

Fee vs. Royalty Licensing in a Differentiated Cournot Duopoly

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Abstract

This paper studies and compares licensing by means of a fixed fee and licensing by means of a royalty in a differentiated Cournot duopoly model where one of the firms has a cost-reducing innovation. It is found that licensing by means of a royalty may be superior to licensing by means of a fixed fee from the viewpoint of the patent-holding firm. However, fixed-fee licensing is always preferred to royalty licensing by consumers.

Running Title: Fee vs. Royalty Licensing
Keywords: Fee licensing; Royalty licensing; Differentiated Cournot duopoly
JEL Classification: D45

* I thank Bill Yang, Editors Kenneth Kopecky and Sherrill Shaffer and two anonymous referees for their helpful comments and suggestions.

1. Introduction

The literature on licensing has developed mainly along two lines. One strand examines the impact of the possibility of licensing on innovation activities. Major contributions are provided by Gallini and Winter (1985), Katz and Shapiro (1985), and Grossman and Shapiro (1987). All of these papers are only concerned with one form of licensing. For example, Gallini and Winter (1985) consider only royalty licensing, while Katz and Shapiro (1985) consider only fixed-fee licensing. The second strand of the literature has examined alternative forms of licensing policies. Kamien (1992) contains an excellent survey of this strand of the literature. The model that has been mostly studied in the early literature is the licensing of a cost-reducing innovation by a patent holder, which is a non-producer, to existing firms with inferior production technologies. A major finding of the early literature has been that licensing by means of a fixed fee is superior to licensing by means of a royalty for the patent holder (e.g., Kamien and Tauman, 1986). In a recent paper, Wang (1998) compares licensing by means of a fixed fee and licensing by means of a royalty in a homogeneous goods duopoly where one of the firms has a cost-reducing innovation, and shows that, for a non-drastic innovation, royalty licensing may be better than fixed-fee licensing for the patent-holding firm. The purpose of the present paper is to extend the homogeneous goods model considered by Wang (1998) to a differentiated goods duopoly. With imperfect substitutes, we can examine how the incentive to license its innovation changes for the patent-holding firm and how the alternative licensing methods are compared for different degrees of differentiation between the two goods. In addition to generalizing results in the homogenous goods model, a particularly interesting finding here is that the patent-holding firm may even license a drastic innovation when the goods are imperfect substitutes and when that occurs royalty is again the preferred choice.

A patent provides an inventor the opportunity to reap a reward on his or her investment in research and development. For a non-producing patent holder, this reward may be realized through licensing its innovation to the producing firms. For a producing firm, its innovation may gain it an

advantage in competing with its competitors if the innovation is kept for its own use, it may also license the innovation to its competitors to generate licensing revenues. Why does a patent-holding firm have an incentive to license its more advanced technology to its competitor(s)? The answer lies in the tradeoff between the cost advantage for the patent-holding firm as one with the more advanced technology and the efficiency gained by the other firm(s) through licensing the more advanced technology. The patent holder can extract some of this efficiency gain through licensing fees. When the efficiency gain for the licensee(s) overwhelms the cost advantage of the licensor, the latter may be able to achieve a larger total income by licensing its new technology to the other firm(s).¹

The remainder of the paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 study fixed-fee licensing and royalty licensing, respectively. Section 5 compares the two alternative licensing methods. Section 6 contains some further discussions. Section 7 concludes the paper.

2. Model of a Differentiated Duopoly

We study a Cournot duopoly producing differentiated products that are substitutes. For simplicity, we assume linear market demand functions given by

$$p_1 = a - q_1 - dq_2 \quad \text{and} \quad p_2 = a - q_2 - dq_1, \quad (1)$$

where p_1 and p_2 denote, respectively, firms 1 and 2's price and q_1 , q_2 represent their outputs. The substitution coefficient d takes values in the interval $(0,1)$. The two goods are closer substitutes as the substitution coefficient d becomes closer to 1.² With the old technology, both firms have the same constant unit production cost of c ($0 < c < a$). The cost-reducing innovation by firm 1 lowers its unit cost by the amount of ϵ and if applied it will also lower firm 2's unit production cost by the amount of ϵ .

