A Positive Theory of Government Land Sales

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Abstract

We analyze the problem of disposing government land in an overlapping generations framework. Our main result is that while complete privatization of government land is desirable; it may be made politically infeasible by a compensation gap, when the losses of current property owners are greater than the government revenue from land sales. We argue that the cross-country diversity of government land ownership owes to historical incidents in some countries (such as the U.S. in the 19th century) that allowed disposal of government land without filling the compensation gap and the absence of such incidents in others (such as Hong Kong). JEL Codes: H00, R14

1 Introduction

The global abandoning of central planning economic system in the last two decades leaves many governments with the problem of how best to dispose large quantities of public lands to the private sector. Should a government release lands quickly to private hands and let the markets take over the development decisions? Or should a government sell lands slowly and remain in control of the real estate development? The former policy in fact is what the U.S. government chose to do in the 19th century with its newly acquired lands to the west of Mississippi. The latter policy however is what Hong Kong government adopted and still keeps in place today.

Conventional economic theory presents an efficiency argument for the US policy of quick land disposal, as the fundamental theorems of welfare economics would suggest that it is Pareto efficient to release a valuable resource, such as land, from the public hand to the private sector. However, economic history of Hong Kong does not present us with a similar clear cut conclusion, as the slow

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§7Cheung and Ni acknowledged a grant supported from Hong Kong Research Grant Council CUHK4128/03H.

land disposal policy of Hong Kong government does not seem to impede its development nor shaken its stability, even though Hong Kong is considered as one of the densest place on earth.

The case of Hong Kong is even more puzzling when the data are examined.[1] Hong Kong has approximately 84% of undeveloped lands, largely remained in the government hand. Even if one takes into account of terrains that are difficult to develop, developable public lands remain sizable. Historically, only small plots of land were released for residential and commercial uses; and prices of land and housing are always stayed at enormously high levels by world standard. Annual proceeds from selling minuet quantity of public lands account for a sizable share of government revenues. Given these facts, how could Hong Kong government resist from selling more lands, especially conventional wisdom seems to suggest that it would be socially efficient to do so?

This paper provides a political economy explanation for this puzzle. We argue that releasing lands slowly is a rational response of Hong Kong government to the potential conflict between the current property owners and other interest parties (such as tenants and new entrants) in the economy. In an overlapping-generation framework which accounts for intertemporal tradeoffs, we highlight this conflict as one between the old and the new generations. The main point we make in this paper is that once initial property rights to real estate are established and new taxes are not allowed, under plausible parametrization, it is unlikely that the government land sale can generate enough revenues to compensate for the losses of the initial property owners. Note that, in establishing this argument, we neither take strong stands on the identity of initial property owners nor hang our story into (say) a median voter interpretation of our model. Instead, we are only making the claim that, the higher is the gap between losses imposed on current property owners and the government revenue on potential land sale, the less likely that (any) government would undertake land sale. We call this key measure a compensation gap, but we remain agnostic on how the actual compensation would take place, given that it is both likely to be politically hard to do and that it depends heavily on the institutional aspects of the economy (namely the ownership structure and the identity of owners).[2]

[1] For more information, see Section 4 for a brief summary of land sale history in Hong Kong.

[2] Compensation gap, in a sense, measures the inability of the government to orchestra a Pareto-improving transfer scheme to the old (existing property owners) when new taxation to the young (non-property owners) is politically infeasible. The assumption on taxes is reasonable as people will generally oppose any new taxes for the benefits of the wealthy (the property owners). Also, people expect selling a public resource such as land ought to be able to raise enough revenue for transfer purpose."

Our result is established from examining the force that drives the extreme property of the equilibrium in the model. In fact, we show that, for any initial distribution of land property right, once established, the government would sell its lands as little as possible; which means, in our model, government land sale would be zero. It does not take an expert in land policy to notice that not every jurisdiction on our globe has similar "extremely-low supply, high land prices" policies, with Hong Kong possibly being one of the few exceptions. To this, we offer several answers: first, one can argue that many jurisdictions (e.g. most of the U.S. cities) face competitive and migration pressures that eliminate the compensation gap. Second, historically disposals of large quantity of government land seemed to have occurred when the compensation gap was minor (e.g. when populating the U.S. West) or when the government did not have to fill the compensation gap in an era of significant political changes (such as collapse of the Soviet bloc.)

The main argument we make in this paper is that compensation gap in an economy with wide housing ownership creates political resistance against efficiency-enhancing government land sale. Consequently government should sell public land at a time when the compensation gap and housing ownership are low (for instance, during the transitional period for a newly formed democracy.) The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides a positive analysis of government land sale policies. Section 4 discusses the implications of compensation gap with reference to past episodes of land disposals in various countries and municipals. The last section concludes.

2 Model

Our starting point is a very simple overlapping generations model. At each time period there are two generations alive: the young and the old. The young individuals supply labor inelastically, consume housing and the single consumption good in the economy. The old individuals only consume the single consumption good, with no supply in labor and no consumption in housing. Consumers are rational and they maximize a well-defined lifetime utility function. It is assumed that population

grows at a fixed rate n and wage at a rate g.[3]

Housing is produced by combining two inputs - construction material m and land l - under a constant return to scale technology $\Delta h = f(m, l)$. Housing is assumed to be perfectly durable and thus housing stock grows in steps with new constructions. As in a small open economy, the interest rate R and the construction material price r in the model are assumed to be given by the world markets.

Land supply is initially fixed at $\bar{L} < \infty$ and is assumed to be controlled by the government. Land is also assumed to have no value in alternative uses, so the optimal policy problem with unlimited land supply would not have an interesting solution.

