MARANGONI AND BUOYANCY EFFECTS ON
DIRECT METAL LASER SINTERING WITH A
MOVING LASER BEAM

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A three-dimensional model describing melting and resolidification of a direct metal laser
sintering process under the irradiation of a moving Gaussian laser beam is developed.
Effects of shrinkage and natural convection driven by the surface tension and buoyancy
force are taken into account. The energy equation is formulated using a temperature-
transforming model and solved by the finite-volume method. The temperature distribution
and velocity field are investigated. The results show that increasing the initial porosity of
the powder bed enlarges the depth of the melt–solid interface and that the laser intensity
has great influence on both the depth and width of the liquid pool. The whole molten pool
shifts toward the direction opposite the laser scanning as the scanning velocity increases.

1. INTRODUCTION

Selective laser sintering (SLS) is a rapid prototyping process that fabricates
parts directly from CAD design. An object is created by selectively fusing a powder
layer with a scanning laser beam [1–4]. SLS is a complex process involving multiple
heat and mass transfer modes that have significant influence on the quality of final
product. Therefore, it is necessary to establish sound physical models to analyze the
effects of various processing parameters on the laser sintering process. Many
semianalytical heat conduction models have been developed to determine the tem-
perature distribution and melt/solid interface for laser-induced heating problems
in the past years. Carslaw and Jaeger [5] considered the melting of a semi-infinite
body with constant thermophysical properties and obtained an analytic solution
for Dirichlet boundary conditions. Hoashi et al. [6] analyzed numerically the heat
transfer with phase change in a material irradiated by an ultrashort pulsed laser.
Chowdhury and Xu [7] solved a simplified one-dimensional, parabolic, two-step
model to predict heating, melting, and evaporation of metal under femtosecond laser
irradiation. Cline and Anthony [8] derived a thermal solution for a moving Gaussian
laser beam with a constant velocity. Lax [9] studied the steady-state heat conduction

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process in a cylindrical medium when the thermal conductivity is strongly temperature-dependent, and solved the problem by linearization with a Kirchoff transformation. Reddy et al. [10] established a three-dimensional transient nonlinear finite-element model to analyze heat transfer in a stainless steel pulsed gas tungsten arc (GTA) welding process. Sensitivity analysis was applied to check the response of the model to constant and temperature-dependent material properties, heat loss due to vaporization of alloying elements, and total number of nodes to model the solution domain. Experimental work on thermal profiles and weld bead dimensions was also carried out to validate the results predicted by the model.

Particle melting by laser irradiation results in formation of convective streams caused by surface tension and buoyancy forces within the melt pool. The fluid flow plays an important role in the temperature distribution and the shape of the liquid pool. To understand quantitatively the effect of fluid flow on pool shape and

