Finite Difference Approximation

1.) Centered:

\[
\begin{align*}
\frac{\partial Q}{\partial x} & \approx \frac{Q_a - Q_b}{2\Delta x} \\
\frac{\partial Q}{\partial y} & \approx \frac{Q_c - Q_d}{2\Delta y} \\
\frac{\partial^2 Q}{\partial x^2} & \approx \frac{Q_a + Q_b - Q_0}{\Delta x^2} \\
|\nabla Q| & = \frac{\partial Q}{\partial n} \approx \frac{Q_a - Q_b}{2\Delta n} \quad \text{(natural coords)} \\
\nabla Q & = \hat{n} \frac{\partial Q}{\partial n}
\end{align*}
\]
2.) Forward:

\[ \frac{\partial Q}{\partial x} \approx \frac{Q_a - Q_o}{\Delta x} \]
3.) Backward:

\[ \frac{\partial Q}{\partial x} \approx \frac{Q_o - Q_b}{\Delta x} \]
Finite Difference Approximation

\[ \frac{\partial Q}{\partial x} \approx \frac{Q_x = +1 - Q_x = -1}{2\Delta x} \]

* Taylor series expansion for \( f(x) \) at \( x=a \)

\[ f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^n(a)}{n!}(x-a)^n \]

\[ f' = \frac{\partial f}{\partial x} \]

\[ f'' = \frac{\partial^2 f}{\partial x^2} \]

finite approximation only got out this far
Objective Analysis

Objective Analysis:
- A method of analyzing data with a min of human input (and error)

Means of fitting irregularly spaced data to an evenly spaced grid
- Useful for calculating derived quantities

Method of successive corrections:
- Grid point type (among others)
  - Cressman
  - Barnes
Objective Analysis

Given:

- First guess at all GRID POINTS, \( x \)
- Observations, \( \bigcirc \)
- Observations w/in scan radius \( R=1 \), \( \bullet \) = A, B, C, D
Objective Analysis

Task:

1. Get an analyzed value at grid point, g using obs w/in scan radius, R.
   - First guess at grid points

Solution:

1. Measure distances dA, dB, dC, dD
2. Assign weight to each station in the scan radius (closer stations \( \alpha \) more weight)
   - Weight of A = \( W_A = R^2 - dA^2/R^2 + dA^2 \)
   - (same for \( W_B \), \( W_C \), and \( W_D \))
Objective Analysis

Solutions Cont.

3. Determine a correction, C, for ob A

CA = T_A (observed @ A) – T_A (calculated @ A), which is found by using surrounding grid points:

\[ T_{AC} = \frac{\alpha + \beta + \gamma + \Delta}{4} \]

where \( \alpha, \beta, \gamma, \Delta \) are first guess temp (in this case) values.
Objective Analysis

Solutions Cont.

4. Adjust temp at g based on A
   - $A_gA = W_A \times C_A$
   - $A_gB = W_B \times C_B$

5. Final adjustment: $C_g = (A_gA + A_gB)/(W_A + W_B)$

6. Final value: $T_{g\text{ new}} = T_{g\text{ guess}} + C_g$