We start our analysis by considering the Cournot duopoly where firm 1 has an unspecified constant unit production cost of c_1 and firm 2 has an unspecified constant unit production cost of c_2 . Results of this model will serve as a reference for deriving results for the alternative licensing models

studied in sections 3 and 4. Throughout the paper subscripts 1 and 2 denote for firm 1 and 2, respectively. Firm 1's profit function is given by $\Pi_1 = (a - q_1 - dq_2 - c_1)q_1$. Choosing q_1 to maximize Π_1 yields firm 1's quantity-reaction function as $q_1 = (a - c_1 - dq_2)/2$. Maximizing firm 2's profit function $\Pi_2 = (a - dq_1 - q_2 - c_2)q_2$ yields firm 2's quantity-reaction function as $q_2 = (a - c_2 - dq_1)/2$. Assuming an interior solution, the intersection of these reaction functions gives the firms' Cournot equilibrium quantities

$$q_1^* = \frac{(2-d)a - 2c_1 + dc_2}{4-d^2} \quad \text{and} \quad q_2^* = \frac{(2-d)a - 2c_2 + dc_1}{4-d^2}. \quad (2)$$

Their equilibrium profits are

$$\Pi_1^* = \left[\frac{(2-d)a - 2c_1 + dc_2}{4-d^2} \right]^2 \quad \text{and} \quad \Pi_2^* = \left[\frac{(2-d)a - 2c_2 + dc_1}{4-d^2} \right]^2. \quad (3)$$

We now go back to consider the model posited at the beginning of this section. We first consider the Cournot equilibrium when firm 1 cannot license its innovation to firm 2. In this case, firm 1 will have in use the new technology while firm 2 will have the old technology. Thus, firm 1's unit production cost is $c-\varepsilon$ and firm 2's unit cost is c . We will need to consider two separate cases: non-drastic and drastic innovations, depending on the magnitude of the innovation (ε). A drastic innovation is one where the innovating firm will become a monopoly if licensing does not occur. In other words, a drastic innovation is one where the monopoly price with the new technology is equal to or less than the unit production cost of the old technology (so that the firm using the old technology is driven out of the market).³ From equation (2), firm 2 will drop out of the market if and only if $\varepsilon \geq (a-c)(2-d)/d$. Hence, any ε that is greater than or equal to $(a-c)(2-d)/d$ corresponds to a drastic innovation.

Consider first a non-drastic innovation (i.e., $\varepsilon < (a-c)(2-d)/d$). In this case, both firms will produce a positive level of output when licensing does not occur. Substituting $c_1 = c-\varepsilon$ and $c_2 = c$ into equations (2) and (3) gives the firms' Cournot equilibrium quantities and their profits (the superscript NL denotes "no licensing")

$$q_1^{NL} = \frac{(2-d)(a-c) + 2\varepsilon}{4-d^2}, \quad q_2^{NL} = \frac{(2-d)(a-c) - d\varepsilon}{4-d^2}, \quad (4)$$

$$\Pi_1^{NL} = \left[\frac{(2-d)(a-c) + 2\varepsilon}{4-d^2} \right]^2, \quad \text{and} \quad \Pi_2^{NL} = \left[\frac{(2-d)(a-c) - d\varepsilon}{4-d^2} \right]^2. \quad (5)$$

Next, consider a drastic innovation (i.e., $\varepsilon \geq (a-c)(2-d)/d$). Equation (2) implies that, with a drastic innovation, firm 2 will drop out of the market, making firm 1 a monopoly. Solving the monopoly problem gives the following quantities and profits

$$q_1^{NL} = \frac{a-c+\varepsilon}{2}, \quad q_2^{NL} = 0, \quad (6)$$

$$\Pi_1^{NL} = \frac{(a-c+\varepsilon)^2}{4}, \quad \text{and} \quad \Pi_2^{NL} = 0. \quad (7)$$

In the following sections, we study two alternative licensing methods: fixed-fee licensing and royalty licensing. We will then compare these alternative licensing methods from the view point of the patent-holding firm.

3. Licensing by Means of a Fee

We consider in this section licensing by means of a fixed fee only. Under a fixed-fee licensing method, firm 1 licenses its cost-reducing technology to firm 2 at a fixed fee F which is invariant of the quantity firm 2 will produce using the new technology. The maximum license fee firm 1 can charge firm 2 is what will make firm 2 indifferent between licensing and not licensing the new technology. In the case that licensing occurs, both firms will produce at constant unit cost $c-\varepsilon$.

The fixed-fee licensing game involves three stages. In the first stage, firm 1 acts as a Stackelberg leader in setting a fixed licensing fee. In the second stage, firm 2 acts as a Stackelberg follower in deciding whether to license the new technology from firm 1 at the fee asked. In the third stage, the two firms choose their outputs simultaneously and noncooperatively.