2.1 First Best Land Policy

The first best land and resource allocation policy is defined as the solution to the following simple problem:

$$\max \sum_{i=0}^{\infty} \lambda_i u(c_i^y, h_i/(1+n)^i, c_i^o) \tag{1}$$

subject to

$$\sum_{i=0}^{\infty} \left(\frac{1+n}{R}\right)^i (c_i^y + c_{i-1}^o/(1+n) + rm_i/(1+n)^i - W(1+g)^i) = 0, \tag{2}$$

$$h_i - h_{i-1} = f(m_i, l_i), \tag{3}$$

$$L_i = L_{i-1} + l_{i-1}, \tag{4}$$

$$L_i \leq \bar{L}, \tag{5}$$

$\forall i > 0$, where λ_i's are the welfare weight of each generation, and c^y, c^o, W are consumption of young, old, and the initial wage of the first generation. Initial values of housing, developed land and the consumption of the current old are fixed. In addition to this list of constraints, there are also constraints on non-negativity of investment, land use and consumption.

[3]We are trying purposefully to keep our model simple and abstract hence from any agglomeration effects that would introduce a interdependence between housing market and labor productivity. For an empirical analysis of the importance of these effects for Chinese cities see Au & Henderson (2006).

The optimal policy for this maximization problem follows a very simple rule: as long as land availability constraint is slack, optimal land investment policy satisfies

$$f_l(m_i, l_i) \times \sum_{j=i}^{\infty} \left(\frac{\partial u(c_j^y, h_j/(1+n)^j, c_j^o)}{\partial \left(h_j/(1+n)^j \right)} \right) / \left(R^j \frac{\partial u(c_j^y, h_j/(1+n)^j, c_j^o)}{\partial c_i^y} \right) = \text{constant.} \quad (6)$$

This condition, derived in the Appendix, implies that the return to land in housing production increases as land becomes more scarce. It follows immediately that if $f(m, l)$ is linear in land (or housing = land) then all land should be released in the initial period. On the other hand, if land and construction material are perfect complements in construction, land should be released at a much slower schedule.

Since an economy without productive capital accumulation is dynamically efficient, the US-style rapid privatization is an optimal land sale policy in the model. Here, the government sells enough land in period 0 and competitive markets result in a Pareto optimal allocation. The schedule of selling the remaining land does not matter and would not affect the present value of government land sale proceeds.[4]

A slower, apparently more active, land disposal policy may also correspond to an optimal policy, as long as it is in accord with condition (6).[5] However, its successful implementation relies on the assumption that the government is able to perform intergenerational transfers, not only though taxation but also through international borrowing and lending that extend across many generations. This is practically not possible, as international financial markets have never reached a stage that allows countries to borrow a sum for a time that is large and long enough to meet the requirements of our model without initiating a currency or financial crisis. This consideration leads us to our next topic, a positive theory of land sales. In the next section, we will argue that a very inefficient (under-utilization) of land policy, once set, can persist over time.[6]

[4]More precisely: As long as private sector holds some non-developed land every period before the total land endowment is privatized the net present value of the land revenue is independent of the particular land sales schedule satisfying the constraint given in (6).

[5]Active government policy on land (zoning, growth control) may also be justified partially for reasons of public good, amenity, environment and congestion.

[6]One interesting alternative theory predicting under-utilization of land (in formal sector) of developing countries is squatting, where in the equilibrium both formal and informal housing sectors exist and the expansion of the formal sector is constrained by the land use of the informal sector (see, e.g. Brueckner and Selod, 2009). For the specific case of

3 Positive Analysis of Government Policies

In this section we analyze a market economy where the government controls land supply. Housing is produced via competitive markets and at the beginning of each time period the current old sell their housing capital to the current young. Our main interest is the political implications of land policy. In an OLG economy with exogenous labor income and interest rate, the timing of government land sale has differential impacts on different generations. In particular, *for any given land sale schedule, an unanticipated land sale in the beginning of period t benefits all generations born after t at the expense of the period t old.* This statement holds because the added housing supply raises the utility of the period t young and all generations born thereafter, but reduces the housing price paid to the period t old.

The question we are asking is how much political pressure there would be for the government not to release any land. Our measure of political pressure is marginal compensation gap, which is defined as the difference between losses of market value by current owners of housing due to increased housing supply and governments revenue from a marginal land sale. Larger (or more positive) compensation gap means more political pressure against land sale. If we are to take our current overlapping generations setting literally, the compensation gap would measure whether (say) a pension transfer program that would be financed entirely by the land revenue would be Pareto-improving.[7] Less literally, we think that marginal compensation gap measures the political feasibility of a land sale regardless of the exact labels (old, young, real estate investors, capitalists, landlords, renters etc.) applied to our model and we postulate that a government policy that fails the compensation test is unlikely to be politically feasible. Thus a 'liberal' government land policy (releasing land to markets quickly) that would be efficient might be politically infeasible due to distributional concerns (either inter-generational distribution, or distribution between renter versus owners).

The key endogenous price variables for our analysis of market economy are the spot prices for

Hong Kong, we do not think that squatting plays an essential role empirically in land sales since Hong Kong's squatter population is very small. Smart (2001) estimates that the squatter population in Hong Kong in 1998 was less than 5% of the population, dues to city's active anti-squatter policy.

[7]Here, we assume side-payments, either from taxing the young or borrowing from the international financial market, are deemed infeasible in the political game and are not included in the measure of compensation gap. Otherwise, the calculation of compensation gap will be reduced to the usual cost-benefit calculus, which bears a foregone efficiency conclusion. See footnote 2 for more discussion.

land $p_{l,t}$ and housing $p_{h,t}$ and the lifetime cost of owning housing $q_{h,t} = p_{h,t} - \frac{p_{h,t+1}}{R}$. The last is the relevant variable for specifying the market clearing conditions for housing markets while the spot prices are relevant for revenue gap considerations. In the absence of bubbles, housing price is determined by the present value of future rents.

$$p_{h,t} = \sum_{s=0}^{\infty} \frac{q_{h,t+s}}{R^S}. \tag{7}$$

The equilibrium we are analyzing are as follows. At time period 0 there is expectation that government will follow a future land sale policy $\{\bar{l}_t\}_{t=0}^{\infty}$ and all economic agents rationally see the corresponding equilibrium values of housing and prices.

