

Price Uncertainty and Consumer Welfare in an Intertemporal Setting

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Abstract

In this paper we examine how increases in intertemporal price uncertainty affect the welfare of a consumer. In the preference structure of the consumer the coefficient of relative risk aversion and the elasticity of intertemporal substitution (EIS) are parametrically independent. We find that under some circumstances, including configurations where parameter values are consistent with much of the empirical literature, for each given degree of risk aversion an increase in price uncertainty reduces consumer welfare if the EIS is lower than a corresponding threshold value. We also find that when saving rates are sensitive to the EIS then there are important counterexamples to this intuitive case.

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1. Introduction

The notion that increased price uncertainty reduces consumer welfare does not always find support from economic models. In particular, analyses that focus on static settings often suggest that price uncertainty is likely to enhance consumer welfare. Waugh (1944) showed that price instability always implies higher expected consumer surplus than that implied by price stabilization. Turnovsky et al (1980) demonstrated that price instability does not always entail higher expected utility but most likely does. Baye (1985) showed that if a population of individuals has a range of preferences that includes the entire set of neoclassical preferences, then price stabilization makes at least one individual worse off. The intuition reflected in the static results cited above is that price instability brings with it both a risk (prices overall will be sufficiently high for lower utility) and an opportunity (prices overall will be sufficiently low for higher utility). Taking preferences, probabilities, and the quasi-convexity of the indirect utility function in the price vector into consideration, the opportunity may well outweigh the risk.

Situated in an intertemporal setting, the analysis of the present paper indicates that the static analysis must be supplemented. Increased price uncertainty reduces consumer welfare over a broader range of cases than the static results intuitively would suggest. The additional results in a dynamic setting derive from two, related aspects of our analysis. First, past actions are irreversible. This introduces a type of constraint on intertemporal substitution among consumption goods (i.e. on choice of the consumption vector) that has no analogue in the static setting (where the entire vector of consumption goods is chosen simultaneously).

Second is our treatment of preferences. We employ a tractable model of preferences, adopted from Weil (1990), (see also Epstein and Zin 1989), in which risk aversion and intertemporal substitution are parametrically disentangled. Intuitively, as is discussed later in the paper, these factors play distinct roles in determining the impact of price uncertainty on consumer welfare. Hence, a choice-theoretic framework allowing separation of these factors is of

interest.¹ (Compare the standard time-separable, constant relative risk aversion framework, in which the CRRA coefficient measures both the consumer's attitude toward risk and the inverse of the elasticity of intertemporal substitution.) Consideration of a more general preference structure is also supported by the empirical studies of Epstein and Zin (1991) and Giovannini and Weil (1989), in both of which the time-separable expected-utility representation is rejected.

The earlier literature on the welfare implications of price dispersion was motivated in part by the government policy of stabilizing agricultural prices (see Waugh 1944). The theory developed in that context focuses on the notion that consumers may find advantage in the fluctuations in food prices, implying that stabilization policy is not warranted. Recent studies (e.g., Bakos 1998, Baye and Morgan 2001, and Smith and Brynjolfsson 2001) on the effect of the internet on price competition indicate that consumers face price dispersion even in the world of e-commerce. Moreover, analyses such as that in the Baye and Morgan paper suggest that there may be sound theoretical arguments for why we should expect this. More importantly, if such dispersion is in fact beneficial to consumers, the “digital divide” implies that the more affluent consumers who have internet access can take advantage of these price fluctuations to a greater extent than those who are not connected. Understanding the implications of price uncertainty for consumer welfare is not only relevant for the macroeconomy but for distributional social welfare issues as well.

¹ The preference specification employed here emerges from an alternative theoretical direction in the paper of Maenhout (1999). He shows that an economic agent who has a CRRA utility function and a preference for “robustness” may behave as if she has the preference structure assumed in the present paper. Robustness is, loosely, a characteristic of the mode of decision-making in circumstances of model uncertainty such that a satisfactory outcome is attained in the worst-case scenario. For more discussion of the concept of robustness, see Anderson, Hansen, and Sargent (2000).

In line with our discussion above, our results involve a set of pivotal conditions -- conditions ensuring curvature properties of a key function in our representation of preferences -- in which the degree of risk aversion and the willingness to substitute intertemporally enter independently. An example of the (generally intuitive) implications flowing from these conditions is the following. Provided that the degree of risk aversion is above a certain cutoff, for any given degree of risk aversion, the lower is a consumer's elasticity of intertemporal substitution the more likely she is to be worse off in the face of a multi-period increase in price uncertainty. We also find cases that are counterintuitive. Although all of our results reflect the intertemporal context in the model, the intertemporal setting is of particular importance to the existence of these less-intuitive results.

We discuss the foregoing and other implications in the body of the paper. We also review some of the empirical work on risk aversion and intertemporal substitution. Overall, our results suggest that for a range of parameter values consistent with much of the empirical literature, increased price uncertainty in an intertemporal setting reduces consumer welfare.

Section 2 below presents the general model, and analysis for the case of a finite horizon. Section 3 provides results for the case of an infinite horizon. Section 4 presents several numerical examples. Section 5 provides a brief discussion of related empirical literature. Section 6 concludes.

2. The Finite Horizon Model

We consider an individual who has a finite horizon of N periods or an infinite horizon, who consumes one type of good, and who faces an uncertain price for that good in every period (except the first period). We assume the consumer has given lifetime income in order to isolate the problem of price uncertainty from those of interest rate and income uncertainty.

Consistent with our earlier discussion, we assume a generalized preference structure with a constant coefficient of relative risk aversion and a constant, parametrically independent elasticity of intertemporal substitution. Following Weil (1990), the resulting recursive relationship representing this individual's preference ordering is written

$$\begin{aligned}
 V_k(A_k, p_k) &= \max U[c_k, E_k V_{k+1}(A_{k+1}, p_{k+1}^*)] \\
 &= \max \frac{\{(1-\beta)(c_k)^{1-\rho} + \beta [1+(1-\beta)(1-\gamma)E_k V_{k+1}(A_{k+1}, p_{k+1}^*)]^{(1-\rho)/(1-\gamma)}\}^{(1-\gamma)/(1-\rho)} - 1}{(1-\beta)(1-\gamma)}. \quad (1)
 \end{aligned}$$

In (1), A_k is wealth at the beginning of period k and p_k is the price in period k , (where no confusion results, the subscripts are dropped); $A_{k+1} \in [0, A_k]$ is wealth at the beginning of the $(k+1)$ st period, optimally chosen in period k , and $c_k \in [0, A_k/p_k]$ is optimal consumption in the k th period, with of course $p_k c_k + A_{k+1} = A_k$. V_k is the value function, mapping the period- k (wealth, price) ordered pair into expected benefit over the remaining periods. The function U is an "aggregator function", (Weil 1990, p.31), the structure of which, (in (1) above), implements the generalization of the usual isoelastic preference structure with expected-utility representation. The notation “ $*$ ” denotes a random variable, (this notation is sometimes dropped in the sequel; no confusion should result), E is the (unconditional) expectation operator, E_k is the expectation conditional on p_k , and $\beta \in (0,1)$, $\rho > 0$, $\rho \neq 1$, and $\gamma > 0$, $\gamma \neq 1$ are parameters.²

Parameter γ measures relative risk aversion, (see (2) below). For deterministic time paths, $1/\rho$ measures the elasticity of intertemporal substitution, (in (1), drop the expectation operator and by means of sequential substitution derive an expression involving successive consumption

² A suitable limiting treatment yields forms for the value functions for the cases of $\rho=1$ and $\gamma=1$. For simplicity we ignore these boundary cases in the development below. Results follow essentially the same pattern as that below but require changes in details because of the differing forms of the value functions.

levels). Where time paths are random, this elasticity is not well-defined. Nonetheless, (see Hall 1988, for example), $1/\rho$ still provides an indication of attitude toward intertemporal fluctuations per se in consumption that can constitute another source, in addition to relative risk aversion, of the adverse effect price uncertainty can have on consumer welfare in an intertemporal setting. The degree to which it represents a potential source of adverse effects on welfare is heightened under uncertainty by the fact that past actions are irreversible, a point emphasized in the introduction.

Prices are independent continuous random variables defined on the finite interval $[a,b]$. Initially prices are identically distributed across periods. We introduce increases in uncertainty, affecting the current and future choices of a consumer and her expected well-being, by means of mean preserving spreads of the distributions of one or more prices.

We consider first the N -period case, where $N \geq 2$ and finite. By induction (the proof making use of the maximization just below), one can show that for any $k \in \{1, \dots, N-1\}$, V_k can be written

$$V_k(A_k, p_k) = \frac{(1-\beta)^{1-\gamma/\rho} A_k^{1-\gamma} \phi_k(p_k) - 1}{(1-\beta)(1-\gamma)}, \quad (2)$$

where $\phi_k(p_k)$ plays the role of a scaling function of the value of the asset, and is positive. For $k = N$, the value function is also given by (2), with ϕ_N taking the simple form $\phi_N(p) = p^{\gamma-1}$. Beginning with ϕ_N , approximate forms for the functions ϕ_k can be derived sequentially, if model parameters and distributions for the prices are specified. However, here we shall be interested simply in an equation that characterizes ϕ_k .