The third-stage equilibrium is that given in section 2 if licensing does not occur in stage two of the game. To find the third-stage equilibrium when licensing occurs in the second stage of the game, substituting $c_1 = c_2 = c - \varepsilon$ into equations (2) and (3) yields the firms' quantities and profits (the superscript F denotes "fee licensing")

$$q_1^F = q_2^F = \frac{(2-d)(a-c+\varepsilon)}{4-d^2}, \quad (8)$$

$$\Pi_1^F = \Pi_2^F = \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2. \quad (9)$$

With a non-drastic innovation: $\varepsilon < (a-c)(2-d)/d$, by equations (5) and (9), the license fee firm 1 charges will be

$$F = \Pi_2^F - \Pi_2^{NL} = \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2 - \left[\frac{(2-d)(a-c) - d\varepsilon}{4-d^2} \right]^2. \quad (10)$$

Under fixed-fee licensing, firm 1's total income is composed of two components: profits from its own production as given in (9) and licensing revenue from licensing its innovation to firm 2 at a fixed fee as given in (10). Since firm 2 becomes more efficient in production under fee licensing, firm 1 takes a reduction in profits by licensing its new technology to firm 2, i.e., Π_1^F in (9) is less than Π_1^{NL} in (5). Hence, when the licensing revenue is more than enough to compensate for its profit loss firm 1 will license its new technology to firm 2.

More specifically, from (9) and (10), firm 1's total income under fee licensing is

$$\Pi_1^F + F = 2 \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2 - \left[\frac{(2-d)(a-c) - d\varepsilon}{4-d^2} \right]^2. \quad (11)$$

Comparing (5) and (11) we obtain that $\Pi_1^F + F > \Pi_1^{NL}$ if and only if the following condition is satisfied:

$$(8d - 4 - d^2)\varepsilon < 2(2-d)^2(a-c). \quad (12)$$

In particular, if $d \leq 0.8284$ then (12) is satisfied and firm 1 will license its innovation to firm 2.⁴ For $d > 0.8284$, firm 1 is less likely to license its innovation as d approaches one since $2(2-d)^2/(8d-4-d^2)$ is a decreasing function of d for $d > 0.8284$, implying that the range of ε that satisfies (12) becomes smaller

as d becomes bigger. Hence, for any substitution coefficient $d > 0.8284$, firm 1 will not license a sufficiently large innovation. That is, for large innovations, the licensing revenue from fee licensing cannot cover the profit loss for firm 1.

With a drastic innovation: $\varepsilon \geq (a-c)(2-d)/d$, by (7) and (9), the licensing fee is given by

$$F = \Pi_2^F - \Pi_2^{NL} = \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2. \quad (13)$$

So, in this case, the licensing fee is equal to firm 2's total profit. This is so because firm 2 would not be producing at all (hence earning zero profits) if firm 1 does not license its new technology. Hence, firm 1's total income is

$$\Pi_1^F + F = 2 \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2. \quad (14)$$

Comparing (7) and (14), we obtain that $\Pi_1^F + F < \Pi_1^{NL}$ if and only if $d > 0.8284$. Hence, if $0 < d \leq 0.8284$ firm 1 will license its drastic innovation and if $d > 0.8284$ firm 1 will not license its drastic innovation and will become a monopoly. It is implied that, under fixed-fee licensing, firm 1 may license a drastic innovation as long as the two firms' products are not very close substitutes.

To summarize the above results, we have the following proposition.

Proposition 1. Under a fee licensing method, firm 1 will license a non-drastic innovation to firm 2 if and only if condition (12) is satisfied. In particular, firm 1 is less likely to license a non-drastic innovation as the substitution coefficient d approaches 1. With a drastic innovation, firm 1 will license its innovation to firm 2 if $0 < d \leq 0.8284$ and will not license if $d > 0.8284$.

Proposition 1 implies that, under fixed-fee licensing, the patent-holding firm (firm 1) is more willing to license its (non-drastic and drastic) innovation as the two goods are more distant substitutes. In particular, if $0 < d \leq 0.8284$ then it is more advantageous for firm 1 to license its drastic innovation to

firm 2 than to keep it for its own use and become a monopoly. The reason for this result is two fold. On one hand, firm 2's existence does not cut too much into firm 1's profit when their products are not close substitutes. On the other hand, licensing a drastic innovation to firm 2 is likely to generate a handsome license fee for firm 1.

4. Licensing by Means of a Royalty

In this section, we consider licensing by means of a royalty only. Under royalty licensing, firm 1 licenses its new technology to firm 2 at a fixed royalty rate r and the amount of royalty firm 2 pays will depend on the quantity firm 2 will produce using the new technology. In this case, firm 1's unit production cost is $c-\varepsilon$, firm 2's unit production cost is $c-\varepsilon+r$ if it licenses from firm 1 and is c if it does not license. Note that the maximum royalty rate firm 1 can set obviously cannot exceed ε (i.e., $0 \leq r \leq \varepsilon$).

Similar to the fixed-fee licensing game studied above, a royalty licensing game also involves three stages. In the first stage, firm 1 sets a royalty rate. In the second stage, firm 2 decides whether to license the new technology from firm 1 at the royalty rate asked. In the third stage, the two firms choose their outputs simultaneously and noncooperatively. Firm 1 sets the royalty rate to maximize its total income.

