Numerical simulation of laminar forced convection heat transfer of Al$_2$O$_3$–water nanofluid in a pipe with return bend

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A B S T R A C T

Laminar forced convection heat transfer of the Al$_2$O$_3$–water nanofluid in a pipe with a return bend is analyzed by using a finite element method. The results show that the average Nusselt number increases with increasing Reynolds number and Prandtl number, and the increment of specific heat in the nanofluid contributes to the heat transfer enhancement. The average Nusselt number in the return bend appears higher than that in the inlet and outlet pipes due to the secondary flows. However, the pressure drop in the pipe largely increases with the increment of nanoparticle volume concentration. The empirical correlations for the average Nusselt numbers are obtained as functions of the Dean number and the Prandtl number.

1. Introduction

Various industries in many countries have made great efforts to reduce the greenhouse emissions that cause global warming. To achieve this, the active endeavor to increase the energy efficiency has been made in industrial settings. The methods to improve the efficiency of heat exchangers can also be one of these many efforts. In this connection, many researchers have been interested in the geometry of the heat exchanger to get higher thermal efficiency [1–3]. Meanwhile, other research groups have been trying to enhance the heat transfer in pipe systems by using nanofluids. The concept of the nanofluids started from the fact that the thermal conductivity of the fluid containing suspended nanoparticles is improved more than that of the conventional heat transfer fluid [4]. Before the nanoparticles appeared, the researches on the fluid and heat transfer problems had been carried out with millimeter- or micrometer-sized particles [5–7]. These particles were not put to practical use because they may have brought about poor suspension stability, channel clogging, system abrasion and so on.

However, the fluids including nanometer-sized particles do not have those serious problems and the measured thermal conductivity of the nanofluid is anomalously greater than the theoretical prediction [8].

The nanofluid technology has been studied and developed by many research groups worldwide [9,10]. Effects of particle volume concentration, particle material, particle size, particle shape, base fluid material, temperature, additive, and acidity on heat transfer enhancement were investigated experimentally by multiple research groups. Although the heat transfer data were partially inconclusive or conflicting, the heat transfer enhancement appeared in the range of 15–40% [11].

Straight pipes have been mainly utilized in the experimental or numerical studies on the convective heat transfer enhancement using nanofluids. Pak and Cho [12] experimentally investigated the turbulent friction and the heat transfer behaviors of the nanofluids such as γ-Al$_2$O$_3$–water and TiO$_2$–water in a circular tube at the Reynolds numbers of $10^4$–$10^5$. The measured viscosities at the volume concentration of 10% were approximately 200 times greater for the γ-Al$_2$O$_3$–water and 3 times greater for the TiO$_2$–water than for the pure water. Also, the Nusselt number of the nanofluids increased with the augmentation of the volume concentration as well as the Reynolds number. However, the results showed that the convective heat transfer coefficient of the nanofluids was smaller than that of the pure water under the condition of same average velocity.
Xuan and Li [13] measured the convective heat transfer coefficient and the friction factor of the Cu–water nanofluid in the turbulent flow with the Reynolds numbers of 10,000–25,000. The nanofluid had the larger convective heat transfer coefficient than the base fluid at the same Reynolds number. However, the friction factor of the nanofluid was almost same as that of the pure water because of the low volume fractions of 0.3–2.0%. Wen and Ding [14] reported an experimental work on the convective heat transfer of the Al₂O₃–water nanofluid flowing through a copper tube in the laminar flow regime with the Reynolds numbers of 500–2100. The nanoparticles enhanced the convective heat transfer with the increment of the Reynolds number and the particle volume concentration. They also concluded that the enhancement of the convective heat transfer could not be solely attributed to the improvement of the effective thermal conductivity.

Maïga et al. [15] numerically studied the turbulent flow of Al₂O₃–water and Al₂O₃–ethylene glycol nanofluids inside the single uniformly heated tube at the Reynolds numbers of 10,000 and 50,000. It showed that the Al₂O₃–ethylene glycol nanofluid offered a better heat transfer enhancement than the Al₂O₃–water nanofluid. However, the wall shear stress in the Al₂O₃–ethylene glycol nanofluid was higher than that in the Al₂O₃–water. Also, Maïga et al. [16] continued to perform the numerical simulations for the laminar flow with the Al₂O₃–water nanofluid at the Reynolds numbers of 250, 500, and 1000 and with the Al₂O₃–ethylene glycol at the Reynolds numbers of 63.1, 63.1, and 631, respectively. The tendencies of the convective heat transfer coefficient and the wall shear stress were similar to the previous results.