We introduce the variable \bar{p}_k , a function of the expectation of the scaling function $\phi_k(p_k)$, through the definition $\bar{p}_k = [E\phi_k(p_k^*)]^{1-\rho/1-\gamma}$ for $k \in \{2, \dots, N\}$, where the expectation is with respect to the distribution of p_k^* . The value function V_k can then be written

$$V_k(A_k, p_k) = \max_{A_{k+1}} \frac{\{(1-\beta)(A_k - A_{k+1})^{1-\rho} p_k^{\rho-1} + (1-\beta)\beta A_{k+1}^{1-\rho} \bar{p}_{k+1}\}^{1-\gamma/1-\rho} - 1}{(1-\beta)(1-\gamma)} \quad (3)$$

for $k \in \{1, \dots, N-1\}$. (For $k = N$, recall that the value function is given by (2) with $\phi_N(p) = p^{\gamma-1}$.)

In (3), $\beta \bar{p}_{k+1}$ plays the role of a discount factor, for the value of next period's initial asset level.

The impact of price uncertainty is reflected in \bar{p}_{k+1} . An increase in \bar{p}_{k+1} raises the value of the asset next period. With extraneous terms deleted, the first-order condition is

$$-(A_k - A_{k+1})^{-\rho} p_k^{\rho-1} + \beta A_{k+1}^{-\rho} \bar{p}_{k+1} = 0. \quad (4)$$

It is straightforward to check that the second-order condition for a maximum holds and that in fact the (interior) solution to (4) yields a global maximum. From (4) one can derive expressions for A_{k+1} and $A_k - A_{k+1}$. Substituting into (3) and going through some tedious but straightforward algebra, one has the following:

$$V_k(A_k, p_k) = \frac{(1-\beta)^{1-\gamma/1-\rho} A_k^{1-\gamma} \{p_k^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+1}^{1/\rho}\}^{\rho(1-\gamma)/1-\rho} - 1}{(1-\beta)(1-\gamma)}. \quad (5)$$

Therefore we have the recursive form for the function ϕ_k :

$$\phi_k(p_k) = \{p_k^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+1}^{1/\rho}\}^{\rho(1-\gamma)/1-\rho}, \quad (6)$$

where $\bar{p}_{k+1} = [E\phi_{k+1}(p^*)]^{1-\rho/1-\gamma}$ for $k = 1, \dots, N-1$, (the expression for $\phi_N(p_N)$ is given above).

Using (6) and the definition of \bar{p}_k , we have

$$\bar{p}_k = \{E[p_k^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+1}^{1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} \quad (7)$$

where the expectation is taken with respect to the distribution of p_k . (This distribution remains fixed when \bar{p}_{k+1} changes.) This equation plays a key role in much of the analysis to follow.

For the optimization problem specified here, the pair (A_k, p_k) is given, and the prices p_{k+1}, \dots, p_N are continuous random variables, where k takes any value in $\{1, \dots, N-1\}$. (Hence, in all cases (A_1, p_1) is nonrandom.) As indicated earlier, these prices are independently distributed with finite support $[a, b]$ in \mathfrak{R}_{++} . They are taken as initially identically distributed with distribution function F . When a price has the distribution function F we shall say that the price has “the initial distribution.”

For any period $k = 2, \dots, N$, we use the notation “ $\hat{}$ ” to indicate that the value $\bar{p}_k = \hat{\bar{p}}_k$ is the value of \bar{p}_k defined when the prices p_k, \dots, p_N are iid with the initial distribution. This same notation in the expression $\hat{\phi}_k, k=1, \dots, N$, indicates that this function is defined by (6) with $\bar{p}_{k+1} = \hat{\bar{p}}_{k+1}$.

Increases in uncertainty are introduced by means of independent mean preserving spreads (mps’s) in the distributions of one or more prices, with the initial distribution maintained for the remaining prices. The mps’s for different prices need not be the same. We identify three different types of increase in uncertainty. The first type is that in which there is an mps for p_N only. The second type is that in which an mps occurs for some single price other than p_N . In the final type, the increase in uncertainty takes the form of mps’s for two or more of the prices.³

³ We could articulate this taxonomy so that the three types of increase in uncertainty are nested. The way in which we have chosen to distinguish the types simplifies the formal exposition.

We make use of the following lemmas in the sequel. The proofs can be found in the appendix.

Lemma 1. *For $k=2,\dots,N-1$, \bar{p}_k is monotonically increasing in \bar{p}_{k+1} .*

To avoid repetition, we note here that generally applications of Lemma 1 in the sequel involve a trivial induction. The goal in each case is to show that an increase (decrease) in \bar{p}_{k+r} implies an increase (decrease) in \bar{p}_{k+q} for some $r>q$.

Lemma 2. *Suppose that p_2,\dots,p_N are iid. Then for $k = 2,\dots,N-1$, $\bar{p}_k > \bar{p}_{k+1}$.*

That the sequence of the \bar{p}_k is decreasing is consistent with our interpretation of $\beta\bar{p}_k$ as the discount scalar for the value of the initial asset level in period k . In period k there is one more period left compared with period $k+1$. Hence the discount factor for asset-holding at the outset of period k is larger than its counterpart for period $k+1$, $\beta\bar{p}_{k+1}$.

For an increase in price uncertainty in the last period of life we have the following result.

Proposition 1. *Suppose that there is an mps for p_N . Then for any given $k \in \{1,\dots,N-1\}$ and corresponding realization (A_k,p_k) : (a) If $\gamma < 2$, $V_k(A_k,p_k)$ increases. (b) If $\gamma = 2$, $V_k(A_k,p_k)$ is unchanged. (c) If $\gamma > 2$, $V_k(A_k,p_k)$ decreases.*

Proof: The cases (a)-(c) are broken down into subcases according to the specified range of values for ρ . Demonstrations for all subcases are very similar, so we restrict attention to $\gamma > 2$ and

$\rho < 1$. From $\gamma > 2$, $p_N^{\gamma-1}$ is strictly convex, so that an mps implies that $E p_N^{\gamma-1}$ increases.⁴ This, together with $(1-\rho)/(1-\gamma) < 0$, implies that \bar{p}_N decreases. Using Lemma 1, it is then easy to see that \bar{p}_k decreases for all k . This and $(1-\gamma)/(1-\rho) < 0$ imply that the numerator in (5) increases for any k . Since $1-\gamma < 0$, $V_k(A_k, p_k)$ decreases. //

By way of expansion on the statement of results in Proposition 1, suppose that there is an mps in period N . Then if the consumer finds herself in some period $k \in \{1, \dots, N-1\}$, with some realization (A_k, p_k) , she is unaffected by the increase in uncertainty if $\gamma = 2$; she is better off (worse off) than she would have been in that period, with that realization, if $\gamma < 2$ ($\gamma > 2$).

Intuitively, this result can be understood by noting that the agent consumes everything in the last period of life regardless of the last period price. When the price in the last period of life is observed the agent cannot reverse the decision she made in the previous period and there are no future periods for which to plan. For this reason the welfare effect of price uncertainty only depends on the risk aversion parameter. The elasticity of intertemporal substitution plays no role.

We focus briefly on the different effect of price instability in a static multi-good setting as opposed to a dynamic multi-period setting. In both the multi-good and multi-period models, price instability creates opportunity but also risk for consumers. Under certain circumstances the benefit of the opportunity outweighs the risk. Intuitively price instability in a static multi-good model tends to be more favorable to consumers than that in a comparable multi-period model. This is because in the static multi-good model, consumption for all goods is decided after the realization of prices, whereas in a multi-period model, consumption of some goods is chosen before the realization of uncertain future prices. In comparison, different goods are better

⁴ That the expectation of a strictly convex (strictly concave) function increases (decreases) with an mps follows from the equivalence results in the paper by Rothschild and Stiglitz (1970).

intratemporal substitutes in the static setting than they are intertemporal substitutes in the dynamic setting, and the demands for the goods with uncertain prices are more elastic in the static multi-good setting. We can support this intuition by comparing the following two-good model with the two-period model. In the two-good model, the price of good one is fixed at 1; the price of good 2, p , is uncertain. Consumption decisions for both goods are made after p is observed.

Suppose the utility function is of the constant relative risk aversion type. The consumer's indirect utility function, (defined in terms of the relative price p), is $W(A,p) = \max (c_1^{1-\gamma} - 1)/(1-\gamma) + \beta(c_2^{1-\gamma} - 1)/(1-\gamma)$, where the budget constraint is $c_1 + pc_2 = A$. We are interested in the condition under which the second order partial derivative with respect to price is greater than 0. One can easily verify that $\partial^2 W / \partial p^2 > 0$ at the mean price $p=1$ if and only if $\gamma(\gamma - 2) < \beta^{-1/\gamma}$. If $\beta = 1$ then the threshold level such that the consumer benefits (suffers) from the instability in p is defined by γ smaller (larger) than 2.414. If $\beta < 1$ then the threshold is even larger. Recall that Proposition 1 states that in a corresponding two-period model, when $\gamma = 2$ the consumer is indifferent to instability in the second period price. This example suggests that price instability in multi-period settings is less favorable than price instability in static multi-good settings.

The size of γ in relation to the benchmark value of 2 and the curvature of $\phi_N(p) = p^{\gamma-1}$ are key elements in the statement and proof of Proposition 1. When we turn to the two other types of increase in uncertainty, the situation is similar. Here is the main result for the case of an mps for a single price other than p_N .

Consider the function

$$\xi_{k+1}(p) = (\gamma - 2) - (1/\rho)(\beta^{1/\rho} \hat{p}_{k+2}^{1/\rho} p^{1-\rho/\rho}). \quad (8)$$

Proposition 2 shows that the sign of the function ξ_{k+1} determines the welfare effect of an mps for p_{k+1}^* for some single $k \in \{1, \dots, N-2\}$. The proof is in the appendix.