The third-stage equilibrium is that given in section 2 if licensing does not occur in stage two of the game. To find the third-stage equilibrium when licensing occurs in the second stage of the game, substituting $c_1 = c-\varepsilon$ and $c_2 = c-\varepsilon+r$ into equations (2) and (3) gives the firms' equilibrium quantities and profits under royalty licensing (the superscript R denotes "royalty licensing")

$$q_1^R = \frac{(2-d)(a-c+\varepsilon)+dr}{4-d^2}, \quad q_2^R = \frac{(2-d)(a-c+\varepsilon)-2r}{4-d^2}, \quad (15)$$

$$\Pi_1^R = \left[\frac{(2-d)(a-c+\varepsilon)+dr}{4-d^2} \right]^2, \quad \text{and} \quad \Pi_2^R = \left[\frac{(2-d)(a-c+\varepsilon)-2r}{4-d^2} \right]^2. \quad (16)$$

Unlike fixed-fee licensing, the licensor is always more efficient than the licensee (firm 2) under royalty

licensing because the royalty rate adds to firm 2's unit cost of production. Still, unless the royalty rate is equal to the value of innovation (i.e., $r = \varepsilon$), firm 2 becomes more efficient in production licensing than not licensing, which implies that firm 1 will incur a loss in profits by licensing its new technology to firm 2, i.e., Π_1^R in (16) is less than Π_1^{NL} in (5) for $r < \varepsilon$. Firm 1 will license its new technology to firm 2 when the royalty income is more than enough to compensate for its profit loss.

More specifically, firm 1's total income is

$$\Pi_1^R + r q_2^R = \left[\frac{(2-d)(a-c+\varepsilon)+dr}{4-d^2} \right]^2 + r \frac{(2-d)(a-c+\varepsilon)-2r}{4-d^2}. \quad (17)$$

From (17), firm 1's profits from own production is an increasing function of r while the licensing revenue is a concave function of r . Accordingly, the total income is either an increasing function of r (implying a corner solution for r) or an inverse U-shaped curve in r (implying an interior solution for r).

Choosing r to maximize firm 1's total income yields the following: If

$$\varepsilon < \frac{8-4d^2+d^3}{8-2d^2-d^3}(a-c), \quad (18)$$

then the optimal $r = \varepsilon$; and if

$$\varepsilon \geq \frac{8-4d^2+d^3}{8-2d^2-d^3}(a-c), \quad (19)$$

then the optimal royalty rate

$$r = \frac{8-4d^2+d^3}{16-6d^2}(a-c+\varepsilon). \quad (20)$$

Consider first a non-drastring innovation (i.e., $\varepsilon < (a-c)(2-d)/d$). If condition (18) is satisfied, substituting $r = \varepsilon$ into (15) and (16) gives the firms' equilibrium quantities and profits under royalty licensing

$$q_1^R = \frac{(2-d)(a-c)+2\varepsilon}{4-d^2}, \quad q_2^R = \frac{(2-d)(a-c)-d\varepsilon}{4-d^2}, \quad (21)$$

$$\Pi_1^R = \left[\frac{(2-d)(a-c)+2\varepsilon}{4-d^2} \right]^2, \quad \text{and} \quad \Pi_2^R = \left[\frac{(2-d)(a-c)-d\varepsilon}{4-d^2} \right]^2. \quad (22)$$

Firm 1's total income is

$$\Pi_1^R + r q_2^R = \left[\frac{(2-d)(a-c) + 2\varepsilon}{4-d^2} \right]^2 + \varepsilon \frac{(2-d)(a-c) - d\varepsilon}{4-d^2}. \quad (23)$$

Comparing (5) and (23), we see that licensing its innovation is better than not licensing for firm 1. Note that from (5) and (22) firm 2 is indifferent between licensing and not licensing from firm 1 in this case.

If condition (19) is satisfied, substituting r in (20) into (16) yields

$$\Pi_1^R = \left[\frac{2-d + d(8-4d^2+d^3)/(16-6d^2)}{4-d^2} \right]^2 (a-c+\varepsilon)^2, \quad \Pi_2^R = \left[\frac{2(1-d)}{8-3d^2} (a-c+\varepsilon) \right]^2. \quad (24)$$

By (17), firm 1's total income is

$$\Pi_1^R + r q_2^R = \left\{ \left[\frac{2-d + d(8-4d^2+d^3)/(16-6d^2)}{4-d^2} \right]^2 + \frac{(1-d)(8-4d^2+d^3)}{(8-3d^2)^2} \right\} (a-c+\varepsilon)^2. \quad (25)$$

It can be shown easily that firm 1's total income given in (25) is greater than that given in (5). Hence, licensing its innovation is better than not licensing for firm 1. Further, it can be verified that firm 2 is better off in this case licensing than not licensing from firm 1.