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimensionless Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>source term</td>
<td>$= u + v + w$</td>
</tr>
<tr>
<td>$B$</td>
<td>dimensionless source term</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>specific heat, J/kg K</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>dimensionless heat capacity</td>
<td>$= p_{c} / (p_{c})_{i}$</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration, m/s$^2$</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>convective heat transfer coefficient, W/m$^2$K</td>
<td></td>
</tr>
<tr>
<td>$h_{fm}$</td>
<td>latent heat of melting, J/kg</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, W/m K</td>
<td></td>
</tr>
<tr>
<td>$Ma$</td>
<td>Marangoni number</td>
<td>$= (\alpha_{d}g_{0}^{1/2})(\pi k_{r}(T_{f} - T_{i}))$</td>
</tr>
<tr>
<td>$N_{j}$</td>
<td>dimensionless moving laser beam intensity</td>
<td>$= \alpha_{d}g_{0} / (\pi R k_{r}(T_{f} - T_{i}))$</td>
</tr>
<tr>
<td>$N_{R}$</td>
<td>radiation number</td>
<td>$= \varepsilon_{s}(T_{f} - T_{i})^{3} R / k_{r}$</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure, N/m$^2$</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure</td>
<td>$= T_{f} / (T_{f} - T_{i})$</td>
</tr>
<tr>
<td>$q_{0}$</td>
<td>laser power, W</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>radius of the laser beam, m</td>
<td></td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
<td>$= \alpha_{d}g_{0}^{1/2}(\pi k_{r}n_{s}a_{s})$</td>
</tr>
<tr>
<td>$s$</td>
<td>location of the melt–solid interface, m</td>
<td></td>
</tr>
<tr>
<td>$s_{0}$</td>
<td>location of surface, m</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>dimensionless location of the melt–solid interface</td>
<td>$= s / R$</td>
</tr>
<tr>
<td>$S_{0}$</td>
<td>dimensionless location of surface</td>
<td>$= s_{0} / R$</td>
</tr>
<tr>
<td>$t$</td>
<td>time, s</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, K</td>
<td></td>
</tr>
<tr>
<td>$u_{b}$</td>
<td>scanning velocity of the laser beam, m/s</td>
<td></td>
</tr>
<tr>
<td>$U_{b}$</td>
<td>dimensionless scanning velocity of the laser beam</td>
<td>$= u_{b} / a_{s}$</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity vector</td>
<td>$= u + v + w$</td>
</tr>
<tr>
<td>$V$</td>
<td>dimensionless velocity vector</td>
<td>$= v R / a_{s}$</td>
</tr>
<tr>
<td>$V_{o}$</td>
<td>volume, m$^3$</td>
<td></td>
</tr>
<tr>
<td>$w_{s}$</td>
<td>velocity induced by shrinkage, m/s</td>
<td></td>
</tr>
<tr>
<td>$W_{s}$</td>
<td>dimensionless velocity induced by shrinkage</td>
<td>$= w_{s} R / a_{s}$</td>
</tr>
<tr>
<td>$x/y/z$</td>
<td>coordinates, m</td>
<td></td>
</tr>
<tr>
<td>$X/Y/Z$</td>
<td>dimensionless coordinates</td>
<td>$= (x, y, z) / R$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity, m$^2$/s</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{a}$</td>
<td>absorptivity</td>
<td></td>
</tr>
<tr>
<td>$\delta T$</td>
<td>one-half of phase-change temperature range, K</td>
<td></td>
</tr>
<tr>
<td>$\delta\theta$</td>
<td>one-half of dimensionless phase-change temperature range</td>
<td>$= \delta T / (T_{f} - T_{i})$</td>
</tr>
<tr>
<td>$\varepsilon_{s}$</td>
<td>emissivity of surface</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{c}$</td>
<td>dimensionless temperature</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity, kg/m/s</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
<td>$= 5.67 \times 10^{-8}$ W/m$^2$K$^4$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>dimensionless time</td>
<td>$= \alpha_{s} t / R^2$</td>
</tr>
</tbody>
</table>

**Subscripts**
- eff effective
- f fusion
- g gas
- l liquid
- p powder material
- s solid
temperature distribution during laser processing of nonporous materials, more complicated models have been reported in the literatures [11–17]. Chan et al. [11] analyzed the transient behavior of heat transfer and fluid flow driven by surface tension on a stationary laser heating process. The effects of dimensionless parameters on the surface temperature, melt pool, and cooling rate were presented. Kou and Wang [12] developed a three-dimensional convection model in the coordinate system that moves with the laser beam. Fluid flow driven by buoyancy force and surface tension gradient was considered. The model demonstrated that the surface-tension temperature coefficient could significantly affect both the convection pattern and the penetration of laser-melted pool. Li et al. [13] studied the convection-diffusion phase-change process during laser melting of ceramic materials. Results by a pure heat conduction model, a heat conduction model incorporating latent heat of fusion, and a model involving both latent heat of fusion and fluid were compared. They demonstrated that the best prediction accuracy for the melt–solid interfaces can be achieved by considering both the latent heat of fusion and fluid flow in the melt pool. Kim et al. [14] investigated the heat transfer and fluid flow of the molten pool in stationary gas tungsten arc welding using argon shielding gas. The effect of driving forces including electromagnetic, surface tension, buoyancy, and impinging plasma arc force for the weld pool convection were considered. Jadidi and Dutta [15] conducted a numerical study of three-dimensional heat transfer and fluid flow in a moving gas metal arc welding process. Chakraborty and co-workers simulated the three-dimensional turbulent weld pool convection in GTW processes and a surface alloying process using a high Reynolds number $k-\varepsilon$ model, which was suitably modified to account for the morphology of the solid–liquid interface [16, 17].