Proposition 2. *If $\xi_{k+1}(p) < 0$ on (a, b) , then for any $j = 1, \dots, k$ and corresponding realization (A_j, p_j) , $V_j(A_j, p_j)$ increases. If $\xi_{k+1}(p) > 0$ on (a, b) , then for any such j , $V_j(A_j, p_j)$ decreases.*

The intuition for this result dovetails with that for Proposition 1. As is noted earlier in the discussion following Proposition 1, when there is an increase in price uncertainty in the last period, the agent is unable to change consumption decisions made in the past and there are no future periods, so the elasticity of intertemporal substitution (EIS) does not affect the condition under which the increase in price uncertainty increases or reduces consumer welfare. Alternatively, consider the effect of a one-time increase in price uncertainty in period $k+1$ when there are one or more periods left in the agent's life. The agent can make changes in (ex ante plans for) future consumption after observing the price for period $k+1$. The welfare implication of this option depends on the EIS. The role of the EIS depends on the number of periods the agent has left after the increase in price uncertainty is resolved.

From Proposition 2, an increase in uncertainty in the period-($k+1$) price makes the consumer worse off if $\gamma - 2 > (1/\rho)(\beta^{1/\rho} \bar{p}_{k+2}^{1/\rho} p^{1-\rho/\rho})$ for $p \in (a, b)$ and better off with the reverse inequality. Note that under the condition $\gamma = 2$, which according to Proposition 1 makes the consumer indifferent to price uncertainty in the last period, the consumer's welfare is increased when there are more periods left in her life after the realization of increased price uncertainty.

Another useful insight the above inequality yields is the following. According to Lemma 2, \bar{p}_k is decreasing in k . Therefore if k is larger, (that is, when the increase in price uncertainty

occurs later), the inequality is more likely to hold, i.e., the consumer is more likely to be hurt by increased price uncertainty. This is because when there are fewer periods left in the consumer's life after the (now increased) price uncertainty is resolved, the consumer's ability to respond is more limited. A practical implication is that in an economy with agents of different ages, old agents' welfare is more likely to be adversely affected by price uncertainty than that of young agents.

To develop further an intuitive understanding for the results in Proposition 2, we introduce a connection between $\xi_{k+1}(p)$ and the consumer's rate of saving out of current wealth. Let $s_{k+1} \in (0,1)$ denote this saving rate for period $k+1$. A glance at the first-order condition (4) reveals that we have the following relation:

$$\beta^{1/\rho} \frac{\hat{p}_{k+2}^{1/\rho} p_{k+1}^{1-\rho/\rho}}{p_{k+1}} = \frac{s_{k+1}}{1-s_{k+1}}, \quad (9)$$

where p_{k+1} , (replacing p in (8)), denotes the period- $(k+1)$ price. The righthand side of equation (9) is simply the ratio of the period- $(k+1)$ saving rate to the period- $(k+1)$ consumption rate, out of period- $(k+1)$ wealth. The viewpoint offered by equation (9) contributes considerably to our understanding of the welfare impact of price uncertainty.

Combining (8) and (9) gives rise to a threshold condition for price uncertainty to hurt consumers: $(\gamma - 2) > (1/\rho)s_{k+1}/(1-s_{k+1})$. This suggests, in accord with intuition, that increased price uncertainty should be welfare reducing if the consumer is risk averse (γ is large), or reluctant to substitute consumption intertemporally (ρ is large). Corresponding to this intuition, as γ increases, the threshold value of ρ , (that is, the value at which $\xi_{k+1}(p) = 0$ holds for some p and beyond which $\xi_{k+1}(p) > 0$ on (a,b)), should decrease, (i.e. the EIS should increase). It turns out that this is not always the case because the saving rate term is also affected by changes in the parameters γ and ρ . To see why, let γ be fixed at a value above 2, and take the current

price p to be fixed, (to simplify the notation, we suppress this fixed p in what follows). There are two scenarios to be considered.

If as ρ increases the change in the saving rate does not overwhelm the change in $1/\rho$, then the inequality is more likely to hold, so that an increase in price uncertainty makes the consumer worse off. This case often holds when $\beta \hat{p}_{k+2} > 1$, that is, when the saving rate is large. The alternative scenario is that in which, as ρ increases, the change in the saving rate overwhelms the change in $1/\rho$. This is more likely to happen when $\beta \hat{p}_{k+2} < 1$, i.e., when the saving rate is small. Consider the case as $\rho \rightarrow 0$, in which case $(\beta \hat{p}_{k+2})^{1/\rho}$ (the saving rate) converges to 0 faster than ρ does. It follows that $\xi_{k+1}(p)$ is positive when ρ is sufficiently small, i.e., for a given value of the risk aversion parameter, an increase in price uncertainty is more likely to make the consumer worse off when she has a larger EIS. This implication is counterintuitive. The apparent puzzle can be explained when we analyze more closely the benefit of price fluctuations in this case. When the EIS is large and the parameter β is small, the consumer finds it is optimal to consume early. Imagine that it is optimal to consume most of her resources in the first period of life, before realization of any price uncertainty. The "front loading" of the consumption profile means that after the realization of price uncertainty there is not much wealth left to consume. When the uncertain price turns out to be low the consumer's gain is limited by her lack of resources. When the consumer consumes her resources early due to a low discount factor, and with an already large EIS, a larger EIS (i.e., smaller ρ) leads to further front-loading. This effect can offset the benefit of intertemporal flexibility and result in a net reduction in welfare.

We have a corollary to Proposition 2. The proof is in the appendix.

Consider the following conditions:

$$\gamma - 2 < (1/\rho)(\beta^{1/\rho} \hat{p}_N^{1/\rho} a^{1-\rho/\rho}) \text{ for } \rho < 1, \gamma - 2 < (1/\rho)(\beta^{1/\rho} \hat{p}_N^{1/\rho} b^{1-\rho/\rho}) \text{ for } \rho > 1; \quad (10)$$

$$\text{and } \gamma - 2 > (1/\rho)(\beta^{1/\rho} \hat{p}_2^{1/\rho} b^{1-\rho/\rho}) \text{ for } \rho < 1, \gamma - 2 > (1/\rho)(\beta^{1/\rho} \hat{p}_2^{1/\rho} a^{1-\rho/\rho}) \text{ for } \rho > 1. \quad (11)$$

Corollary 1. *Suppose that there is an mps for a price p_{k+1}^* , for some single $k \in \{1, \dots, N-2\}$. Then for any $j = 1, \dots, k$, and corresponding realization (A_j, p_j) , $V_j(A_j, p_j)$ increases if (10) holds, decreases if (11) holds.*

Conditions (10) and (11) are more restrictive than analogous conditions involving the function ξ_{k+1} defined in (8). The main significance of the corollary is that the conditions (10) and (11) are independent of the timing, in addition to the other characteristics, of the particular mps being considered.

Suppose now the third type of increase in uncertainty: independent mps's for two or more prices. Let $h+1$ denote the earliest period in which there is an mps, for p_{h+1}^* , where h takes some value in $\{1, \dots, N-2\}$. If the mps for p_{h+1}^* were the only mps, then we would be in the context of Proposition 2. In that context, the post-mps value of \bar{p}_{h+2} is \hat{p}_{h+2} and the post-mps function ϕ_{h+1} is $\hat{\phi}_{h+1}$. This configuration reflects the fact that there is no increase in uncertainty beyond that in period $h+1$.

In the current context, the post-mps value of \bar{p}_{h+2} , denote it \bar{p}_{h+2}^r , is not equal to \hat{p}_{h+2} . (In the present discussion and in Proposition 3, the superscript "r" denotes post-mps values and functions.) This is because there is at least one period $k \geq h+2$ in which there is an mps for p_k^* . That is, \bar{p}_{h+2}^r differs from the initial value \hat{p}_{h+2} because of a change directly in the distribution of p_{h+2}^* due to an mps and/or because there is an mps for some $k > h+2$, the latter then affecting the value of \bar{p}_{h+2} through the recursive relation (7), (or equivalently, through recursive application

of the definition of \bar{p}_k with ϕ_k determined in (6), $k \geq h+2$). The value $\bar{p}_{h+2} = \bar{p}_{h+2}^r$, and hence also the post-mps function $\phi_{h+1} = \phi_{h+1}^r$, thus depend on the entire set of mps's other than that for p_{h+1}^* . They reflect that part of the increase in uncertainty represented by the set of mps's for periods $k \geq h+2$. The dichotomy here between the increase in uncertainty in period $h+1$ and that in some periods k later than $h+1$ arises because period $h+1$ marks the boundary between periods where there is no increase in uncertainty ($k=1, \dots, h$) and periods in some of which an increase in uncertainty occurs (the boundary period $h+1$ and some of the periods $k \geq h+2$).

In light of the dichotomy just noted, a similarity here to the case of a single mps is the pivotal role played by ϕ_{h+1}^r , with the additional element now of how this function is related to $\hat{\phi}_{h+1}$. The curvature of $\hat{\phi}_{h+1}$ is still important and so the function $\xi_{h+1}(p)$ defined in (8), (with the subscript h replacing k), is important. We have the following result. The proof can be found in the appendix.