Ding et al. [17] experimentally studied the heat transfer behavior of the CNT(carbon nanotubes)-water nanofluid flowing through a horizontal tube at the Reynolds numbers of 800–1200. Over 350% of the maximum heat transfer enhancement was obtained in the weight concentration of 0.5% at the Reynolds number of 800. It was proposed that the observed large enhancement could be caused by the particle re-arrangement, reduction of thermal boundary layer thickness, very high aspect ratio of CNT, as well as the thermal conductivity. Chun et al. [18] gained the thermal boundary layer thickness, very high aspect ratio of CNT, as of 800. It was proposed that the observed large enhancement was explained to be the high concentration of the CNT(carbon nanotubes)-water nanofluid. The convective heat transfer enhancement was explained to be the high concentration of the CNT because the nanoparticles are ultrafine and thermal characteristics of the Al₂O₃–water nanofluid in the pipes including a return bend with relatively low Reynolds numbers, which are similar to the pipe shape in a heat exchanger system. The reason why the numerical investigation is conducted at relatively low Reynolds is that the effects of the heat transfer in the return bend advance more noticeably in this range than those in the inlet pipe and the outlet pipe. The FEM (finite element method) has been used in solving the governing equations for the fluid and thermal flows. To figure out the heat transfer behaviors of the nanofluid, the average Nusselt numbers will be calculated with the nanoparticle volume concentrations of 0.0, 2.5, 5.0, 7.5, and 10.0% and the Reynolds numbers of 10, 25, 50, 75, and 100. The velocity and temperature distributions will be obtained to understand the role of the return bend, and the pressure drop will be considered to realize the effect of the nanofluid viscosity. In addition, the correlations which can be applied to the heat transfer analysis of the heat exchanger system will be provided in the inlet pipe, the return bend, and the outlet pipe, respectively.

2. Numerical analysis

2.1. Governing equations

The nanofluid consists of Al₂O₃ and water, and the nanoparticles (Al₂O₃) are assumed to be well dispersed within the base fluid (water). Furthermore, the nanofluid can be regarded and analyzed as a single-phase fluid because the nanoparticles are ultrafine and they can be fluidized easily [25]. Also, the motions of the nanoparticles can be neglected and the thermal equilibrium state can be assumed to be predominant [16,22]. Therefore, the following general conservation equations can be used to compute the flow and thermal fields with the effective physical properties of the nanofluid.

\[
\frac{\partial u_k}{\partial t} + u_j \frac{\partial u_k}{\partial x_j} = -\frac{\partial p}{\partial x_k} + \frac{1}{Re} \frac{\partial^2 u_k}{\partial x_i \partial x_i} \tag{1}
\]

\[
\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{1}{Pe} \frac{\partial^2 T}{\partial x_j^2} \tag{2}
\]
The dimensionless conservation equations are presented in equations (1)–(3), which are continuity, momentum, and energy equations, respectively. These equations written in the Cartesian tensor form are for the 3-dimensional incompressible laminar flow. The viscous dissipation and the gravity terms are not considered because these terms make little effects on the heat transfer analysis in the pipe at relatively low Reynolds number.

2.2. Finite element method

The FEM to solve the governing equations can be classified into the mixed interpolation methods, the penalty methods, and the segregated scheme adopted in the present study requires much less execution time and storage than the other two methods, particularly for the three-dimensional configurations. Also, the pressure is computed only once in the stage of the pressure correction without iterative calculations.

Unlike the basic FVM (Finite Volume Method), the FEM can be easily applied to the arbitrary shapes with multi-blocks because it includes the concepts of the coordinate transformation for each element in the local coordinate system. On the other hand, the FEM code used in the present study has a limitation that the stability requirement is needed due to the explicit scheme. Therefore, the time step should be decided taking the Courant number and the limit of the explicit scheme into consideration.

The numerical algorithm by FEM is shown in Fig. 1. The velocity and the pressure are obtained from the continuity and momentum equations through the four stages of the solution procedures. The segregated approaches consist of convective approximation, viscous prediction, pressure correction, and velocity correction [27–29]. The temperature is acquired from the energy equation through the two stages: the convective approximation and the diffusive prediction [30]. When the temperature is calculated, the velocity is also required in the energy equation.