During the metal SLS process, when the laser beam scans and melts a row of powder particles, the melted powder grains stick to each other via surface tension forces, thereby forming a series of spheres with diameters approximately equal to the diameter of the laser beam; this is referred to as balling. In order to overcome the balling phenomenon, several methods, including melting of low-melting-point powder in a two-component metal powder system [18, 19], partial melting of single-component metal powders [20–22], and direct metal laser sintering (DMLS) [23–26] have been proposed. For the case of two-component metal powders, only binders with lower melting points are melted by the laser beam to bind the powder particles and build up metallic parts. For the case of partial melting of single-component powders, only the surface of the powder particles is melted, and the nonmelted solid cores of particles are joined together by resolidification of the liquid layer on the surface. The window for processing parameters is very narrow in order to ensure the partial melting of articles. The parts produced by partial melting of single-component or two-component metal powders usually are not fully densified and require time-consuming postprocessing. In contrast, DMLS can fabricate fully densified metal parts, and postprocessing can be eliminated. By applying lower scanning velocity and high laser intensity with a protective gas to fully melt each single track, the entire track of the laser scanning is completely melted without forming a spherical structure [23, 25].

During the SLS process, the powder bed shrinks because the powder bed, with as much as 60% porosity, is molten and resolidified to full densification as the laser
moves away. The importance of shrinkage phenomena, which generate a considerable effect on the temperature distribution and the shape of the liquid pool, has been considered by some researchers [27–29]. Zhang and Faghri [27] solved analytically a one-dimensional melting problem in a powder bed containing a mixture of powders under a boundary condition of the second kind. The result showed that the shrinkage effect on the metal laser sintering is not negligible. Chen and Zhang [28] investigated numerically the sintering process of two-component metal powders under irradiation of a moving Gaussian laser beam. The liquid velocities caused by capillary and gravity forces and the solid velocities caused by shrinkage were considered. Xiao and Zhang [29] solved partial melting and resolidification of a single-component metal powder bed with a Gaussian laser beam.

For better understanding the influences of the processing parameters on the pool dynamics and geometry as well as surface temperature distribution, the convective heat transfer and fluid flow during the sintering process of loose powders are analyzed numerically in the present study. The continuity, momentum, and energy equations are solved in coupled manner using a fixed-grid, finite-volume methodology. Shrinkage phenomena of the powder bed and the fluid flow driven by buoyancy force and surface tension gradient are taken into account.

2. PHYSICAL MODELS AND MATHEMATICAL FORMULATION

2.1. Problem Statement

Figure 1 shows a schematic diagram for melting and resolidification of metal powders under a moving Gaussian laser beam at a constant scanning speed, \( u_b \), along the positive \( x \) direction. A fraction of the laser power is absorbed by the powders and leads to the formation of a melt pool. After the laser source moves away, the liquid pool cools and resolidifies to form the fully densified part. The problem under consideration is a typical moving-heat-source problem [30]. Due to axisymmetry with respect to the center plan (\( y = 0 \)), the velocity and temperature fields were computed on only half of the domain (\( y > 0 \)). Since the size of the heat source (of the order of 0.1 mm) is much smaller than the size of the powder bed, the sintering

![Figure 1. Physical model for direct metal laser sintering with a moving laser beam.](image)
process appears to be quasi-steady-state from the standpoint of the observer located in and traveling with the heat source.

2.2. Governing Equations

A temperature-transforming model using a fixed-grid method [31] is employed to describe the melting and resolidification problem in the present study. This model assumes that the solid–liquid phase change occurs over a range of phase-change temperature, and it can also be successfully applied to phase-change problems at a fixed melting point. Since the molten pool moves with the laser beam, the problem is more conveniently studied in a reference frame \((x, y, z)\) fixed with the laser beam. The governing equations in the moving coordinate system \((x, y, z)\) can be written as [32]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0
\]

\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} [\rho u(u - u_b)] + \frac{\partial}{\partial y} (\rho uv) + \frac{\partial}{\partial z} [\rho u(w + w_s)]
\]

\[
= - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right)
\]

\[
= - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right)
\]

\[
\frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial x} [\rho v(u - u_b)] + \frac{\partial}{\partial y} (\rho vv) + \frac{\partial}{\partial z} [\rho v(w + w_s)]
\]

\[
= - \frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left( \mu \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial w}{\partial z} \right) + \rho g \beta (T - T_f)
\]

\[
\frac{\partial}{\partial t} (pcT) + \frac{\partial}{\partial x} [pcT(u - u_b)] + \frac{\partial}{\partial y} (pcvT) + \frac{\partial}{\partial z} [pcT(w + w_s)]
\]

\[
= \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right)
\]

\[
- \left\{ \frac{\partial}{\partial t} (pb) + \frac{\partial}{\partial x} [pb(u - u_b)] + \frac{\partial}{\partial y} (pBV) + \frac{\partial}{\partial z} [pb(w + w_s)] \right\}
\]

