**Introduction**

In order to understand many atmospheric processes, we need to know how the temperature and water vapor content of air changes as it is transported horizontally and vertically. What will today’s maximum temperature be? Will it rain? Do we expect thunderstorms? When can we expect air pollution situations? (These events occur with opposite conditions.) More generally, how can we judge atmospheric stability?

As air ascends or descends, its temperature and humidity change greatly. An understanding of many atmospheric processes depends upon accounting for these changes and knowing why they occur.

The theory behind this understanding is greatly simplified by the use of graphs called “thermodynamic diagrams.” A basic understanding of simple physical laws allows us to understand above processes. We call this area of physics “thermodynamics.” Many meteorological processes can be **simulated** by simply calculations on thermodynamic diagrams.
I. Meteorological Thermodynamics Diagrams

A. Background

1. Coordinates – Isolines – Process Curves

Consider this p vs. T diagram:

A single point on a p-T diagram uniquely describes the thermodynamic variables $p, \alpha, T$ of a sample of air.

2. Moisture Content Expression

a. Saturation or Equilibrium Vapor Press ($e_s$)

Since $e_s = e_s(T)$, a particular value of $T$ or isotherm determines a unique value of $e_s$ and isotherms are isolines of saturation-equilibrium vapor pressure.

For unsaturated air, $e_{\text{actual}} < e_s$, so by definition we express the relative humidity (RH) as a fraction:

$$RH = \frac{e_{\text{act}}}{e_s}.$$  

So,
e_{act} = RH \times e_s.

b. Dew Point Temperature ($T_d$)

The dew point is the temperature to which air must be cooled isobarically to produce a saturated condition (i.e. the temperature for which the actual vapor pressure is a saturation vapor pressure.

Thus two points on a p-T diagram:

i. at $p_i$ & $T_i$ yield $p_i$, $T_i$, $\alpha_i$, $\rho_i$, $e_{ai}$

ii. at $p_i$ & $T_{di}$ yield $p_i$, $T_{di}$, $e_{ai}$

c. Mixing Ratio ($w$), saturation mixing ratio ($w_s$)

$$w_s = \frac{water - vapor - mass}{unit - mass - of - dry - air}$$ (saturated sample)

Units are non dimensional: $\frac{gm}{gm}$, $\frac{kg}{kg}$ or $\frac{gm}{kg}$
Although $e_s$ is strictly a function of $T$, $w_s$ is a function of $T$ and $p$ and linearly dependent on the latter (see derivations in Wallace and Hobbs, p.71; Bluestein, p.206):

$$w_s \approx \frac{622}{p} e_s$$

Note: A particular value of $p_i$, $T_i \rightarrow$ unique value of $e_s \rightarrow$ a unique value of $w_s$, but isolines of $w_s$ slope

Isolines of $w_s$ passing thru $T_d$ yields actual mixing ratio, $w$ or $w_{act}$. So,

$$\text{RH} \approx \frac{\omega_{act}}{\omega_s}$$

Most meteorological thermodynamic diagrams have isopleths of $w$ labeled in units of

$$(\text{grams of H}_2\text{O}) / (\text{kilograms of dry air})$$

Thus, $w_s$ is a function of the actual temperature, whereas $w$ is a function of the dew point temperature.
3. Unsaturated or Dry Adiabatic Process Curves and Potential Temperature

a. Unsaturated or Dry Adiabatic Processes

From the adiabatic form of the First Law of Thermodynamics, we know:

$$\delta Q = 0 = c_p dT - \alpha dp$$

Using the Equation of State

$$p \alpha = RT$$

we substitute to get

$$c_p dT - \frac{RT}{p} dp = 0$$

or

$$\frac{dT}{T} = \frac{R}{c_p} \frac{dp}{p}$$

Integrating the last expression, we get:

$$\int_{T_i}^{T} d(\ln T) = R/c_p \int_{p_i}^{p} d(\ln pP)$$

or

$$\ln\left(\frac{T}{T_i}\right) = \frac{R}{c_p} \ln\left(\frac{P}{P_i}\right)$$

$$\ln\left(\frac{T}{T_i}\right) = \ln\left(\frac{P}{P_i}\right)^{\frac{R}{c_p}}$$

and

$$T = T_i \left(\frac{p}{p_i}\right)^{\frac{R}{c_p}}$$

where \(\frac{R}{c_p} \approx \frac{2}{7} = .286\)

which holds during “dry” or unsaturated adiabatic process.
Isoline describing such a process or unsaturated adiabatic process curve is linear on a diagram with $p^{\frac{R}{cp}}$ as a coordinate and goes towards decreasing $T$ with decreasing $p$.