Proposition 3. *Suppose that there is a set of independent mps's, not necessarily the same, for two or more prices p_{k+1}^* , where $k \in \{1, \dots, N-1\}$. Let $h+1$ denote the earliest period in which there is an mps, and $h+M$ the last period in which there is an mps, where h takes some value in $\{1, \dots, N-2\}$, M some value in $\{2, \dots, N-h\}$. If $\xi_{h+1}(p) > 0$ on (a, b) , then for any $j=1, \dots, h+M-1$ $V_j^r(A, p) < V_j(A, p)$ for any $A > 0$ and $p \in [a, b]$ (here, V_j is the initial, pre-mps value function for period j). That is, for any such j and corresponding realization (A_j, p_j) , the consumer is worse off.*

The discussion following Proposition 2, (as it relates to the second part of that proposition), carries over to Proposition 3. Part of the intuition that carries over takes the following form. If the consumer is worse off due to an increase in period-($h+1$) price uncertainty (when there are $N-h-1$ periods left in her life), she will be worse off with an increase in price

uncertainty after period $h+1$, when there are fewer periods left in her life for adjustment to the increased uncertainty. Therefore the combination of increases in price uncertainty at and after period $h+1$ should make the consumer worse off if the increase in price uncertainty in period $h+1$ makes her worse off.

Symmetrically, if the consumer benefits from an increase in price uncertainty in period $h+1$, then she benefits from increases in price uncertainty in periods earlier than $h+1$, and from the combination of increases in price uncertainty in periods before and at period $h+1$. The counterpart of Proposition 3 is stated as follows with proof omitted.

Proposition 4. *Suppose that there is a set of independent mps's, not necessarily the same, for two or more prices p_{k+1}^* , where $k \in \{1, \dots, N-1\}$. Let $h+1$ denote the latest period in which there is an mps, where h takes some value in $\{2, \dots, N-1\}$. If $\xi_{h+1}(p) < 0$ on (a, b) , then for any $j=1, \dots, h$, $V_j^r(A, p) > V_j(A, p)$ for any $A > 0$ and $p \in [a, b]$ (here, V_j is the initial, pre-mps value function and V_j^r the post-mps value function for period j). That is, for any such j and corresponding realization (A_j, p_j) , the consumer is better off.*

(In the statement of Proposition 4, we define $\xi_N(p) = \gamma - 2$, i.e. if $h+1=N$, we must assume $\gamma < 2$.) Following upon the result in Proposition 3, another question comes to mind. Suppose initially a single mps in period $h+1$, for some $h \in \{1, \dots, N-2\}$, and suppose that this makes the consumer worse off than she would have been, in any given period $j = 1, \dots, h$, for any given associated realization (A_j, p_j) . That is, for any $j=1, \dots, h$, and corresponding realization (A_j, p_j) , $V_j(A_j, p_j)$ decreases. Is there a further decrease in these $V_j(A_j, p_j)$ when one or more mps's are added for periods later than $h+1$? It turns out that the condition $\xi_{h+1}(p) > 0$ on (a, b) is sufficient for an affirmative answer here. The proof is in the appendix.

Proposition 5. *Suppose that initially there is a single mps in period $h+1$, for some $h \in \{1, \dots, N-2\}$, and suppose that $\xi_{h+1}(p) > 0$ on (a, b) . Denote the resulting value functions $V_j^s, j=1, \dots, h$. (From Proposition 2, we know that $V_j^s(A, p) < V_j(A, p)$ for $A > 0$ and $p \in [a, b]$.) Suppose that one or more further mps's are added for periods $h+i$, for some values of $i \in \{2, \dots, N-h\}$. For the full configuration of mps's, denote the resulting value functions V_j^t . Then for $j=1, \dots, h$, $V_j^t(A, p) < V_j^s(A, p)$ for $A > 0$ and $p \in [a, b]$.*

There are immediate corollaries to Propositions 3-5, analogues to the corollary to Proposition 2. (There is the obvious difference that these corollaries involve conditions (10) and (11) separately. Also, for Proposition 4, the indicated corollary holds only if the restriction $h+1 < N$ is added.) The significance of these corollaries is the same as for the earlier one: conditions (10) and (11) are independent of the timing of the increase in uncertainty being considered. In the interest of brevity, we omit explicit presentation of these additional results.

3. The Infinite Horizon Model

For the case of an infinite horizon, we denote the current period as period 1 with realization (A_1, p_1) . Initially, the prices for all future periods are iid with distribution function F and support $[a, b]$, just as for the N -period case. Additionally, as for the N -period case, independence and the support $[a, b]$ are maintained when prices are not identically distributed. We introduce the subscript $k = 1, 2, \dots$ to index all periods.

The maximization here is strictly analogous to that for the N -period case. For the initial specification, we can drop the subscripts and obtain the resulting value function

$$\begin{aligned}
V(A, p) &= \frac{(1 - \beta)^{1-\gamma/1-\rho} A^{1-\gamma} \phi(p) - 1}{(1 - \beta)(1 - \gamma)} \\
&= \frac{(1 - \beta)^{1-\gamma/1-\rho} A^{1-\gamma} \{p^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}^{1/\rho}\}^{\rho(1-\gamma)/1-\rho} - 1}{(1 - \beta)(1 - \gamma)}, \tag{12}
\end{aligned}$$

with ϕ thus given by

$$\phi(p) = \{p^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}^{1/\rho}\}^{\rho(1-\gamma)/1-\rho}, \text{ where } \bar{p} = [E\phi(p^*)]^{1-\rho/1-\gamma}. \tag{13}$$

Using a suitable limiting argument, we can show that there exists a continuous function ϕ defined on $[a, b]$ satisfying (13). (It is clear from (13) that ϕ is in fact differentiable in p .)

From (13), it is immediate to derive the analogue to (7):

$$\bar{p} = \{E[p^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}^{1/\rho}]\}^{\rho(1-\gamma)/1-\rho} \tag{14}$$

If future prices are not identically distributed, or generally if we want to distinguish among periods when considering value functions or other variables, then (5)-(7), with the corresponding definition of \bar{p}_k , apply.

As there is no final period in the infinite-horizon case, we consider just the second and third types of increase in uncertainty distinguished in the finite-horizon case. That is, we consider the case in which an mps occurs in a single future period and the case in which (independent) mps's occur in multiple future periods, finite in number.⁵

Suppose that there is an mps in some single period $k+1$, where $k \in \{1, 2, \dots\}$. Consider the function ξ defined by

$$\xi(p) = (\gamma - 2) - (1/\rho)(\beta^{1/\rho} \bar{p}^{1/\rho} p^{1-\rho/\rho}). \tag{15}$$

The following proposition for a single mps and an infinite horizon is identical to Proposition 2 except that (15) replaces (8).

⁵ We conjecture that the restriction to a finite number of mps's is not necessary but have not proved this result.

Proposition 6. *If $\xi(p) < 0$ on (a,b) , then for any $j = 1, \dots, k$ and corresponding realization (A_j, p_j) , $V_j(A_j, p_j)$ increases. If $\xi(p) > 0$ on (a,b) , then for any such j , $V_j(A_j, p_j)$ decreases.*

The mps for p_{k+1}^* does not affect \bar{p}_{k+2} and hence does not alter ϕ_{k+1} ; $\bar{p}_{k+2} = \bar{p}$ defined above and $\phi_{k+1}(p) = \phi(p)$ on $[a,b]$, where ϕ is defined in (13). As in the finite-horizon setting, the curvature properties of ϕ are established using the sign of $\xi(p)$ on (a,b) and the size of γ . It is also immediate that Lemma 1 holds for $j = 1, \dots, k$. With these facts in hand, the proof of Proposition 6 is essentially identical to that for Proposition 2. Note that the function ξ is independent of the timing of the mps.

There is a corollary to Proposition 6 that is analogous to that to Proposition 2, the only change being that \bar{p} replaces \hat{p}_N (in condition (10)) and \hat{p}_2 (in condition (11)). Since $\xi(p)$ is already independent of the timing of the mps, the only force of the corollary is to replace the sufficient conditions in Proposition 6 by simpler conditions involving the values a and b .

We now take the case of an increase in uncertainty represented by a set of two or more (but finite in number) mps's in the context of an infinite horizon. For this case, in contrast to the finite horizon, we have results for increases and decreases in consumer well-being that involve the same timing of the mps's. Formally, this is because there is no role in the current context for anything like Lemma 2. Intuitively, the consumer always has infinitely many periods remaining and the initial condition against which the impact of increased price uncertainty is assessed is the same in every period. Hence, beginning with increased price uncertainty in some period k that is beneficial to the consumer, the increased uncertainty in period k combined with increased uncertainty in either earlier periods or later periods will still benefit the consumer. The proof of Proposition 7 is in the appendix.

Proposition 7. *Suppose that there is a finite set of independent mps's, not necessarily the same, for two or more prices p_{k+1}^* . Let $h+1$ denote the earliest period in which there is an mps, and $h+M$ the last period in which there is an mps. Denote the post-mps value functions by V_j^r . If $\xi(p) < 0$ on (a,b) , then for any $j = 1, \dots, h+M-1$, $V_j^r(A, p) > V(A, p)$ for $A > 0$ and $p \in [a, b]$. If $\xi(p) > 0$ on (a,b) , then for any such j , $V_j^r(A, p) < V(A, p)$ for $A > 0$ and $p \in [a, b]$.*

Finally, Proposition 8 below provides an answer to the symmetric version of the question addressed in Proposition 5: will a consumer, already made better off (worse off) due to a single mps in some period, be made still better off (still worse off) with the addition of further mps's in later periods? Because the elements of the proof of Proposition 8 can be easily provided on the basis of the proof of Proposition 5 and that of Proposition 7, we state the proposition without proof.

