An integral approximate solution of heat transfer in the grinding process

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Abstract—An integral approximation solution of heat transfer in the grinding process is presented in this paper. Heat transfer models for the abrasive grain, fluid and workpiece were developed by using the integral approximation method. For the case in which film boiling does not occur in the grinding zone during the grinding process, the workpiece background temperature rise calculated by the present model agreed very well with previous theoretical analysis. The present model can also correctly simulate the grinding process when film boiling occurs in the grinding zone. Furthermore, a simplified model, which is applied only for cases where film boiling does not occur in the grinding zone, has also been developed.

1. INTRODUCTION
In any grinding process, thermal damage is of serious concern when considering the quality of the final workpiece. In comparison with other machining processes, such as milling and turning, the grinding process requires a very high energy input. This energy is dissipated as heat in the grinding zone [1]. Heat transfer in grinding has been intensively studied by many researchers and many papers have been published. The earlier works have been compiled in detailed reviews by Snoeys et al. [2] and Malkin [3]. All of the early studies of heat transfer in grinding processes addressed the workpiece only, with appropriate thermal boundary conditions at its surface. The fraction of total grinding power that enters the workpiece had to be assumed.

Lavine and Jen [4, 5] presented a model to simulate the heat transfer to the workpiece, wheel, and fluid, which eliminated the need to specify the fraction of the total grinding power that enters the workpiece or the convection heat transfer coefficient of the grinding fluid. They assumed that the heat flux into the workpiece, wheel, grains and fluid were uniformly distributed in the grinding zone. However, this assumption would result in a contradiction of the temperature equations. They resolved this problem by allowing the heat flux into the grains, \( q'_g \), to vary in the grinding zone, but this approximation resulted in an error in the temperature distribution.

This model was modified by Jen and Lavine [6] to allow the heat flux to vary with location. The grinding fluid boiling and workpiece burn were also considered in ref. [6]. The drawback of Jen and Lavine's work [6] is that the heat transfer coefficient for various materials used were for uniform heat flux. When film boiling occurs in the grinding zone, the heat flux into the grinding fluid is suddenly reduced to zero. The heat fluxes into the workpiece and abrasive grain have to be adjusted in order to satisfy the energy balance in the grinding zone. Therefore, the heat flux into various materials is significantly changed when the film boiling occurs in the grinding zone. However, the heat transfer coefficient for uniform heat flux was still employed by Jen and Lavine [6] to study the grinding process with film boiling. Thus, the physical model of ref. [6] needs to be verified by other models.

Jen and Lavine [7] proposed an improved model, which accounted for the variation of heat flux in the grinding zone by using Duhamel's theorem. However, film boiling in the grinding process was not considered in their paper. In order to obtain the heat flux and temperature distribution in the grinding zone, an integral-differential equation had to be solved, which required significant computer time since the grid number was 40,001. Their results showed that the physical model proposed by Jen and Lavine [6] was accurate enough for the uniform grinding power input.

In this paper, a new model to simulate the heat transfer to the workpiece, wheel, and fluid will be presented. The solution is based on the direct relation between temperature and arbitrarily varied heat flux, which can be obtained by an integral approximated solution. The temperature and heat flux distribution in the grinding zone with and without film boiling will be presented.

2. PHYSICAL MODEL
The physical model and the coordinate system for a typical grinding wheel and workpiece are shown in Fig. 1. The heat flux and temperature distribution in
the grinding zone, which is the contact area of wheel and workpiece with length \(l\) and width \(b\), will be studied in this paper. Many grains of the wheel surface cut into the workpiece with a very high speed and a lot of heat is generated in the grains–workpiece interface and their vicinity. In order to simplify the problem, it is assumed that all of the heat is generated at the grain–workpiece interface and that the heat generated in the vicinity (such as grain–chip interface, and the shear plane between the workpiece and chip) can be neglected [4–7].

