ and } p_1 \in (\underline{p}, v] \\ \widehat{\rho}(\widehat{\mu}(p_1)), & \text{if } q_1 = 0 \text{ and } p_1 \in (\underline{p}, v] \\ E[\lambda], & \text{if } q_1 = 1 \text{ and } p_1 \leq \underline{p}. \end{cases}$$

Note that strategic rejections in the Bad PBE for  $p_1 \in (\underline{p}, v]$  and in all PBE for  $p_1 \in (v, 1]$  imply that  $D_1(p_1)$  is strictly lower than the expected demand of a non-strategic buyer at these prices. In particular, Proposition 1 indicates that a non-strategic buyer accepts  $p_1 \leq v$  with probability one and accepts  $p_1 \in (v, 1]$  with probability  $E[\lambda]$ . Because strategic rejections also impede learning, it follows immediately that S is weakly worse off when facing a strategic buyer.

S’s continuation payoff from offering  $p_1$  to a strategic buyer is

$$\Pi_S(p_1) = D_1(p_1)(p_1 + \max\{E[\lambda|p_1, 1], v\}) + (1 - D_1(p_1)) \max\{E[\lambda|p_1, 0], v\}.$$

The final step in deriving a PBE of the entire game is to determine the value of  $p_1$  that maximizes  $\Pi_S(p_1)$ . Of course, the solution to this problem depends on which PBE obtains for continuation games with  $p_1 \leq v$ . The Good and the Bad PBE are investigated, respectively, in the next two sections.

### 5 The good equilibrium

In this section, the Good PBE in which B always accepts  $p_1 \leq v$  is fully characterized. The main finding is that in this equilibrium S never offers a first-period price that yields her valuable information about B’s structural demand parameter  $\lambda$ .

To ease notation, for any  $\mu \in [0, 1]$  define

$$I(\mu) \equiv \int_0^\mu \lambda f(\lambda) d\lambda.$$

First, suppose S is considering offering  $p_1 \in (\bar{p}, 1]$ . Observe that the marginal type  $m$  that accepts such an offer is calibrated so that the value of the information conveyed to S is zero. In particular, if  $v > E[\lambda]$ , then

$$E[\lambda|p_1, 1] = \alpha(m) = v > \rho(m) = E[\lambda|p_1, 0].$$

Thus, setting  $p_2 = v$  (which is optimal under the prior) maximizes S’s expected second-period return following a first-period acceptance *and* rejection. Similarly, if  $v < E[\lambda]$ , then

$$E[\lambda|p_1, 1] = \alpha(m) > v = \rho(m) = E[\lambda|p_1, 0].$$

Setting  $p_2 = 1$  (which is optimal under the prior) maximizes S’s expected second-period return following a first-period acceptance *and* rejection. Hence, S’s continuation payoff from offering  $p_1 \in (\bar{p}, 1]$  can be written as

$$\Pi_S(p_1) = I(m)p_1 + \max\{E[\lambda], v\},$$

which is maximized at  $p_1 = 1$ .

Second, suppose S is considering offering  $p_1 \in (\nu, \bar{p}]$ . The information conveyed by B’s purchasing decision is valuable to S for prices in this range because

$$E[\lambda|p_1, 1] = \alpha(\mu(p_1)) \geq \nu \geq \rho(\mu(p_1)) = E[\lambda|p_1, 0],$$

with strict inequalities when  $p_1 \in (\nu, \bar{p})$ . In the second period, S optimally sets  $p_2 = 1$  following acceptance and  $p_2 = \nu$  following rejection. Hence, her continuation payoff is

$$\Pi_S(p_1) = I(\mu(p_1))(p_1 + E[\lambda|p_1, 1]) + (1 - I(\mu(p_1)))\nu.$$

Finally, suppose S is considering offering  $p_1 \leq \nu$ . Because B always accepts such an offer, S’s continuation payoff is

$$\Pi_S(p_1) = p_1 + \max\{E[\lambda], \nu\},$$

which is maximized at  $p_1 = \nu$ .