2.2.1. Continuity and momentum equations

In the stage of the convective approximation, the first term and the second convective term on the left-hand side are considered in the momentum equation (2). After the Taylor series expansion, the Galerkin method, and the Green’s theorem are applied to these terms, the discretized equation comes out as shown in equation (4). Here, the first term on the left-hand side is approximated by a diagonal form and the integrations are calculated by the Gaussian quadrature.

\[
\int_{\Omega} N_j \mu D_{ij} d\Omega = \int_{\Omega} N_j \mu D_{ij} d\Omega - \Delta t \int_{\Omega} \frac{\partial \rho}{\partial x_i} d\Omega + \frac{1}{Re} \int_{\Omega} \frac{\partial \rho}{\partial x_i} n_k d\Gamma
\]  

(4)

In the stage of the viscous prediction, the first term on the left-hand side and the second viscous term on the right-hand side are used in the momentum equation (2). The discretized equation is obtained by using the Galerkin method and the Green’s theorem, expressed in equation (5).

\[
\int_{\Omega} N_j \mu D_{ij} d\Omega = \int_{\Omega} N_j \mu D_{ij} d\Omega - \frac{\Delta t}{Re} \int_{\Omega} \frac{\partial N_j \mu D_{ij}}{\partial x_i} d\Omega + \frac{\Delta t}{Re} \int_{\Omega} N_j \frac{\partial \rho}{\partial x_i} n_k d\Gamma
\]  

(5)

In the stage of the pressure correction, the continuity equation is employed to compute the pressure values. At first, only the two terms, which are composed of the first term on the left-hand side and the first pressure term on the right-hand side in the momentum equation (2), are considered. When the gradient of the two terms is taken and then the continuity equation is substituted to this equation, the pressure equation is obtained. The Galerkin method and the Green’s theorem are also applied to the pressure equation. The discretized equation is:

\[
\int_{\Omega} \frac{\partial N_j \rho}{\partial x_j} d\Omega = -\frac{1}{\Delta t} \int_{\Omega} N_j \frac{\partial \rho}{\partial x_j} d\Omega + \int_{\Omega} N_j \frac{\partial \rho}{\partial x_j} n_j d\Gamma
\]  

(6)

On the other hand, the velocity values computed in the viscous prediction stage do not generally satisfy the continuity equation. Therefore, the two terms of the momentum equation used in the pressure correction stage are again taken into account in the stage.
of the velocity correction. When the Galerkin method is applied to this equation, the discretized equation is obtained as:

$$
\int_\Omega N_j u_j^i d\Omega = \int_\Omega N_j u_j^i d\Omega - \Delta t \int_\Omega \frac{\partial T^0}{\partial x_j} d\Omega
$$

(7)

2.2.2. Energy equation

In the stage of the convective approximation, the two terms of the energy equation (3), the first term and second convective term on the left-hand side, are used to calculate the temperature. The discretized equation obtained by the Taylor series expansion, the Galerkin method, and the Green’s theorem is:

$$
\int_\Omega NT^4 d\Omega = \int_\Omega NT^0 d\Omega - \Delta t \int_\Omega Nu_k^0 \frac{\partial T^0}{\partial x_k} d\Omega + \frac{(\Delta t)^2}{2} \int_\Gamma Nu_k^n \frac{\partial T^0}{\partial x_k} n_k d\Gamma
$$

(8)

In the stage of the diffusive prediction, the first term on the left-hand side and the thermal diffusive term on the right-hand side of the energy equation (3) are considered to calculate the final temperature values. When the Taylor series expansion, the Galerkin method, and the Green’s theorem are also applied to this equation, the discretized equation is obtained as:

$$
\int_\Omega NT^4 d\Omega = \int_\Omega NT^0 d\Omega - \frac{\Delta t}{P_c} \int_\Omega \frac{\partial T^0}{\partial x_j} \frac{\partial T^0}{\partial x_j} d\Omega + \frac{\Delta t}{P_c} \int_\Gamma N_j^0 \frac{\partial T^0}{\partial x_j} n_j d\Gamma
$$

(9)

2.3. Validation of FEM code

Before the above finite element method is applied to the hydrodynamic and thermal analyses in the pipes including a return bend, the numerical analysis is carried out for a straight pipe in order to validate the FEM code because it has analytic solutions [31] at steady state. Furthermore, at unsteady state, the results by the FEM code are compared with those by the CFX commercial software [32] based on FVM. Another reason why the straight pipe in the validation is employed is that the straight pipe and the pipe with the return bend have the same configuration and boundary conditions in the computational space. Fig. 2 (a) and (b) show the geometric shapes of the straight pipe and the pipe with the return bend, respectively. These geometric shapes are different in the physical space. However, they have the same shape, which is obtained by conducting the coordinate transformations in order to solve the governing equations, in the computational space as shown in Fig. 2 (c). In addition, the same types of boundary conditions are used for the two geometric shapes. In other words, these calculations are carried out with the same configuration and boundary conditions in the computational space. Only the Jacobian of transformation is different, which is interpreted as the ratio of the volumes in the physical space to that in the computational space. Therefore, the validations by the FEM code in the straight pipe are still available for the pipe with the return bend.