Proposition 8. *Suppose that initially there is a single mps in period $h+1$, for some $h \in \{1, 2, \dots\}$. Denote the resulting value functions by V_j^s . Suppose that one or more, but a finite number, of further mps's are added for periods $h+i$, with i taking values in some subset of $\{2, 3, \dots\}$. For the full configuration of mps's, denote the resulting value functions by V_j^t . If $\xi(p) < 0$ on (a,b) , then for $j=1, \dots, h$, $V_j^s(A, p) < V_j^t(A, p)$ for $A > 0$ and $p \in [a, b]$. If $\xi(p) > 0$ on (a,b) , then the inequalities are reversed.*

We turn now to consideration of the same general question as that considered in the preceding section: How do the coefficient of risk aversion γ and EIS $1/\rho$ influence the welfare impact of an increase in price uncertainty? As with the finite horizon case, where the relation in (9) holds, we have for an infinite horizon

$$\xi(p) = (\gamma - 2) - (1/\rho)(\beta^{1/\rho} \bar{p}^{1/\rho} p^{1-\rho/\rho}) = (\gamma - 2) - (1/\rho)[s/(1-s)]. \quad (16)$$

As in the finite horizon case, the saving rate is in general a complicated function of γ , ρ , β , and the character of the price uncertainty the consumer faces. In the infinite horizon case the saving rate may be more sensitive to the factors listed above than in the finite horizon case. As a result, the welfare effects of increases in price uncertainty may be counterintuitive. To illustrate this point, consider the case where the support of the initial price distribution is sufficiently small that we can approximate it with a degenerate initial distribution. Given this, without (further) loss of generality, we can then simply set price equal to one. In this case, the saving rate is $\beta^{1/\rho}$, i.e. $\beta^{1/\rho} \hat{p}^{1/\rho} = \beta^{1/\rho} / (1 - \beta^{1/\rho})$. When ρ becomes larger (the EIS decreases), the saving rate increases sufficiently so that the term $(1/\rho)[s/(1-s)]$ is increasing in ρ . Hence, for a given degree of risk aversion, a decrease in the EIS makes it more likely that the consumer will benefit from an increase in price uncertainty. Making use of the “threshold value” concept, employed in discussing the finite horizon case, (see page 10 above), the behavior of the threshold value reflects the counterintuitive nature of the result at hand: the threshold value of γ is increasing in ρ . That is, to maintain a given impact on the consumer of an increase in price uncertainty, a decrease in willingness to substitute consumption intertemporally must be offset by an increase in aversion to risk.

However, such an effect may not be empirically important. For example, setting $\beta = 0.95$, when $\rho = 2$, the threshold γ is 21.2. When $\rho = 10$ the threshold γ is 21.4. In this example, the welfare effect of price uncertainty appears to be determined by the risk aversion factor. We stress though that the result here is based on a very special assumption regarding the character of price uncertainty. Whether it holds for more general settings is a topic for future research.

It is instructive to consider briefly the time-separable case, where $\rho = \gamma$ and the preference representation reduces to the standard expected utility representation. Here, the single

parameter represents simultaneously the degree of risk aversion and the reciprocal of the EIS. For this standard case, the two are inseparable: an increase in the degree of risk aversion is simultaneously a decrease in the EIS. We restrict attention to the case of an infinite horizon and to the case that is most relevant in the present context: $\gamma (= \rho) > 2$. Again making use of the degenerate-distribution approximation, we have the following result: as γ increases, the consumer is more likely to be worse off with an increase in price uncertainty. The most straightforward proof is the following. (It will be easy to see that our proof is logically equivalent to beginning with the standard Bellman equation, assuming time separability and CRRA utility function, and proceeding from there.)

We begin with the function in (15), initially taking γ and ρ as independent parameters, with a general distribution for price, (as in our basic definitions and general results, continuous with support $[a,b]$), and with p in (15) being any price in the support. The next steps are to take partial derivatives in (15) with respect to γ and ρ , then impose $\gamma = \rho$ and the degenerate-distribution approximation, and then add the resulting expressions for the partial derivatives. After some tedious algebra, one finds that the resulting derivative is positive for $\gamma > 2$ and $\beta \in (0,1)$ if the following inequality holds:

$$\beta^{1/\gamma} \ln \beta^{1/\gamma} + \gamma^2 (1 - \beta^{1/\gamma})^2 + \beta^{1/\gamma} (1 - \beta^{1/\gamma}) > 0. \quad (17)$$

Now fix γ , and denote the lefthand side in (17) by $g(\beta)$. The first expression on the lefthand side of (17) converges to 0 as $\beta \rightarrow 0$, so that $g(\beta) \rightarrow \gamma^2$ as $\beta \rightarrow 0$. Also, $g(1) = 0$. Hence, if one can show that $g'(\beta) < 0$, for all $\beta \in (0,1)$, then there can be no zero's for g on $(0,1)$, and the result follows. The derivative $g'(\beta)$ is given by:

$$[\ln \beta^{1/\gamma} + (1 - \beta^{1/\gamma})(1 - 2\gamma^2)](-1/\gamma^2)\beta^{1/\gamma} \ln \beta, \quad (18)$$

which is negative on the domain of interest.

This is the result we expect, consistent with simple intuition for both the impact of an increased degree of risk aversion, (more likely to be made worse off when γ is larger), and simultaneously, inseparably, for the impact of a decreased willingness to substitute intertemporally, (more likely to be made worse off when $1/\rho = 1/\gamma$ is smaller). In a precise fashion, this highlights the potential for increased understanding gained from moving from the standard framework to the non-expected utility framework, with the degree of risk aversion and the EIS parametrically disentangled. The sort of configuration obtained earlier within the latter framework – where simultaneously, the effect with an increase in risk aversion is adverse, while the effect with a decrease in the EIS is favorable – will be missed within the standard framework.

Having said this, however, we note that the outcome for the time-separable case is somewhat suggestive. At least for certain sorts of “small” changes in the degree of risk aversion and the EIS, namely, those where the change in the degree of risk aversion is equal to that in the reciprocal of the EIS, the impact of the change in risk aversion dominates the impact resulting from the (implied) change in the willingness to substitute intertemporally, and the consumer is more likely to be worse off with an increase in price uncertainty. Viewing this from the threshold-value perspective, for the case of a degenerate-distribution approximation for price, and with $\beta=0.95$, the unique value of ρ such that the threshold value for γ is equal to ρ , is 21.48; and a small increase in γ makes the consumer worse off.

We conclude this section of the paper with some brief comments concerning the question of increased uncertainty versus stabilization as it applies to other variables affecting a consumer’s budget constraint. We restrict attention to the case of an infinite horizon.

Consider, first, interest rate uncertainty, (in a setting now with no consumer-goods price uncertainty). It is not difficult to show, (details available upon request), that movement to the non-expected utility framework does not alter, (under otherwise standard assumptions), the

familiar result: increased interest rate uncertainty makes the consumer worse off. The intuition for this result broadly resembles that for the standard model employed in the asset-pricing literature. Should interest rate stabilization become policy-relevant in the context, for example, of partially-privatized social security, any recommendations emerging from the standard model would, at least in broad outline, remain unchanged with movement of the analysis to a non-expected utility framework.

Consider next a more likely candidate with regard to which stabilization recommendations might be forthcoming: income, (excluding here portfolio income). Again assume away consumer-goods price uncertainty. It has turned out, somewhat unexpectedly, that increased income uncertainty constitutes a much more difficult subject for analysis within our framework than interest rate uncertainty. The problem is this: treating income uncertainty in a natural way, (a) there appears to be no useful specification of the value function such that it is separable in (a function of) the state variable wealth and (a function of) income unless the preference setting is changed to constant absolute risk aversion (as in Weil 1993) instead of constant relative risk aversion; and (b) we have not yet found a fruitful analytical route that does not make use of such separability. This remains a subject for ongoing and future research. Awaiting future research is also the analysis, within the present framework, of the case where there is simultaneously price uncertainty, interest rate uncertainty, and, say, labor income uncertainty. Our view is that this is, of course, an ultimate goal, but that a great deal can be learned from treating the three sorts of uncertainty one at a time, in isolated fashion.

4. Numerical Examples

In this section we present three numerical examples. The first involves a single mps and the second multiple mps's. The third involves quantitative measurement, in terms of equivalent consumption, of the potential short-run welfare cost of increasing price uncertainty.

In the first two examples we find combinations of the parameters (γ, ρ) for which mps's in prices make the consumer worse off, using the condition that the function defined in (8), (in this section of the paper denoted simply ξ), is positive, (Propositions 2 and 3).

For simplicity, in the examples, the initial and post-mps distributions are discrete, rather than continuous as assumed in the use made of ξ in the proofs of Propositions 2 and 3. In the numerical examples we have examined, ξ being positive on the relevant discrete set of prices remains a sufficient condition for the consumer to be worse off. One can also think of approximating the discrete distributions arbitrarily closely with continuous distributions.

Example 1. $N=3$, $\beta = .95$. The relevant support is $\{.98, 1, 1.02\}$. The initial distribution assigns probability one to $p_1 = p_2 = p_3 = 1$. An mps is introduced in period 2 with probability of .5 assigned to $p_2 = .98$ and to $p_2 = 1.02$.

(Insert Table 1 here.)