Summarizing the above, in the Good PBE, S charges some  $p_1 \in \{1, \nu\} \cup (\nu, \bar{p})$ , depending on the primitives of the model  $\nu$  and  $F(\lambda)$ . In fact, Propositions 3 and 4 below show that S never sets a first-period price  $p_1 \in (\nu, \bar{p})$  that yields valuable information.

**Proposition 3** (Optimal prices when  $\nu$  is high) *Suppose  $\nu \geq E[\lambda]$ . Then, S offers  $p_1 = p_2 = \nu$  in the Good PBE.*

Recall from Proposition 1 that when B is not strategic, it is optimal for S to experiment by charging  $p_1 = 1$  if  $\nu \in [E[\lambda], \nu')$ . When B is strategic, however, S runs a substantially increased risk that B will reject any offer  $p_1 > \nu$ . Specifically, for any informative offer  $p_1 \in (\nu, \bar{p})$ , Proposition 3 shows that the information rent S must yield to B dominates the value of the information obtained, and S, therefore, opts not to experiment. Because S learns nothing by setting  $p_1 = \nu$  in the Good PBE, she also sets  $p_2 = \nu$  when  $\nu \geq E[\lambda]$ . This, of course, maximizes welfare because B buys the good in both periods with probability one.

**Proposition 4** (Optimal prices when  $\nu$  is low) *Suppose  $\nu < E[\lambda]$ . Then in the Good PBE:*

- (i) *If  $\nu < \nu''$ , S offers  $p_1 = p_2 = 1$ .*
- (ii) *If  $\nu \in [\nu'', E[\lambda])$ , S offers  $p_1 = \nu$  and  $p_2 = 1$ , where  $\nu''$  is implicitly defined by*

$$I(\rho^{-1}(\nu'')) = \nu''.$$

The story behind this result is similar to the previous one. The information rent outweighs the value of the information obtained, and S, therefore, prefers prices that yield no valuable information. In the Good PBE, there are two potentially optimal prices for which this is true,  $p_1 = \nu$  and  $p_1 = 1$ . Propositions 3 and 4 indicate that

S always sets one of these prices, and hence, she never obtains valuable information about B’s demand parameter  $\lambda$  from his first-period purchasing decision.

Proposition 4 indicates that if  $v < E[\lambda]$ , then S always sets  $p_2 = 1$  on the equilibrium path. The fact that she is not committed to do this, however, causes serious erosion in her first-period expected profit. If S could commit to offer  $p_2 = 1$ , then B would not strategically reject offers of  $p_1 = 1$ . This would raise S’s profit whenever  $v < E[\lambda]$  and would raise total welfare when  $v < v''$ .

Finally, while it seems plausible that S might be able to commit not to raise prices, a commitment not to lower them is not renegotiation proof. Note, however, that it is the commitment not to lower prices that has strategic value to S when  $v < E[\lambda]$ . In particular, if S commits to  $p_2 = 1$  and B rejects  $p_1 = 1$ , then S regrets her commitment to a high second-period price (and so does B).

### 6 The bad equilibrium

In this section, the Bad PBE described in Lemma 5 is investigated. While it is not possible to provide a full characterization for this equilibrium, some general results are obtained. Also, two examples are presented that illustrate a variety of first-period equilibrium pricing behavior. Example 1 shows that—unlike in the Good PBE—S may offer a first-period price that reveals valuable information about B’s demand parameter. Example 2, however, shows that this is not necessarily the case and that S may even elect to set the uninformative first-period price  $p_1 = \underline{p}$  which is below  $v$ , B’s lowest possible valuation for the good.

