NON-FOURIER HEAT CONDUCTION EFFECT ON
LASER-INDUCED THERMAL DAMAGE IN
BIOLOGICAL TISSUES

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To ensure personal safety and improve treatment efficiency in laser medical applications, one of the most important issues is to understand and accurately assess laser-induced thermal damage to biological tissues. Biological tissues generally consist of nonhomogeneous inner structures, in which heat flux equilibrates to the imposed temperature gradient via a relaxation phenomenon characterized by a thermal relaxation time. Therefore, it is naturally expected that assessment of thermal damage to tissues could be inaccurate when a classical bioheat conduction model is employed. However, little attention has been given to studying the impact of the bioheat non-Fourier effect. In this article, a thermal wave model of bioheat transfer, together with a seven-flux model for light propagation and a rate process equation for tissue damage, is presented to investigate thermal damage in biological tissues. It is shown that the thermal damage assessed with the thermal wave bioheat model may differ significantly from that assessed with the classical bioheat model. Without including the bioheat non-Fourier effect, the assessment of thermal damage to biological tissue may not be reliable.

1. INTRODUCTION

Lasers have been widely used in medical applications for more than three decades. A great part of these applications involves thermal effects such as laser-induced hyperthermia, coagulation, caries therapy, surgery, etc. No matter which application is involved, accurate thermal damage assessment is indispensable in order to ensure personal safety and improve treatment efficiency.

Welch [1] proposed a general three-step model of laser–tissue interactions, including (1) laser energy deposition in tissue, (2) heat transfer, and (3) tissue protein denaturation. Since then, many researchers have followed his approach by applying different models or methods to solve the problems involved in the process. For example, Beacco et al. [2] presented a beam-broadening model for light propagation in tissues and a bioheat transfer equation for thermal response. Prahl et al. [3] developed a thermal damage model for a laser tissue welding process, in which a simple Beer’s law and the rate process equation were used to estimate light energy absorption and thermal damage, respectively. For heat conduction, a lump capacity approach was employed before the tissue was vaporized and a one-dimensional analytical solution after it was.
vaporized. Zhu et al. [4] applied diffusion theory to evaluate laser energy absorption, as well as the classical bioheat equation and the rate process model, to predict thermal response and tissue damage, respectively. Diaz et al. [5] used a finite-element method to solve the heat conduction equation for laser irradiated cartilage. Recently, Zhou et al. [6] performed a parametric study on thermal damage in laser-irradiated tissues using a seven-flux model and the classical bioheat transfer equation. To our knowledge, all the bioheat conduction equations used in assessing laser-induced damage in biological tissues are based on the classical Fourier law.

Rastegar [7], Kaminski [8], and Mitra et al. [9] showed that for biomaterials with nonhomogeneous inner structures, the heat flux equilibrates to the imposed temperature gradient via a relaxation phenomenon characterized by a thermal characteristic (relaxation) time. To more accurately describe thermal transport in biomaterials, the thermal wave effect [10] has been incorporated into the classical

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$[A], [B]$</td>
<td>Jacobian matrices</td>
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<tr>
<td>$c$</td>
<td>speed of thermal wave, m/s</td>
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<tr>
<td>$c_b$</td>
<td>specific heat of blood, J/kg · K</td>
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<tr>
<td>$c_p$</td>
<td>specific heat of the tissue, J/kg · K</td>
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<td>$E$</td>
<td>energy of activation of denaturation reaction, J</td>
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<td>$F$</td>
<td>light flux, W/m$^2$</td>
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<tr>
<td>$g$</td>
<td>internal heat sources due to laser light absorption in the tissue, W/m$^3$</td>
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<td>$k$</td>
<td>thermal conductivity of tissue, W/m · K</td>
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<tr>
<td>$L(\vec{r},\vec{s})$</td>
<td>intensity of laser light at position $\vec{r}$ in the $\vec{s}$ direction, W/m$^2$ · sr</td>
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<td>$p(\vec{s},\vec{s'})$</td>
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<td>$q$</td>
<td>heat flux, W</td>
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<td>$q_r, q_z$</td>
<td>radial and axial components of heat flux, respectively, W/m$^2$</td>
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<td>${Q}, {E}, {F}, {S}$</td>
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<td>$g_m$</td>
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<tr>
<td>$R$</td>
<td>universal gas constant</td>
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<td>$\left(= 8.314 \text{J/mol} \cdot \text{K}\right)$</td>
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<td>$[R_x], [R_z]$</td>
<td>right eigenmatrices, given by Eq. (12)</td>
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### Subscripts