The heat generated at the grain–workpiece interface, \(q_{\text{grind}}\), is assumed to be uniform in the present study and will conduct into the workpiece or grain. Therefore,

\[
q_{\text{grind}} = q_w(x) + q_g(x) \tag{1}
\]

where, \(q_w(x)\) and \(q_g(x)\) are the heat fluxes into the workpiece and grain, respectively. It is noticed that the heat flux into the workpiece and grain vary with the location in the grinding zone although \(q_{\text{grind}}\) is uniform [6].

The heat flux into the workpiece, \(q_w(x)\), will be
Heat transfer in the grinding process

Heat transfer in an abrasive grain.

Fig. 2. Heat transfer in an abrasive grain.

divided into two parts: one part remains in the workpiece; the other part is removed by the grinding fluid. Assuming that the fractional grain–workpiece contact area is $A$, we can write

$$Aq_{ww}(x) = q_{wb}(x) + (1 - A)q_f(x)$$ (2)

where $q_{ww}(x)$ and $q_f(x)$ are the heat flux that remains in the workpiece and the heat flux into the grinding fluid, respectively.

Heat transfer to the abrasive grain, the fluid and the workpiece will be studied separately. These models will then be coupled in order to obtain the heat flux and temperature distributions in the grinding zone.

2.1. Heat transfer to the abrasive grain

Consider an individual abrasive grain moving along the workpiece with velocity $v_s$. The abrasive grain can be considered as a semi-infinite frustum of cone with a cross-sectional area

$$A_c = \pi r^2 = \pi (r_0 + 2\gamma)^2$$ (3)

where $\gamma = dr/dz \approx 1$ [4]. The heat flux into the grain at the grain–workpiece surface is denoted as $q_g$. The schematic diagram of the heat transfer in an abrasive grain is shown in Fig. 2. In the cylindrical coordinate system fixed to the grain, the grain temperature depends on $r$, $z$ and time $t$ since the grain is in contact with the workpiece. Since the thermal conductivity of the grain is much larger than that of the grinding fluid, it is assumed that the heat that enters the grain simply conducts further into the grain without any heat being removed by the fluid. Therefore the grain has a uniform temperature across cross-sectional area [4, 8]. It is noticed that the time that the grain is in contact with the workpiece can be expressed as $t = x/v$, and therefore the energy equation for the grain can be expressed as

$$\frac{\partial \theta_g}{\partial x} = \frac{A_c}{A_c v_s} \frac{\partial}{\partial z} \left( \frac{\partial \theta_g}{\partial z} \right)$$ (4)

where $\theta_g$ is the grain temperature rise relative to the initial grain temperature at $x = 0$. Substituting equation (3) into equation (4), it can be re-written as follows

$$\frac{\partial \theta_g}{\partial x} = \frac{A_c}{v_s} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta_g}{\partial r} \right).$$ (5)

It is noticed that equation (5) is the heat conduction equation in the spherical coordinate system. The initial condition and the boundary conditions of equation (5) can be obtained by re-writing those of equation (4), which can be found in ref. [8]. They can be expressed as

$$\theta_g = 0 \quad x = 0$$ (6)

$$-k_g \frac{\partial \theta_g}{\partial r} = q_g^*(x) \quad r = r_0$$ (7)

$$\theta_g \text{ finite, } \quad r \to \infty.$$ (8)

The physical problem described by equations (5)–(8) can be considered as a heat conduction problem in a spherical shell with an inner radius, $r_0$, and an infinite outer radius. This problem is a non-linear heat conduction problem because equation (7) is a non-linear boundary condition. An integral approximation method will be employed to solve this problem.

Assuming that the thermal boundary thickness is $r_s - r_0$, then integrating equation (5) on the interval $(r_0, r_s)$, and using the boundary condition, equation (7), the integrated energy equation of the grain will become

$$\frac{\partial \Theta_g}{\partial x} = \frac{q_g^* r_s^2}{(\rho c_p)_{gr}}$$ (9)

where

$$\Theta_g = \int_{r_0}^{r_s} r^2 \theta_g \, dr.$$ (10)