2.3.1. Calculation conditions

The straight pipe, which has the dimensionless diameter of 1 and the dimensionless length of 10, is considered in the numerical analysis. The grids of the straight pipe consist of the nodal points of 5226 and the elements of 4875 for the FEM, and the nodal points of 292,800 and the elements of 301,701 for the CFX, respectively. The fine meshes are used in the CFX for more accurate comparison, which are approximately 60 times as much element as in the FEM.

The boundary conditions for the straight pipe are represented in Table 1. The dimensionless velocity in the axial direction ($w$) and the dimensionless temperature ($T$) are set up to 1 and 0 each at the inlet, and the dimensionless pressure ($p$) is given 0 at the outlet. The dimensionless velocities ($u, v, w$) of 0 and the uniform heat flux ($q_w$) of 0.5 (W/m²) are granted at the wall boundary. The numerical
analysis is performed at the Reynolds number of 14.3 and the Prandtl number of 0.7, which result in a Pecllet number of 10. The calculation is conducted until the iteration is performed up to 10,000 times with the dimensionless time step of 0.001, which is the calculation condition that reaches to the steady state.

2.3.2. Validation results

The dimensionless velocity and temperature profiles calculated by the FEM, the CFX, and the analytic equations are observed in Fig. 3 and Fig. 4. These values are at the middle cross section of the straight pipe, which is the region of the fully developed flow. The dimensionless velocity and the dimensionless temperature predicted by the FEM almost coincide with the results from the CFX with time. Also, the results from the FEM in the steady state are in good agreement with the analytic solutions. Here, the analytic solutions for the straight pipe are obtained by [31]:

\[
\frac{W'}{W_c} = 1 - \left( \frac{r}{r_0} \right)^2
\]

\[
\frac{(T_w - T)}{(T_w - T_c)} = 1 - \frac{4}{3} \left( \frac{r}{r_0} \right)^2 + \frac{1}{3} \left( \frac{r}{r_0} \right)^4
\]

The Nusselt number calculated by the FEM is compared with that by the analytic solutions as illustrated in Table 2. The Nusselt numbers of 4.434 and 4.364 are obtained by the FEM and the analytic solutions, respectively. An error of 1.6% is appeared in the numerical solution and this difference is believed to have occurred due to the grid size effect. Consequently, the numerical solutions by the FEM are acceptable and the FEM code can be used in the numerical analysis for the nanofluid flow in the pipes including a return bend.

3. Flow and heat transfer characteristics of nanofluids

3.1. Geometric shape and calculation conditions

The geometrical configuration of the pipes with the return bend is shown in Fig. 5. The dimensionless diameter \(d\) of the pipe is given 1.0 and the lengths \(l\) of inlet and outlet pipes are set up to 10 times the length of the diameter. Also, the curvature radius \(R\) of the return bend is 1.5 times of the diameter.

The inlet boundary conditions are the dimensionless velocities in the axial direction, \(w = 1\), and the dimensionless temperature, \(T = 0\), as illustrated in Fig. 6. The dimensionless velocities, \(u = v = w = 0\), and the uniform heat flux, \(q_u = 10,000\) \(\text{W/m}^2\), are assigned at the wall boundary of the pipe. Also, the reference pressure, \(p = 0\), is set up at the outlet boundary. Meanwhile, the geometric coordinate values of the pipes with the return bend are obtained from a straight pipe by using coordinate transformations. The relatively fine grids are used in the regions of the inlet and the return bend. The grid points of 11,256 and the elements of 10,450 are used in the entire pipes. The numerical solutions are calculated until the iteration is conducted up to 50,000 times with the dimensionless time step of 0.002 so as to compare with the numerical results of the same computational conditions in the steady state. Here, the dimensionless time step is decided by the convergence conditions of the Courant number and the diffusion number [33].

The physical properties of water and Al2O3 are shown in Table 3, and the effective physical properties of the nanofluids are obtained from the physical properties of the base fluid (water) and the nanoparticles (Al2O3), respectively. The Al2O3—water nanofluid is expected to have the enhanced heat transfer characteristics since the thermal conductivity of Al2O3 is much higher than that of water.