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<tr>
<td>$b$</td>
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<td>$c$</td>
<td>collimated light</td>
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<td>$(i, j)$</td>
<td>control-volume indices</td>
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### Superscripts

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<td>$k$</td>
<td>kth wave</td>
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<td>time levels $n$ and $n+1$</td>
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<td>$^1$</td>
<td>inverse matrix</td>
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<td>$^*$</td>
<td>dimensionless variable</td>
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heat conduction equations [7–9] and bioheat conduction equations [11–15]. Recently, Mitra and his associates [16] have further demonstrated experimentally and theoretically that for both short-pulse and continuous-wave laser irradiations, the hyperbolic heat conduction equation is a better approximation than the parabolic heat conduction equation for modeling temperature response in tissues.

Since most of the bioheat conduction equations employed in assessing laser-induced damage in biological tissues are based on the classical Fourier law [1–6], it is naturally expected that assessment of thermal damage to tissues might be different when a non-Fourier bioheat conduction model is considered. In this article, we present a two-dimensional (2-D), axisymmetric thermal wave model of bioheat transfer (TWMBHT) to investigate laser-induced damage in biological tissues. The other two processes, laser energy deposition in tissue and tissue protein denaturation, are simulated using the seven-flux model and the rate process equation, respectively. To avoid fictitious numerical oscillations in the vicinity of the sharp thermal wave fronts and reflective boundaries, the TWMBHT is solved numerically using a highly accurate, oscillation-free, total variation diminishing (TVD) method [17, 18]. The results, including temperature and thermal damage, are compared with those calculated with the Pennes bioheat transfer (PBHT) model [19].

2. PROBLEM FORMULATION

Consider a cylindrical biological tissue of radius $r_0$ and thickness $z_0$ irradiated by a continuous-wave laser beam for a period of time $t_p$. The beam is applied normally to the upper surface ($z = 0$) of the tissue, as shown in Figure 1. It has a Gaussian spatial profile of radius $r_p$, defined as the radial distance at the $1/e^2$ points of the pulse intensity distribution. Let the center of the incident laser beam coincide with the origin $O$ of the cylindrical coordinate system. The other boundary surfaces are thermally insulated to the environment. Assume that the tissue is initially at a

![Figure 1. Schematic model of a cylindrical tissue under laser irradiation.](image-url)
uniform temperature $T_0$. Due to the axisymmetry of the tissue geometry and the thermal load, a 2-D axisymmetric cylindrical domain $OABC$ (Figure 1) is analyzed for laser light propagation, heat transfer, and thermal damage in the tissue.

2.1. 2-D Axisymmetric Thermal Wave Model of Bioheat Transfer

Though numerous bioheat transfer models have been proposed for different tissue structures in the past decades (interested readers are referred to those review articles [20–22]), the PBHT model [19], with its mathematical simplicity, is still a powerful theoretical tool for various biomedical applications [23–26]. In this work, a general type of tissue is considered, and all the blood vessels in the tissue are assumed to be much smaller than the bulk tissue. Under these circumstances, it is reasonable to choose the PBHT model as the basis for formulation of the TWMBHT.

The 2-D axisymmetric TWMBHT includes the PBHT equation [19] and two heat flux constitutive relations as follows:

$$\rho_c \frac{\partial T}{\partial t} = -\frac{1}{r} \left[ \frac{\partial (r \cdot q_r)}{\partial r} + \frac{\partial (r \cdot q_z)}{\partial z} \right] + w_b \rho_b c_b (T_b - T) + g_m + g$$  \hspace{1cm} (1)

$$\tau \frac{\partial q_z}{\partial t} + q_z = -k \frac{\partial T}{\partial z}$$  \hspace{1cm} (2)

$$\tau \frac{\partial q_r}{\partial t} + q_r = -k \frac{\partial T}{\partial r}$$  \hspace{1cm} (3)

The relaxation time $\tau$ can be interpreted as the time scale at which the intrinsic length scale of thermal diffusion [$\lambda_D = (\alpha t)^{1/2}$] is equal to the intrinsic length scale of the thermal wave ($\lambda_W = ct$) [27], where $\alpha$ is the thermal diffusivity and $c$ denotes the thermal propagation speed of biological tissues. Setting $\lambda_W = \lambda_D$ leads to the relaxation time $\tau = \alpha/c^2$. The thermal wave speed $c$ becomes finite when $\tau > 0$ and approaches infinity as $\tau \to 0$. Equations (1)–(3) reduce to the PBHT model when $\tau = 0$. For biological tissues, the value of $\tau$ was estimated to be 20–30 s [8]. Mitra et al. [9] found $\tau \approx 16$ s from the measurement for a processed meat. In this study, the thermal relaxation time of $\tau \approx 16$ s is used.