The table shows that, for instance, with risk aversion coefficient γ set at 2.5, the increase in uncertainty in the second period price makes the consumer worse off (in period 1) when the parameter ρ is either larger than 1.968 (i.e., with elasticity of intertemporal substitution (EIS) less than 0.508) or smaller than 0.005 (i.e., with EIS greater than 200).⁶ As the consumer becomes more risk averse (γ increases), the two threshold values of the parameter ρ become closer to each other. Column two shows that the threshold value of ρ decreases as γ increases. An intuitive discussion related to this case was given following Proposition 2. In the present example $\bar{p}_3 = 1$. Hence by Proposition 2 and definition (8) of ξ the increase in price uncertainty lowers consumer welfare if $(1/\rho) \beta^{1/\rho}$ is small enough or $(\gamma-2)$ is large enough. In the case at

⁶ The threshold values for ρ are defined for $p_2 = 1.02$ in both examples. For each parameter pair, such as (2.5, 1.968) in the first example, the value of ξ is positive for $p_2 = 0.98$ and 1.

hand, there is a tradeoff between γ and ρ . As the consumer becomes more risk averse, she is more likely to be made worse off by the increase in price uncertainty unless she has a larger EIS (smaller ρ).

An interesting feature of Table 1 is that the threshold value of ρ is not necessarily decreasing in γ and is not unique. The third column shows that when ρ is close to 0 (EIS is large), then for a given level of risk aversion the consumer is more likely to be worse off with the increase in price uncertainty when she has a larger EIS. As was discussed following Proposition 2, this counterintuitive result is due to the fact that when $\beta\bar{p}_3 < 1$, if her EIS is large the consumer chooses to consume most of her resources early in her life. The "front-loading" consumption profile reduces the potential benefit of increased price uncertainty. For instance, consider the first row of Table 1 with $\gamma=2.5$. Our calculations show that when $\rho=1.968$ expected consumption in the last period of life is about 32.4% of the consumer's lifetime resources; in contrast when $\rho=0.005$ expected consumption in the last period of life is less than 10^{-7} of her lifetime resources. Because third period consumption is so low, even if p_2 turns out to be low the consumer cannot take much advantage of the low price. As ρ gets smaller, consumption becomes more front-loaded and the benefit of a potentially low price is further reduced. Given the degree of risk aversion the consumer becomes more likely to be worse off due to increased price uncertainty when her EIS is larger.

The third column also shows a positive relationship between γ and the threshold value of ρ . When γ becomes larger, the consumer leans toward less front-loading, and so, (connecting with the preceding argument), a given, large EIS contributes less to the likelihood that the consumer is made worse off by an increase in price uncertainty. Thus, the threshold value of the EIS is smaller (the value of ρ is larger).

It should be noted that our example shows that the EIS has to be very large for the unintuitive case to hold. The smallest EIS in the third column is about 100, which is much larger than what most empirical studies find.

Example 2. β , the support, and the initial distribution are as in Example 1. The same mps as in Example 1 is introduced in every period j after the first period ($j=2,3,..N$). Several lengths of lifespan are considered: $N=3,5,7,10$.

(Insert Table 2 here.)

The second columns of Tables 1 and 2 both report the threshold values of ρ for given values of γ ; the former pertains to a single-period mps in second period price 2, the latter to mps's in both second and third period prices. The results are in accord with Proposition 5: if the consumer is made worse off by an mps in price in a single period k , then she is (even) worse off with the addition of mps's for prices after period k . The results suggest that for small increases in price uncertainty the threshold values for the case of multiple mps's are quite similar to those for a single mps. Unlike the case studied in example 1 (where $N=3$), for the cases $N=5,7$, and 10, $(\beta \hat{p}_3)$ is larger than 1, hence the "low-remaining-wealth" scenario does not appear and the threshold value of ρ is decreasing in γ .

Table 2 provides a numerical assessment of the likelihood that an increase in price uncertainty reduces consumer welfare. For example, if $N=7$ (that is, there are six years left in the consumer's life) and if the risk aversion coefficient γ is 2.5, then price uncertainty reduces consumer welfare if ρ is larger than 10.029 (or the EIS is less than 0.1). As just noted above, for a given N , the threshold value of ρ decreases as γ increases. The intuition carries over from the single-mps case. As risk aversion increases, an increase in price uncertainty is more likely to

make the consumer worse off unless the consumer is more willing to substitute consumption intertemporally, (that is, unless the EIS is higher (ρ lower)).

Comparison of different columns of Table 2 shows that as N increases the threshold value of ρ increases for a given value of γ . Intuitively: For a larger value of N there are more periods left after the realization of a given period's price, for instance, the period 2 price. The agent has more time to respond to the observed price. For the consumer to be worse off given the level of risk aversion, the EIS must be lower (ρ must be larger).

Our discussion thus far concerns the qualitative welfare effects of price uncertainty. It is useful to compute the welfare cost of price uncertainty in terms of equivalent consumption. In a related literature of macroeconomics, economists are concerned with the welfare cost of business cycles. Lucas (1987) calculated the welfare cost of unexpected fluctuations in consumption streams based on the standard expected utility model with CRRA. He found that, given the standard deviation of U.S. consumption growth of 1.3%, the representative consumer would trade consumption in the magnitude of 0.4% of GDP for elimination of consumption fluctuations. Dolmas (1998) and Tallarini (2000) suggested that this sort of welfare analysis should be based on a model in which, as in the present paper, the degree of relative risk aversion and the elasticity of intertemporal substitution are parametrically independent. They argued that the primary determinant of the welfare cost of consumption fluctuations is the risk aversion coefficient γ . In the following, maintaining a fixed, "reasonable" value for ρ , we compute the short-run welfare cost of price uncertainty for a "reasonable" range of values for γ . We do this by computing the short-run welfare gain of removing shocks to price with standard deviations of, for example, 5%, over the range of parameter values for γ . Our results indicate that the short run welfare cost of a significant degree of price uncertainty may be substantial but that a moderate degree of price uncertainty is not particularly costly. The following table reports welfare loss

measured by that of a reduction of consumption under various degrees of risk aversion and standard deviation of prices.

(Insert Table 3 here.)

5. Empirical Literature on the Values of Parameters ρ and γ

The foregoing development focuses on the role of risk aversion, measured by the parameter γ , and intertemporal substitution, measured by the inverse of the structurally distinct parameter ρ , in determining the impact of price uncertainty on consumer welfare. It is therefore of interest to briefly review the findings of the empirical literature regarding these parameters.

Studies that focus on elements of asset return data find that γ has to be large. The simulations by Kocherlakota (1990a), Cecchetti et al (1993), and Kandel and Stambaugh (1990), employing a standard expected-utility representation of preferences, conclude that γ should be around 13.7, 29, and 55 respectively, in order to explain the observed equity premium. Weil (1989) derives asset-pricing implications of a model embodying the preference structure used in this paper. With β set at 0.95, he finds that to explain both the observed equity-premium and the risk-free rate in the U.S. economy the risk aversion coefficient γ should be about 45 and the elasticity of intertemporal substitution $1/\rho$ should be about 0.1 (i.e., ρ should be about 10).

Estimation of the parameters γ and ρ is a common exercise in the literature of consumption-based asset pricing theory. Much of the earlier empirical work involves tests of consumption-based asset-pricing models using the state-separable utility function,

$$\sum_{t=0}^{\infty} \beta^t (c_t^{1-\gamma} - 1)/(1-\gamma),$$
 with γ serving the dual roles of coefficient of relative risk aversion and

the inverse of the elasticity of intertemporal substitution. Estimation using the Generalized Method of Moments (GMM) often produces low estimates for γ . For example, Mankiw et al

(1985) find γ to be close to 0.3 and Hansen and Singleton (1982) find γ to be close to 0.8.⁷ Hall (1988) argues that the estimated parameter, γ , of the state-separable utility function actually measures the inverse of the elasticity of intertemporal substitution ρ . He concludes that empirical evidence supports the view that $1/\rho$ is quite small, (in the neighborhood of 0.1). Neely et al. (2002) show that the previous estimates of γ are based on weak instruments, and their estimate of γ is relatively small. Weil (1990) points out that, within the context of the preference structure of the present paper, Hall's interpretation is valid if asset returns are lognormal. Kocherlakota (1990b) argues that consumption growth and asset returns cannot be jointly lognormal if consumer preferences have a non-expected utility representation. He argues that Hall's estimated parameter γ likely reflects risk aversion. Some recent studies employ preference structures that explicitly separate the parameters ρ and γ . Epstein and Zin (1991) find that $1/\rho$ is about 0.2 to 0.9 and γ is close to 1. However, Jorion and Giovannini (1993) find that ρ and γ cannot be precisely estimated. Their estimates of ρ and γ vary considerably, with estimates for ρ as large as 13, and estimates for γ as large as 12, depending on whether the estimation method is maximum likelihood or GMM, and for the latter case, the instruments used for estimation.⁸

In summary, in this literature there is considerable disagreement regarding estimates of the coefficient of relative risk aversion and the elasticity of intertemporal substitution. What can

⁷ Dunn and Singleton (1986), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1990), and Ogaki and Reinhart (1998) show that the estimated EIS depends on, in addition to consumption of nondurables, whether and how consumption of leisure and durable goods are introduced into the utility function.

⁸ Some of Jorion and Giovannini's estimates for ρ and γ are even negative.

be said, from the perspective of the current paper, is that some of the estimates are consistent with increases in intertemporal price uncertainty being welfare reducing, and others, welfare enhancing.

