Two- and three-dimensional numerical simulations of natural convection in a cylindrical envelope with an internal concentric cylinder with slots

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\textbf{A B S T R A C T}

Two-dimensional and three-dimensional numerical simulations were carried out to simulate the natural convection in a cylindrical envelope with an internal concentric cylinder with slots. For the case of steady flow, the numerical solutions in 2D and 3D simulations are in good agreement with the experimental results. When the convection in experiment becomes unsteady convection at the larger slot degree, the oscillated solutions in 3D simulation differ essentially from steady solutions in 2D simulation. The critical Rayleigh numbers from steady to unsteady flow in the 3D simulations are lower than those in the 2D simulations.

1. Introduction

The enclosed isolated-phase busbar used for transmitting large electric current consists of two horizontal concentric metal cylinders. The heat generated in the busbar from the Joule heating is transferred to the outer envelope by radiation and natural convection. The unsteady natural convections are well studied in horizontal concentric cylindrical annuli. Cheddadi et al. \cite{1} studied the two-dimensional natural convection bifurcation in a horizontal annulus, and showed that flow pattern was not unique and depended on the initial conditions at high Rayleigh number. Liu et al. \cite{2} investigated the critical Rayleigh number dictates the transition from steady to unsteady. Yoo \cite{3} investigated the bifurcation sequences to the chaos for the natural convection in horizontal concentric annuli using two-dimensional simulation. Adachi and Imai \cite{4} investigated three-dimensional linear stability of natural convective flow between concentric horizontal cylinders. Ridouane et al. \cite{5} numerically studied the multiple 3-D flow regimes including conduction, steady convection, and unsteady convection in a loop as the Rayleigh number was increased.

Comparatively, little works have been reported on unsteady natural convection in more complex domain, such as in a cylindrical envelope with an internal concentric cylinder with slots. Kuleek \cite{6} assumed that the heat transfer enhancement of a cylindrical envelope with an internal slotted cylinder was expected to be about 30–40% higher than that of the concentric cylindrical annuli. Wang et al. \cite{7} experimentally investigated natural convection in a cylindrical envelope with an internal slotted cylinder and found that the convective heat transfer coefficient with slots could be enhanced by as much as 50%. Zhang et al. \cite{8–9} investigated nonlinear characteristics of natural convection in two-dimensional numerical simulation and experiment. The results indicated that the oscillatory flow undergoes several bifurcations and ultimately evolves to a chaotic flow. However, their results covered only the two-dimensional cases. It is not known whether the results in two-dimensional simulations are able to predict the non-linear behavior obtained in real three-dimensional domain.

The objective of this paper is to study natural convection heat transfer in a cylindrical envelope with an internal concentric cylinder with slots using two-dimensional and three-dimensional approaches. Two- and three-dimensional numerical simulations of the natural convection at the same parameters will be performed in order to study the differences between them. The numerical solutions from both approaches will be compared in detail for steady and unsteady flow and heat transfer.

2. Problem formulation

A schematic diagram of the physical model under consideration is shown in Fig. 1. The inner and outer cylinders are kept at uniform but different temperatures \( T_i \) and \( T_o \), respectively, with \( T_i > T_o \).
As a result of the temperature difference between the two cylinders, density gradient occurs which leads to natural convection. It is assumed that the fluid in the enclosure is of a Boussinesq type.

When it is assumed that the flow and heat transfer are two-dimensional, the physical model and governing equations of two-dimensional numerical simulations were introduced by Ref. [8]. The dimensionless governing equations of three-dimensional numerical simulations can be written as:

\[
\frac{\partial}{\partial R} \left( \frac{V}{R} \right) + \frac{1}{R} \frac{\partial U}{\partial \theta} + \frac{\partial W}{\partial Z} = 0
\]

(1)

\[
\frac{\partial U}{\partial R} + \frac{U}{R} \frac{\partial U}{\partial \theta} + \frac{W}{R} \frac{\partial U}{\partial Z} - \frac{\partial P}{\partial R} = \frac{Pr}{(RePr)^{1/2}} \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial Z^2} \right] + \frac{S_u}{R}
\]

