
YUWEN ZHANG and AMIR FAGHRI
Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269-3139

(Received January 29, 1995; in final form July 9, 1995)

The heat transfer enhancement in the latent heat thermal energy storage system by using an externally finned tube is presented in this paper. The melting in the phase change materials and heat conduction in the tube wall and fins are described by a temperature transforming model coupled to the heat transfer from transfer fluid. The forced convection inside the tube was solved by an analytical method. The results show that the local Nusselt number inside the tube cannot be simply given by the Graetz solution. The effects of the tube wall and initial subcooling on the heat transfer performance are also studied.

Key Words: Thermal energy storage; Finned tube

The thermal energy storage system using a Phase Change Material (PCM) is an attractive project because a large amount of latent heat can be stored or released during the melting or solidification process of the PCM [Viskanta, 1983]. An effective latent heat storage system is the shell-and-tube exchanger with the PCM filling the shell-side and the transfer fluid flowing inside the tube. This type of latent heat storage system has been studied by a number of authors employing numerical and analytical methods [Ismail and Alves, 1986; Yimmer and Adami, 1989; Cao and Faghri, 1991; Cao and Faghri, 1992; Zhang and Faghri, 1995]. The drawback of this type of thermal energy storage system is that the thermal conductivity of the PCM is usually very low. In order to overcome this drawback, the use of finned tubes is considered to be especially effective and reliable. Although the thermal energy storage system with finned tubes has been studied by a few researchers [Padmanabhan and Murthy, 1986; Sasaguchi, 1988], their solution of the phase change problem did not link to the heat transfer from the transfer fluid inside the tube.

Lacroix [1993] studied the heat transfer behavior of a latent heat thermal energy storage unit with a finned tube. In this study, the phase change problem of the PCM around a finned tube was analyzed in terms of both the radial and axial directions and linked to the heat transfer from the transfer fluid inside the tube. The range of Reynolds number was 20 to 6000. For the laminar flows, i.e., Reynolds number ≤ 2000, it was assumed that the velocity profile of the transfer fluid was fully developed at the inlet while the temperature profile was developing under the special wall temperature condition. Therefore, the local convection heat transfer coefficient inside the tube was estimated at each axial node with a correction for the local Nusselt number of the Graetz solution [Kays and Crawford, 1980]. In fact, the Graetz solution is only applied to the
thermally developing convection heat transfer inside a tube with a Constant Wall Temperature (CWT) boundary condition. Since the convection inside the tube is coupled with the melting process of the PCM outside the tube, the temperature along the axial direction of the tube could not be maintained at a constant value. The present authors have studied the heat transfer in a latent heat thermal energy storage system with conjugate laminar forced convection inside the tube without the transverse fins [Zhang and Faghri, 1995]. The results showed that the convection inside the tube could not be simply treated as a Graetz problem. It should instead be treated as a forced convection problem with an arbitrarily varied wall temperature.

In this paper, the heat transfer enhancement in the latent heat thermal energy storage system using a finned tube will be studied. The heat conduction in the tube wall, fins, and the melting process of the PCM outside the tube will be solved by the temperature transforming model which was proposed by Cao and Faghri [1990], and the convection inside the tube will be solved analytically [Zhang and Faghri, 1995]. The effect of the tube wall, initial subcooling and the height of the fins on the heat transfer will also be investigated.

PHYSICAL MODEL OF THE MELTING PROBLEM

The physical model of the problem is shown in Figure 1. The PCM fills the annular shell space of inner radius \( r_i \) and outer radius \( r_o \), while the transfer fluid flows inside the tube of inner radius \( r_i \) and outer radius \( r_o \). The PCM is adiabatic at the outer radius \( r_o \). In order to solve this problem, the following assumptions are necessary.

1. The thermophysical properties of the PCM and transfer fluid are independent of temperature, but the properties of the PCM can be different in the solid and liquid phases.
2. The PCM is homogeneous and isotropic.
3. The heat conduction is axisymmetric around the tube.
4. The melting of the PCM occurs at a single temperature \( T_m^0 \).