### Table 1

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet</td>
<td>(u = v = 0, w = 1, T = 0)</td>
</tr>
<tr>
<td>Outlet</td>
<td>(p = 0)</td>
</tr>
<tr>
<td>Wall</td>
<td>(u = v = w = 0, qw = -k(\partial T/\partial r))</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Nusselt numbers</th>
<th>Nusselt numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Numerical Solution</strong></td>
<td>4.434</td>
</tr>
<tr>
<td><strong>Analytic Solution</strong></td>
<td>4.364</td>
</tr>
<tr>
<td><strong>Error</strong></td>
<td>1.604 (%)</td>
</tr>
</tbody>
</table>
The effective physical properties of the nanofluid are obtained as functions of nanoparticle volume concentration ($\phi$) [22]:

$$\rho_{\text{eff}} = (1 - \phi) \rho_f + \phi \rho_s$$

(12)

$$\mu_{\text{eff}} = \left(123 \phi^2 + 7.3 \phi + 1\right) \mu_f$$

(13)

$$(C_p)_{\text{eff}} = \frac{(1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s}{(1 - \phi)\rho_f + \phi \rho_s}$$

(14)

$$k_{\text{eff}} = \frac{k_f + (n-1)k_f - (n-1)\phi(k_f-k_s)}{k_s}$$

(15)

The equation (13) of the effective viscosity was obtained by performing a least-square curve fitting of experimental data available for the mixtures, and presented in the literature [15]. Also, the equation (15) of the effective thermal conductivity was introduced by Hamilton and Crosser [34], and was shown to be appropriate for the nanofluids in the literature [35]. Here, the concentration of 0% means the pure water in which the nanoparticles are not included within. The values of effective properties used in this work are tabulated in Table 4.

3.2. Results and discussions

The numerical analysis for the hydrodynamic and thermal flows of nanofluid in the pipes with the return bend has been performed with different concentrations and Reynolds numbers. In total, the 25 cases have been simulated in the conditions of the concentrations of 0.0%, 2.5%, 5.0%, 7.5%, and 10.0% and the Reynolds numbers of 10, 25, 50, 75, and 100, respectively. For better understanding the thermal and fluid flows in the return bend, the dimensionless velocity and temperature distributions at the concentration of 0% are shown in Fig. 7 and Fig. 8 when the Reynolds numbers are 10 and 100, respectively. The velocity profiles at the Reynolds number of 10 keep nearly parabolic shapes in the entire pipes. However, at the Reynolds number of 100, the dimensionless velocity in the outer region of the return bend appears higher than that in the inner region. This flow can affect the heat transfer characteristics in the return bend and in the outlet pipe. Also, the heat transfer is expected to be improved between the inside and the outside of the pipe because the cold flow in the core of the inlet pipe shifts to the outer region of the return bend as shown in Fig. 8.

The dimensionless velocity profiles for the minus x-direction at the cross section (A–A) are represented in Fig. 9. The position of the maximum velocity is moved towards the outer region of the return bend as the Reynolds number increases. The dimensionless temperature at the wall of the return bend is higher than that at the center as shown in Fig. 10 because the heat flux is uniformly supplied at the wall boundary. However, as the Reynolds number increases, the dimensionless temperature declines gradually in the outer region of the return bend due to the cold flow from the inlet pipe.

The secondary flow occurs only in the return bend because of the centrifugal force which acts in the vertical direction of the main flow as shown in Fig. 11. In other words, the two rotating flows at the top and the bottom of the return bend occur due to the different centrifugal forces between in the inner region and in the outer regions of the return bend. Therefore, the secondary flow can be defined as the rotating flow in the circumferential direction. It can be seen in detail that the cold flow moves to the outer region with the increase of the Reynolds number. The dimensionless temperature distribution at the Reynolds number of 10 is different from

| Physical properties of water and Al$_2$O$_3$. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Density (kg/m$^3$) | Viscosity (Ns/m$^2$) | Specific heat (J/kgK) | Conductivity (W/mK) | Shape factor (n) |
| Water           | 998.0           | 1.0020E-03       | 4182.0          | 0.5984          | –               |
| Al$_2$O$_3$     | 3960.0          | –               | 753.0           | 46.0            | 3               |

Fig. 5. Schematic representation for cross section of pipes with return bend.

Fig. 6. Boundary conditions for pipes with return bend.