6. Conclusion

In this paper we examine how intertemporal price uncertainty affects consumer welfare. We employ a preference structure in which the coefficient of relative risk aversion and the elasticity of intertemporal substitution (EIS) are parametrically independent. This allows us to highlight the distinct roles they play in determining the impact of increased intertemporal price uncertainty on consumer welfare.

The coefficient of risk aversion (γ) being larger than 2 is a necessary condition for the consumer to suffer from increases in intertemporal price uncertainty. No comparable necessary condition exists for the EIS ($1/\rho$). Given that γ is greater than 2, for an arbitrary pair γ and ρ , younger consumers are less likely to be worse off due to a given increase in price uncertainty than are older consumers. Under empirically plausible circumstances, for a given value of γ if ρ is larger than a corresponding threshold value, then the consumer is worse off with an increase in price uncertainty. In other words, for a given degree of risk aversion, increased price uncertainty reduces consumer welfare if the EIS is sufficiently low. For higher γ the threshold value of ρ is lower; if an increase in the degree of risk aversion is not offset by a large enough increase in the EIS, the consumer suffers from an increase in price uncertainty. However, if the EIS is very large, the relationship can change: as the degree of risk aversion increases, it is possible that an increase in the EIS makes it more likely that increased price uncertainty reduces consumer welfare.

Appendix: Proofs of lemmas and propositions.

Proof of Lemma 1.

This result follows immediately from equation (7), and a case-by-case approach with cases defined by combinations of $\rho > 1$ or < 1 and $\gamma > 1$ or < 1 . The arguments are analogous across cases, so we restrict attention to just one combination of values, $\rho < 1$ and $\gamma > 1$.

We reproduce (7) from the body of the paper,

$$\bar{p}_k = \{E[p_k^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+1}^{1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} \quad (7)$$

where the reader will recall that the expectation is with respect to the distribution of p_k , and that this distribution remains fixed when \bar{p}_{k+1} changes. Suppose that \bar{p}_{k+1} increases. Then the expression in square brackets increases. Since $1 - \rho > 0$ and $1 - \gamma < 0$, the expectation decreases and \bar{p}_k increases. //

Proof of Lemm2.

The proof is by induction. (Implicitly, the proof here covers the four cases defined in the proof of Lemma 1.) Consider first $k = N-1$. Using the definition of \bar{p}_{N-1} and \bar{p}_N in terms of ϕ_{N-1} and ϕ_N , respectively, and making use of equation (7), we obtain

$$\begin{aligned} \bar{p}_{N-1} &= \{E[p_{N-1}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_N^{1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} \\ &> \{E[p_N^{\rho-1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} = \{E p_N^{\gamma-1}\}^{1-\rho/1-\gamma} = \bar{p}_N. \end{aligned}$$

Note that in the term after the inequality, we have substituted p_N for p_{N-1} . Only the notation differs here; the two variables both represent the iid price random variable that is assumed in the present lemma.

Turning now to the general case, for purposes of the formal proof, switch to subscript j , $j = 4, \dots, N$, and suppose that $\bar{p}_{j-1} > \bar{p}_j$. Then we need to show that $\bar{p}_{j-2} > \bar{p}_{j-1}$. The proof is simply the following:

$$\begin{aligned}\bar{p}_{j-2} &= \{E[p_{j-2}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{j-1}^{1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} \\ &> \{E[p_{j-1}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_j^{1/\rho}]^{\rho(1-\gamma)/1-\rho}\}^{1-\rho/1-\gamma} = \bar{p}_{j-1}. //\end{aligned}$$

Proof of Proposition 2.

Proof. Begin with $\phi_{k+1}(p_{k+1})$ defined as in (6). The second derivative with respect to p_{k+1} is

$$\begin{aligned}\phi_{k+1}''(p_{k+1}) &= [\rho(1-\gamma)/(1-\rho)][\rho(1-\gamma)/(1-\rho)-1]\{p_{k+1}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+2}^{1/\rho}\}^{\frac{\rho(1-\gamma)-2}{1-\rho}} [(\rho-1)/\rho]^2 p_{k+1}^{-2/\rho} \\ &\quad + [\rho(1-\gamma)/(1-\rho)]\{p_{k+1}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+2}^{1/\rho}\}^{\frac{\rho(1-\gamma)-1}{1-\rho}} [(\rho-1)/\rho](-1/\rho)p_{k+1}^{\frac{-1}{\rho}-1}.\end{aligned}$$

This expression is <0 (>0) on (a,b) as

$$(\gamma-1)[\rho(1-\gamma)/(1-\rho)-1][(\rho-1)/\rho] + (\gamma-1)(-1/\rho)\{p_{k+1}^{\rho-1/\rho} + \beta^{1/\rho} \bar{p}_{k+2}^{1/\rho}\} p_{k+1}^{1-\rho/\rho} <0$$
 (>0) on (a,b).

We have, after some further manipulation, that this is equivalent to $(\gamma-1)\xi_{k+1}(p_{k+1}) <0$ (>0) on (a,b). It follows that $\phi_{k+1}(p_{k+1})$ is strictly concave (strictly convex) on [a,b] as $(\gamma-1)\xi_{k+1}(p_{k+1}) <0$ (>0) on (a,b). From the recursive structure in the current setting, it follows that the mps for p_{k+1}^* does not affect \bar{p}_{k+2} . Hence, $\bar{p}_{k+2} = \hat{\bar{p}}_{k+2}$ in the definition of ξ_{k+1} , and also ϕ_{k+1} is the function $\hat{\phi}_{k+1}$.

As with Proposition 1, there are a number of subcases. Here, the subcases are defined by the size of γ , the size of ρ , and the sign of $\xi_{k+1}(p_{k+1})$ on (a,b). Again, as with Proposition 1, demonstrations for the various subcases are similar. We therefore restrict attention to a single subcase. We take $\gamma > 2$, $\rho < 1$, and $\xi_{k+1}(p_{k+1}) > 0$, (for which $\gamma > 2$ is a necessary condition).

For this subcase, $\hat{\phi}_{k+1}(p_{k+1})$ is strictly convex in p_{k+1} . Then $E\hat{\phi}_{k+1}(p_{k+1}^*)$ increases with the mps for p_{k+1}^* . With $(1-\rho)/(1-\gamma) < 0$, it follows that \bar{p}_{k+1} falls with the mps. Consequently, from (6), for any given p_k , $\phi_k(p_k)$ increases since $(1-\gamma)/(1-\rho) < 0$. Finally, the fact that $1-\gamma < 0$ implies that for any realization (A_k, p_k) , $V_k(A_k, p_k)$ decreases with the mps.

If $k > 1$, take any $j = 1, \dots, k-1$. Since \bar{p}_{k+1} decreases, Lemma 1 implies that \bar{p}_{j+1} falls for any j . (Note that the random variables p_{j+1}^* , $j=1, \dots, k-1$, remain iid with the initial distribution.) Then as above, for any given p_j , $\phi_j(p_j)$ increases, and for any realization (A_j, p_j) , $V_j(A_j, p_j)$ decreases with the mps.//

Proof of Corollary 1.

From Lemma 2, $\hat{p}_N \leq \hat{p}_{k+2} < \hat{p}_2$. It follows that condition (10) implies $\xi_{k+1}(p) < 0$ for $p = a$, (if $\rho < 1$), and for $p = b$, (if $\rho > 1$). Similarly, condition (11) implies $\xi_{k+1}(p) > 0$ for $p = b$, (if $\rho < 1$), and for $p = a$, (if $\rho > 1$). Because all values of p lie in $[a, b]$, this means that $\xi_{k+1}(p) < 0$ on $[a, b]$ if condition (10) holds and $\xi_{k+1}(p) > 0$ on $[a, b]$ if condition (11) holds. The results for V_j then follow from Proposition 2.//

Proof of Proposition 3.

The proofs for the cases of $\rho < 1$ and $\rho > 1$ are very similar, so we present in detail only the proof for $\rho < 1$. The strategy of the proof is this. First we show that $\bar{p}_{h+k}^r < \hat{p}_{h+k}$ for $k=1, \dots, M$. Then we use $\bar{p}_{h+1}^r < \hat{p}_{h+1}$ to show that $\bar{p}_j^r < \hat{p}_j$ for $j=2, \dots, h$. Finally, from these results we obtain the desired conclusion regarding the value functions. It should perhaps be noted that there is some redundancy of implications in the proof, but we proceed as we do for the sake of a unified logical pattern.

We show by induction that $\bar{p}_{h+k}^r < \hat{p}_{h+k}$ on the set $K = \{1, \dots, M\}$, (note that for some elements $k \in K$, there may not be an mps for p_{h+k}^*). If $M=N-h$, so that $p_{h+M}^* = p_N^*$, then the strict convexity of $\hat{\phi}_N$, (implied by $\gamma > 2$, which is in turn implied by $\xi_{h+1}(p) > 0$), along with $\rho < 1$ and $\gamma > 2$, imply that the mps for p_N^* leads to $\bar{p}_{h+M}^r = \bar{p}_N^r < \hat{p}_N = \hat{p}_{h+M}$. Alternatively,

suppose that $M < N-h$. Then using the definition of M and, as in Proposition 2, the recursive structure in the current setting, we have $\bar{p}_{h+M+1}^r = \hat{p}_{h+M+1}$. From (6), this implies $\phi_{h+M}^r = \hat{\phi}_{h+M}$. Also, from Lemma 2, $\hat{p}_{h+M+1} < \hat{p}_{h+2}$. Since $\xi_{h+1}(p) > 0$ on (a,b), it follows that $\xi_{h+M}(p) > 0$ on (a,b). Thus, just as in Proposition 2, $\hat{\phi}_{h+M}$ is strictly convex on [a,b]. Consequently, using $(1-\rho)/(1-\gamma) < 0$, the mps for p_{h+M}^* leads to $\bar{p}_{h+M}^r < \hat{p}_{h+M}$.