The government announces a deviation from this expected land policy by announcing that it is selling rights to additional land Δl_s for $s \geq 0$ periods into the future.[8]

The equilibrium for housing markets, for any time period t and for any land policy, is described by:

$$h_t = D(q_{h,t}), \tag{8}$$

$$h_{t+1} = h_t + f(m_{t+1}, l_{t+1}), \tag{9}$$

$$p_{h,t} f_m = r, \tag{10}$$

$$p_{h,t} f_l = p_{l,t}. \tag{11}$$

Our compensation gap (for additional land sale in time period s) is defined as

$$\chi(\Delta l_s; s; \bar{l}) = (\bar{p}_{h,0} - p_{h,0})\bar{h}_0 + \sum_{t=0}^{\infty} \frac{(\bar{p}_{l,t} - p_{l,t})\bar{l}_t}{R^t} - \frac{p_{l,s}\Delta l_s}{R^s}, \tag{12}$$

where quantities marked by a bar on top (e.g. $\bar{h}$) refer to the original equilibrium. The first term in this expression is the initial loss for the property owner at period t (when policy is announced), the

[8]We also make an assumption that the announced land sale is sufficiently small that every unit of land sold in each period is being immediately converted to housing by the competitive housing sector. This allows us not to consider the complicated situation where the private sector holds some undeveloped land. We also assume, for simplicity of notation, that population growth is zero while still allowing for technological growth.

second term is the effect of the announcement on the net present value of all planned land sales by the government and the third term is the revenue from additional land sales.

Now we can use the constant returns to scale assumption in housing production and write $p_{l,s}\Delta l_s = \alpha_s p_{h,s}(h_s - \bar{h}_s)$ i.e. the payment to land in housing production is a share $0 < \alpha < 1$ of the total revenue of the construction industry in the period where the actual land sale takes place. Note that α like everything else, is defined as an implicit function of the government's land policy. A notable special case is constant α that corresponds to Cobb-Douglas production.[9]

Taking this into account and using the key pricing relation for housing we can calculate the marginal compensation gap as:

$$\begin{aligned}\frac{\partial \chi(\Delta l_s; s; \bar{l})}{\partial \Delta l_s} = & -\bar{h}_0 \frac{\partial p_{h,0}}{\partial \Delta l_s} \\ & - \sum_{t=0}^{\infty} \frac{\bar{l}_t}{R^t} \frac{\partial p_{l,t}}{\partial \Delta l_s} \\ & - \frac{\partial \alpha_s}{\partial \Delta l_s} \times \frac{p_{h,s}(h_s - \bar{h}_s)}{R^S} \\ & - \alpha_s \left(\frac{\frac{\partial p_{h,s}}{\partial \Delta l_s} \times (h_s - \bar{h}_s) + p_{h,s} \frac{\partial h_s}{\partial \Delta l_s}}{R^s} \right). \end{aligned} \tag{13}$$

The second line of the previous expression is the hardest to evaluate explicitly. It incorporates all the effects of land sales in period s to land sales revenue (through land prices) in all future periods. This effect will be a fully dynamic effect that depends on both production technology of housing (substitution between capital and land) and the demand for housing. Without any substitution between land and materials (Leontieff production technology for housing) this term is unambiguously positive by simple economic intuition.

We will instead use our equation to analyze a much simpler situation: we will (informally) introduce a game where government land policy is governed by compensation gap. Assume that government can deviate from pre-announced policy $\bar{l}$ only if the compensation gap is negative. We will analyze below the condition on when a deviation from a "Hong Kong type"-policy is possible, i.e. when it is possible to deviate from "$\bar{l}_t = 0 \; \forall t$" type policy. This means, we want to ask when is the

[9]Thornes (1997) finds using data from Portland, Oregon metropolitan area that Cobb-Douglas (in land and materials) is a reasonable approximation for production function for *single* family housing.

compensation gap positive (Hong Kong policy is self-reinforcing) or negative (Hong Kong policy is not self-reinforcing) when starting with (near) zero land sales.

Evaluating the compensation gap at $\bar{l} = 0$, assuming Cobb-Douglas production, using the fact that the change in housing in every period after $t + s$ is the same, and applying the pricing formula for land, we get the marginal compensation gap for small land sale (with derivation in the appendix) as:

$$\begin{aligned} \left.\frac{\partial \chi(l_s)}{\partial \Delta l_s}\right|_{\bar{l}=0,\Delta l_s \to 0} &= -\sum_{t=s}^{\infty} R^{-t} \left(\alpha_s q_{h,t} + \frac{\overline{h}_t}{D'_t}\right) \frac{\partial h_t}{\partial \Delta l_s} \\ &= -\frac{\partial h_s}{\partial \Delta l_s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \left(\alpha_s + \frac{1}{\epsilon_{h,t}}\right) \end{aligned} \tag{14}$$

where $\epsilon_{h,t}$ is the housing demand elasticity and D'_s is the housing demand derivative for time period s. To simplify the discussion, let us assume that the elasticity of demand is constant across time periods. Then this results means that, if (the absolute value of) the inverse of elasticity of housing demand is smaller than the land share in production in the period where the land is to be used, then marginal compensation gap is negative and land sale are politically feasible. Notice also that the (sign) of political feasibility is independent of the timing of the land sale.[10]

To put this in perspective we first note that for Cobb-Douglas demand (unit elasticity), land sale are politically feasible only if land share in production is one. This in our view highlights the political infeasibility of land sale policy: the estimates of housing price elasticities are typically significantly less than one (making $1/\epsilon_h$ large) and while the land share in housing production varies across markets it is certainly less than one.[11]

The following observation suggests that without binding constraints in land supply, the political economy of the government land sale is biased against efficient land allocations. In the case of Hong Kong, the land share of the cost of residential construction is about 0.5, and the estimate of the demand elasticity of housing service by Peng and Wheaton (1994) is roughly -1. This means

[10]Note that in the discussion from now on we will be referring to the magnitudes of demand of elasticity and the inverse of the demand elasticity without the explicit mention of the sign of the quantity of question.