The temperature distribution in the grain is assumed to be a cubic polynomial function. The constants in the cubic polynomial function can be determined by using equations (5)–(8). Finally, the temperature distribution can be expressed as

$$\theta_g = \frac{q_g^* r_s^2 (r_s - r)^3}{k_g \left( 1 + \frac{3}{r_s/r_0 - 1} \right)}.$$ (11)

Substituting equation (11) into equation (9), we can obtain a differential equation about the thermal boundary layer radius, $r_s$.

$$\frac{d}{dx} \left[ \frac{\eta^3/4 + \eta^1/20}{\eta + 3} q_g^* \right] = \frac{A_c q_g^*}{r_0^2 v_s}$$ (12)

where
An iteration equation of $\eta$ can be obtained by integrating equation (12)

$$\eta = \left[ \frac{20(\eta + 3)}{\eta + 5} \cdot \int_{0}^{\infty} q'_{b} \, dx \right]^{1/2}. \quad (14)$$

The grain temperature at the grain–workpiece surface can be expressed as

$$\theta_{g} = \frac{q'_{g} \eta}{k_{g}} \eta - 1. \quad (13)$$

2.2. Heat transfer in grinding fluid and workpiece

The grinding fluid is introduced to the grinding zone and is distributed throughout the grinding zone by the wheel. Since the contact area between the workpiece and the grain is typically only a few percent of the grinding zone, the workpiece is covered by the grinding fluid over most of the grinding zone. It can be assumed that the grinding fluid fills the space around the abrasive grain to a depth that is larger than the thickness of the thermal boundary layer [9]. Therefore, the grinding fluid flow over the workpiece surface can be considered as slug flow with a uniform velocity, $v_{w}$, at its surface. The governing equation and its initial condition and boundary conditions are similar to ref. [9] but the heat flux into the grinding fluid depends on the location in the grinding zone. The temperature rise of the grinding fluid, $\theta_{g}$, can be obtained by using an integral approximation solution [10].

$$\theta_{g} = \frac{q'_{g} \eta}{k_{g}} \eta + 3. \quad (15)$$

An individual grain is modeled as a band heat source with width $l_{g}$, causing a heat flux, $q'_{w}$, into workpiece surface [5]. It is noticed that the individual grain heat sources modeled here are not circular which are used in the analysis of heat conduction in an abrasive grain. The workpiece moves with a velocity $v_{w} - v_{a} \approx v_{g}$, relative to an individual grain surface. The average workpiece surface temperature rise underneath a grain due to an individual heat source can be expressed as follows

$$\theta_{w} = \frac{4}{3(kpc_{p})_{w} v_{w}} \int_{0}^{\infty} q_{w} \, dx \, q'_{w}. \quad (17)$$

Although the heat flux, $q'_{w}$, is assumed to be uniform by Lavine and Jen [5], equation (18) is also valid when the heat flux is a function of $x$ [7].

2.3. Coupling the model

The individual heat transfer models for abrasive grain, fluid and workpiece have been developed. In order to obtain the heat flux and temperature distribution, the above separate models have to be coupled by requiring that the surface temperature match. In other words, at a point on the workpiece exposed to the fluid, the workpiece background temperature rise, $\theta_{w}$, must equal the grinding fluid temperature rise, $\theta_{g}$.

$$\theta_{w} = \frac{4}{3(kpc_{p})_{w} v_{w}} \int_{0}^{\infty} q_{w} \, dx \, q'_{w}. \quad (19)$$

Equation (19) can also be written as

$$\frac{q'_{w} \int_{0}^{\infty} q_{w} \, dx}{q_{w} \int_{0}^{\infty} q_{w} \, dx} = \frac{(kpc_{p})_{w} v_{w}}{(kpc_{p})_{w} v_{w}} \frac{q'_{w}}{q_{w}}. \quad (20)$$

It is observed that the right-hand side of equation (20) does not depend on the grinding zone location and is a constant for a specific case. The value of the constant depends on the properties of grinding fluid and workpiece, and the velocity of the wheel and workpiece. In order to maintain the left-hand side of equation (20) as a constant, the following equation has to be satisfied.

$$\frac{q'_{w}}{q_{w}} = C. \quad (21)$$

where the constant, $C$, is

$$C = \left[ \frac{(kpc_{p})_{w} v_{w}}{(kpc_{p})_{w} v_{w}} \right]^{1/2}. \quad (22)$$
Equation (21) confirms the relation between \( q''_g \) and \( q''_{wb} \) obtained by Duhamel's theorem as found by Jen and Lavine [7].