Table 3
the distributions at the other Reynolds numbers. More heat is supplied into the outer region because the curved length in the outer region is relatively longer than the length in the inner region. Thus, the dimensionless temperature in the outer region at the Reynolds number of 10 appears higher than that in the inner region. However, at the other Reynolds numbers, the dimensionless temperature in the outer region is lower than that in the inner region because the intensity of the secondary flow increased gradually. The velocity ratios of the secondary flow to the main flow rise with the increase of the Reynolds number as represented in Fig. 12.

To investigate the role of the return bend, the entire pipes are divided into three parts: the inlet pipe, the return bend, and the outlet pipe. The average Nusselt numbers at the concentration of 0% are obtained with the increase of the Reynolds number as shown in Fig. 13. Although the slopes appear different, the average Nusselt numbers rise in all parts because the inlet velocity increases with increasing Reynolds number. At the Reynolds numbers of 10 and 25, the average Nusselt numbers of the inlet pipe are relatively higher than those of the return bend and the outlet pipe. However, at the Reynolds numbers of 50, 75, and 100, the average Nusselt numbers of the return bend are larger than those of the inlet and outlet pipes because the cold flow in the inlet pipe moves to the outer region of the return bend as shown in Figs. 7 and 8. In other words, the heat transfer in the return bend improves with the increase of the Reynolds number due to the secondary flow. On the other hand, the pressure drops are compared in each of three parts as shown in Fig. 14. The pressure drops in the return bend are lower than those in the inlet and outlet pipes because the pressure drop depends mainly on the pipe length. Here, the length ratio of the return bend to the straight pipe is about 0.47. The effect of the secondary flow is relatively small on the pressure loss. The pressure drops in the inlet pipe are higher than those in the outlet pipe. It is because the back pressure at the end of the inlet pipe is larger than that at the end of the outlet pipe due to the return bend. As a result, the return bend plays a major role in enhancing the heat transfer without the large pressure loss.

Fig. 15 shows the average Nusselt numbers at different Reynolds numbers of 10, 25, 50, 75, and 100 and concentrations of 0.0%, 2.5%, 5.0%, 7.5%, and 10.0%, respectively. The average Nusselt numbers increase with the rise of the Reynolds number and the concentration. In general, the heat transfer of the nanofluid is improved with the augmentation of the concentration because the thermal conductivity of the nanofluid grows with the concentration as shown in Table 4. In addition, the increment of the concentration is accompanied by the rise of the inlet velocity at the same Reynolds number as expressed in Table 5. Therefore, these conditions cause the increase of the average Nusselt number. It is noted that the results at the concentrations of 7.5% and 10.0% might not be correct in reality because the stable nanofluid could not be found for such high volume concentrations.

For the entire pipes, the increasing rates of the average Nusselt number with the effect of the Reynolds number and the concentration are illustrated in Fig. 16. These increments mean the ratios of the average Nusselt numbers at the nanofluids to the average Nusselt number at the pure water of the concentration of 0.0%. Of course, the average Nusselt numbers continue to increase at the entire pipes with increasing Reynolds number as shown in Fig. 13. Also, at the inlet pipe, the increasing rates continue to be on the rise with increasing Reynolds number as represented in Fig. 17. However, the slope of the average Nusselt numbers between the Reynolds numbers of 25 and 50 is steeper than the slopes at the other Reynolds numbers as shown in Fig. 13. Because the average Nusselt number increases noticeably in the return bend at the Reynolds number of 50. Although the average Nusselt numbers have high values in the return bend at the Reynolds numbers of 75 and 100, they have relatively low values in the inlet and outlet pipe.
pipes. Therefore, the increasing rate of the average Nusselt number at the Reynolds of 50 is higher than the increasing rates at the other Reynolds numbers. Those rates decrease gradually at the Reynolds numbers of 75 and 100. As a result, it is noted that the increasing rate of the average Nusselt number is affected by the return bend. On the other hand, the highest increment, approximately 112%, appears at the Reynolds number of 50 and the concentration of 10.0%. However, this value does not reach up to the increasing rate, 115.2%, for the thermal conductivity of the nanofluid as represented in Table 4.

The pressure drop between inlet and outlet of the pipe is depicted in Fig. 18. Here, the dimensional pressure is used because the dimensionless pressure has the same value at the same Reynolds number. The pressure drop increases dramatically with the increment of the Reynolds number. The increasing rate of the pressure drop at the concentration of 10.0% with respect to the concentration of 0.0% appears nearly 6 times higher at all the Reynolds numbers. This value is considerably high in comparison with the increasing rate of the heat transfer. Consequently, the high pressure drop for the nanofluid should be given careful consideration in designing heat exchangers although the nanofluid does not have the characteristics such as poor suspension stability, channel clogging, system abrasion and so on.