(2)

\[
\frac{\partial W}{\partial R} + \frac{U}{R} \frac{\partial W}{\partial \theta} + \frac{W}{R} \frac{\partial W}{\partial Z} = - \frac{\partial P}{\partial Z} + \frac{Pr}{(RePr)^{1/2}} \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial W}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 W}{\partial \theta^2} + \frac{\partial^2 W}{\partial Z^2} \right]
\]

(3)

\[
\frac{\partial}{\partial R} \left( \frac{\partial \Theta}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( R \frac{\partial \Theta}{\partial \theta} \right) + \frac{\partial^2 \Theta}{\partial Z^2} = \frac{k}{(RePr)^{1/2}} \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Theta}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 \Theta}{\partial \theta^2} + \frac{\partial^2 \Theta}{\partial Z^2} \right]
\]

(4)

\[
\frac{\partial}{\partial R} \left( \frac{\partial \Theta}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( R \frac{\partial \Theta}{\partial \theta} \right) + \frac{\partial^2 \Theta}{\partial Z^2} = K \frac{1}{(RePr)^{1/2}} \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Theta}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 \Theta}{\partial \theta^2} + \frac{\partial^2 \Theta}{\partial Z^2} \right]
\]

(5)

where the source terms are

\[
S_u = -\frac{UV}{R} + \Gamma \left( \frac{U}{R} + \frac{2}{R^2} \frac{\partial V}{\partial \theta} \right) - \Theta \sin \theta
\]

(6)

\[
S_R = \frac{U^2}{R} + \Gamma \left( \frac{V}{R} + \frac{2}{R^2} \frac{\partial U}{\partial \theta} \right) + \Theta \cos \theta
\]

(7)

The Rayleigh number \( Ra \), Prandtl number \( Pr \) and thermal conductivity \( K \) are defined as

\[
Ra = \frac{\beta g L (T_i - T_0)}{\alpha v}, \quad Pr = \frac{v}{\alpha}, \quad K = \frac{k_s}{k_f}
\]

(8)

where \( \beta, a, v, k_s \) and \( k_f \) are thermal expansion coefficient, thermal diffusivity, kinematic viscosity of the fluid, thermal conductivity for solid and fluid, respectively. \( L \) is the gap width of annulus, and \( g \) is the gravitational acceleration.

The boundary conditions for Eqs. (1)–(5) are as following:

\[
\frac{\partial W}{\partial R} = 0, \quad \frac{\partial \Theta}{\partial R} = 0 \quad \text{at} \quad R = 0
\]

(9)

\[
U = V = W = 0, \quad \Theta = 0 \quad \text{at} \quad R = R_o
\]

(10)

\[
U = V = W = 0, \quad \frac{\partial \Theta}{\partial Z} = 0 \quad \text{at} \quad Z = 0
\]

(11)

\[
U = V = W = 0, \quad \frac{\partial \Theta}{\partial Z} = 0 \quad \text{at} \quad Z = LM
\]

(12)

The following periodical boundary conditions are given at \( \theta = 0 \) and \( \theta = 2\pi \):

\[
U(\theta = 0) = U(\theta = 2\pi), \quad V(\theta = 0) = V(\theta = 2\pi)
\]

\[
W(\theta = 0) = W(\theta = 2\pi), \quad \Theta(\theta = 0) = \Theta(\theta = 2\pi)
\]

(13)

Apart from the above boundary conditions, the following boundary conditions for the internal solid cylinder must be satisfied

\[
U = V = W = 0, \quad \Theta = 1
\]

(14)

The initial conditions are

\[
Fo = 0, \quad U = V = W = 0, \quad \Theta = 0
\]

(15)

To observe the total heat transfer effect, the average dimensionless equivalent thermal conductivity based on the entire outer circle \( (K_{eq}) \) is defined as

\[
K_{eq} = \frac{1}{2\pi LM} \int_0^LM \int_0^{2\pi} \frac{\partial \Theta}{\partial R} R_o \ln \frac{R_o}{R_i + \delta} \, dR \, d\theta
\]