At a point underneath a grain, the grain temperature rise, \( \theta_{gs} \), must equal the sum of the workpiece background temperature rise, \( \theta_{wb,s} \), and the workpiece temperature rise due to an individual grain:

\[
\frac{l_g}{\sqrt{\pi k_s}} \eta + 3 \eta q''_g = \frac{4}{3(k \rho c_p)_{w,v}} q''_{wb},
\]

\[
+ \frac{2}{3} \left( \frac{4 l_g}{\pi (k \rho c_p)_{w,v}} \right)^{1/2} q''_{wb},
\]

(23)

It is noticed that different geometries of grain–workpiece contact areas were assumed in this study: a circle of radius \( r_0 \) and a band of width \( l_g \). The relation between \( r_0 \) and \( l_g \) is:

\[
l_g^2 = \pi r_0^2
\]

(24)

and this relation has been used in equation (23) to express the grain temperature \( \theta_{gs} \). The parameter \( \eta \) in equation (23) can be obtained by substituting equation (24) into equation (14) to obtain the following:

\[
\eta = \left[ \frac{20(n + 3) \pi \alpha_x}{\eta + 5} \frac{q''_r}{q''_b} \right]^{1/2}.
\]

(25)

Substituting equations (1), (2) and (21) into equations (23) and (25), an equation representing the heat flux into the workpiece, \( q''_{wb} \), can be obtained:

\[
q''_{wb} = \frac{l_g}{\sqrt{\pi k_s}} \eta + 3 q''_{grind} \left\{ \frac{1 + (1 - A)C}{A} \right\} \times \left[ \frac{l_g}{\sqrt{\pi k_s}} \eta + 3 \left( \frac{4 l_g}{3 \pi (k \rho c_p)_{w,v}} \right)^{1/2} \right] + \left[ \frac{20(n + 3) \pi \alpha_x}{\eta + 5} \frac{q''_r}{q''_b} \right]^{1/2}.
\]

(26)

where

\[
\eta = \left[ \frac{20(n + 3) \pi \alpha_x}{\eta + 5} \frac{q''_{grind}}{q''_{wb} - (1 + (1 - A)C)} \right]^{1/2}.
\]

(27)

It is noticed that the heat flux into the workpiece, \( q''_{wb} \), is the only unknown variable in equation (26). Since \( q''_{wb} \) also appears on the right-hand side of equation (26), iteration is needed during the solution procedure.

After the distribution of \( q''_{wb} \) is obtained, the distribution of the other heat fluxes can be obtained by solving equations (1), (2) and (21). The workpiece background temperature rise, \( \theta_{wb,s} \), which is a most important parameter from the viewpoint of engineering, can be calculated from equation (17).

It should be pointed out that equation (26) can also be used to simulate the heat transfer in the grinding process with film boiling in the grinding zone. When film boiling occurs, the grinding fluid and the workpiece are separated by a vapor film. Therefore, the heat amount removed by the grinding fluid can be neglected [4–6]. Equations (26) and (27) can be easily applied to simulate the grinding with film boiling by setting the constant, \( C \), to zero when the workpiece background temperature is greater than the boiling point of the grinding fluid.

### 3. RESULTS AND DISCUSSION

The present model will be used to predict the heat flux and temperature distributions in the grinding zone. The thermophysical properties of the abrasive grains (Al₂O₃), workpiece (steel), and grinding fluid (water) can be found in refs. [4–7]. The grinding power input, \( q''_{grind}(x) \), is assumed to be uniform in the grinding zone. The ambient temperature is taken to be 25°C.