In order to exclude the influence of the increasing inlet velocity caused by the concentration, the numerical analysis has been conducted at the same volumetric flow rate in the pipe. That is, the average Nusselt numbers are obtained in the condition of the same dimensionless inlet velocity at the concentrations of 0.0% and 10.0% as expressed in Fig. 19. The results show that the average Nusselt numbers at the concentration of 0.0% are higher than those at the
concentration of 10.0%. This means that the increment of the inlet velocity can improve the heat transfer in the pipe without using the nanofluids. Meanwhile, the specific heat of the nanofluid decreases with the increase of the concentration as shown in Table 4. For the purpose of investigating the effect of the specific heat, the additional numerical calculation is carried out with the increasing specific heat of the nanofluid. To achieve this, the specific heat, 3132.8 (J/kgK), of the nanofluid at the concentration of 10.0% is increased up to 4182.0 (J/kgK), which is the value of the specific heat at the concentration of 0.0%. The results show that the average Nusselt numbers increase with the specific heat as represented with the symbol ◦ in Fig. 19. It means that the specific heat can play an important role in the heat transfer enhancement of the nanofluid. The pressure drop with respect to the dimensionless inlet velocity is illustrated in Fig. 20. The pressure drop at the concentration of 10.0% is larger than that at the concentration of 0.0% because of the kinematic viscosity of the nanofluid.

In the meantime, the convective heat transfer coefficient is generally required when the temperature distribution is calculated to design heat exchangers. If the average Nusselt number is offered...
with the equation form based on the mean temperature instead of the center temperature in the pipe system, the convective heat transfer coefficient can be easily applied to the numerical analysis. The correlation equations below are obtained by the multiple linear regression analysis as functions of the Dean number and the Prandtl number:

\[
N_{u_i} = 2.2999 \cdot De^{0.1906} \cdot Pr^{0.1950} \quad \text{(inlet pipe)} \tag{16}
\]

\[
N_{u_b} = 1.4039 \cdot De^{1.7533} \cdot Pr^{1.9463} \quad \text{(return bend)} \tag{17}
\]

\[
N_{u_o} = 1.6519 \cdot De^{0.2760} \cdot Pr^{0.2073} \quad \text{(outlet pipe)} \tag{18}
\]

Table 5

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>Nanoparticle volume concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>2.5%</td>
</tr>
<tr>
<td>10</td>
<td>1.0040E-03</td>
</tr>
<tr>
<td>25</td>
<td>2.5100E-03</td>
</tr>
<tr>
<td>50</td>
<td>5.0200E-03</td>
</tr>
<tr>
<td>75</td>
<td>7.5301E-03</td>
</tr>
<tr>
<td>100</td>
<td>1.0040E-02</td>
</tr>
</tbody>
</table>

Fig. 14. Comparisons of pressure drops in various parts.

Fig. 15. Dependence average Nusselt number on concentration at various Reynolds numbers.

Fig. 16. Average Nusselt number ratios in entire pipes.

Fig. 17. Average Nusselt number ratios in inlet pipe.
\[ \text{Nu}_e = 1.8330 \text{De}^{0.2675} \text{Pr}^{0.1900} \text{ (entire pipes)} \] (19)

Here, the distinctions by concentrations are not necessary in the above functions because the Nusselt number, the Dean number, and the Prandtl number are decided by the concentration of the nanofluid with the effective physical properties, regardless of the nanofluid. The adjusted \( R^2 \) squares at the confidence level of 95% in equations (16)–(19), which mean the coefficients of determination, are 0.981, 0.952, 0.960, and 0.968, respectively. The standard deviations are 0.136, 0.509, 0.273, and 0.246. These equations are effective in the range of the Dean numbers of 6–58 and the Prandtl numbers of 7–12.

4. Conclusions

The hydrodynamic and thermal analyses have been performed on the \( \text{Al}_2\text{O}_3 \)–water nanofluid in the pipes with return bend. The results show that the average Nusselt numbers increase with increasing Reynolds number and the nanoparticle volume concentration. Especially, the heat transfer enhancement in the return bend appears larger than that in the inlet and outlet pipes due to the effect of the secondary flow. However, the concentration increment of the nanofluid is accompanied by the high pressure drop in the pipe. Also, the increasing rate of the average Nusselt number is less than that of the thermal conductivity of the nanofluid with the augmentation of the concentration.