Consider next a drastic innovation (i.e., $\varepsilon \geq (a-c)(2-d)/d$). It is easy to check that condition (19) is always satisfied when $\varepsilon \geq (a-c)(2-d)/d$. Substituting r in (20) into (17) gives firm 1's total income as given by (25). Straightforward calculations show that firm 1's total income given by (25) is greater than that given by (7) for all d : $0 < d < 1$. Hence, licensing is better than not licensing for firm 1. Further, as in the non-drastic case above, firm 2 is better off in this case licensing than not licensing from firm 1.

We summarize the above results in the following proposition.

Proposition 2. Under a royalty licensing method, firm 1 will always license its (non-drastic and drastic) innovation to firm 2.

Wang (1998) shows that, in a homogeneous goods duopoly, licensing does not occur under royalty licensing when the innovation is drastic since the licensor produces zero output regardless of

whether or not it licenses from the licensor. Proposition 2 demonstrates that this result is an extreme outcome that breaks down if the goods are imperfect substitutes.

5. Comparison: Fee vs. Royalty Licensing

In this section, we evaluate the superiority of fixed-fee licensing versus royalty licensing by allowing the patent-holding firm (firm 1) the freedom to choose either fixed-fee licensing or royalty licensing when it licenses its innovation. As noted above, the licensor's total income is composed of profits from its own production and licensing revenue. Which licensing method is superior for the licensor depends on comparisons between these two sources of income. Since the licensee (the licensor's competitor in the output market) is less efficient under royalty licensing than under fee licensing, the licensor's profit is always higher under royalty licensing, as can be seen by comparing Π_1^R in (16) and Π_1^F in (9). This distortion also implies that the licensing revenue from royalty licensing to be smaller than that under fee licensing. However, as shown below, the net result of this distortion works in licensor's advantage to generate a higher total income under royalty licensing in most instances. The exceptions occur when the two goods are distant substitutes, in which case the distortion is the smallest.

Consider first a non-drastic innovation (i.e., $\varepsilon < (a-c)(2-d)/d$). It has been shown above that, with a non-drastic innovation, firm 1 may or may not license its innovation to firm 2 under fee licensing and it always licenses under royalty licensing. However, there are two optimal royalty rates depending on whether condition (18) or condition (19) is satisfied. Hence, there are four scenarios to analyze in order to compare the two alternative licensing methods.

If condition (12) does not hold (i.e., firm 1 does not license its innovation under fee licensing), firm 1's profit under fee licensing is given by Π_1^{NL} in (5). If (18) is satisfied, firm 1's total income under royalty licensing is given by (23) which is obviously greater than Π_1^{NL} in (5). If (19) is satisfied, firm 1's

total income under royalty licensing is given by (25) which is greater than Π_1^{NL} in (5) by straightforward numerical calculations. Hence, royalty licensing is better than fee licensing for firm 1 if condition (12) does not hold.

If condition (12) holds (i.e., licensing occurs under fee licensing), the comparison between fee licensing and royalty licensing is tedious. We relegate the details to the appendix and summarize the results in the following proposition.

Proposition 3. With a non-drastic innovation, if licensing does not occur under fee licensing then licensing by means of a royalty is superior to licensing by means of a fee for firm 1. If licensing occurs under fee licensing, we have the following:

- (i) if (19) holds then licensing by means of a royalty is superior to licensing by means of a fee for firm 1;
- (ii) if (18) holds and $d > 0.7280$ then licensing by means of a royalty is superior to licensing by means of a fee for firm 1;
- (iii) if (18) holds and $d \leq 0.7280$ then licensing by means of a fee is superior to licensing by means of a royalty for firm 1 for $\varepsilon < (a-c)d(2-d)^2/[(1-d)(4+d^2)]$ and licensing by means of a royalty is superior to licensing by means of a fee for firm 1 for $\varepsilon \geq (a-c)d(2-d)^2/[(1-d)(4+d^2)]$.

Proof. In the appendix.

This proposition demonstrates that, in the case of non-drastic innovation, licensing by means of a royalty is superior to licensing by means of a fee for firm 1 in most instances. The exception (case (iii) in Proposition 3) occurs when both the substitution coefficient and the magnitude of the innovation are small (more specifically, when $d \leq 0.7280$ and $\varepsilon < (a-c)d(2-d)^2/[(1-d)(4+d^2)]$).

The conclusion in Proposition 3 extends the main result in Wang (1998) to a more general differentiated goods model and is in contrast with the result by Kamien and Tauman (1986) which

purports that licensing by means of a fixed fee is superior to licensing by means of a royalty for the non-producing patent holder. The reason for these contrasting results lies in the key difference between the present model and the model of Kamien and Tauman. In the present model the patent holder is also a producer in the industry while the patent holder is an outsider to the oligopolistic industry in their model. An outside patent holder is only interested in the total licensing revenue while a patent-holding firm is interested in its total income (i.e., its licensing revenue as well as its profit).