(16)

The slot width is defined as

\[
S = \frac{2\pi}{\pi}
\]

(17)

3. Results and discussions

The governing equations were discretized by the finite volume method. The simple algorithm with quick scheme was used for handling the pressure velocity coupling. The numerical simulations have been performed in two-dimensional and three-dimensional cases at experimental parameters in Ref. [8]. The grid-indepen-
dence of the numerical results is studied for $Ra = 10^5$. The $52 \times 102$ and $52 \times 52 \times 102$ grid points are adequate to yield accurate results for 2D and 3D case, respectively. The time step is selected as 0.01 for the present computations. The average dimensionless equivalent thermal conductivities $K_{\text{eq}}$ at different Rayleigh numbers are calculated numerically and shown in Fig. 2. The computational results indicate that for the case that the flow is steady at $Ra = 10^4$, the numerical results obtained by both 2D and 3D models agreed very well with those obtained experimentally by Wang et al. [7], and the differences between the two models are negligible. However, the computational time for two- and three-dimensional simulations is 7.8 and 112.5 s, respectively. Obviously, two-dimensional simulation is satisfactory and gives a good compromise between precision and CPU time when the flow remains steady. The differences of thermal conductivities $K_{\text{eq}}$ obtained by two-dimensional simulation and experiment become larger with the increasing Rayleigh numbers, while the numerical results obtained by three-dimensional simulations still agreed very well with experimental results, especially when the flow and heat transfer is unsteady at $Ra = 10^5$. It is believed that the reason is related to the discrepancy of non-linear characteristics in two- and three-dimensional systems. The flow loses its stability and allows the non-periodic solution to appear at $Ra = 10^6$ in three-dimensional simulation. However, the solutions oscillate periodically in two-dimensional simulation.

On further investigations, the temperature fields are obtained by using laser holographic interferometry technique for $R_s = 1.712$, $R_t = 0.712$, $\delta/L = 0.159$, $Pr = 0.701$, $LM = 9.5$, $S = 0.472$, $Ra = 5.64 \times 10^4$. Because the experimental results are only used to be compared with numerical results, the experimental apparatus and method are not depicted and is available elsewhere [9]. The measured temperature fields at different time are shown in Fig. 3. Although the boundary conditions are steady, the experimental temperature fields oscillated due to non-linear characteristic when time is sufficiently long. The oscillatory phenomenon occurs obviously at the top of the outer cylindrical envelope, and the amplitude of the temperature decreases gradually from top to bottom. Considering the outer cylindrical envelope, the isotherms are closely packed at the top. The local heat transfer in this region is highest while that at the bottom is lowest. Both two-dimensional and three-dimensional models are used to simulate the non-linear behavior in experiment. Fig. 3(b) shows the temperature fields in two-dimensional simulation for $Ra = 5.64 \times 10^4$. The isotherm is different from that in our experiment. The two-dimensional numerical simulation revealed that the steady velocity and temperature field can be reached from the initial zero velocity and uniform temperature field. The solutions in two-dimensional numerical simulation can also not predict the non-linear phenomenon in our experiment. Numerical temperature fields in three-dimensional simulation at different time for $Ra = 5.64 \times 10^4$ are shown in Fig. 3(c). The most obvious oscillation occurs also above the inner cylinder. These behaviors are in good agreement with those observed in our experiment. The results in three-dimensional simulations conform to experimental results and the two-dimensional simulations do not conform to experimental results. Therefore, the three-dimensional computations were necessary for a better simulation of non-linear phenomena.