The effect of natural convection in the liquid PCM was also taken into account by introducing the effective thermal conductivity \( k_e \) for the liquid region using the following empirical correlation [Lacroix, 1993]:

\[
k_e/k_r = 0.099 Ra^{1/4}
\]  

(1)

The heat conduction in the tube wall, fins and the PCM are described by a temperature transforming model using a fixed grid numerical method [Cao and Faghri, 1990]. This model assumes that the melting process occurs over a range of phase change temperatures from \( (T_m^0 - \delta T^0) \) to \( (T_m^0 + \delta T^0) \), but it also can be successfully used to simulate the melting process occurring at a single temperature by taking a small range of phase change temperature \( 2\delta T^0 \). This model has the advantage of eliminating the time step and grid size limitations that are normally encountered in other fixed grid methods. The governing equation for the tube wall, fins and the PCM are:

\[
\frac{\partial (C^0 T^0)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( k^r T^0 \frac{\partial T^0}{\partial r} \right) + \frac{\partial}{\partial x} \left( k^x \frac{\partial T^0}{\partial x} \right)
\]

\[- \frac{\partial S^0}{\partial t} \quad 0 < x < l, \quad r_i < r < r_o \]  

(2)

For the tube wall and the fins

\[
C^0 = C_w^0 \quad k = k_w \quad S^0 = 0
\]  

(3)

For the PCM

\[
C^0(T^0) = \begin{cases} 
C^o & T^0 < T_m^0 - \delta T^0 \\
\frac{1}{2}(C^o + C_l^0) + \frac{\rho H}{2\delta T^0} & T_m^0 - \delta T^0 \leq T^0 \leq T_m^0 + \delta T^0 \\
C_l^0 & T^0 > T_m^0 + \delta T^0 
\end{cases}
\]  

(4)
By defining the following dimensionless variables:

\[
\begin{align*}
X &= \frac{x}{r_i} \\
L &= \frac{l}{r_i} \\
R &= \frac{R}{r_i} \\
R_w &= \frac{r_w}{r_i} \\
R_t &= \frac{r_t}{r_i} \\
F_o &= \frac{\alpha_f}{r_i} \\
C &= \frac{C_p}{C_f} \\
C_s &= \frac{C_s}{C_f} \\
K &= \frac{k}{k_f} \\
K_w &= \frac{k_w}{k_f} \\
K_f &= \frac{k_f}{k_f} \\
S &= \frac{S^0(T^0)}{C_p(T_{in} - T_m^0)} \\
P_e &= \frac{2U_w r_i}{\alpha_f} \\
\delta T &= \frac{T_{m} - T_{in} - T_m}{T_{m} - T_{in} - T_m} \\
St_e &= \frac{C_i}{C_s(T_{in} - T_m)} \\
T &= \frac{T_{m} - T_{in} - T_m}{T_{m} - T_{in} - T_m} \\
T_i &= \frac{T_{m} - T_{in} - T_m}{T_{m} - T_{in} - T_m} \\
T_o &= \frac{T_{m} - T_{in} - T_m}{T_{m} - T_{in} - T_m}
\end{align*}
\]

the governing equations become:

\[
\begin{align*}
\frac{\partial (CT)}{\partial F_o} &= \frac{1}{R} \frac{\partial}{\partial T} \left( K R \frac{\partial T}{\partial R} \right) + \frac{\partial}{\partial X} \left( K \frac{\partial T}{\partial X} \right) \\
- \frac{\partial S}{\partial F_o} &= 0, \quad X < X_s, \quad 1 < R < R_s
\end{align*}
\]

For the tube wall and fins:

\[
C = C_w \quad K = K_w \quad S \equiv 0
\]