Consider next a drastic innovation (i.e., $\varepsilon \geq (a-c)(2-d)/d$). It has been shown above that, with a drastic innovation, firm 1 licenses its innovation under royalty licensing for any $0 < d < 1$; under fee licensing, it licenses if $0 < d \leq 0.8284$ and does not license if $d > 0.8284$. Consider first $0 < d \leq 0.8284$. Comparing firm 1's total income under fee licensing, given by (14), with its total income under royalty licensing, given by (25), we obtain that the former is greater than the latter if $0 < d \leq 0.7878$ and the former is less than the latter if otherwise. Consider now $d > 0.8284$. Under fee licensing, firm 1 will not license its innovation and its total income is given by Π_1^{NL} in (7) which is less than its total income under royalty licensing, given by (25), since firm 1 could obtain Π_1^{NL} in (7) by choosing not to license under royalty licensing.

To summarize the preceding results, we have the following proposition.

Proposition 4. With a drastic innovation, licensing by means of a fee is superior to licensing by means of a royalty for firm 1 if $0 < d \leq 0.7878$, licensing by means of a royalty is superior to licensing by means of a fee for firm 1 if $d > 0.7878$.

Proposition 4 demonstrates that in the case of a drastic innovation fee licensing is better than royalty licensing for firm 1 for small values of the substitution coefficient d , royalty licensing is better than fee licensing for large values of d . This proposition also shows that, when both fee licensing and

royalty licensing are considered, firm 1 will always license its drastic innovation.

To conclude this section, we examine consumers' preferences between fixed-fee and royalty licensing. Based on Singh and Vives (1984), the appropriate measure of consumer surplus (CS) for the demand system (1) is

$$CS = [a(q_1 + q_2) - (q_1^2 + 2dq_1q_2 + q_2^2)/2] - [p_1q_1 + p_2q_2],$$

where the terms in the first brackets denote utility while the terms in the second brackets denote consumer expenditure. Applying (1), we obtain

$$CS = (q_1^2 + 2dq_1q_2 + q_2^2)/2. \quad (26)$$

From (8) and (15), the difference between consumer surplus under royalty licensing and consumer surplus under fixed-fee licensing is

$$\begin{aligned} CS^R - CS^F &= \frac{r(4 - 3d^2)}{2(2 - d^2)^2} \left[r - \frac{2(1 + d)(2 - d)^2}{4 - 3d^2} (a - c + \varepsilon) \right] \\ &< \frac{r(4 - 3d^2)}{4(2 - d^2)^2} \left[1 - \frac{4(1 + d)(2 - d)^2}{4 - 3d^2} \right] (a - c + \varepsilon) \\ &< 0, \end{aligned}$$

where the first inequality is obtained by applying the inequality that the optimal royalty rate $r < (a - c + \varepsilon)/2$, which can be derived easily from (18)-(20), and the second inequality is obtained by using straightforward numerical calculations to show that the term in the brackets in the middle line above is less than zero for all d in $[0,1]$. Hence, we have shown that consumers always prefer fixed-fee licensing to royalty licensing. The reason for this conclusion is due to the fact that the licensee is less efficient under royalty licensing compared to fixed-fee licensing. Consequently, the total output produced under royalty licensing is less than that under fixed-fee licensing, which causes consumer surplus to be smaller under royalty licensing.⁵

6. Further Discussions

We may use two distinctive effects involved in a licensing game to offer an explanation of the relevant results in the literature and those found in this paper. The first may be called the strategic effect. Based on Bulow et al (1984), firms' quantity variables in a Cournot competition are strategic substitutes in the sense that each firm's marginal profit from its own output is a decreasing function of the other firm's quantity, while their prices in a Bertrand competition are strategic complements in the sense that each firm's marginal profit from its own price is an increasing function of the other firm's price. Because of the strategic effect, firms are willing to pay more than the direct savings to obtain a lower marginal cost faced with a strategic substitute, while they are willing to pay less than their direct savings for an innovation when faced with a strategic complement. The other effect may be called the cost effect. Relative to royalty licensing, the licensee has a lower marginal cost of production under fee licensing since only a fixed cost is incurred under fee licensing while a royalty adds to the marginal cost of production. In a model of licensing by an independent research unit, the cost effect always works in favor of fee licensing. In Cournot competition, the strategic effect also works in favor of fee licensing, from which follows the result by Kamien and Tauman (1986) that fee licensing is superior to royalty licensing in Cournot competition with an outside licensor. In a Bertrand competition, the strategic effect is in favor of royalty licensing. Furthermore, it can dominate the cost effect for small innovations, from which follows the Muto (1993) result that royalty licensing can be superior to fee licensing for small innovations in Bertrand competition.⁶ In the present model of licensing by an innovating firm, the cost effect works for the licensor under royalty licensing. This effect is large when the innovation is large and/or the goods are close substitutes. This leads to our result that royalty licensing is preferred to fee licensing by the innovating firm when the innovation is large and/or the coefficient of substitution is large.