![Diagram showing solid contours as unsaturated adiabatic process curves](image)

**Note:** During adiabatic expansion $\rightarrow$ pressure decreases and temperature falls (and vice versa for adiabatic compression) at a predictable rate determined by the First Law of Thermodynamics. Moisture content is invariant ($w_{\text{actual}}$ is const), but RH and $T_d$ will vary.

b. Potential Temperature

From the relationship for $p$ and $T$ during an unsaturated adiabatic process

$$T = T_i \left( \frac{p}{p_i} \right)^{\frac{R}{cp}}$$

let the final pressure $p = p_0 \equiv 1000\text{mb}$ at which time the temperature $T \equiv 0$

$$\theta = T_i \left( \frac{p_0}{p_i} \right)^{\frac{R}{cp}}$$
OR

\[ \theta = T_i \left( \frac{1000}{p_i} \right)^{\frac{R}{c_p}} \]

which is the expression for the potential temperature. **Potential temperature is defined as the temperature that air at any initial temperature,** \( T_i \), **and pressure,** \( p_i \), **would have after unsaturated adiabatic compression to** \( p = 1000 \text{ mb} \).

Note: Final temperature at 1000mb (\( \theta \)) is uniquely determined at \((T_i, p_i)\), so for each point on the diagram, a unique value of \( \theta_i \) exists. During unsaturated adiabatic processes, the value of \( \theta \) is invariant, or conserved, thus unsaturated adiabatic process curves are also isolines of \( \theta \) and are also called “isentropes” (convention is to label in units of Kelvins, abbreviated K, not °K).
4. Saturated or pseudoadiabatic process curves and equivalent potential temperature

a. The Saturated Adiabatic Process Curve

If unsaturated air is expanded towards decreasing pressure, its temperature decreases (which means its saturation mixing ratio will also decreases) until finally $w_{\text{sat}}$ decreases to $w_{\text{act}}$ and the air is saturated. The air is now at its adiabatic condensation level (ACL) or lifting condensation level (LCL). Any additional press decrease lowers $\omega_k$ below $\omega_{ac}$ and requires condensation of excess water vapor into liquid. Condensation of each gram of H$_2$O releases $\sim 600$ cal of latent heat of condensation to the air, which will not continue to decrease its temperature with decreasing pressure at the unsaturated adiabatic rate, but at a lesser saturated adiabatic process rate. The saturated adiabatic process rate will depart from the unsaturated in proportion to how fast $\omega_k$ is decreasing.

Since every point on the diagram determines a unique value of T, p, $\omega_k$, and subsequent expansion reduces $\omega_k$ at a known rate (if $\omega_k$ is completely determined by T & p) there can be constructed through all points on the diagram a process curve governing saturated adiabatic process.
This can be obtained numerically by integration of the saturated form of the First Law:

\[-L \, d\omega_s \cong C_p \, dT - \alpha \, dp\]

Note: Once condensation and latent heat release begin, \( \theta \) is no longer constant or conserved but increases in proportion to latent heat released (large change at high temperature and pressure – small changes at low temp; small \( \omega_s \)).

As \( T \to 0 \) \( \omega \to 0 \) and \( \left( \frac{dT}{dp} \right)_{sat} \to \frac{dT}{dp} \).

b. The Pseudo-equivalent Temperature (\( T_e \))\(^1\)

The pseudo-equivalent temperature is that temperature a parcel of air would have if, starting at some initial value of \( T_i \), \( p_i \) & \( Tdi \) or \( \omega_{acti} \), it is expanded in an unsaturated adiabatic process to its ACL or LCL and then, by a saturated adiabatic process, to \( p = 0 \) or until all its water vapor has condensed out and latent heat has been released, followed lastly by subsequent unsaturated adiabatic \textit{compression} to its original pressure (\( p_i \)).