To ease notation, for any  $\hat{\mu} \in [0, 1]$  define

$$\hat{I}(\hat{\mu}) \equiv 1 - \int_{\hat{\mu}}^1 (1 - \lambda)f(\lambda) d\lambda.$$

For first-period prices  $p_1 > v$ , B’s purchasing behavior and S’s expected profit are the same in the Good and the Bad PBE. Suppose S is considering offering  $p_1 \in (\underline{p}, v]$ . The information revealed by B’s purchasing decision over this range of prices is valuable to S in the Bad PBE. Specifically, in the second period, S optimally sets  $p_2 = 1$  following acceptance and  $p_2 = v$  following rejection because

$$E[\lambda|p_1, 1] = \hat{\alpha}(\hat{\mu}(p_1)) > v > \hat{\rho}(\hat{\mu}(p_1)) = E[\lambda|p_1, 0].$$

Given this, S’s continuation payoff from offering  $p_1 \in (\underline{p}, v]$  is

$$\Pi_S(p_1) = \hat{I}(\hat{\mu}(p_1))(p_1 + \hat{\alpha}(\hat{\mu}(p_1))) + (1 - \hat{I}(\hat{\mu}(p_1)))v.$$

Finally, suppose S is considering offering  $p_1 \leq \underline{p}$ . Because B always accepts such an offer, S’s continuation payoff is

$$\Pi_S(p_1) = p_1 + \max\{E[\lambda], v\},$$

which is obviously maximized at  $p_1 = \underline{p}$ .

Summarizing the above, in the Bad PBE, S charges some  $p_1 \in \{\underline{p}, 1\} \cup (\underline{p}, \bar{p})$ , depending on the primitives of the model  $v$  and  $F(\lambda)$ . Before exploring  $S$ 's first-period pricing decision any further, it is worth establishing that the Bad PBE is indeed bad from  $S$ 's perspective.

**Proposition 5** (The Bad PBE vs. the Good PBE) *The expected payoff to S is (weakly) lower in the Bad PBE than in the Good PBE.*

This result is easily understood. For  $p_1 > v$ ,  $S$ 's continuation payoff  $\Pi_S(p_1)$  is the same in the Bad PBE and in the Good one. Also,  $S$  does strictly worse by offering  $p_1 = \underline{p}$  in the Bad PBE than by offering  $p_1 = v$  in the Good one because  $\underline{p} < v$  and  $B$  purchases with probability one in both cases. The only remaining question is whether  $S$  can do better by charging an informative price  $p_1 \in (\underline{p}, v]$  in the Bad PBE than by charging the uninformative price  $p_1 = v$  in the Good one. It turns out that the value of the information obtained by charging  $p_1 \in (\underline{p}, v]$  in the Bad PBE never makes up for the lower probability of sale, and hence,  $S$  does strictly worse at these prices.

In fact, from  $S$ 's perspective, the Good PBE dominates not just the Bad PBE, but any other PBE as well. It is worth noting that if  $S$  is considered as a mechanism designer without the power to commit (but with the power to select the equilibrium she pleases), then the Good PBE outcome will be the optimal implementable one for her.

Propositions 3 and 4 of the previous section established that  $S$  never sets an informative first-period price in the Good PBE. The following example shows that this is not true for the Bad PBE.

*Example 1* (Maximal learning) Suppose  $F(\lambda) = 1 - \sqrt{1 - \lambda}$ . In this case  $E[\lambda] = 2/3$ ,  $\underline{E} = 2/5$ , and  $\bar{E} = 4/5$ . Consider  $v = 2/3$ . Straightforward algebra shows that  $S$ 's continuation payoff over the set of informative prices  $p_1 \in (\underline{p}, \bar{p})$  is maximized at  $p_1 = v$ . Moreover, setting  $p_1 = v$  dominates the uninformative prices  $p_1 = \underline{p}$  and  $p_1 = 1$ . Because  $B$  accepts  $p_1 = v$  if and only if  $v_1 = 1$ , maximal learning occurs on the equilibrium path.

The next proposition demonstrates that the Bad PBE may be either worse or better for  $B$  than the Good PBE depending on parameter values. In particular, the set of parameters for which  $S$  sets  $p_1 = 1$  is larger in the Bad PBE. On the other hand, there exist parameter values for which  $S$  finds it optimal to induce complete pooling by setting a price lower than  $v$ , something she would never do in the Good PBE.