[11]A vast empirical literature has been developed on housing elasticities of U.S. and other countries, e.g., Polinsky and Ellwood (1978), Rosen (1979), Hanushek and Quigley (1980), Horioka (1988), Peng and Wheaton (1994), and studies referenced therein.

Table 1: Compensation Gap

	Demand Elasticity						
Share of land	-0.4	-0.5	-0.8	-1	-2	-5	-10
0	2.5	2	1.25	1	0.5	0.2	0.1
0.1	2.4	1.9	1.15	0.9	0.4	0.1	0
0.2	2.3	1.8	1.05	0.8	0.3	0	-0.1
0.3	2.2	1.7	0.95	0.7	0.2	-0.1	-0.2
0.4	2.1	1.6	0.85	0.6	0.1	-0.2	-0.3
0.5	2	1.5	0.75	0.5	0	-0.3	-0.4
0.6	1.9	1.4	0.65	0.4	-0.1	-0.4	-0.5
0.7	1.8	1.3	0.55	0.3	-0.2	-0.5	-0.6
0.8	1.7	1.2	0.45	0.2	-0.3	-0.6	-0.7
0.9	1.6	1.1	0.35	0.1	-0.4	-0.7	-0.8
1	1.5	1	0.25	0	-0.5	-0.8	-0.9

Note to Table 1: Marginal compensation gap values assuming constant demand elasticity across time periods. Interpretation of the values: if the land sales result in a 1% increase in total housing stock, then the compensation gap will be 1% times (value from the table) times (value of the current housing stock). The row corresponding to zero share of land gives the absolute elasticity of value of existing housing with respect to new housing.

that each percentage increase in Hong Kong's housing stock creates a compensation gap of more than 0.5% of the value of housing market. Most estimates of demand for housing service cited in footnote 4 are below unity, so the 0.5% of the value of housing market is a conservative estimate of the compensation gap. The value of private residential housing, according to the May 2001 issue of Quarterly Bulletin published by the Hong Kong Monetary Authority, is 2.8 times the GDP in 1997 (a peak year for the Hong Kong housing market) and 1.4 times for 2000 (a near trough for the market). The government spending is 14.2% of GDP in 1997 and is 17.7% for 2000. So, if by our conservative estimates that 0.5% of housing stock is the compensation gap to increase housing supply by 1% in Hong Kong, then to fully compensate the existing owners it would take 4.0% to 9.8% of the government budget in addition to the proceeds of the land sale.

Note that what makes it politically difficult for the Hong Kong government to substantially increase land supply is that the housing market value is high relative to GDP. As we will discuss in the next section, in countries where government sold substantial quantities of public land (or housing) the total value of private housing stock was small relative to GDP or the elasticity of demand for

housing service from the new land was large.

4 Policy Implications of the Compensation Gap

In this section we will discuss how in a number of countries historical incidents influence the compensation gap and government land sales or giveaways. In a market economy with a substantial fraction of population being homeowners, the compensation gap may limit government land disposal. The political resistance to land disposal is larger when land price is higher. We will extend our discussion started in the introduction section on Hong Kong's land sale policy.

We also find in historical cases that disposal of large quantity of public land seemed to have occurred only when the government was not pressured to fill the compensation gap or the existing housing ownership was low. Specially, (1) the 18th and 19th century U.S. government land disposal was made easy because it did not need to compensate property owners west of Mississippi, and the land was a poor substitute for the east coast properties. (2) East Bloc privatization of public housing through a whole-sale change of ownership during a transition period was aided by the fact that there was no need to compensate the original owners. (3) Singapore's low housing price policy was announced in the 1960s upon independence when there was little pressure to compensate the existing homeowners and the policy has been sustained since then. (4) The compensation gap faced by local governments in China was initially small because of low housing ownership and incomplete market. (5) The municipal competition for migration makes the demand for housing elastic and compensation gap negative. More detailed discussions of country-specific government land policy are given below.

Explaining the Land Sale Experiences in Hong Kong, U.S., and Transitional Economies

The system of land and housing policy in Hong Kong originated from the date of the colonial period. Following the land and housing policy in Britain, the colonial government adopted and maintained a freehold system. All lands in Hong Kong were owned by the government on behalf of the royal,

and after 1997 turnover the freehold land interest was passed onto the Special Administrative Region Government. As such, land buyers obtain only the rights for development and occupation under specific terms. Hong Kong is not large in land mass, about $1,100$ km^2 as a total. Surprisingly, only 16% of the total area was developed. This low percentage number is partially attributed to Hong Kong's hilly topography. Land usages are strictly regulated and land conveyances are through public auction, tender and Letter A/B (which entitles the holder to be granted land with building status at an unspecified future date with the stipulated ratio exchange.) As such, the property market is closely linked to governmental attitude and land policies. Table 2 summarizes new land supply in Hong Kong for recent years.