The most commonly used means of licensing are fixed-fee licensing, per unit royalty licensing,

and a combination of these. According to Rostoker (1984), royalty alone was used thirty-nine percent of the time, fixed fee alone thirteen percent, and royalty plus fixed fee forty-six percent, among the firms surveyed. While this paper has focused on comparing fixed-fee licensing and royalty licensing by a patent-holding firm, the model framework can also be used to study other forms of licensing methods. Here we briefly discuss the widely used fee (F) plus royalty (r) method. Since this includes fixed fee only and royalty only as special cases, from a theoretical point of view, the licensor cannot do any worse using fee plus royalty than using either alone.⁷ Obviously, the fee F does not change the firms' second stage decisions. Hence, their output and profits are the same as those given by (15) and (16) (excluding the fee transfer). The licensor's (firm 1) problem in the first stage is to choose F and r to maximize $\Pi_1^R + r q_2^R + F$, where Π_1^R is given by (16) and q_2^R is given by (15). The maximum fee is to make firm 2 (the licensee) indifferent between licensing and not licensing. That is, $F = \Pi_2^R - \Pi_2^{NL}$, where Π_2^R is given by (16) and Π_2^{NL} is given by (5).⁸ Solving the above maximization problem for firm 1 gives the optimal royalty rate as $r = (a - c + \varepsilon)d(2 - d)^2 / [2(4 - 3d^2)]$. It can be verified that if the innovation is small, in particular, if $\varepsilon \leq (a - c)d(2 - d)^2 / (8 - 2d - 2d^2 - d^3)$ then the optimal royalty rate is $r = \varepsilon$. In this case, the fee must be equal to zero since the licensee's marginal cost of production does not change by licensing. Note that since the optimal royalty rate above is never equal to zero, fee alone can never be optimal under a fee plus royalty system, while royalty alone may still be optimal for small innovations.⁹

Finally, we comment on licensing in an oligopoly. Although we have studied the duopoly model in this paper, the intuitions (especially the two effects mentioned above) should apply similarly to an oligopolistic model with more than two firms and one of them has a cost-reducing innovation. In addition to more complex algebra, the major additional factor that needs to be considered is that the licensor will have to choose how many licenses to sell to its competitors instead of the simple decision of licensing or no licensing in the duopoly model. Another licensing method that has drawn much attention in the literature is auctions. Obviously, licensing by auction and fee licensing are equivalent in the duopoly

model. The same reason and same result as in Kamien (1992) also apply here, namely auction licensing is at least as good as fee licensing for the patent-holder.

7. Conclusion

This paper has studied and compared licensing by means of a fixed fee and licensing by means of a royalty in a differentiated Cournot duopoly where one of the firms has a cost-reducing innovation. A major finding of this paper is that licensing by means of a royalty can be superior to licensing by means of a fixed fee from the viewpoint of the patent-holding firm. Specifically, it is found that either licensing method may be superior to the other for the patent-holding firm and royalty licensing is preferred to fee licensing by the patent-holding firm when goods are close substitutes. It is also found that the patent-holding firm may even license a drastic innovation.

Appendix

Proof of Proposition 3:

The first part of the proposition is straightforward. That firm 1 does not license its innovation under fee licensing implies that firm 1's profit under fee licensing is given by Π_1^{NL} in (5) which is what firm 1 would get if it chose not to license under royalty licensing. The fact that firm 1 always licenses under royalty licensing implies immediately the conclusion that licensing by means of a royalty is superior to licensing by means of a fee for firm 1 if licensing does not occur under fee licensing.

Now consider the second part of the proposition, i.e., firm 1 licenses under fee licensing. If (19) is satisfied, by (11) and (25), the difference between firm 1's total income under fee licensing and that under royalty licensing is given by

$$\begin{aligned}
 (\Pi_1^F + F) - (\Pi_1^R + r q_2^R) &= \left\{ 2 \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2 - \left[\frac{(2-d)(a-c) - d\varepsilon}{4-d^2} \right]^2 \right\} - \\
 &\quad \left\{ \left[\frac{2-d + d(8-4d^2+d^3)/(16-6d^2)}{4-d^2} \right]^2 + \frac{(1-d)(8-4d^2+d^3)}{(8-3d^2)^2} \right\} (a-c+\varepsilon)^2 \\
 &= v(a-c)^2 + \left[\frac{2(2-d)}{(4-d^2)^2} + v \right] 2(a-c)\varepsilon + \left[\frac{4(1-d)}{(4-d^2)^2} + v \right] \varepsilon^2
 \end{aligned} \tag{A1}$$

where

$$v = - \frac{8-4d^2+d^3}{4(4-d^2)^2(8-3d^2)^2} [4(2-d)(8-3d^2) + d^2(8-4d^2+d^3) + 4(1-d)(4-d^2)^2]$$

It is straightforward to check numerically that all coefficients in (A1) are negative for any d in the interval $(0,1)$. This implies result (i) in the proposition.