For the PCM:

\[
C(T) = \begin{cases} 
C_s & T < -\delta T \\
\frac{1}{2} (1 + C_s) + \frac{1}{2St_e} & -\delta T \leq T \leq \delta T \\
1 & T > \delta T
\end{cases}
\]

\[
K(T) = \begin{cases} 
K_s & T < -\delta T \\
\frac{K_s}{2\delta T} & -\delta T \leq T \leq \delta T \\
1 & T > \delta T
\end{cases}
\]

the initial condition and boundary condition of Eq. (11) are:

\[
\begin{align*}
T_f^0 &= T_i^0 \quad t = 0 \\
S(T) &= \begin{cases} 
C_s \delta T & T < -\delta T \\
\frac{1}{2} (1 + C_s) \delta T + \frac{1}{2St_e} & -\delta T \leq T \leq \delta T \\
C_s \delta T + \frac{1}{St_e} & T > \delta T
\end{cases}
\end{align*}
\]

where \( T_f^0 \) in Eq. (8) is the transfer fluid temperature. It can be determined by the following equation which represents the energy balance on a fluid control volume.

\[
\frac{d}{d t} \left( C_s \pi r_i^2 \frac{\partial T_f^0}{\partial t} \right) = 2 \pi r_i h(T_f^0_{,\infty} - T_f^0) - \pi r_i^2 U_m C_f \frac{\partial T_f^0}{\partial x}
\]

The initial condition and boundary conditions can be written as

\[
T^0 = T_i^0 \quad 0 \leq x \leq l, \quad r_i \leq r \leq r_w, \quad t = 0
\]

\[
-k \frac{\partial T^0}{\partial r} = h(T_f^0 - T^0) \quad r = r_i
\]

\[
\frac{\partial T^0}{\partial r} = 0 \quad r = r_w
\]

\[
\frac{\partial T^0}{\partial x} = 0 \quad x = 0, l
\]
\[ T = T_i, \quad 0 \leq X \leq L, \quad 1 \leq R \leq R_s, \quad Fo = 0 \quad (20) \]

\[ -K_v \frac{\partial T}{\partial R} = \frac{1}{2} K_f Nu(T_f - T) \quad R = 1 \quad (21) \]

\[ \frac{\partial T}{\partial R} = 0 \quad R = R_s \quad (22) \]

\[ \frac{\partial T}{\partial X} = 0 \quad X = 0, L \quad (23) \]

\[ \frac{C_f \partial T_f}{K_f \partial Fo} = Nu(T|_{R = 1} - T_f) - \frac{1}{2} Pe \frac{\partial T_f}{\partial X} \quad (24) \]

\[ T_f = T, \quad Fo = 0 \quad (25) \]

\[ T_f = 1, \quad X = 0 \quad (26) \]

which is described by Kays and Crawford [1980]. The final result is rewritten as [Zhang and Faghri, 1995):

\[ Nu(X) = \sum_{i=1}^{j} \Delta T_i \sum_{s=0}^{\infty} \frac{G_s}{\lambda_s} \exp \left( -\frac{\lambda_s^2}{Pe} \frac{(X - (i - 1)\Delta X)}{\Delta X} \right) \]

\[ = \sum_{i=1}^{j} \Delta T_i \sum_{s=0}^{\infty} \frac{G_s}{\lambda_s} \exp \left( -\frac{\lambda_s^2}{Pe} \frac{(X - (i - 1)\Delta X)}{\Delta X} \right) \quad (27) \]

where the values of constant \( G_s \) and eigenvalues \( \lambda_s \) can be found in Kays and Crawford [1980], and the value of \( j \) in Eq. (27) can be determined by

\[ j = \text{int} \left( \frac{X}{\Delta X} \right) + 1 \quad (28) \]

where \( \text{int} \) in Eq. (28) is an integer function.

**LOCAL NUSSELT NUMBER INSIDE THE TUBE**

The local Nusselt number in Eq. (24) can be obtained analytically [Zhang and Faghri, 1995]. In order to obtain its analytical expression, the following assumptions are needed.

1. The transfer fluid flow inside the tube is laminar. The inlet velocity is fully developed but heat transfer occurs in the thermal entry region.
2. Axial heat conduction in the transfer fluid is neglected.
3. The quasi-steady assumption is applied to convection heat transfer inside the tube. In other words, transient convection inside the tube is treated as a series of steady state forced convection problems.