Table 2: New land supply in Hong Kong

Year	New Land (square kilometers)	Year	New Land (square kilometers)
1979	0.1705	1992	0.2303
1980	0.4323	1993	0.2820
1981	0.6514	1994	0.2799
1982	0.4551	1995	0.6625
1983	0.1542	1996	0.6412
1984	0.4345	1997	0.6496
1985	0.4117	1998	0.7518
1986	0.2130	1999	0.7649
1987	0.2647	2000	0.7772
1988	0.3545	2001	0.6332
1989	0.3212	2002	0.8920
1990	0.1907	2003	0.0007
1991	0.3309	2004	0.3046

Note that the total area of Hong Kong is 1,104 square kilometers.

At the speed of land sale implied by these numbers, it would easily take another century or even longer for Hong Kong to develop all land held by government. Hong Kong is a prominent example that the compensation gap is a key factor in the decision of government land disposal. There is direct evidence that past attempts to deviate from the high housing price policy failed because of the compensation gap. For instance, right after Hong Kong's turnover in 1997 from Britain to China, the new administration established a new land sale policy that aimed to boost up land and housing

supply. Although the policy was presented with a clear deliberation that it would benefit the general public, given the large compensation gap, the uncompensated losses in property value to existing owners had created a tremendous political turmoil and the policy, failed miserably, was abandoned later.

Hong Kong is not the only jurisdiction that has recently have to deal with the issue of significant land and real estate holdings by the government. Below, we will discuss some important episodes of land sales and land giveaways by governments in the U.S., and transitional economies. In light of these instances, we argue that public land sale policies that result in disposal of large quantity of land seemed to have occurred only when the compensation gap was small or when government did not have to fill the compensation gap.

Our theory suggests that the sale and give-away of the government land in the U.S. history was made politically feasible by the fact that the majority of the citizens in the nation were landless immigrants. The most prominent example of liberal land policy can be observed in the U.S. before the mass-populating of the country by Europeans. In a period of one century, from 1783 to 1883, the U.S. population increased from below 3 millions to over 50 millions and the land that U.S. government acquired increased from 820 thousands square miles to 3.5 million square miles. During the same period, over 680 million acres (or more 1 million square miles) were disposed (see appendix of Hart (1887), Tables 1 and 2). Selling public land could have generated an important source of government revenue but there is scant evidence that the U.S. attempted to maximize the proceeds of the land sale. While about a fourth (192 million acres) of the public land disposition were through sales, most of the public land disposition were made through grant to institutions (e.g. universities) and individuals. In a short period of time (1847-1855), 61 million acres of public land were granted to soldiers of all Indian wars and the Mexican war (Gates 1941, page 61). As for land sale conducted by the government, Gates noted that "[t]he various preemption acts, the prospective preemption act of 1841 and its subsequent amendments, weakened the revenue principle of public land policy since they permitted settlers to squat upon public lands not officially opened for sale, and subsequently to buy land on which their improvement were located for minimum price of $\$1.25$ per acre without having to bid for it against competitors." Although public land was a source of revenue, the U.S.

government adopted a policy of making it almost free for settlers instead of keeping most of the land publicly owned. The land policy served the purpose of keeping the newly acquired land occupied and under the control of the U.S. Since the land given away was in undeveloped areas of the country, this policy can be easily understood by our formula because the new housing was likely to be an imperfect substitute for old housing and hence the elasticity of demand should be really small.[12] This policy, which was taken more than a hundred years ago, shaped the land and housing market of the U.S. to the present day.

Like Hong Kong, Singapore was a former British Colony with a small land mass and large population and its land was regulated and controlled by a colonial system. But, unlike Hong Kong, Singapore has successfully reshaped its land and housing policy at the time of its independence in 1965. Under the reformed policy that remains effective today (see Yuen 2005), a new national housing authority was created with a mandate to build enough public housing to meet the government-established housing needs. At the same time, a new land acquisition legislation was enacted to empower the government to acquire any private land at un-inflated prices for the purposes. This policy change in Singapore was in fact much more drastic than the one undertaken in Hong Kong in 1997. What makes Singapore succeeded in overcoming the compensating gap while Hong Kong failed? Aside from the fact that Singapore's housing policy is subsumed under its national growth program, we speculate that instability at the time of policy change has helped to overcome the compensation gap issue. At the time of independence, Singapore's survival as an independent country was very much doubtful and Singapore economy was in dismal condition. These are more overriding concerns for the newly independent country than the compensation gap of existing real estate owners. In contrast, the smooth and well planned transition and the healthy economic conditions at the time of its turnover from Britain to China makes compensation gap very much an issue in Hong Kong.

The political economy considerations of the government land sale is most relevant for economies that makes transition from state ownership of land and housing to private ownership. The recent distribution of land and housing stock in some former socialist countries was possible because the

[12]This calls for a slightly extended version of our model allowing for two types of housing, but the extension needed is very straightforward.

citizens were not property owners at the time. Following the collapse of the communist bloc in Europe the transitional economies followed the policy path of selling it to the current occupants often at heavily subsidized rates (Stryuk, 1996; Turner and Vitorin, 1996).[13] Note that these housing market reforms were often parallel to the reforms in financial sector and developing legal and regulatory framework for private ownership of housing.[14]

Unlike the one-time change in land ownership in the former Eastern Bloc countries and the persistent high land price policy in Hong Kong, China has adopted a policy of gradual land releasing amid rising land price. The starting point of China's housing and land market is state-ownership of land as well as other properties. Housing was provided with nominal rent as part of the fringe benefit. The expenditure on rent in urban areas was less than two percent of household income in 1978 and even dropped to below one percent in 1990 (National Bureau of Statistics of China, 1993) as income rose. Development of housing market became a necessity as the consequence of development of labor market in China (so employees will not be tied to the current job by the provision of cheap housing) and the shortage of housing service caused by the distorted low price became obvious.