Under the same volumetric flow rate, the average Nusselt numbers of the pure water are greater than those of the nanofluid with the concentration of 10%. Also, the increasing rate of the pressure drop in the nanofluid appears larger than the rate in the pure water. Meanwhile, the heat transfer characteristics of the nanofluid are improved with the increase of the specific heat.

The empirical correlation equations for the average Nusselt number are obtained as functions of the Dean number and the Prandtl number in the inlet pipe, the return bend, the outlet pipe, and the entire pipes, respectively. These equations will be expected to be useful in the temperature analysis for designing heat exchangers. In the future study, the hydrodynamic and thermal analyses will be conducted with the consideration of the various parameters of the nanofluid and also the nanoparticles will be simulated from the viewpoint of the molecular dynamics without using the effective physical properties.

Acknowledgments

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Nomenclature

- \( C_p \): specific heat, J/(kgK)
- \( d \): dimensionless diameter of pipe
- \( d^* \): diameter of pipe, m
- \( \text{De} \): Dean number \( (-Re(d/2R)^{1/2}) \)
\( \bar{T}_m \) average convective heat transfer coefficient using mean temperature, W/(m²K) 
\( k \) thermal conductivity, W/(mK) 
\( l \) dimensionless length of inlet pipe or outlet pipe 
\( n \) shape factor of nanoparticle (=3 for spherical nanoparticle) 
\( n_{i,fi}, n_{i,fl} \) unit vectors in Cartesian tensor form \((i,j,k = 1, 2, 3)\) 
\( N \) interpolation function 
\( N_j \) interpolation functions in Cartesian tensor form \((j = 1, 2, 3)\) 
\( Nu \) average Nusselt number (= \( \bar{T}_m d/\bar{k} \)) 
\( p \) dimensionless pressure (= \( p/p_w^* \)) 
\( p^* \) pressure, Pa 
\( Pe \) Peclet number (= \( Re Pr \)) 
\( Pr \) Prandtl number (= \( \bar{C}_p \bar{k}/\bar{\mu} \)) 
\( q_w \) heat flux at wall, W/m² 
\( r \) coordinate in radial direction, m 
\( r_0 \) radius of pipe, m 
\( R \) dimensionless curvature radius of the return bend 
\( Re \) Reynolds number (= \( \bar{p} \bar{w} \bar{d}/\bar{\mu} \)) 
\( \tau \) dimensionless time (= \( \bar{w}/\bar{d} \)) 
\( \tau^* \) time, s 
\( \Delta \tau \) dimensionless time step 
\( \bar{T} \) dimensionless temperature (= \( (T - T_i^*)/(T_{w}^* - 273.15) \)) 
\( T_{w}^* \) temperature, K 
\( T_i^* \) temperature at inlet, K 
\( T_e \) temperature at center, K 
\( T_{w} \) temperature at wall, K 
\( u_{x,y}, w \) dimensionless velocities in \( x,y,z \) directions (=velocity/\( \bar{w} \)) 
\( u_{i,fi}, u_{i,fl} \) dimensionless velocities in Cartesian tensor form (=velocity/\( \bar{w} \), \( i,j,k = 1, 2, 3 \)) 
\( \bar{w} \) axial velocity, m/s 
\( \bar{w}_c \) axial velocity at center, m/s 
\( x, y, z \) dimensionless coordinates (=coordinate/\( \bar{d} \)) 
\( x_i, y_i, z_i \) dimensionless coordinates in Cartesian tensor form (=coordinate/\( \bar{d} \), \( i,j,k = 1, 2, 3 \))

**Greek symbols**

\( \Gamma \) boundary of the domain \( \Omega \) 
\( \mu \) dynamic viscosity, (Ns)/m² 
\( \xi, \eta, \zeta \) coordinates in computational space 
\( \rho \) density, kg/m³ 
\( \phi \) nanoparticle volume concentration, % 
\( \Omega \) bounded domain

**Superscripts**

\( c \) refers to result from convective approximation step 
\( n \) refers to result from pressure correction step and velocity correction step 
\( o \) refers to result from previous time step 
\( v \) refers to result from viscous prediction step

**Subscripts**

\( b \) refers to return bend 
\( \text{eff} \) refers to effective physical property 
\( f \) refers to base fluid (water) 
\( i \) refers to inlet pipe 
\( nf \) refers to nanoparticle flow 
\( o \) refers to outlet pipe 
\( s \) refers to nanoparticle 
\( t \) refers to entire pipes

**References**