Forced convective heat transfer in the tube has neither a constant wall temperature nor constant heat flux boundary condition at the tube wall. The most appropriate boundary condition is an arbitrarily varied wall temperature. The full length of the tube can be divided into \( N \) sections. It is assumed that every section has a uniform temperature. In other words, the wall temperature variation along the axial direction is treated as discontinuous variations. For example, if the dimensionless tube wall temperature at the section \( i \) and \( i + 1 \) are \( T_w \) and \( T_{w+1} \), respectively, the dimensionless tube wall temperature variation from section \( i \) to section \( i + 1 \) is \( \Delta T_{w+1} = T_{w+1} - T_w \). The local Nusselt number can be obtained by an analytical method.

**NUMERICAL SOLUTION PROCEDURE**

The melting problem has been specified mathematically by Eqs. (15)-(26). These equations can be solved by a finite difference method described by Patankar [1980]. In this methodology, the discretization equations are obtained by applying the conservation laws over a finite size control volume surrounding the grid node and integrating the equation over the control volume. The resulting scheme has the form:

\[ a_p T_p = a_e T_e + a_w T_w + a_N T_N + a_S T_S + b \quad (29) \]

where the factors in Eq. (29) can be found in Patankar [1980]. For the dimensionless thermal conductivity \( K \), it is important to use the harmonic mean at the faces of the control volume.

The calculation is started from \( Fo = 0 \). The computational domain of the heat conduction includes the PCM, the tube wall and fins. For any time step, the problem can be solved by the following procedure:

1. Assumed the temperature distribution in the computational domain \( T(X, R) \), while the assumed temperature distribution can be obtained by using solution of previous time step or previous iteration.
2. Calculate the local Nusselt number \( Nu(X) \) along the axial direction by Eq. (27).
3. Calculate the transfer fluid temperature along axial direction \( T_f(X) \) by the discretization form of Eq. (24).
4. Calculate the factors in Eq. (29), and account for the boundary conditions of the computational domain.
5. Solve Eq. (29) by Alternating Direction Implicit (ADI) method [Patankar, 1980] and obtain the temperature distribution in the calculated region \( T'(X, R) \).
6. Compare the calculated temperature field with the assumed value in step 1; if the maximum difference is less than \( 10^{-3} \), calculate the next time step, if not, return to step 1.

During the iteration process, some underrelaxation is necessary. The relaxation factor used in this method is \( 0.1 \sim 0.2 \).

RESULTS AND DISCUSSION

Before studying the effect of transverse fins on the heat transfer in the PCM thermal storage system, the code was first used to calculate the melting of PCM in a latent heat thermal energy storage system with a bare tube. This problem has been studied analytically by the present authors [Zhang and Faghri, 1995]. For simplicity, however, the initial temperature was assumed to be at the freezing temperature of the PCM. In order to simulate the melting process occurring at a single temperature, a very small dimensionless phase-change temperature range \( \delta T = 0.001 \) is used in the calculation and the initial temperature of the system is set to \( T_i = -0.001 \) instead of \( T_i = 0 \). The calculations were carried out for a grid size of 102 nodes in the axial direction and 42 nodes in the radial direction with a dimensionless time step of \( \Delta Fo = 0.1 \). Finer grid sizes and smaller time steps were also used in the calculations, but their results did not show noticeable differences with the present grid size and time step.

Figure 2 shows the comparison of the melting front obtained by the present numerical method and the analytical method developed by Zhang and Faghri [1995]. The dimensionless parameters in Figure 2 are same as Zhang and Faghri [1995] for comparative purpose. Clearly, the difference between present numerical results and analytical results is very small, especially for \( Fo = 1.0 \) and 2.0. For \( Fo = 4.0 \), the melting front reached the shell of the thermal energy storage system. The melting front obtained by the present numerical method is slightly faster than the analytical solution of Zhang and Faghri [1995]. This is because the axial conduction in the PCM was not accounted for in the analytical model. Therefore, even though the melting front in the left side of the thermal energy storage system had reached the shell of the system, the heat would not conduct from the left to the right. However, even for \( Fo = 4.0 \), the difference between the present numerical model and analytical model is very small. Therefore, for the thermal energy storage system without initial subcooling, the analytical model proposed by Zhang and Faghri [1995] provides an efficient means to predict the system performance.