#### 3.1. The grinding process without film boiling

The results obtained by the present model are compared to the results of Jen and Lavine's model [6, 7] for creep feed grinding. Figure 3 shows the comparison of the workpiece background temperature rise obtained by using the present integral approximation solution and Jen and Lavine's model [6, 7] for convenience of comparison. The grinding heat flux, \( q''_{grind} \), is held constant at \( 6.5 \times 10^8 \text{ W m}^{-2} \). The wheel and workpiece speeds are 18 m s⁻¹ and 0.6 mm s⁻¹, respectively. The fractional grain–workpiece contact area, \( A \), is taken to be 0.01. The length of the grinding zone for creep feed grinding is \( l = 17.5 \text{ mm} \). The effect of the grid size on \( \theta_{wb,s} \) is very small and Fig. 3 shows that the workpiece background temperature rise is the result of \( Ax = 0.01 \text{ mm} \) (grid number is 175). Although a finer grid size (\( Ax = 0.001 \text{ mm} \)) is also used to solve equation (26), the maximum difference between the two grid sizes is less than 0.02°C. It can be seen that the agreement between the results of the present model and previous models are very good. The workpiece background temperature rise, \( \theta_{wb,s} \), of the present model and that of ref. [7] shows a maximum difference of less than 3°C. The difference between the present result and the result of ref. [6] is smaller than 2°C. Figure 4 shows the various heat flux distributions for creep feed grinding. The parameters in Fig. 4 are the same as that of Fig. 3. As can be seen in Fig. 4, for creep feed grinding with uniform grinding power input, the various heat fluxes are almost unchanged except for the beginning of the grinding zone.

#### 3.2. A simplified model

The grid number of 175 used in the calculation is much smaller than the grid size of 40 001 used by Jen and Lavine [7]. Therefore, the computer time used in the present study is much less than ref. [7]. However,
an iteration is still needed during the solution procedure of equation (26) because \( q_{\omega b}^{r} \) appears on the right-hand side of equation (26). An integration of \( q_{\omega b}^{r} \) has to be calculated during the solution procedure.

It can be seen from Fig. 4 that \( q_{\omega b}^{r} \) is almost unchanged in the grinding zone. A simplified model is therefore proposed according to this behavior of \( q_{\omega b}^{r} \). The main idea of the simplified model is assuming \( q_{\omega b}^{r} \) is a constant when integrating \( q_{\omega b}^{r} \) in the right-hand side of equation (26). Equations (26) and (27) can be simplified as follows due to this assumption:

\[
q_{\omega b}^{r} = \frac{I_{s}^{r}}{\sqrt{\pi k_{g}}} \frac{\eta}{\eta + 3} q_{\text{wind}} \left[ \frac{L_{s}}{A} \frac{\eta}{\sqrt{\pi k_{g}}} \right]^{\frac{1}{2}} + 3 \left( \frac{4L_{s}}{3(\kappa c_{p_{w}})_{v_{w}}} \right)^{\frac{1}{2}} \left( \frac{4x}{3(\kappa c_{p_{w}})_{v_{w}}} \right)^{\frac{3}{2}} (28)
\]

\[
\eta = \left( \frac{20(\eta + 3) \pi x_{r}^{2}}{\eta + 5} \right)^{\frac{1}{2}} (29)
\]

It is observed that \( q_{\omega b}^{r} \) does not appear on the right-
hand side of equation (28). Therefore, iteration is no longer needed during the solution procedure of equation (28) and a closed form solution of \( q_{wb} \) is obtained. After \( q_{wb} \) is obtained, the workpiece background temperature, \( \theta_{wb,b} \), can be obtained by the following equation.

\[
\theta_{wb,b} = \left[ \frac{4x}{3(kpc_p)_{wb}} \right]^{1/2} \quad q_{wb}.
\] (30)

It should be noted that equation (30) is obtained by assuming a uniform \( q_{wb} \) when integrating \( q_{wb} \) in equation (17).

Figure 5 shows the comparison of the workpiece background temperature rise and abrasive grain temperature rise obtained by the different methods. It can be seen that the results obtained by the simplified model agree very well with the results obtained by the model described in the previous section of this paper. Figure 6 shows the comparison of various heat fluxes obtained by the two different models. It can be seen that the differences between the results obtained by the two models are hardly noticeable in Figure 6 except at the beginning of the grinding zone. Therefore, for creep feed with uniform heat input, the simplified model is a very reasonable substitute of the model proposed in the previous section.