A distinct feature in China's land market is the separation between land ownership and user right[15]. In principle, all land is owned by the central government, but the local governments assign right for using land and benefit from the sale of land use right. With respect to the development of land market, the initial impact of the land sale on housing market was not as transparent as that in Hong Kong. The demand for ownership of commercial housing was not strong for employees who were paying nominal rents. The legitimacy of such sale was established gradually and unevenly, following the successful experiments in coastal regions (see Yu 2006). In light of the model we discussed in this paper, the compensation gap of the land sale did not cause political problems in China for three reasons. First, the number of initial home owners was small. Second, the absence of a well-functioning housing market made it less transparent how the land sale harmed the existing owners. Third, the interests of the existing home owners are not the main concern of the local

[13] A major exception to this is the Baltic States (Estonia, Latvia and Lithuania) who also pursued restitution of pre-Soviet occupation ownership rights, probably motivated by the fact that many current occupants were ethnic Russians who were not immediately granted full citizenship rights.

[14] For an analysis of interdependence of housing markets and financial markets in transformations, see Renaud (1996).

[15] Separation of land ownership and user right is not unique in China. In fact, Hong Kong and Israel adopt a similar system.

governments. These factors have been changing. The home ownership rate in China has grown from the teens in 1990 to thirty percent in 1995, and then to seventy percent in 2000 (Yu 2006). Private housing has become an important part of the household asset and the price of housing has become one of the most closely followed economic indicators. With these changes, we predict that as China's housing market matures, the political economy of land sale will become an eminent policy issue.

Implications on Present Land Policy of Municipal Government in Most Countries

The issue of compensation gap is still relevant in places where government does not sell public land but can influence land supply through zoning and other regulations. In many communities, the debate between advocates of expansion and 'way of life' is common place. It is well understood in the urban economics literature that this debate is often motivated by the political economy of housing price.[16]

At first glance it would seem that our model would have a very strong policy prediction for local land policy: no land should ever get released by the local government. So in our attempt to explain Hong Kong's restrictive housing policy we have a model explaining too much: it would seem to no jurisdiction ever should sell any land because of the political pressure from the land owners.

Our answer to this puzzle comes from a reinterpretation of the demand elasticity in our model. If we believe the argument presented (among others) in Aura & Davidoff (2008), the relevant demand elasticity for finding housing market equilibrium should include also the effect of in-migration into housing demand. This can explain why say small U.S. municipalities can follow liberal land market policies: the smaller the jurisdiction is in comparison to the economy of as whole, the larger the effect of the in-migration elasticity to the housing market. In Aura & Davidoff the reasonable values for $1/\epsilon_h$ for a very small jurisdiction were calculated to be potentially as small as .05 making potential land sale policies very feasible.[17] Note that this analysis here is almost certainly an over-

[16]Some examples of analysis growth controls and zoning are Brueckner (1990), Turnbull (2004) and Quigley & Swoboda (2005). Examples of analysis that highlight the issue of local decision making and political economy are Hamilton (1978), Brueckner (1998) and Glaeser & Ward (2006).

[17]Note that the argument here does not hinge on the government getting the sales revenue, a simple lobbying model could easily be built where the political lobbying efforts of the land owners for zoning changes would depend on the potential land revenue while the lobbying effort of the existing homeowners would depend on the effect of new development on housing prices. The point of Aura & Davidoff estimates in this context is that gains for land developers in small

simplification of the complicated decision-making relating to land use policies in a small jurisdiction that (in the case of US) provide property tax funded public goods in a world where Tiebout-sorting considerations are also relevant.

5 Conclusions

One implication of our analysis is that sub-federal control of land-policy could be efficient. Since smaller jurisdiction are not facing the more elastic demand curves (due to migration pressures) they are more likely to follow "liberal" land policies while a single federal government faces the market demand curve as a monopoly and is much more likely to follow a restrictive land policy. In China, the land policy for the central government is considerably more conservative than that of the regional government. The political economy issue discussed here is one of the contributors to the difference in policy.

Another implication of our result on compensation gap casts doubt on a justification of government land holding. Although the argument that government land holding may deter housing bubble is intuitively appealing, the data on Hong Kong's housing prices do not lend support to it. The practical value of using government held land as a deterrence of housing bubble is questionable because regardless of cause of rising housing price, whenever government releases land it must deal with the compensation gap, especially when housing price is high. Government land holding designed for maximizing land sale revenue is also politically constrained by the fact that the higher land price the stronger is the resistance from the existing owners to government land sales.

In summary, this short paper notes that the policy decision on whether and when to sell government held land is in many countries as important as fiscal and monetary policy. Unlike fiscal and monetary policies which can be altered over time, it is difficult for the government to reverse its decision once ownership or right of usage of land is transferred from government to the private sector. We argue that a major reason for the remarkable diversity in land policy across countries today is due to the path dependency of land sale policy. When a large portion of the land is in government hand and

jurisdiction are likely to be significantly higher than the losses to the existing property owners.

the majority of the households are homeowners, it is politically difficult for the government to release large amount of land because of the compensation gap. The 'tight land supply' policy leads to high housing price and high compensation gap. This self-sustaining high compensation gap handicaps the government's ability to ever sell land for productive use if the government needs to answer to the existing property owners. Less regard to the initial owners may give rise to substantial benefit for the future generations. Commitment to selling public land should be made as early and as predictable as possible. Our highly stylized model can be extended for quantitative analysis of government land policy. For instance, simulation of an extended model with realistic parameter values can help find a land sale policy that minimizes the compensation gap, which will guide the government to privatize public land with the least political resistance.