**Proposition 6** (Prices in the Bad PBE) *Suppose the Bad PBE obtains, then:*

- (i) *There exists  $\epsilon > 0$  such that if  $v < v'' + \epsilon$ , then  $S$  offers  $p_1 = p_2 = 1$ .*
- (ii) *There exists  $\xi > 0$  such that if  $v > \bar{E} - \xi$ , then  $S$  offers  $p_1 = \underline{p}$  and  $p_2 = v$ .*

Recall from Proposition 4 that when  $v < v''$ ,  $S$  charges  $p_1 = 1$  in the Good PBE. This price also maximizes  $S$ 's continuation payoff in this region of the parameter space when the Bad PBE obtains. Indeed, because charging  $p_1 = v$  is strictly worse for  $S$  in the Bad PBE than in the Good PBE, the region over which it is optimal for her to set  $p_1 = 1$  is somewhat larger, i.e.,  $v < v'' + \epsilon$ . The most striking aspect of Proposition 6

is that if  $v$  is sufficiently high, then it is optimal for S to set  $p_1 = \underline{p}$ , which is always strictly less than B's valuation for the good.

Proposition 6 shows that for arbitrary prior beliefs  $F(\lambda)$  there exist values of  $v$  for which S sets an uninformative first-period price (either  $p_1 = 1$  or  $p_1 = \underline{p}$ ) in the Bad PBE. The following example demonstrates that for the uniform prior  $\bar{S}$  sets an uninformative first-period price for any value of  $v \in (\underline{E}, \bar{E})$ .

*Example 2 (No learning)* Suppose  $F(\lambda) = \lambda$ . In this case  $E[\lambda] = 1/2$ ,  $\underline{E} = 1/3$ , and  $\bar{E} = 2/3$ . Straightforward algebra shows that if  $v \geq 1/2$ , S's continuation payoff is maximized at  $p_1 = \underline{p}$ . If  $v < 1/2$ , there exists  $v'''$  such that S offers  $p_1 = 1$  if  $v \in (1/3, v''')$ , and she offers  $p_1 = \underline{p}$  if  $v \in [v''', 1/2)$ . Thus, learning does not occur on the equilibrium path for any value of  $v$ .

## 7 Conclusion

This paper investigated a new wrinkle in an old question. In particular, it built on the monopoly price-experimentation literature pioneered by Rothschild (1974) by considering the impact of strategic buyers on the ability of firms to learn about demand. When market demand is composed of many anonymous individuals or when each individual purchases the good only once, strategic considerations are not relevant. Increasingly, however, firms are able to use innovations in information technology to track customers and make them personalized offers that depend on their history of prior purchases. In such settings, consumers have incentives to manipulate the beliefs of sellers in order to induce them to offer low prices.

In the context of the model presented above, it was shown that a buyer may strategically reject a seller's first-period offer for one of two reasons. First, in order to conceal information (i.e., to pool), a high-valuation buyer may reject high prices that would never be accepted by a low-valuation buyer. Second, in order to reveal information (i.e., to signal), a low-valuation buyer may reject low prices that would always be accepted by a high-value buyer. Given this behavior, the seller often finds it optimal to post prices that generate no useful information. Indeed, in the Good equilibrium (where there is no signaling), it was shown that the seller never charges a first-period price that yields valuable information. Learning may occur, however, in the Bad equilibrium (where there is maximal signaling). Nevertheless, the scope for learning by the seller appears to be quite limited even in this case. Indeed, it was shown that in order to preempt signaling, in the Bad equilibrium the seller may actually set a first-period price strictly below the buyer's lowest possible valuation.

The model presented here is probably the simplest one in which learning by a monopolist in the context of a strategic buyer can be meaningfully addressed. It would, therefore, be interesting both theoretically and practically to explore the robustness of the findings presented here to other specifications such as longer time horizons or more sophisticated parameterizations of demand. While such generalizations will undoubtedly involve serious technical hurdles, the value of such research appears large and growing.

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## Appendix

*Proof of Proposition 1:*  $B_t$  accepts if and only if  $p_t \leq v_t$ . Given this, S would never charge  $p_t \neq v$  or 1.