The present simplified model is then used to simulate the conventional grinding process, in which the workpiece speed is faster but the length of grinding zone is much shorter. Figure 7 shows the comparison of workpiece temperature rise, \( \theta_{wb,b} \), obtained by the present simplified model and Jen and Lavine's model [6]. The grinding power input, \( q_{\text{grind}} \), is held at a constant \( 1.4 \times 10^5 \text{ W m}^{-2} \). The wheel and workpiece speed are \( 20 \text{ m s}^{-1} \) and \( 0.033 \text{ m s}^{-1} \) respectively. It can be seen that the workpiece background temperature rise obtained by the present simplified model is slightly lower than that obtained by Jen and Lavine's model [6], but the maximum difference between the two models is less than 2°C. Therefore, the simplified model is also a useful tool in simulating conventional grinding for uniform grinding power input.

3.3. The grinding process with film boiling

In the case of larger grinding power inputs, the workpiece background temperature will be larger than the boiling point of the grinding fluid and flow boiling will occur in the grinding zone. The workpiece background temperature is monotonically increased by the increasing of \( x \). Therefore, the heat transfer mechanism of the fluid in the grinding zone can be a combination of the single phase forced convection in the beginning and flow boiling in the end. It is well known that the flow boiling has two different states: nucleate boiling and film boiling. The result of ref. [6] showed that the transition from single phase forced convection to film boiling is very fast and the effect of the nucleate boiling can be neglected. It can be assumed that the heat transfer mechanism in the grinding zone is a combination of the single phase forced convection and the film boiling. The transition point from the single phase forced convection to the film boiling is a point that its temperature reaches that of the critical workpiece temperature. This temperature has been determined by ref. [6] and its value was 130°C. As mentioned in the previous section, the constant, \( C \), in equations (26) and (27) can be set to zero after the transition point.

Figure 8 shows the heat flux distribution in the grinding zone for creep feed grinding when film boiling occurs in the grinding zone. It can be seen that heat flux into the grinding fluid is suddenly reduced to zero and the heat flux into the workpiece is suddenly increased. The other heat fluxes in Fig. 8, such as \( q_{wb} \),
and \( q''_{v} \), also have discontinuous changes at the transition point. Therefore, in the case of film boiling, heat fluxes into various materials no longer can be assumed to be uniform even though the grinding power input is uniform. The physical model proposed by Jen and Lavine [6], which employed the heat transfer coefficient for uniform heat flux, is not suitable for the case of film boiling occurring in the grinding zone. Figure 9 shows the workpiece background temperature for creep feed grinding when film boiling occurs. As can be seen in Fig. 9, the physical model proposed by Jen and Lavine [6] obtained higher workpiece background temperatures when film boiling occurred. The difference of the workpiece background temperature is very large when film boiling begins to occur but it becomes smaller at the end of the grinding zone. The present physical model can correctly handle the variation of heat fluxes (even for discontinuous variations) by employing the integral approximation method. Therefore, the present model is a very useful tool in simulating the grinding process with film boiling.

4. CONCLUSIONS

Heat transfer in the grinding process has been investigated in this paper. Heat conduction models in abrasive grain, grinding fluid and workpiece are
developed by employing integral approximation methods and coupled by satisfying the requirement matching temperatures at the workpiece-fluid interface and grain-workpiece interface. The heat transfer model developed in the present paper overcame the contradiction of temperature match in refs. [4, 5] and is simpler than the model of ref. [7]. A simplified model is proposed to simulate the heat transfer in the grinding process with uniform grinding power input. The agreement of the workpiece background temperature rise obtained by the physical models presented in the present paper and Jen and Lavine’s model [6, 7] are very good. The present model can also be used to simulate the grinding process when the film boiling occurs in the grinding zone, in which the heat fluxes have discontinuous changes.

REFERENCES
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