As porosity equals zero within the molten pool after melting, the shrinkage velocity of the melting zone can be determined in the moving coordinate as

\[
w_s = \begin{cases} 
0 & z > s \\
\varepsilon \left( \frac{\partial}{\partial t} - u_b \frac{\partial}{\partial x} \right) & z < s
\end{cases}
\]
By applying a temperature-transforming model, the effective specific heat of the powder bed can be expressed as

\[
c = \begin{cases} 
  c_s & T < T_f - \delta T \\
  c_m + \frac{h_d}{28T} & T_f - \delta T \leq T \leq T_f + \delta T \\
  c_f & T > T_f + \delta T 
\end{cases}
\]  

(7)

and \( b \) in Eq. (5) is

\[
b = \begin{cases} 
  0 & T < T_f - \delta T \\
  \frac{h_d}{28T} & T_f - \delta T \leq T \leq T_f + \delta T \\
  h_{sl} & T > T_f + \delta T 
\end{cases}
\]  

(8)

The thermal conductivity in Eq. (5) in the two-phase region can be assumed as a linear function of temperature, i.e.,

\[
k = \begin{cases} 
  k_{eff} & T < T_f - \delta T \\
  k_{eff} + \frac{k_p - k_g}{28T}(T - T_f + \delta T) & T_f - \delta T \leq T \leq T_f + \delta T \\
  k_f & T > T_f + \delta T 
\end{cases}
\]  

(9)

where \( k_{eff} \) is the effective thermal conductivity of the unsintered powder bed. The effective thermal conductivity of the powders depends on the arrangement of the particles in the powder bed and the order of magnitude of the thermal conductivity of the gas(es) and the particle. Contact between the particles also plays a significant role on the value of the effective thermal conductivity. For the case of a randomly packed powder bed with large thermal conductivity ratio, \( k_p/k_g \), the empirical correlation proposed by Hadley [33] appears to be the best correlation since it agrees the experimental data very well. Therefore, the effective thermal conductivity of the powder bed before sintering is calculated by the following correlation [33]:

\[
\frac{k_{eff}}{k_g} = (1 - \alpha_0) \frac{\varepsilon f_0 + (k_p/k_g)(1 - \varepsilon f_0)}{1 - \varepsilon f_0 + (k_p/k_g)\varepsilon(1 - f_0)} + \alpha_0 \frac{2(k_p/k_g)^2(1 - \varepsilon) + (1 + 2\varepsilon)(k_p/k_g)}{(2 + \varepsilon)(k_p/k_g) + 1 - \varepsilon}
\]  

(10)

where

\[
f_0 = 0.8 + 0.1\varepsilon
\]  

(11)

\[
\log \alpha_0 = \begin{cases} 
  -4.898\varepsilon & 0 \leq \varepsilon \leq 0.0827 \\
  -0.405 - 3.154(\varepsilon - 0.0827) & 0.0827 \leq \varepsilon \leq 0.298 \\
  -1.084 - 6.778(\varepsilon - 0.298) & 0.298 \leq \varepsilon \leq 0.580
\end{cases}
\]  

(12)

At the solid–liquid interface or in the mushy region, the velocities are zero. A commonly used procedure is to prescribe a fluid viscosity that is equal to the liquid viscosity in the liquid region and increases gradually over the mushy region to a large
value \( N \) in the solid. Therefore the viscosity is expressed as

\[
\mu = \begin{cases} 
N & T < T_f - \delta T \\
\mu_e + \frac{\mu_e - N}{257} (T - T_f - \delta T) & T_f - \delta T \leq T \leq T_f + \delta T \\
\mu_e & T > T_f + \delta T 
\end{cases}
\]  

(13)