*Case 1*  $v < \underline{E}$ . If S sets  $p_1 = 1$ , then the expected values of  $\lambda$  conditional on acceptance and rejection are  $\overline{E}$  and  $\underline{E}$ , respectively. In either case, S sets  $p_2 = 1$  by Lemma 1. If S sets  $p_1 = v$ , then the expected value of  $\lambda$  conditional on acceptance is  $E[\lambda]$ , and the expected value conditional on rejection is immaterial. Hence, S sets  $p_2 = 1$ . Offering  $p_1 = p_2 = 1$  yields higher payoff to S than offering  $p_1 = v$  and  $p_2 = 1$  because

$$2E[\lambda] > v + E[\lambda]$$

holds for  $v < \underline{E}$ .

*Case 2*  $v \in [\underline{E}, E[\lambda])$ . If S sets  $p_1 = 1$ , then she optimally sets  $p_2 = v$  following rejection and  $p_2 = 1$  following acceptance. If S sets  $p_1 = v$ , then she optimally sets  $p_2 = 1$ . Offering  $p_1 = 1$  and  $p_2 = q_1 + (1 - q_1)v$  yields higher payoff to S than offering  $p_1 = v$  and  $p_2 = 1$  because

$$E[\lambda](1 + \overline{E}) + (1 - E[\lambda])v > v + E[\lambda]$$

holds for  $v \in [\underline{E}, E[\lambda])$ .

*Case 3*  $v \in [E[\lambda], \overline{E})$ . If S sets  $p_1 = 1$ , then she optimally sets  $p_2 = v$  following rejection and  $p_2 = 1$  following acceptance. If S sets  $p_1 = v$ , then she optimally sets  $p_2 = v$ . Offering  $p_1 = 1$  and  $p_2 = q_1 + (1 - q_1)v$  yields higher payoff to S than offering  $p_1 = p_2 = v$  if and only if

$$E[\lambda](1 + \overline{E}) + (1 - E[\lambda])v > 2v,$$

or  $v < v'$ .

*Case 4*  $v \geq \overline{E}$ . If S sets  $p_1 = 1$ , then she optimally sets  $p_2 = v$  following rejection as well as acceptance. If S sets  $p_1 = v$ , then she optimally sets  $p_2 = v$ . Offering  $p_1 = p_2 = v$  yields higher payoff to S than offering  $p_1 = 1$  and  $p_2 = v$  because

$$2v > E[\lambda] + v$$

holds for  $v \geq \overline{E}$ . □

*Proof of Lemma 3:* The proof consists of three steps.

*Step 1.* By way of contradiction, suppose  $\theta(p_1, 1) = \theta(p_1, 0)$  and  $E[\lambda|p_1, 1] < E[\lambda|p_1, 0]$ . It follows from Lemma 2 that if  $p_1 > 1$ , then B always rejects the first-period offer.  $D_1(p_1) = 0$  in this case. Next, if  $p_1 \in (\nu, 1]$  then B accepts if and only if  $v_1 = 1$ . Hence,  $E[\lambda|p_1, 1] = \bar{E} > E[\lambda|p_1, 0] = \underline{E}$ , which contradicts the supposition. Finally, if  $p_1 \leq \nu$  then B always accepts the first-period offer.  $D_1(p_1) = 1$  in this case.

*Step 2.* By way of contradiction, suppose  $\theta(p_1, 1) > \theta(p_1, 0)$ . If  $p_1 > 1$ , then B accepts if and only if  $v_1 = 1$  and  $\lambda \geq \lambda'$ , where

$$\lambda' = \frac{p_1 - 1}{(\theta(p_1, 1) - \theta(p_1, 0))(1 - \nu)}.$$

(If calculated  $\lambda' \geq 1$ , then B always rejects the first-period offer,  $D_1(p_1) = 0$ .) Hence,  $E[\lambda|p_1, 1] \geq \bar{E} > E[\lambda] \geq E[\lambda|p_1, 0]$ . This along with Lemma 1 implies that  $\theta(p_1, 1) = 0 \leq \theta(p_1, 0)$  must hold, which contradicts the supposition.