Suppose now that $\bar{p}_{h+k}^r < \hat{p}_{h+k}$ for $k \in \{m, \dots, M\}$, for some m satisfying $M \geq m \geq 2$. Then we want to show that $\bar{p}_{h+m-1}^r < \hat{p}_{h+m-1}$. We know that $\bar{p}_{h+m}^r < \hat{p}_{h+m}$, implying, (using (6) and $(1-\gamma)/(1-\rho) < 0$), that $\phi_{h+m-1}^r(p) > \hat{\phi}_{h+m-1}(p)$ on [a,b]. Also, using Lemma 2, $\xi_{h+1}(p) > 0$ on (a,b) implies $\xi_{h+m-1}(p) > 0$ on (a,b). The now familiar argument then yields $\hat{\phi}_{h+m-1}$ strictly convex on [a,b]. It follows that

$$E\hat{\phi}_{h+m-1}(p^*) \leq E\hat{\phi}_{h+m-1}(p_{h+m-1}^{*r}) < E\phi_{h+m-1}^r(p_{h+m-1}^{*r}), \quad (\text{A1})$$

where p^* is the period-(h+m-1) price with the initial distribution and p_{h+m-1}^{*r} is the post-mps period-(h+m-1) price. (The equality in (A1) holds if there is no mps in period h+m-1.) Using the definition of \bar{p}_{h+m-1}^r and the fact that $(1-\rho)/(1-\gamma) < 0$, one concludes that $\bar{p}_{h+m-1}^r < \hat{p}_{h+m-1}$. Hence, by induction, $\bar{p}_{h+k}^r < \hat{p}_{h+k}$ for $k=1, \dots, M$, and in particular, $\bar{p}_{h+1}^r < \hat{p}_{h+1}$.

The prices for $j=2, \dots, h$ remain iid with the initial distribution. (If $h=1$, skip this step in the proof.) Given this and $\bar{p}_{h+1}^r < \hat{p}_{h+1}$, Lemma 1 implies that $\bar{p}_j^r < \hat{p}_j$ for $j=2, \dots, h$.

To summarize, for any $j=2, \dots, h+M$, $\bar{p}_j^r < \hat{p}_j$. Using (6) and $(1-\gamma)/(1-\rho) < 0$, it follows that for $j=1, \dots, h+M-1$, $\phi_j^r(p) > \hat{\phi}_j(p)$ on [a,b]. Finally, $1-\gamma < 0$ implies that for any such j , $V_j^r(A, p) < V_j(A, p)$ for $A > 0$ and $p \in [a, b]$.

For the case of $\rho > 1$, the proof is strictly analogous to that for $\rho < 1$, the changes being that $(1 - \rho)/(1 - \gamma) > 0$ is used, and that $\bar{p}_j^r > \hat{p}_j$ for $j=2, \dots, h+M$. This completes the proof of the proposition.//

Proof of Proposition 5.

As with the value functions in the statement of the proposition, denote values and functions resulting from the initial mps by a superscript “s” and those resulting from the full configuration of mps’s by a superscript “t”.

The first part of the proof is parallel to the first part of the proof for Proposition 3. The execution -- establishing that $\bar{p}_{h+k}^t < \bar{p}_{h+k}^s = \hat{p}_{h+k}$, (for $\rho < 1$), and that $\bar{p}_{h+k}^t > \bar{p}_{h+k}^s = \hat{p}_{h+k}$, (for $\rho > 1$), but for k only running over 2, ..., M -- is essentially unchanged and so is not repeated here. (The equalities $\bar{p}_{h+k}^s = \hat{p}_{h+k}$, $k=2, \dots, M$, hold, because, as in Proposition 2, the (isolated) mps for p_{h+1}^* does not affect \bar{p}_{h+k} for $k \geq 2$. However, again as in Proposition 2, $\bar{p}_{h+1}^s \neq \hat{p}_{h+1}$.) Also, as for Proposition 3, the pattern of the proof for the case of $\rho > 1$ is strictly analogous to that for $\rho < 1$, so we attend only to the latter case. Finally, we omit many details of implications that are analogous to those in the proofs of Propositions 2 and 3.

Thus, take $\bar{p}_{h+2}^t < \bar{p}_{h+2}^s = \hat{p}_{h+2}$. It follows that $\phi_{h+1}^t(p) > \phi_{h+1}^s(p) = \hat{\phi}_{h+1}(p)$ on [a,b]. This implies that $E\phi_{h+1}^t(p_{h+1}^{*s}) > E\phi_{h+1}^s(p_{h+1}^{*s})$, where p_{h+1}^{*s} is the price in period h+1 after the initial mps. From this last inequality, we have $\bar{p}_{h+1}^t < \bar{p}_{h+1}^s$. This in turn implies that $\phi_h^t(p) > \phi_h^s(p)$ on [a,b]. We then have $V_h^t(A, p) < V_h^s(A, p)$ for $A > 0$ and $p \in [a, b]$.

If $h > 1$, consider periods $j=1, \dots, h-1$. Given that $\bar{p}_{h+1}^t < \bar{p}_{h+1}^s$, Lemma 1 implies that $\bar{p}_{j+1}^t < \bar{p}_{j+1}^s$ for any such j. Then as before, $\phi_j^t(p) > \phi_j^s(p)$ on [a,b], implying that $V_j^t(A, p) < V_j^s(A, p)$ for $A > 0$ and $p \in [a, b]$.//

Proof of Proposition 7.

For the case of $\xi(p) > 0$ on (a,b), the proof is very similar to that for Proposition 3, the changes being that \bar{p} everywhere replaces the \hat{p} values and that in establishing $\bar{p}_{h+k}^r < \bar{p}$, (for $\rho < 1$), and $\bar{p}_{h+k}^r > \bar{p}$, (for $\rho > 1$), no analogue to Lemma 2 is required. Hence, we omit the proof for this case.

The case of $\xi(p) < 0$ on (a,b) has four subcases, (rather than the two for $\xi(p) > 0$ on (a,b)), defined by $\rho > 1$ ($\rho < 1$) and $\gamma > 1$ ($\gamma < 1$). The proofs for these subcases are strictly parallel, with the obvious subcase-specific modifications; and these proofs are also very much parallel to those for $\xi(p) > 0$ on (a,b). So we restrict attention to $\rho < 1$ and $\gamma > 1$, and provide merely a sketch of the proof.

Given the definition of M, $\bar{p}_{h+M+1}^r = \bar{p}$. We then have $\phi_{h+M}^r(p) = \phi(p)$, and we know that ϕ is strictly concave on [a,b], so consequently $\bar{p}_{h+M}^r > \bar{p}$.

An induction on the set $K = \{1, \dots, M\}$ now establishes that $\bar{p}_{h+k}^r > \bar{p}$ for $k \in K$. The induction is analogous to that in the proof of Proposition 3, with (i) \bar{p} replacing the various \hat{p} values, (ii) ϕ replacing the functions $\hat{\phi}_{h+k}$, and with ϕ strictly concave whereas the latter functions are strictly convex in Proposition 3, (iii) no use of a result such as Lemma 2, and (iv) reversal of inequalities where appropriate.

Focusing on the case of $h > 1$, the prices for periods $j = 2, \dots, h$ remain iid with the initial distribution. It then follows from $\bar{p}_{h+1}^r > \bar{p}$ that $\bar{p}_j^r > \bar{p}$ for such j . Thus, $\bar{p}_j^r > \bar{p}$ for $j = 2, \dots, h+M$ and consequently, using $(1-\gamma)/(1-\rho) < 0$, $\phi_j^r(p) < \phi(p)$ on [a,b] for $j = 1, \dots, h+M-1$. Hence, using $1-\gamma < 0$, for all of these $h+M-1$ periods, $V_j^r(A, p) > V(A, p)$ for $A > 0$ and $p \in [a, b]$ //

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Table 1: The combinations of parameters for which the mps in p_2 makes the consumer worse off.

γ	Consumer is worse off with the mps in p_2 when $\rho >$	Consumer is worse off with the mps in p_2 when $\rho <$
2.5	1.968	0.005
5.5	0.247	0.009
7.5	0.143	0.011

Table 2: The combinations of parameters for which the set of multiple mps's makes the consumer worse off.

γ	N=3, consumer is worse off if $\rho >$	N=5, consumer is worse off if $\rho >$	N=7, consumer is worse off if $\rho >$	N=10, consumer is worse off if $\rho >$
2.5	1.968	5.999	10.029	15.074
5.5	.246	.754	1.274	2.069
7.5	.142	.446	.748	1.212

Table 3. Welfare cost of mps of price, in terms of equivalent consumption (N=3, $\rho = 10$)

γ	s.d.(p)=2%	s.d.(p)=5%	s.d.(p)=10%	s.d.(p)=20%
2.5	0.005%	0.039%	0.116%	0.472%
5.5	0.035%	0.214%	0.838%	3.080%
7.5	0.054%	0.335%	1.382%	4.422%