2.3. Boundary Conditions

The boundary condition at the top surface \((z = s_0)\) is given by

\[
-\frac{q_0}{\pi R^2} \exp\left(-\frac{x^2 + y^2}{R^2}\right) + h(T - T_\infty) + \sigma_e \varepsilon(T^4 - T_\infty^4) = -k \frac{\partial T}{\partial z} \bigg|_{z=s_0}
\]  

(14)

where \( R \) is the radius of the Gaussian laser beam. Also, to model the Marangoni convection due to temperature gradients at the top surface, we balance the shear force and surface tension at the free surface,

\[
\mu_e \left( \frac{\partial v_s}{\partial n} + \frac{\partial v_n}{\partial s} \right) = \frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial s}
\]  

(15)

where \( v_s \) and \( v_n \) in Eq. (15) are the tangential and normal velocity components at the heating surface.

The boundary of pressure at the surface of the molten pool is assumed to be the atmospheric pressure, and the pressure at the liquid–solid interface is determined by the Laplace-Young equation.

\[
p = p_\infty \quad z = s_0
\]  

(16)

\[
p_e = p_v - \frac{2\sigma}{r_{\text{eff}}} = p_\infty - \frac{2\sigma}{r_{\text{eff}}} \quad z = s
\]  

(17)

Considering that the curvature of the radius of the free surface (of the order of the radius of the laser beam) is much larger than the curvature of the radius at the boundary of the liquid pool and unsintered region (of the order of powder particle radius), the effect of the curvature at the free surface on the liquid pressure was not considered in this article.

At the bottom surface \((z = z_{\text{max}})\), the boundary conditions for velocity and temperature are as follows:

\[u = v = w = 0\]  

(18)

\[-k \frac{\partial T}{\partial z} = h(T - T_\infty)\]  

(19)

Symmetric conditions at the center surface \((y = 0)\) are valid:

\[v = 0\]  

(20)
\[
\frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} = \frac{\partial T}{\partial y} = 0
\]  
(21)

At the side surface \((y = y_{\text{max}})\), the boundary conditions are
\[
u = v = w = 0
\]  
(22)

\[-k \frac{\partial T}{\partial y} = h(T - T_{\infty})
\]  
(23)

At \(x = -x_{\text{max}}\) (far ahead of the heat source),
\[
\nu = v = w = 0
\]  
(24)

\[T = T_i\]
(25)

At \(x = x_{\text{max}}\) (far behind the heat source),
\[
u = v = w = 0
\]  
(26)

\[\frac{\partial T}{\partial x} = 0
\]  
(27)

### 2.4. Dimensionless Governing Equations

Introducing the dimensionless variables

\[
(X, Y, Z) = \frac{(x, y, z)}{R} \quad (U, V, W, U_b) = \frac{(u, v, w, u_b)}{\alpha_t} \quad S = \frac{s}{R} \quad \tau = \frac{\alpha_t t}{R^2}
\]

\[
\theta = \frac{T - T_f}{T_f - T_i} \quad \delta \theta = \frac{\delta T}{T_f - T_i} \quad C = \frac{\rho c}{(pc)_t} \quad C_{st} = \frac{(pc)_s}{(pc)_t} \quad B = \frac{b}{c_i(T_f - T_i)}
\]

\[
K = \frac{k}{k_t} \quad K_{\text{eff}} = \frac{k_{\text{eff}}}{k_t} \quad \text{Pr} = \frac{\mu}{\rho \alpha_t} \quad \text{St} = \frac{c_i(T_f - T_i)}{h_{st}} \quad \delta \theta = \frac{\delta T}{T_f - T_i}
\]

\[
P = \frac{(p + \rho g)R^2}{\rho \alpha_t^2}
\]  
(28)

the governing equations can be rewritten as

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0
\]  
(29)

\[
\frac{\partial U}{\partial \tau} + \frac{[U(U - U_b)]}{\partial X} + \frac{\partial (UV)}{\partial Y} + \frac{\partial [U(W + W_s)]}{\partial Z}
\]

\[
= - \frac{\partial P}{\partial X} + \frac{\partial}{\partial X} \left( \text{Pr} \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \text{Pr} \frac{\partial U}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \text{Pr} \frac{\partial U}{\partial Z} \right)
\]  
(30)
\[
\frac{\partial V}{\partial \tau} + \frac{\partial [V(U - U_b)]}{\partial X} + \frac{\partial (VV)}{\partial Y} + \frac{\partial [V(W + W_s)]}{\partial Z} = - \frac{\partial P}{\partial Y} + \frac{\partial}{\partial X} \left( \Pr \frac{\partial V}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \Pr \frac{\partial V}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \Pr \frac{\partial V}{\partial Z} \right)
\]