The effect of slot degrees on the heat transfer was studied for different Rayleigh numbers. The equivalent thermal conductivity $K_{\text{eq}}$ at different slot degrees were obtained by three-dimensional simulations, as is shown in Table 1. When the value of thermal conductivity oscillates with time, the numerical result of $K_{\text{eq}}$ in Table 1 is the time-averaged value. The numerical results indicated that when the natural convection is steady, the value of $K_{\text{eq}}$ first increases and then decreases with the increasing slot degree. For dif-
different Rayleigh numbers, all the maximum values of $K_{eq}$ occur near the slot degree $S = 0.2$. These trends were broken at certain parameters when the unsteady solutions occur. The reason is that the heat transfer rate is related to the flow structure and the transitions from steady to unsteady solutions leads to the dramatic change of the flow structure.

There exists a critical Rayleigh number $R_{ac}$, which can dramatically change the flow structure. When the Rayleigh number is lower than $R_{ac}$, the natural convection is steady. When the Rayleigh number is greater than $R_{ac}$, the flow turns to oscillated solution after the first Hopf bifurcation. However, for the case of $S = 0.4$, the oscillated flow becomes steady solution with increasing Rayleigh numbers and then to oscillated solution again. The critical Rayleigh number $R_{ac}$ in the three-dimensional simulation is lower than that in the two-dimensional simulation. Author reason for this phenomenon is that the simulations in three-dimensional case can reflect the three-dimensional flow structure when the Rayleigh numbers are higher than the critical value from 2D to 3D solutions.

**Table 1**

<table>
<thead>
<tr>
<th>S</th>
<th>$10^0$</th>
<th>$3 \times 10^4$</th>
<th>$5 \times 10^4$</th>
<th>$7 \times 10^4$</th>
<th>$10^5$</th>
<th>$3 \times 10^5$</th>
<th>$5 \times 10^5$</th>
<th>$7 \times 10^5$</th>
<th>$10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2.526</td>
<td>3.413</td>
<td>3.895</td>
<td>4.246</td>
<td>4.651</td>
<td>6.130</td>
<td>6.984</td>
<td>7.596</td>
<td>8.279</td>
</tr>
<tr>
<td>0.3</td>
<td>2.498</td>
<td>3.333</td>
<td>3.803</td>
<td>4.145</td>
<td>4.537</td>
<td>5.968</td>
<td>6.771</td>
<td>7.346</td>
<td>8.042</td>
</tr>
<tr>
<td>0.4</td>
<td>2.408</td>
<td>3.232</td>
<td>3.740</td>
<td>4.074</td>
<td>4.467</td>
<td>5.794</td>
<td>6.579</td>
<td>7.147</td>
<td>7.761</td>
</tr>
<tr>
<td>0.5</td>
<td>2.359</td>
<td>3.173</td>
<td>3.643</td>
<td>3.987</td>
<td>4.335</td>
<td>5.688</td>
<td>6.443</td>
<td>6.914</td>
<td>7.513</td>
</tr>
</tbody>
</table>

$^a$ the flow and heat transfer is self-oscillated in simulations for the three-dimensional case.

$^b$ the flow and heat transfer is self-oscillated in simulations for the two-dimensional case.
In some ways, the three-dimensional flow structure can promote the dynamic bifurcations from steady to unsteady flow. By comparison, the simulations in two-dimensional case only can reflect the oscillated solutions with time but cannot reflect the change of solutions along the z-direction. Therefore, the onset of the oscillated solutions with time is delayed in the two-dimensional simulations.

4. Conclusions

Two- and three-dimensional numerical simulations were carried out for the natural convection in a cylindrical envelope with an internal concentric cylinder with slots. The effects of the slot degree on the flow and heat transfer are considered. The results indicated that when the flow is steady, the discrepancy of the simulations in two- and three-dimensional cases is negligible. Two-dimensional simulation should be selected for saving computational time. When the flow and heat transfer are unsteady, the solutions in three-dimensional simulation differ essentially from that in two-dimensional simulation. The critical Rayleigh numbers from steady to unsteady flow in the 3D simulations are lower than those in the 2D simulations. Therefore, three-dimensional numerical simulations can reflect the oscillation phenomenon of those systems, which cannot be simulated by two-dimensional numerical methods with limits. Three-dimensional computations were necessary for a better simulation of non-linear phenomena.

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References