\(^1\) Modern parlance usually refers to this variable as simply the ‘equivalent temperature.’
Note: Liquid condensation falls out is lost. So this is “pseudo-adiabatic” process.

\[ T = \theta_e = 1000 \omega_i T_i \]

Notes:
1. At low temperatures and pressures, the saturated adiabatic process curve → unsat
2. The temperature of the parcel has simply been increasing by releasing latent heat in water vapor content and converting it to sensible heat
3. This release could be visualized as happening isobarically to give an isobaric equivalent temperature, \( T_{ie} \)

\[ T_{ie} = T_i + L \omega \]

\( c. \) Pseudo-equivalent Potential Temperature \((\theta_e)^2\)

If the dry adiabatic compression mentioned above is continued to \( p = 1000 \text{mb} \), then the temperature of the parcel at that pressure is known as its pseudo-equivalent potential temperature.

\( ^2 \) Modern parlance usually refers to this variable as simply the ‘equivalent potential temperature.’
Notes: 1. Any particular saturated adiabatic process curve is associated with a unique final value of $\theta_e$, so the saturated adiabatic process curves are also isolines of $\theta_e$ and may be so labeled.

2. The particular saturated adiabatic process curve upon which the parcel becomes saturated and follows is uniquely determined by its initial $p$, $T$ and $\omega_{act}$, or $T$ and $T_d$, so that during the entire process its $\theta_e$ is invariant or conserved.

d. Adiabatic Wet Bulb Temperature ($T_{wb}$)

If a parcel of air is lifted and then returned to its original pressure by a saturated adiabatic process its final temp is its adiabatic wet bulb temp. For all practical purposes $T_{wb} = $ Isobaric Wet Bulb Temperature.

e. Wet Bulb Potential Temperature ($\theta_{wb}$)

If a compression process along a saturated adiabatic process curve is continued to $p = 1000$mb, the final temperature is the wet bulb potential temperature.
Note: 1. A particular saturated adiabatic process curve is associated with a unique $\theta_{wb}$ . . . saturated adiabatic process curves are isolines of $\theta_{wb}$ and may be so labeled.
2. $\theta_{wb}$ is conserved during saturated and unsaturated processes.

5. Graphical Determination of $e_s$

Since $w_s \cong \frac{622}{p} e_s$

If $p = 622\text{mb}$, then

$$w_s \left(\frac{g}{g}\right) = 0.001 \times e_s (\text{mb})$$

$$w_s \left(\frac{g}{kg}\right) = e_s (\text{mb})$$

During isothermal process $e_s$ is constant, so graphical movement of air isothermally to $622\text{mb}$ will give its $e_s$, since at that pressure, $w_s$ in $g \text{ kg}^{-1} = e_s (\text{mb})$

6. Virtual Temperature

The virtual temperature is the temperature that dry air would have if its pressure and density were equal to those of a given sample of moist air.

We begin with the equation of state

$$p = \rho RT$$
For dry air,

\[ p = \rho_d R_d T_d \]

For moist air,

\[ p = \rho_m R_m T_m \]

Now let

\[ R_m = R_d (1 + 0.61 q) \approx R_d (1 + 0.61 w) \]

and

\[ p = \rho_m R_d (1 + 0.61 w) T \]

Also, let

\[ (1 + 0.61 w) T = T_v \]

\[ p = \rho_m R_d T_v \]

To a fair approximation, we may thus form

\[ T_v = T + \frac{1}{6} w \left( \frac{g}{kgm} \right) \]

or

\[ T_v = T + \frac{w}{6} \]

Where \( w \) is expressed in \( g \ kg^{-1} \).
7. Summary: Graphical Determination of Moisture Variables

Shaded circles with bold margins are the values \((T_i, T_{wb}, T_d)\) with which we begin. Hollow, thin circles represent the derived variables \((T_e, \theta, \theta_w, \theta_e)\).
8. The Conservatism And Non-Conservatism of Thermodynamic Variables