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Appendix

Derivation of Equation (6)

The planner problem is

$$\max_{c_i^y, c_i^o, m_i, l_i, h_i, L_i} \sum_{i=0}^{\infty} \lambda_i u(c_i^y, \frac{h_i}{(1+n)^i}, c_i^o) \tag{A1}$$

subject to

$$\sum_{i=0}^{\infty} \left(\frac{1+n}{R}\right)^i \left(c_i^y + \frac{c_{i-1}^o}{1+n} + \frac{rm_i}{(1+n)^i} - W(1+g)^i\right) = 0 \tag{A2}$$

$$h_i - h_{i-1} = f(m_i, l_i) \tag{A3}$$

$$L_i = L_{i-1} + l_{i-1} \tag{A4}$$

$$L_i \leq \overline{L}. \tag{A5}$$

The Lagrangean is

$$\begin{aligned}
\mathcal{L} \;=\; & \sum_{i=0}^{\infty} \lambda_i u(c_i^y, \frac{h_i}{(1+n)^i}, c_i^o) \\
& + A\left[\sum_{i=0}^{\infty} \left(\frac{1+n}{R}\right)^i \left(-c_i^y - \frac{c_{i-1}^o}{1+n} - \frac{rm_i}{(1+n)^i} + W(1+g)^i\right)\right] \\
& + \sum_{i=0}^{\infty} B_i \left[f(m_i, l_i) - h_i + h_{i-1}\right] \\
& + \sum_{i=0}^{\infty} C_i \left[L_i - L_{i-1} - l_{i-1}\right] \\
& + \sum_{i=0}^{\infty} D_i \left[\overline{L} - L_i\right],
\end{aligned}$$

and the first order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_i^y} = \lambda_i \frac{\partial u}{\partial c_i^y} + A\left(\frac{1+n}{R}\right)^i = 0 \tag{A6}$$

$$\frac{\partial \mathcal{L}}{\partial c_i^o} = \lambda_i \frac{\partial u}{\partial c_i^o} + \frac{A}{R}\left(\frac{1+n}{R}\right)^i = 0 \tag{A7}$$

$$\frac{\partial \mathcal{L}}{\partial m_i} = -\frac{Ar}{R^i} + B_i \frac{\partial f}{\partial m_i} = 0 \tag{A8}$$

$$\frac{\partial \mathcal{L}}{\partial l_i} = B_i \frac{\partial f}{\partial l_i} - C_{i+1} = 0 \tag{A9}$$

$$\frac{\partial \mathcal{L}}{\partial h_i} = \lambda_i \frac{\partial u}{\partial (h_i/(1+n)^i)} \cdot \frac{1}{(1+n)^i} - B_i + B_{i+1} = 0 \tag{A10}$$

$$\frac{\partial \mathcal{L}}{\partial L_i} = C_i - C_{i+1} - D_i = 0. \tag{A11}$$

Using (A6) to express $\lambda_i = -\frac{A}{\frac{\partial u}{\partial c_i^y}}\left(\frac{1+n}{R}\right)^i$, substituting it into (A10), and simplifying give

$$\frac{\frac{\partial u}{\partial (h_i/(1+n)^i)}}{R^i \frac{\partial u}{\partial c_i^y}} = \frac{1}{A}\left(B_i - B_{i+1}\right).$$

Summing this last equation forward gives

$$\sum_{j=i}^{\infty}\left(\frac{\frac{\partial u}{\partial (h_j/(1+n)^j)}}{R^j \frac{\partial u}{\partial c_j^y}}\right) = \frac{1}{A}\left(\sum_{j=i}^{\infty} B_j - \sum_{j=i+1}^{\infty} B_j\right) = \frac{B_i}{A}. \tag{A12}$$

If land availability constraint is slack, then $D_i = 0$ and, by (A11), $C_i = C_{i+1} = C$, implying $B_i = \frac{C}{\frac{\partial f}{\partial l_i}}$ from (A9). Replacing B_i in (A12) and rearranging gives Equation (6) in the text:

$$\frac{\partial f}{\partial l_i} \times \sum_{j=i}^{\infty}\left(\frac{\partial u(c_j^y, h_j/(1+n)^j, c_j^o)}{\partial (h_j/(1+n)^j)}\right) / \left(R^j \frac{\partial u(c_j^y, h_j/(1+n)^j, c_j^o)}{\partial c_j^y}\right) = \frac{C}{A} = \text{constant}.$$

Derivation of Equation (14)

Let $\overline{L} \equiv \{\bar{l}_t = 0 \text{ for all } t > 0\}$ be the original land sale policy and $L \equiv \{l_t = 0 \text{ for all } t \neq s \text{ and } l_s = \Delta l_s\}$ be the new policy. Under L, the equilibrium quantity of housing is

$$h_t = \begin{cases} h_0 & \text{if } t < s \\ h_s & \text{if } t \geq s \end{cases} \tag{A13}$$

making

$$\frac{\partial h_t}{\partial \Delta l_s} = \begin{cases} 0 & \text{if } t < s \\ \partial h_s / \partial \Delta l_s & \text{if } t \geq s. \end{cases} \tag{A14}$$

Since rental market rate is given by

$$h_t = D(q_{h,t}), \tag{8}$$

it follows that

$$\frac{\partial q_{h,t}}{\partial \Delta l_s} = \frac{\partial q_{h,t}}{\partial h_t}\frac{\partial h_t}{\partial \Delta l_s} = \frac{1}{D'}\frac{\partial h_t}{\partial \Delta l_s} = \begin{cases} 0 & \text{if } t < s \\ \frac{1}{D'}\frac{\partial h_s}{\partial \Delta l_s} & \text{if } t \geq s \end{cases}. \tag{A15}$$