Next, if  $p_1 \in (\nu, 1]$  then B rejects if and only if  $v_1 = \nu$  and  $\lambda < \lambda'$ , where

$$\lambda' = \frac{p_1 - \nu}{(\phi(p_1, 1) - \phi(p_1, 0))(1 - \nu)}.$$

(If calculated  $\lambda' \geq 1$ , then B accepts  $p_1$  if and only if  $v_1 = 1$ .) Hence,  $E[\lambda|p_1, 1] \geq E[\lambda] > \underline{E} \geq E[\lambda|p_1, 0]$ . This implies  $\theta(p_1, 1) \leq \theta(p_1, 0) = 1$ , which contradicts the supposition.

Finally, if  $p_1 \leq \nu$ , then B always accepts the first-period offer.  $D_1(p_1) = 1$  in this case.

*Step 3.* By way of contradiction, suppose  $\theta(p_1, 1) \neq \theta(p_1, 0)$  and  $E[\lambda|p_1, 1] < E[\lambda|p_1, 0]$ . This implies either  $E[\lambda|p_1, 1] < \nu \leq E[\lambda|p_1, 0]$  or  $E[\lambda|p_1, 1] \leq \nu < E[\lambda|p_1, 0]$ . Thus, it must be  $\theta(p_1, 1) > \theta(p_1, 0)$ , which cannot happen in equilibrium by step 2. □

*Proof of Lemma 4:* Each part is proven in turn.

- (i) Consider  $p_1 \in (\nu, \bar{p}]$ . Given B's strategy to accept  $p_1$  if and only if  $v_1 = 1$  and  $\lambda \leq \mu(p_1)$ , S's posterior beliefs are  $E[\lambda|p_1, 0] = \rho(\mu(p_1)) \leq \nu$  and  $E[\lambda|p_1, 1] = \alpha(\mu(p_1)) \geq \nu$ , with strict inequalities when  $p_1 \in (\nu, \bar{p})$ . It then follows from Lemma 1 that setting  $\theta(p_1, 0) = 1$  and  $\theta(p_1, 1) = 0$  is optimal. By Lemma 2, B with  $v_1 = 1$  accepts  $p_1$  if and only if

$$1 - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - \nu) = \lambda(1 - \nu),$$

or  $\lambda \leq \mu(p_1)$ .

- (ii) Consider  $p_1 \in (\bar{p}, 1]$  and suppose  $\nu > E[\lambda]$ . Given B's strategy to accept  $p_1$  if and only if  $v_1 = 1$  and  $\lambda \leq m$ , S's posterior beliefs are  $E[\lambda|p_1, 0] = \rho(m) < \nu$

and  $E[\lambda|p_1, 1] = \alpha(m) = v$ . Hence, setting  $\theta(p_1, 0) = 1$  and any  $\theta(p_1, 1) \in [0, 1]$  is optimal. Mixing probability  $\theta(p_1, 1)$  is calibrated to make B with  $v_1 = 1$  and  $\lambda = m$  indifferent between accepting and rejecting,

$$1 - p_1 = m(1 - \theta(p_1, 1))(1 - v).$$

Now suppose  $v < E[\lambda]$ . Given B's strategy to accept  $p_1$  if and only if  $v_1 = 1$  and  $\lambda \leq m$ , S's posterior beliefs are  $E[\lambda|p_1, 0] = \rho(m) = v$  and  $E[\lambda|p_1, 1] = \alpha(m) > v$ . Hence, setting  $\theta(p_1, 1) = 0$  and any  $\theta(p_1, 0) \in [0, 1]$  is optimal. Mixing probability  $\theta(p_1, 0)$  is calibrated to make B with  $v_1 = 1$  and  $\lambda = m$  indifferent between accepting and rejecting,

$$1 - p_1 = m\theta(p_1, 0)(1 - v).$$

□

*Proof of Lemma 5:* First, consider the Good PBE. Given B's strategy to accept  $p_1 \leq v$ , S's beliefs remain unchanged,  $E[\lambda|p_1, 1] = E[\lambda]$  and  $E[\lambda|p_1, 0] = E[\lambda]$  (immaterial). It then follows from Lemma 1 that setting  $\theta(p_1, 1) = \theta(p_1, 0) = 0$  if  $v < E[\lambda]$  and  $\theta(p_1, 1) = \theta(p_1, 0) = 1$  if  $v > E[\lambda]$  is optimal. By Lemma 2, B with  $v_1 = v$  has no incentives to reject  $p_1$  as

$$v - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - v) = 0$$

holds for all  $\lambda \in [0, 1]$ .