Lacroix [1993] studied the heat transfer behavior of a latent heat thermal storage unit with bare tubes as well as finned tubes by the enthalpy method. The present computational code is also validated by the computational results of Lacroix [1993]. In order to achieve this goal, the local Nusselt number is calculated by the Graetz solution [Kays and Crawford, 1980] just like Lacroix [1993]. The grid size and dimensionless time step is the same as Figure 2. The dimensionless phase-change temperature range \( \delta T \) is also set to 0.001. Figure 3 shows the comparison of the molten volume fraction (MVF) obtained by the present temperature transform model and Lacroix's [1993] enthalpy model. The dimensionless parameters in Figure 3 are obtained from the dimensional parameter in Lacroix [1993]. As evident, for both the bare tube as well as finned tube, the agreement between present result and Lacroix's [1993] result is good. Therefore, the present code can accurately simulate the melting process in a latent heat thermal energy storage system with a bare or finned tube.

The heat transfer in the latent heat thermal energy storage system was then calculated by the present method, and the local Nusselt number inside the tube was calculated by Eq. (27) instead of using the Graetz solution. Figure 4 shows the variation of the tube wall
The Nusselt number obtained from the Graetz solution is not suitable to be used in calculating the local Nusselt number inside the tube. Figure 5 shows the comparison of the local Nusselt number obtained by the present model and the Graetz [Kays and Crawford, 1980]. Compared to the local Nusselt number obtained by the Graetz solution, the local Nusselt number obtained by the present model is very complex because very complex variation of tube wall temperature. The local Nusselt number at the position of the transverse fins is much higher than at other positions. The local Nusselt number at the other positions is also higher than the value obtained from Graetz solution.

Figure 6 shows the comparison of the melting front calculated by using different local Nusselt numbers. As shown in Figure 6, the radii of the melting fronts calculated by using the present formulation are slightly
larger than those calculated by using the Graetz solution, especially at the positions between transverse fins. However, the effect of the local Nusselt number on the melting front is not very significant even though the local Nusselt number obtained by the present model is significantly different from the value obtained from the Graetz solution.

In Lacroix's [1993] experiments, the thickness of tube wall was not zero. Since the thermal conductivity of the tube wall is very high, the thermal resistance of the tube wall along the radial direction is very small [Zhang and Faghri, 1995]. However, the axial conduction in the tube wall will have a significant effect on the variation of tube wall temperature along the axial direction because the thermal conductivity of the tube wall is very high. Figure 7 shows the variation of tube wall temperature along axial direction with a varying thickness of the tube wall. It can be seen that the tube wall temperature curves in Figure 7 are smoother than the tube wall temperature curves in Figure 4. Figure 8 shows the variation of the local Nusselt number along the axial direction at different times. It can be seen that the local Nusselt number curves in Figure 8 are smoother than the local Nusselt number curves in Figure 5 and the value of the local Nusselt number is higher than the local Nusselt number obtained by the Graetz solution. Figure 9 shows the effect of tube wall thickness on the melting front. Melting fronts for zero tube wall thickness and certain non-zero tube wall thickness are illustrated in Figure 9. As can be seen, the radius of the melting front with a tube wall is larger than the radius of the melting front without a tube wall. This occurs because the melting front with a tube wall is started from \( R = R_m \) instead of starting from \( R = 1.0 \). Figure 10 shows the effect of a tube wall on the
MVF. It can be seen that the effect of a tube wall on the MVF is very small. Therefore, the thickness of the tube wall will only have a significant effect on tube wall temperatures and local Nusselt numbers, but will have no significant effect on the volume of the molten liquid.

Figure 11 shows the effect of initial subcooling on the position of the melting fronts. It can be seen that the effect of initial subcooling on the radius of the melting fronts at the axial position between the transverse fins are very significant. Otherwise, the initial subcooling has almost no effect on the melting front on the two sides of the transverse fins. This means that the transverse fins are more efficient if initial subcooling exists in the PCM. Figure 12 shows the effect of initial subcooling on the MVF. It can be seen that the initial subcooling in the PCM results in a slow melting process.