Conservative With Respect To…

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unsaturated Adiabatic Process</th>
<th>Saturated Adiabatic Process</th>
<th>Evaporation From Falling Rain</th>
<th>Diabatic Heating Radiation-Conduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (T)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Dew Pt (T_d)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sat Mixing Ratio</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Act Mixing ratio</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Rel Humidity</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Wet Bulb (T_wb)</td>
<td>No</td>
<td>No</td>
<td>Yes (if $p = \text{const}$)</td>
<td>No</td>
</tr>
<tr>
<td>Equiv Temp (T_e)</td>
<td>No</td>
<td>No</td>
<td>Yes (if $p = \text{const}$)</td>
<td>No</td>
</tr>
<tr>
<td>Virtual Temp (T_v)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pot. Temp ($\theta$)</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pseudo Equiv Pot. Temp. ($\theta_e$)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes (Quasi)</td>
<td>No</td>
</tr>
<tr>
<td>Wet Bulb Pot. Temp ($\theta_{wb}$)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes (Quasi)</td>
<td>No</td>
</tr>
</tbody>
</table>

9. Plotting & Analysis of Atmospheric Soundings

A. The Standard Atmosphere (Non-typical)

<table>
<thead>
<tr>
<th>Pressure (mb)</th>
<th>Height (meters)</th>
<th>Height (km, approx.)</th>
<th>Height (feet)</th>
<th>Height (kfeet, approx.)</th>
<th>Temp (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1013</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>111</td>
<td>364</td>
<td></td>
<td></td>
<td>14.3</td>
</tr>
<tr>
<td>900</td>
<td>988</td>
<td>1.0</td>
<td>3243</td>
<td></td>
<td>9.6</td>
</tr>
<tr>
<td>850</td>
<td>1475</td>
<td>1.5</td>
<td>4781</td>
<td>5000</td>
<td>5.6</td>
</tr>
<tr>
<td>800</td>
<td>1949</td>
<td>6394</td>
<td></td>
<td></td>
<td>2.3</td>
</tr>
<tr>
<td>700</td>
<td>3012</td>
<td>3.0</td>
<td>9882</td>
<td>10000</td>
<td>-4.6</td>
</tr>
<tr>
<td>600</td>
<td>4206</td>
<td>13801</td>
<td></td>
<td></td>
<td>-12.4</td>
</tr>
<tr>
<td>500</td>
<td>5574</td>
<td>5.5</td>
<td>18289</td>
<td>18000</td>
<td>-21.2</td>
</tr>
<tr>
<td>400</td>
<td>7185</td>
<td>23574</td>
<td></td>
<td></td>
<td>-31.7</td>
</tr>
<tr>
<td>300</td>
<td>9164</td>
<td>9.0</td>
<td>30065</td>
<td>30000</td>
<td>-44.6</td>
</tr>
<tr>
<td>250</td>
<td>10363</td>
<td>33999</td>
<td></td>
<td></td>
<td>-52.5</td>
</tr>
<tr>
<td>225 (tropo.)</td>
<td>11000</td>
<td>11.0</td>
<td>36000</td>
<td>36000</td>
<td>-56.6</td>
</tr>
<tr>
<td>200</td>
<td>11784</td>
<td>38662</td>
<td></td>
<td></td>
<td>-56.6</td>
</tr>
<tr>
<td>150</td>
<td>13608</td>
<td>44647</td>
<td>45000</td>
<td></td>
<td>-56.6</td>
</tr>
<tr>
<td>100</td>
<td>16180</td>
<td>16.0</td>
<td>53083</td>
<td></td>
<td>-56.6</td>
</tr>
<tr>
<td>75</td>
<td>18000</td>
<td>59040</td>
<td></td>
<td></td>
<td>-56.6</td>
</tr>
<tr>
<td>50</td>
<td>20575</td>
<td>67503</td>
<td>-56.0 approx.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>24971</td>
<td>25.0</td>
<td>81976</td>
<td>-53.0 approx</td>
<td></td>
</tr>
</tbody>
</table>
Notes: A few rules of thumb

1. For every increase of 16 km the pressure decreases by an order of magnitude.

<table>
<thead>
<tr>
<th>Km</th>
<th>P(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>~ 1000</td>
</tr>
<tr>
<td>16</td>
<td>~ 100</td>
</tr>
<tr>
<td>32</td>
<td>~ 10</td>
</tr>
<tr>
<td>48</td>
<td>~ 1</td>
</tr>
</tbody>
</table>