Second, consider the Bad PBE.

- (i) Suppose  $p_1 \in (p, v]$ . Given B's strategy to reject  $p_1$  if and only if  $v_1 = v$  and  $\lambda > \hat{\mu}(p_1)$ , S's beliefs are  $E[\lambda|p_1, 1] = \hat{\alpha}(\hat{\mu}(p_1)) > v$  and  $E[\lambda|p_1, 0] = \hat{\rho}(\hat{\mu}(p_1)) < v$ . Hence, setting  $\theta(p_1, 1) = 0$  and  $\theta(p_1, 0) = 1$  is optimal. B with  $v_1 = v$  accepts  $p_1$  if and only if

$$v - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - v) = \lambda(1 - v),$$

or  $\lambda \leq \hat{\mu}(p_1)$ .

- (ii) Suppose  $p_1 \leq p$ . Signaling is not possible in this case, all types of B must accept  $p_1$ . S's beliefs remain unchanged, she optimally sets  $\theta(p_1, 1) = \theta(p_1, 0) = 0$  if  $E[\lambda] > v$  and  $\theta(p_1, 1) = \theta(p_1, 0) = 1$  if  $E[\lambda] < v$ . Obviously, B with  $v_1 = v$  has no incentives to reject  $p_1$ . □

*Proof of Proposition 3:* Offering  $p_1 = v$  dominates  $p_1 = 1$ . Indeed,

$$2v > I(m) + v,$$

which follows directly from  $v \geq E[\lambda] > I(m)$ .

It is left to show that  $p_1 = v$  dominates all  $p_1 \in (v, \bar{p}]$ . That is, for all  $\mu \in [m, 1)$

$$2v > I(\mu)(1 - \mu(1 - v) + \alpha(\mu)) + (1 - I(\mu))v,$$

or

$$(v - I(\mu)(1 - v(1 - \mu))) + (I(\mu)\mu - I(\mu)\alpha(\mu)) > 0.$$

The first term of this expression is positive. The second term is also positive because

$$I(\mu)\mu - I(\mu)\alpha(\mu) = \int_0^\mu (\mu - \lambda)\lambda f(\lambda) d\lambda > 0$$

for all  $\mu \in [m, 1)$ . □

*Proof of Proposition 4:* Offering  $p_1 = v$  dominates all  $p_1 \in (v, \bar{p}]$ . Indeed, for all  $\mu \in [m, 1)$

$$v + E[\lambda] > I(\mu)(1 - \mu(1 - v) + \alpha(\mu)) + (1 - I(\mu))v,$$

or

$$(E[\lambda] - I(\mu)) + I(\mu)v(1 - \mu) + (I(\mu)\mu - I(\mu)\alpha(\mu)) > 0.$$

The first two terms are non-negative, the third term is positive.

It is left to compare S's continuation payoffs from charging  $p_1 = 1$  and  $p_1 = v$ ,

$$\Pi_S(1) - \Pi_S(v) = I(m) + E[\lambda] - (v + E[\lambda]) = I(\rho^{-1}(v)) - v.$$

Substituting  $v = \underline{E}$  into the above expression yields  $E[\lambda] - \underline{E} > 0$ . Therefore, when  $v$  is close to  $\underline{E}$ , S optimally offers  $p_1 = 1$ . Substituting  $v = E[\lambda]$ , on the other hand, yields  $I(m_{\min}) - E[\lambda] < 0$ . Therefore, when  $v$  is close to  $E[\lambda]$ , S optimally offers  $p_1 = v$ . Because  $I(\rho^{-1}(v)) - v$  is a decreasing function of  $v$ , there is a cut-off value  $v''$  such that S's continuation payoff is higher under  $p_1 = 1$  if  $v < v''$ , and it is higher under  $p_1 = v$  if  $v \in (v'', E[\lambda])$ . □

*Proof of Proposition 5:* Suppose that in the Bad PBE, S offers  $p_1 > v$ , then she can get the same payoff in the Good PBE by adopting the same strategy. Hence, consider  $p_1 \in \{\underline{p}\} \cup (\underline{p}, v]$ .