Figure 13 shows the effect of the fin height on the melting fronts. It can be seen that the melting front at the axial position between fins is affected very little by the height of the fins. Otherwise, the melting fronts on the two sides of the transverse fins are significantly affected by the height of the fins. Figure 14 show the effect of fin height on the MVF. It can be seen that the MVF can be significantly increased if the fin height is increased. Therefore, the melting process can be accelerated by increasing the heights of the fins.

**CONCLUSIONS**

The heat transfer enhancement in the latent heat thermal energy storage system by using the finned tube was studied numerically. The results show that the local Nusselt number calculated by the present model is significantly different from the Graetz solution because the tube wall temperature cannot be treated as a con-
The variation of the tube wall temperatures and local heat transfer if the initial subcooling exists in the PCM. The height of the fins only have significant effect on the melting front between the transverse fins. The MVF can be significantly increased by increasing the heights of fins.

Nomenclature

- \( a \): Factors of algebraic Eq. (29)
- \( b \): Term in Eq. (29)
- \( C \): Heat capacity, \( J/(m^3K) \)
- \( C_0 \): Dimensionless heat capacity, \( C_0/C_0 \)
- \( D \): Internal diameter of the tube, \( 2r_i \)
- \( F_0 \): Fourier number, \( \alpha L/r_i^2 \)
- \( G \): Constant in Eq. (27)
- \( h \): Local heat transfer coefficient, \( W/(m^2K) \)
- \( K \): Local heat of melting, \( J/kg \)
- \( k \): Thermal conductivity, \( W/(mK) \)
- \( K_0 \): Dimensionless thermal conductivity, \( k/k_0 \)
- \( l \): Length of the thermal energy storage system, \( m \)
- \( L \): Dimensionless length of the thermal energy storage system, \( L/r_i \)
- \( M \): Grid numbers along radial direction
- \( N \): Grid numbers along axial direction
- \( Nu \): Local Nusselt number, \( \alpha D/k \)
- \( P \): Peclet number, \( U D/\alpha \)
- \( r \): Radial coordinate, \( m \)
- \( R \): Dimensionless radial coordinate, \( r/r_i \)
- \( R_i \): Dimensionless radius of transverse fins, \( r_f/r_i \)
- \( r_e \): Internal radius of the tube, \( m \)
- \( r_w \): External radius of the tube, \( m \)
- \( R_w \): Dimensionless external radius of the tube, \( r_w/r_i \)
- \( R_s \): Radius of the shell, \( m \)
- \( R_\text{sh} \): Dimensionless radius of the shell, \( r_s/r_i \)
- \( R_a \): Rayleigh number, \( g \beta H(T_0^2 - T_\infty^2)D^3/(\rho \nu) \)
- \( S \): Term in Eq. (2), \( J/kg \)
- \( S^\prime \): \( S^\prime/\alpha (T_\infty^2 - T_0^2) \)
- \( Ste \): Stefan number, \( C_0^2(T_\infty^0 - T^2)/(\rho H) \)
- \( T^\circ \): Temperature, \( K \)
- \( T^\circ \): Dimensionless temperature, \( (T^\circ - T_\infty^2)/(T_0^2 - T_\infty^2) \)
- \( U_\text{av} \): Average velocity of the transfer fluid inside the tube, \( m/s \)
- \( x \): Axial coordinate, \( m \)
- \( X \): Dimensionless axial coordinate, \( x/r_i \)

Greek Letters

- \( \alpha \): Thermal diffusivity, \( m^2/s \)
- \( \delta T \): Phase-change temperature range, \( K \)
- \( \lambda_\text{s} \): Eigenvalues
- \( \rho \): Density

Subscripts

- \( E \): East neighbor of grid \( P \)
- \( f \): Transfer fluid
- \( i \): Initial condition, or internal radius of the tube
- \( in \): Inlet
- \( l \): Liquid phase
- \( m \): Melting point
- \( N \): North neighbor of grid \( P \)
- \( o \): Shell
- \( p \): Grid point
- \( s \): Solid phase
- \( S \): South neighbor of grid \( P \)
- \( w \): Tube wall and fins
- \( W \): West neighbor of grid \( P \)

References