(31)

\[
\frac{\partial W}{\partial \tau} + \frac{\partial [W(U - U_b)]}{\partial X} + \frac{\partial (WV)}{\partial Y} + \frac{\partial [W(W + W_s)]}{\partial Z} = - \frac{\partial P}{\partial Z} + \frac{\partial}{\partial X} \left( \Pr \frac{\partial W}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \Pr \frac{\partial W}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \Pr \frac{\partial W}{\partial Z} \right) + \frac{Ra}{N_i} \Pr \theta
\]

(32)

\[
\frac{\partial (C0)}{\partial \tau} + \frac{\partial [C0(U - U_b)]}{\partial X} + \frac{\partial (C0 V)}{\partial Y} + \frac{\partial [C0 (W + W_s)]}{\partial Z} = \frac{\partial}{\partial X} \left( K \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( K \frac{\partial \theta}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( K \frac{\partial \theta}{\partial Z} \right)
\]

\[
- \left[ \frac{\partial B}{\partial \tau} + \frac{\partial [(U - U_b)B]}{\partial X} + \frac{\partial (VB)}{\partial Y} + \frac{\partial [(W + W_s)B]}{\partial Z} \right]
\]

(33)

where

\[
W_s = \begin{cases} 
0, & Z \geq S \\
\frac{\partial \xi}{\partial X} - U_b \frac{\partial \xi}{\partial X}, & Z < S 
\end{cases}
\]

(34)

\[
Pr = \begin{cases} 
N, & 0 < \delta_0 \\
Pr_t + \frac{(Pr_t - N)}{2\delta_0} (0 - \delta_0), & -\delta_0 \leq 0 \leq \delta_0 \\
Pr_t, & 0 > \delta_0 
\end{cases}
\]

(35)

\[
C = \begin{cases} 
\frac{C_{st}}{4} (1 + C_{st}) + \frac{1}{2 \delta_0 \delta_0}, & 0 < \delta_0 \\
\frac{1}{4}, & -\delta_0 \leq 0 \leq \delta_0 \\
0, & 0 > \delta_0 
\end{cases}
\]

(36)

\[
K = \begin{cases} 
K_{eff}, & 0 < \delta_0 \\
K_{eff} + \frac{(1 - K_{eff})}{2\delta_0} (0 + \delta_0), & -\delta_0 \leq 0 \leq \delta_0 \\
1, & 0 > \delta_0 
\end{cases}
\]

(37)

\[
B = \begin{cases} 
0, & 0 < \delta_0 \\
-\delta_0 \leq 0 \leq \delta_0 \\
\frac{1}{\delta_0}, & 0 > \delta_0 
\end{cases}
\]

(38)

The boundary condition Eq. (14), written in dimensionless form, is

\[
-K \frac{\partial \theta}{\partial Z} = N_i \exp[-X^2 - Y^2] \\
- N_R[(0 + N_i)^4 - (0 + N_i)^4] - B_i (0 - 0) \\
Z = S_0(X, Y)
\]

(39)
And the dimensionless form of Marangoni convection at the heating surface is

\[
\frac{\partial V_s}{\partial n} + \frac{\partial V_n}{\partial s} = -\frac{Ma}{N_l} \frac{\partial \theta}{\partial s} \quad Z = S_0(X, Y)
\]  

The dimensionless forms of other boundary equations can also be obtained using the dimensionless variables defined in Eq. (28).

3. NUMERICAL SOLUTIONS

The melting and resolidification problem specified by Eqs. (29)–(33) is a steady-state, three-dimensional, nonlinear problem. Since the locations of the solid–liquid interface and the heating surface are unknown \textit{a priori}, a false transient method is employed to locate various interfaces. Steady-state solution is obtained when the temperature distribution and locations of various interfaces do not vary with the false time. Equation (33) with the false transient term can be solved by the finite-volume method [34]. The computational domain is the whole powder bed, and a block-off technique [34] is employed to simulate the existence of the empty space created by shrinkage of the powder bed after melting. The density and thermal conductivity in the empty space are set to zero.