If (18) is satisfied, by (11) and (23), the difference between firm 1's total income under fee licensing and that under royalty licensing is given by

$$\begin{aligned}
& (\Pi_1^F + F) - (\Pi_1^R + r q_2^R) = \\
& \left\{ 2 \left[\frac{(2-d)(a-c+\varepsilon)}{4-d^2} \right]^2 - \left[\frac{(2-d)(a-c)-d\varepsilon}{4-d^2} \right]^2 \right\} - \left\{ \left[\frac{(2-d)(a-c)+2\varepsilon}{4-d^2} \right]^2 + \varepsilon \frac{(2-d)(a-c)-d\varepsilon}{4-d^2} \right\} \\
& = \frac{-d(2-d)^2(a-c)\varepsilon + (1-d)(4+d^2)\varepsilon^2}{(4-d^2)^2}. \tag{A2}
\end{aligned}$$

It can be verified numerically that for $d > 0.7280$ condition (18) implies that $\varepsilon < (a-c)d(2-d)^2/[(1-d)(4+d^2)]$. Hence, if $d > 0.7280$ then (18) implies that (A2) is negative. This proves result (ii) of the proposition.

Finally, we prove result (iii). It can be numerically verified that $d(2-d)^2/[(1-d)(4+d^2)] < (8-4d^2+d^3)/(8-2d^2-d^3)$ for $0 < d \leq 0.7280$. This means that condition (18) does not imply that $\varepsilon < (a-c)d(2-d)^2/[(1-d)(4+d^2)]$. (Note that condition (12) is either automatically satisfied or implied by condition (18) for $0 < d \leq 0.7280$.) Hence, for $0 < d \leq 0.7280$, if $\varepsilon < (a-c)d(2-d)^2/[(1-d)(4+d^2)]$ then (A2) is positive implying that licensing by means of a fee is superior to licensing by means of a royalty for firm 1, and if $\varepsilon \geq (a-c)d(2-d)^2/[(1-d)(4+d^2)]$ then (A2) is negative implying that licensing by means of a royalty is superior to licensing by means of a fee for firm 1. This completes the proof of the proposition.

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Footnotes

1. There are other reasons for licensing by a patent-holding firm as well. For example, Gallini (1984) shows that a firm may want to license its technology to reduce its competitor's incentive to develop its own, possibly better, technology.
2. Note that as d increases the degree of substitution increases and both demand curves shift in. So results concerning changes in d reflect both of these two effects.
3. Our definition of drastic innovation is the same as the one provided by Kamien and Tauman (1986), both refer to an oligopoly in which a drastic innovation may alter the number of active firms if no licensing occurs. This definition is slightly different from the one provided by Arrow (1962) who deals with licensing to a perfectly competitive industry. In Arrow (1962), an innovation is drastic if the post-innovation monopoly price is below the pre-innovation competitive price.
4. If $d \leq 0.5358$ then $8d - 4 - d^2 \leq 0$, hence (12) is automatically satisfied. For $0.5358 < d \leq 0.8284$, it is easy to verify that $(2-d)/d < 2(2-d)^2/(8d-4-d^2)$; hence (12) is implied by that the innovation is non-drastic.
5. From (8) and (15), $q_1^R + q_2^R = [2(a-c+\epsilon)-r]/(2+d) < 2(a-c+\epsilon)/(2+d) = q_1^F + q_2^F$. Note that since the firms produce equal quantities under fixed-fee licensing but unequal quantities under royalty licensing and the consumer surplus expression in (26) is convex in quantities, it is while intuitive but not obvious that higher total output under fixed-fee licensing implies higher consumer surplus.
6. Wang and Yang (1999) also consider fee and royalty licensing in a differentiated Bertrand duopoly model. In contrast to Muto's (1993) model, the patent-holder in Wang and Yang (1999) is a competitor in the output market. Their results further strengthen and complement those by Muto (1993) in a Bertrand setting.
7. The fact that many firms choose not to use fee plus royalty is due to possibly many other factors that are not considered here. For example, difficulty in monitoring the licensee's output may prevent the licensor to use an output based royalty scheme. On the other hand, as shown by Gallini and Wright (1990), asymmetric information between licensor and licensee about the economic value of the innovation may cause the licensor to use a royalty method.
8. Our consideration here is restricted to the case of non-drastic innovation. The drastic innovation case can be considered in a similar fashion.
9. Note that it is an open question whether fee plus royalty can be an optimal licensing method to achieve the highest possible benefits for the patent-holding firm. Kamien et al (1992) study optimal licensing of a cost-reducing innovation by an independent research lab.