2. In the troposphere an increase of approximately 5 km halves the pressure.

<table>
<thead>
<tr>
<th>Km</th>
<th>P(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>~ 1000</td>
</tr>
<tr>
<td>5</td>
<td>~ 500</td>
</tr>
<tr>
<td>10</td>
<td>~ 250</td>
</tr>
<tr>
<td>15</td>
<td>~ 125</td>
</tr>
</tbody>
</table>

B. Typical Soundings

<table>
<thead>
<tr>
<th>p (mb)</th>
<th>T (°C)</th>
<th>T_d (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>30</td>
<td>21</td>
</tr>
<tr>
<td>950</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>900</td>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td>800</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>700</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>600</td>
<td>+1</td>
<td>-8</td>
</tr>
<tr>
<td>500</td>
<td>-9</td>
<td>-20</td>
</tr>
<tr>
<td>400</td>
<td>-20</td>
<td>-35</td>
</tr>
<tr>
<td>300</td>
<td>-32</td>
<td>-45</td>
</tr>
<tr>
<td>200</td>
<td>-48</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>-64</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-60</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p (mb)</th>
<th>T (°C)</th>
<th>T_d (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>-25 or (-12)</td>
<td>-25</td>
</tr>
<tr>
<td>950</td>
<td>-15</td>
<td>-18</td>
</tr>
<tr>
<td>900</td>
<td>-15</td>
<td>-19</td>
</tr>
<tr>
<td>800</td>
<td>-17</td>
<td>-20</td>
</tr>
<tr>
<td>700</td>
<td>-21</td>
<td>-24</td>
</tr>
<tr>
<td>600</td>
<td>-27</td>
<td>-31</td>
</tr>
<tr>
<td>500</td>
<td>-35</td>
<td>-42</td>
</tr>
<tr>
<td>400</td>
<td>-44</td>
<td>-50</td>
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<tr>
<td>300</td>
<td>-54</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-56</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-58</td>
<td></td>
</tr>
</tbody>
</table>
10. Comparison Of Meteorological Thermodynamic Diagrams

A. Criteria for judging usefulness of diagrams

1. How large is the angle between isotherms and unsaturated adiabatic process curves?
2. How many isolines or process curves are straight?
3. Is work proportional to area?

B. Work or energy on a thermodynamic diagram

\[
dW = p\,d\alpha
\]

\[
W = \int_{\alpha_1}^{\alpha_2} p\,d\alpha
\]

\[p = 0\]

\[p \downarrow\]

\[p_1\alpha_1 \quad p_2\alpha_2\]

\[\alpha\]

On a p, \(\alpha\) diagram, work is proportional to area over curve

Net work in a cyclic process

\[W = \int_{\alpha_1}^{\alpha_2} p\,d\alpha\]

\[W = \int_{\alpha_1}^{\alpha_2} p\,d\alpha + \int_{\alpha_1}^{\alpha_2} p\,d\alpha\]

\[W \propto A - B = C\]

Net work depends on path
C. Stüve Diagram

\[ T \text{ vs. } p^{R/c_p} \quad \text{(pseudo-adiabatic diagram)} \]

Criteria evaluation

1. \( \sim 45^0 \)
2. \( T, p, \theta, w_s \)
3. No, depends slightly on \( p \)
D1. Skew T–ln p Diagram

Criteria evaluation:  1.  $\sim 90^\circ$  2.  T, p;  $\theta$ are slightly curved  3. Yes

D2. Emagram (Energy/Unit Mass Diagram)   ln p vs. T

Criteria evaluation:  1.  $\sim 45^\circ$  2.  T, p;  $\theta$ are curved  3. Yes
E. Tephigram (T – ln θ Diagram)

Criteria evaluation: 1. ~ 90°  2. T, θ; p curved  3. Yes

E – 1 AROWAgaram (Tephigram With Skewed T)
Skew lnθ vs T

Criteria evaluation: 1. ~ 45°  2. θ, T; p curved  3. Yes
II. Analysis of Hydrostatic Stability of Soundings

A. Parcel Method

Vertical displacements of parcels of air with no interaction with the environment.

A region of the atmosphere is unstable if vertical displacement of a parcel from its initial position leads to a continuing acceleration away from that initial position.