The governing equations (29)–(33) were discretized and solved using the SIMPLER algorithm [34]. The convection and diffusion terms were discretized using the power-law scheme. The boundary conditions are merged into energy and momentum equations for the appropriate nodes using the additional source term method [34]. A nonuniform 122 × 52 × 52 (in the \(X, Y, Z\) directions, respectively) grid number is used in computation, and the false time step is 0.001. A fine grid within the melt pool and a coarse grid in the unsintered region were used. The iterative procedure was continued until the following convergence criterion was satisfied:

\[
\sum_P \frac{|\phi - \phi^{old}|}{\sum_P |\phi|} < 0.001
\]  

where \(\sum_P\) denotes summation over all grid points and \(\phi\) is the variables being computed, e.g., \(U, V, W, \) and \(T\).

4. RESULTS AND DISCUSSION

The computer program developed is first used to simulate laser melting of a nonporous 6063 aluminum sheet with dimensions of \(229 \times 152 \times 3.2\) mm, which was studied experimentally by Kou and Wang [12]. The nominal beam power indicated on the Spectra Physics 971 continuous-wave CO\(_2\) laser was 1.3 kW. The travel speed of the workpiece was 4.23 mm/s. The power absorbed by the workpiece was measured calorimetrically. A calorimeter was made of a \(38 \times 38\) mm square tube of the workpiece material. An 86% heat loss from the surface area irradiated by the laser beam includes those by reflection, radiation, and convection. The beam
Diameter, 0.6 mm, was measured using a “split anode method.” More detail about the experimental procedure and treatment of the data can be found in [12]. Figure 2 shows the comparison of the fusion boundary obtained by the present study and Kou and Wang’s experimental results in [9]. It can be seen that the simulated and measured fusion boundaries are in good agreement with each other except at the edge of the heat-affected zone. The main reason may be due to deviations in the modeled Gaussian model from the actual laser energy distribution.

Numerical calculation is then performed for sintering of AISI 304 stainless steel powder. Table 1 lists the material physical properties for AISI 304 [14, 35]. The average powder size is 64 μm and the thermal conductivity of interstitial gas in the powder bed is $k_g = 3.7 \times 10^{-4}$ W/m K. Figure 3a illustrates the surface temperature distribution of the powder bed subjected to a moving laser beam in the moving coordinates system. The peak temperature at the powder bed surface is near the trailing edge of the laser beam rather than at the center of the laser beam, due to motion of the laser beam. Because the thermal conductivity in the melt pool is much larger than that in the unsintered zone, the temperature changes smoothly in the

![Figure 2. Comparison of experimental and calculation results for laser fusion of 6063 aluminum sheet.](image)

**Figure 2.** Comparison of experimental and calculation results for laser fusion of 6063 aluminum sheet.

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### Table 1. Thermophysical properties of AISI 304

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho_p$</td>
<td>kg/m$^3$</td>
<td>7.200</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$k_p$</td>
<td>W/m K</td>
<td>14.9</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$c_p$</td>
<td>J/kg K</td>
<td>462.6</td>
</tr>
<tr>
<td>Melting point</td>
<td>$T_f$</td>
<td>K</td>
<td>1.670</td>
</tr>
<tr>
<td>Latent heat of fusion</td>
<td>$h_{sf}$</td>
<td>kJ/kg</td>
<td>247</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu$</td>
<td>kg/s m</td>
<td>0.05</td>
</tr>
<tr>
<td>Surface tension at melting point</td>
<td>$\gamma$</td>
<td>N/m</td>
<td>1.943</td>
</tr>
<tr>
<td>Dependence of surface tension on temperature</td>
<td>$\partial \gamma / \partial T$</td>
<td>N/m K</td>
<td>$-4.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>$\beta$</td>
<td>1/K</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
molten pool but sharply in the unsintered zone near the melt pool. Figure 3b shows the three-dimensional shape of the powder bed surface, melt pool, and heat-affected zone at the same conditions as Figure 3a.