*Case 1*  $v \geq E[\lambda]$ . S's expected payoff in the Good PBE is  $2v$ . First, suppose S offers  $p_1 = \underline{p}$  in the Bad PBE, then her expected payoff is  $\underline{p} + v < 2v$ .

Next, suppose S offers  $p_1 \in (\underline{p}, v]$ . It can be shown that

$$\widehat{I}(\widehat{\mu})(v - \widehat{\mu}(1 - v) + \widehat{\alpha}(\widehat{\mu})) + (1 - \widehat{I}(\widehat{\mu}))v < 2v,$$

for all  $\widehat{\mu} \in [0, \widehat{m})$ . Indeed, the inequality can be rewritten as

$$\widehat{I}(\widehat{\mu})\widehat{\mu}(1 - v) + (v - E[\lambda]) + \int_{\widehat{\mu}}^1 \lambda(1 - \lambda)f(\lambda) d\lambda > 0.$$

The first two terms are non-negative, the third term is positive.

Case 2  $v < E[\lambda]$ . S's expected payoff in the Good PBE is at least  $v + E[\lambda]$ . First, suppose S offers  $p_1 = \underline{p}$  in the Bad PBE, then her expected payoff is  $\underline{p} + E[\lambda] < v + E[\lambda]$ .

Next, suppose S offers  $p_1 \in (\underline{p}, v]$ . It can be shown that

$$\widehat{I}(\widehat{\mu})(v - \widehat{\mu}(1 - v) + \widehat{\alpha}(\widehat{\mu})) + (1 - \widehat{I}(\widehat{\mu}))v < v + E[\lambda],$$

for all  $\widehat{\mu} \in [0, \widehat{m})$ . Indeed, the inequality can be rewritten as

$$\widehat{I}(\widehat{\mu})\widehat{\mu}(1 - v) + \int_{\widehat{\mu}}^1 \lambda(1 - \lambda)f(\lambda) d\lambda > 0.$$

The first term is positive, the second term is non-negative. □

*Proof of Proposition 6:* Each part is proven in turn.

- (i) Combined with Proposition 4, Proposition 5 implies that if  $v \leq v''$ , then offering  $p_1 = 1$  strictly dominates  $p_1 = \underline{p}$  and all  $p_1 \in (\underline{p}, \overline{p})$ . Thus, there exists  $\epsilon > 0$  such that when  $v < v'' + \epsilon$ , offering  $p_1 = 1$  is optimal in the Bad PBE.
- (ii) Suppose  $v > E[\lambda]$ . Offering  $p_1 = \underline{p}$  dominates all  $p_1 \in (\underline{p}, v]$  if

$$2v > \widehat{I}(\widehat{\mu})(v - \widehat{\mu}(1 - v) + \widehat{\alpha}(\widehat{\mu})) + (1 - \widehat{I}(\widehat{\mu}))v$$

for all  $\widehat{\mu} \in [0, \widehat{m})$ . Taking the limit as  $v$  goes to  $\overline{E}$  yields  $\overline{E} > E[\lambda]\overline{E}$ . Next, offering  $p_1 = \underline{p}$  dominates all  $p_1 \in (v, \overline{p}]$  if

$$2v > I(\mu)(1 - \mu(1 - v) + \alpha(\mu)) + (1 - I(\mu))v$$

for all  $\mu \in [m, 1)$ . Taking the limit as  $v$  goes to  $\overline{E}$  yields  $\overline{E} > E[\lambda]\overline{E}$ . Finally, offering  $p_1 = \underline{p}$  dominates  $p_1 = 1$  if

$$2v > I(\mu) + v.$$

Taking the limit as  $v$  goes to  $\overline{E}$  yields  $\overline{E} > E[\lambda]$ . Therefore, there exists  $\xi > 0$  such that if  $v > \overline{E} - \xi$ , then offering  $p_1 = \underline{p}$  is optimal in the Bad PBE. □

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