1. Unsaturated Air

If the observed environment lapse rate, $\gamma_E$, is greater than the unsaturated adiabatic process lapse rate $\Gamma_d = \frac{g}{c_p}$, then the atmospheric region is unstable:

Notice that in each scenario, the adiabatic process curves (in brown) do not change. The plotted temperature lapse rate from the free atmosphere (in red) will change from observation to observation, however. In effect, we are testing each new data set against known behaviors for the atmosphere, in this case, one which is unsaturated. These known behaviors, for a specific pressure, temperature, and moisture content, allow the various process contours on a thermodynamic diagram to exist. They are our guide that allow us to test and explore each new set of upper air data (usually from a balloon flight) that we encounter.

In terms of potential temperature for unsaturated air, the following conditions
may exist

\[
\begin{align*}
\left( \frac{\partial \theta}{\partial z} \right) &< 0 \quad \text{unstable} \quad \gamma_E > \Gamma_d \\
\left( \frac{\partial \theta}{\partial z} \right) &= 0 \quad \text{neutral} \quad \gamma_E = \Gamma_d \\
\left( \frac{\partial \theta}{\partial z} \right) &> 0 \quad \text{stable} \quad \gamma_E < \Gamma_d
\end{align*}
\]

2. For saturated air

unstable \quad \gamma_{env} > \Gamma_{sat}

stable \quad \gamma_{env} < \Gamma_{sat}

3. Conditional instability

Note that if

\[ \Gamma_d > \gamma_{env} > \Gamma_{sat} \]

Then conditional instability is present --- the atmosphere will be stable if unsaturated, but unstable if saturated.

4. Level of Free Convection (LFC)

After a rising parcel has ascended to the level where it becomes saturated and moisture begins to condense (its LCL; defined previously), it may need to continue its ascent until arriving at a level where its temperature is warmer than the ambient environment. That level is known as the level of free convection (LFC).
5. Positive and Negative Energy Areas and Latent Instability

![Diagram illustrating positive and negative energy areas]

Positive Energy Area $\propto$ CAPE
(Parcel Warmer Than Environ)

$\Delta$ Accelerate Upward $\frac{dw}{dt} = g\left(\frac{T_p - T_e}{T_e}\right)$

Negative Area $\propto$ CIN
(Parcel Colder Than Environ)

Note: If the positive energy area below the equilibrium level exceeds the negative area, then we say that the sounding has latent instability.

a. Convective Available Potential Energy (CAPE)

This is the numerical expression for the positive energy area on a thermodynamic diagram. It is expressed as

$$CAPE = R_d \int_{p_{LFC}}^{p_e} \left[ T_p(p) - T_e(p) \right] d\ln p$$

- $p_e$ = pressure at equilibrium level
- $p_{LFC}$ = pressure at LFC
- $T_p(p)$ = temp. of parcel
- $T_e(p)$ = temp. of ambient environment

b. Convective Inhibition (CIN)

This is the numerical expression for the negative energy area on a thermodynamic diagram. It is expressed as

$$CIN = R_d \int_{p_o}^{p_{LFC}} \left[ T_p(p) - T_e(p) \right] d\ln p$$

- $p_o$ = surface pressure
6. Convective Condensation Level (CCL) and the Convective Temperature (CT)

A method for determining what amount of surface heating (due mainly to solar-diurnal heating cycle) is necessary to allow convective elements from the surface to produce convective clouds.

**Procedure:** From surface dew point proceed upward along the constant mixing ratio line until it intersects the temperature sounding (this is the CCL). For CT, proceed from CCL to the surface along an unsaturated adiabat. The temperature at the surface is the CT.

Thus, the CCL and the CT can be used to help forecast convection. Specifically, if a forecaster has already made her/his prediction for the maximum temperature for the day, and that forecast value falls below the CT, then convection is less likely.

An improved and physically more realistic CCL may be alternately determined as follows:

1. Graphically determine the *mean* mixing ratio of the lowest 50mb (1/2 km) layer.
2. Project this mean boundary layer mixing ratio value upward alone a constant mixing ratio line until it intersects the sounding
3. The intersection is the CCL
4. From the CCL proceed downward along an unsat adiabatic proc curve to the surface
5. The temp at the surface is the CCT
7. Hydrostatic Stability Indices

a. The Showalter Index (SI)

This may be thought of as an examination of the latent instability of the 850 to 500 mb layer. *Assume* the 850 mb air is representative of the low level air involved in deep convective overturning.