Figures 4a–4c show the velocity vector plots in three different views. During the melting process, thermocapillary flow induced by surface tension variations along a liquid–gas interface caused by nonuniform heating of the free surface is an important phenomenon. Thermocapillary flow is determined by the dependence
of surface tension on temperature, $\frac{\partial \gamma}{\partial T}$, and surface temperature distribution. Since the surface tension is a decreasing function of temperature, i.e., $\frac{\partial \gamma}{\partial T} < 0$, the higher surface tension of the cooler liquid metal near the edge of the liquid pool and the thermal-capillary force induced by the surface temperature gradient tends to pull the liquid metal away from the center of the liquid pool, where the liquid metal is hotter and the surface tension is lower. Therefore, fluid flow on the surface of liquid pool is radially outward, as can be seen in Figure 4a. The liquid metal flow is also driven by buoyancy force as illustrated in Figures 4b and 4c. The hotter liquid metal near the central region of the molten pool floats up to the surface, while the cooler liquid metal near the pool boundary sinks along the melt–solid interface to the bottom of the pool. This circulation of fluid flow induced by the surface tension gradient and buoyancy force is consistent with the typical natural-convection pattern found in the literature.

Figures 5a–5c present the temperature contours in three different views. The top and longitudinal views show that the temperature contours are stretched along the scanning direction of the laser. This stretching is due to the effect of the moving heat source, which causes advection heat flow in the moving direction. That is also the
reason the peak temperature at the surface is near the trailing edge of the laser beam instead of near the beam center. The Marangoni convection (radically outward) at the top surface and the shrinkage phenomenon in the Z direction induce a significant amount of heat flow from the hotter region to the colder region, which in turn results in the melt pool becoming wider and deeper, respectively. Another observation is that the isotherms near the melting front are more closely spaced compared with those far away from the melt–solid interface.

The effect of initial porosity on the sintering process is illustrated by the longitudinal and cross-sectional plots of the heating surface, molten pool, and heat-affected zone as shown in Figures 6a and 6b. With increasing initial porosity, the degree of shrinkage increases, which in turn means the top surface after melting becomes lower. The thermal conductivity in the unsintered zone decreases with increasing porosity so that more heat is available to melt the powder particles, which results in a deeper bottom of the liquid pool. The increase of the surface and bottom of

Figure 5. Dimensionless temperature contour ($N_t = 1.0$, $U_b = 0.05$, $\varepsilon = 0.4$, $Ra = 46.42$, $Ma = 2825.9$).
the molten pool are much more significant than the increases of the molten pool sizes in the $X$ and $Y$ directions.

Figures 7a and 7b illustrate the effect of laser intensity on the sintering process. When the laser intensity increases, an increased amount of liquid phase is formed due to an increased amount of energy input to the powder bed. The fluid at higher temperature flows from the center of the liquid pool outward toward the solid–liquid interface, tending to increase the size of the melt pool. Compared with Figures 6b and 8b, the widening trend with increasing laser intensity is more

Figure 6. Effect of initial porosity of metal powders on the sintering process.
significant than the trend caused by increasing porosity or decreasing scanning velocity. This trend is consistent with the laser surface processing experiments conducted by He et al. [36].

Figures 8a and 8b show the effect of scanning velocity on the sintering process. As can be seen, the depths of the surface, molten pool, and heat-affected zone decrease and the molten pool becomes narrower with increasing scanning velocity because the interaction time between the moving heat source and the powder surface is decreased when the heat source moves faster. Figure 8a also illustrates that, when the scanning velocity increases, the whole melt pool shifts toward the direction
opposite the laser scanning, due to the enhanced advection flow caused by the moving of the laser beam relative to the powder bed.

5. CONCLUSIONS

A three-dimensional numerical model for the convection-diffusion phase-change process during laser sintering of a loose powder bed has been proposed. The numerical model has been validated by comparing the predicted cross-sectional profile for the melt–solid interface for laser processing with nonporous metal experiments.
Temperature and velocity profiles, as well as the shapes of the heating surface, liquid pool, and heat-affected zone have been obtained. The results show that the melting and fluid flow are dominated mainly by the thermocapillary force, shrinkage phenomenon, and buoyancy force. The thermocapillary force pulls the liquid from the melt pool center to the edge, while the shrinkage phenomenon and buoyancy force help the hot fluid downward. The parametric analysis shows that increasing initial porosity of the powder bed or laser intensity and decreasing scanning velocity enlarge the depth of the melt–solid interface. The increasing scanning velocity results in the shift of the melt pool toward the direction opposite the laser beam.

REFERENCES


