Cournot Duopoly

The industry consists of two firms who produce a homogenous product which sells at a uniform price. Denote firm outputs by $Q_1$ and $Q$, so that industry output is $Q = Q_1 + Q_2$. Assume that the duopoly face market demand given by

$$P(Q) = 120 - Q.$$ 

Costs are given by

$$TC_1 = 20Q_1 \quad & \quad TC_2 = 40Q_2, \quad MC_1 = 20 \quad & \quad MC_2 = 30.$$ 

Each firm maximizes it profit assuming that its rival’s output is fixed.

Firm 1: \hspace{1cm} \text{Max } \pi_1 = P(Q_1 + Q_2)Q_1 - TC_1(Q_1) = (120 - Q_1 - Q_2)Q_1 - 20Q_1

Firm 2: \hspace{1cm} \text{Max } \pi_2 = P(Q_1 + Q_2)Q_2 - TC_2(Q_2) = (120 - Q_1 - Q_2)Q_2 - 40Q_2

Profit-maximization requires that each firm equates its marginal revenue to its marginal cost.

$$MR_1 = MC_1 \quad \sim \quad 120 - 2Q_1 - Q_2 = 20 \hspace{1cm} (1.1)$$

$$MR_2 = MC_2 \quad \sim \quad 120 - Q_1 - 2Q_2 = 40 \hspace{1cm} (1.2)$$

Solving the pair of equations gives the profit-maximizing outputs

$$Q_1 = 40, \quad Q_2 = 20, \quad P = P(Q_1 + Q_2) = 60.$$ 

$$\pi_1 = 60(40) - 20(40) = 1600$$

$$\pi_2 = 60(20) - 20(20) = 800.$$ 

The first-order equations (1) can be written as

$$Q_1 = R_1(Q_2) = 50 - \frac{1}{2} Q_2 \hspace{1cm} (2.1)$$

$$Q_2 = R_2(Q_1) = 40 - \frac{1}{2} Q_1 \hspace{1cm} (2.2)$$
In this form, the first-order conditions define each firm’s best response function (BRF).

A firm BRF gives the profit-maximizing output of one firm for each quantity of output of the other firm. The Cournot equilibrium is at the intersection of the BRF’s.