**Procedure:** The 850-mb parcel is lifted to its LCL and then on up to the 500-mb level. Its temperature at that level is then compared with the 500-mb environmental temperature.

\[
SI = T_{e500} - T_{p500} \quad \text{Whole degrees}
\]

If parcel is cooler than env. → stability → SI is positive
If parcel is warmer than env → unstable → SI is negative

The magnitude of SI is roughly proportional to the positive energy area. The intensity of vertical acceleration and motions are generally more intense the more negative the SI.

b. The Lifted Index (LI) (SELS – NSSFC)

Recognizing that the layer mean 50-mb parcel from a morning sounding may be highly non-representative of the low-level air actually involved in afternoon or late afternoon convection, an attempt is made to forecast the low-level afternoon conditions.

**Procedure:**
1. Find mean mixing ratio or lowest 50-mb layer
2. Forecast the mean $w$ and $\theta$ of the lowest 50-mb layer at the time of maximum temperature

3. Allow this low level parcel with ($\bar{w}$ and $\bar{\theta}$) to ascend to 500 mb

$$LI = T_{c500} - T_{p500}$$

If parcel is cooler than env. $\rightarrow$ stability $\rightarrow$ LI is positive
If parcel is warmer than env $\rightarrow$ unstable $\rightarrow$ LI is negative

**Note:** Advantages and Disadvantages with SI and LI

**SI:**

A1. Readily determined – Need only $T_{850}$, $T_{d850}$, $T_{E500}$

A2. Completely Objective - $T_{p500}$ uniquely determined by $T_{850}$ and $T_{d850}$ &

**D1.** 850-mb air may be non-representative of low level air, frequently just above the moist layer

**LI**

D1. Non-objective $\rightarrow$ requires some forecast skill or judgement

---

c. K Index

$$KI = (T_{850} - T_{500}) + [T_{d850} - (T_{700} - T_{d700})]$$

*Implies that 700 mb dryness detracts from air mass type thunderstorm probability*

<table>
<thead>
<tr>
<th>KI</th>
<th>Thunderstorm Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 15</td>
<td>0%</td>
</tr>
<tr>
<td>15 – 20</td>
<td>0 – 20%</td>
</tr>
<tr>
<td>21 – 25</td>
<td>20 - 40</td>
</tr>
<tr>
<td>26 – 30</td>
<td>40 – 60</td>
</tr>
<tr>
<td>31 – 35</td>
<td>60 – 80</td>
</tr>
<tr>
<td>30 – 40</td>
<td>80 – 90</td>
</tr>
<tr>
<td>&gt; 40</td>
<td>~100%</td>
</tr>
</tbody>
</table>
d. Total Totals (TT)

The Total Totals value is a combination of two other metrics

\[
\text{Vertical Total (VT)} = T_{850} - T_{500} \\
\text{Cross Total (CT)} = T_{d(850)} - T_{500}
\]

Total Totals (TT) \(\equiv (T_{850} + T_{d850} - 2T_{500})\)

<table>
<thead>
<tr>
<th>TT</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>Isolated to few thunderstorms approaching severe levels</td>
</tr>
<tr>
<td>48</td>
<td>Isolated severe storms</td>
</tr>
<tr>
<td>52</td>
<td>Scattered severe storms and a few tornado bearing thunderstorms</td>
</tr>
<tr>
<td>56</td>
<td>Numerous thunderstorms with scattered severe storms and tornadoes</td>
</tr>
</tbody>
</table>

e. USAF – AWS SWEAT Index

\[
I = 12 T_{d850} + 20(TT - 49) + 2 V_{850} + V_{500} + 125(S + 0.2)
\]

where:

\[
TT = \text{Total Totals} \\
S = \sin (500\text{-mb wind direction} - 850\text{-mb wind direction})
\]

Notes: 1. If \(T_{d850}\) is negative, then set the first term = 0.
2. If (TT - 49) is negative set second term = 0.