Given that housing price is the present value of rents

$$p_{h,0} = \sum_{t=0}^{\infty} R^{-t} q_{h,t}, \tag{7}$$

(A15) implies

$$\begin{aligned} \frac{\partial p_{h,0}}{\partial \Delta l_s} &= \sum_{t=0}^{\infty} R^{-t}\frac{\partial q_{h,t}}{\partial \Delta l_s} \\ &= \sum_{t=s}^{\infty} R^{-t}\frac{\partial q_{h,t}}{\partial \Delta l_s} \\ &= \frac{\partial p_{h,s}}{\partial \Delta l_s} \end{aligned} \tag{A16}$$

and

$$\begin{aligned} \frac{\partial p_{h,s}}{\partial \Delta l_s} &= \sum_{t=s}^{\infty} R^{-t}\frac{\partial q_{h,t}}{\partial \Delta l_s} \\ &= \sum_{t=s}^{\infty} R^{-t}\frac{1}{D'}\frac{\partial h_s}{\partial \Delta l_s}. \end{aligned} \tag{A17}$$

Now, starting from (13), the marginal compensation gap (before taking limit on Δl_s) can be

written as

$$
\begin{aligned}
& \left.\frac{\partial \chi(\Delta l_s; s; \bar{l})}{\partial \Delta l_s}\right|_{\bar{l}=0} \\
= & -\overline{h}_0 \frac{\partial p_{h,0}}{\partial \Delta l_s} - \sum_{t=0}^{\infty} \frac{\bar{l}_t}{R^t} \frac{\partial p_{l,t}}{\partial \Delta l_s} \\
& -\frac{\partial \alpha_s}{\partial \Delta l_s} \frac{p_{h,s}(h_s - \overline{h}_s)}{R^s} - \frac{\alpha_s}{R^s} \left(\frac{\partial p_{h,s}}{\partial \Delta l_s}(h_s - \overline{h}_s) + p_{h,s} \frac{\partial h_s}{\partial \Delta l_s} \right) \\
= & -\overline{h}_0 \frac{\partial p_{h,0}}{\partial \Delta l_s} - \frac{\partial \alpha_s}{\partial \Delta l_s} \frac{p_{h,s}(h_s - \overline{h}_s)}{R^s} - \frac{\alpha_s}{R^s} \left(\frac{\partial p_{h,s}}{\partial \Delta l_s}(h_s - \overline{h}_s) + p_{h,s} \frac{\partial h_s}{\partial \Delta l_s} \right) \\
= & -\overline{h}_s \frac{\partial p_{h,s}}{\partial \Delta l_s} - \frac{\partial \alpha_s}{\partial \Delta l_s} \frac{p_{h,s}(h_s - \overline{h}_s)}{R^s} - \frac{\alpha_s}{R^s} \left(\frac{\partial p_{h,s}}{\partial \Delta l_s}(h_s - \overline{h}_s) + p_{h,s} \frac{\partial h_s}{\partial \Delta l_s} \right) \\
= & -\overline{h}_s \frac{\partial p_{h,s}}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \left(\frac{\partial p_{h,s}}{\partial \Delta l_s}(h_s - \overline{h}_s) + p_{h,s} \frac{\partial h_s}{\partial \Delta l_s} \right) \\
= & -\overline{h}_s \frac{\partial p_{h,s}}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} p_{h,s} \frac{\partial h_s}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \\
= & -\overline{h}_s \sum_{t=s}^{\infty} R^{-t} \frac{1}{D'} \frac{\partial h_s}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \frac{\partial h_s}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \\
= & -\sum_{t=s}^{\infty} R^{-t} \frac{\overline{h}_t}{D'} \frac{\partial h_t}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \frac{\partial h_t}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \\
= & -\sum_{t=s}^{\infty} R^{-t} \left(\alpha_s q_{h,t} + \frac{\overline{h}_t}{D'} \right) \frac{\partial h_t}{\partial \Delta l_s} - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \\
= & -\frac{\partial h_s}{\partial \Delta l_s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \left(\alpha_s + \frac{\overline{h}_t}{h_t} \frac{1}{\varepsilon_{h,t}} \right) - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \\
= & -\frac{\partial h_s}{\partial \Delta l_s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \left(\alpha_s + \frac{\overline{h}_t}{\overline{h}_t + f(m_s, \Delta l_s)} \frac{1}{\varepsilon_{h,t}} \right) - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s)
\end{aligned}
$$

where the second equality comes from the fact that $\bar{l}_t = 0$ for all $t > 0$, the third equality from (A16) and $\overline{h}_s = \overline{h}_0$ as $\bar{l}_t = 0$ for all t, the fourth from $\frac{\partial \alpha_s}{\partial \Delta l_s} = 0$ for Cobb-Douglas housing production function, the fifth from housing production function and rearranging terms, the sixth from (A17), (7) and (9), the seventh from (A13) and (A14), the eighth from grouping terms, the ninth from applying the definition of elasticity ($\varepsilon_{h,t} = \frac{q_{h,t}}{h_t} \frac{dh_t}{dq_{h,t}} = \frac{q_{h,t}}{h_t} D'(q_{h,t})$) and the last from (9).

Finally, taking limit on Δl_s gives Equation (14) in the text:

$$
\begin{aligned}
& \left.\frac{\partial \chi(\Delta l_s; s; \bar{l})}{\partial \Delta l_s}\right|_{\bar{l}=0, \Delta l_s \to 0} \\
= & \lim_{\Delta l_s \to 0} \left\{ -\frac{\partial h_s}{\partial \Delta l_s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \left(\alpha_s + \frac{\overline{h}_t}{\overline{h}_t + f(m_s, \Delta l_s)} \frac{1}{\varepsilon_{h,t}} \right) - \frac{\alpha_s}{R^s} \frac{\partial p_{h,s}}{\partial \Delta l_s} f(m_s, \Delta l_s) \right\} \\
= & -\frac{\partial h_s}{\partial \Delta l_s} \sum_{t=s}^{\infty} R^{-t} q_{h,t} \left(\alpha_s + \frac{1}{\varepsilon_{h,t}} \right)
\end{aligned}
$$