Introducing the dimensionless quantities

$$\begin{align*}
z^* &= \frac{cz}{2\alpha} & r^* &= \frac{cr}{2\alpha} & t^* &= \frac{c^2 t}{2\alpha} & T^* &= \frac{T}{T_{ref}} & q^*_z &= \frac{\alpha q_z}{c k T_{ref}} \\
q^*_r &= \frac{\alpha q_r}{c k T_{ref}} & g^*_m &= \frac{4\alpha^2 g_m}{c^2 k T_{ref}} & g^* &= \frac{4\alpha^2 g}{c^2 k T_{ref}} & D^*_b &= \frac{2\alpha^2 w_b \rho_b c_b}{c^2 k}
\end{align*}$$  \hspace{1cm} (4)

Eqs. (1)–(3) become

$$\frac{\partial T^*}{\partial t^*} + \frac{1}{r^*} \frac{\partial (r^* q^*_r)}{\partial r^*} + \frac{\partial q^*_z}{\partial z^*} = D^*_b (T_b^* - T^*) + \frac{1}{2} g^*_m + \frac{1}{2} g^*$$  \hspace{1cm} (5)
\[ \frac{\partial q^*_z}{\partial t^*} + \frac{\partial T^*}{\partial z^*} = -2q^*_z \]  
\[ \frac{\partial q^*_r}{\partial t^*} + \frac{\partial T^*}{\partial r^*} = -2q^*_r \]  

For convenience, the superscript \( ^* \) is eliminated in the following derivations. Equations (5)–(7) can be reexpressed in vector form as

\[ \frac{\partial \{ Q \}}{\partial t} + [A] \frac{\partial \{ Q \}}{\partial z} + [B] \frac{\partial \{ Q \}}{\partial r} = \{ S \} \]  

where

\[ \{ Q \} = \begin{bmatrix} T \\ q_z \\ q_r \end{bmatrix} \quad [A] = \frac{\partial \{ E \}}{\partial \{ Q \}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad [B] = \frac{\partial \{ F \}}{\partial \{ Q \}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \]  

\[ \{ S \} = \begin{pmatrix} \frac{1}{2} g' - \frac{q_r}{r} \\ -2q_z \\ -2q_r \end{pmatrix} \quad \{ E \} = \begin{pmatrix} q_z \\ T \\ 0 \end{pmatrix} \quad \{ F \} = \begin{pmatrix} q_r \\ 0 \\ T \end{pmatrix} \]  

with \( g' = 2D_b(T_b - T) + g_m + g \). The Jacobian matrices \([A]\) and \([B]\) can be diagonalized through the eigenmatrices \([R_z]\) and \([R_r]\),

\[ [A] = [R_z][\Lambda_z][R_z]^{-1} \quad [B] = [R_r][\Lambda_r][R_r]^{-1} \]  

in which \([\Lambda_z]\) and \([\Lambda_r]\) are the diagonal matrices consisting of three eigenvalues of \([A]\) and \([B]\), respectively. The superscript \( ^{-1} \) represents the inversion of matrix. The diagonal matrices and the eigenmatrices are given by

\[ [\Lambda_z] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad [\Lambda_r] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \]  

\[ [R_z] = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \end{pmatrix} \quad [R_z]^{-1} = \begin{pmatrix} 0 & 0 & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \quad [R_r] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \]  

\[ [R_r]^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} \]
2.2. Light Propagation

Light propagation in soft biological tissues is much more complicated, because of the strong scattering effect in the therapeutic window (visible and infrared wavelengths) [28]. A radiation-transfer integral equation for laser light propagation in biological tissues is given by [29]

\[
\hat{s} \cdot \nabla L(\hat{r}, \hat{s}) + \gamma L(\hat{r}, \hat{s}) = \frac{\gamma}{4\pi} \int_{4\pi} p(\hat{s}, \hat{s}') L(\hat{r}, \hat{s}') \, d\omega'
\]

(13)

where \( \hat{r} \) is a position vector, \( \hat{s} \) and \( \hat{s}' \) are the direction vectors, \( L(\hat{r}, \hat{s}) \) is the intensity of laser light at position \( \hat{r} \) in the \( \hat{s} \) direction, \( \omega' \) is the solid angle, and \( p(\hat{s}, \hat{s}') \) is the phase function representing the scattering contribution from \( \hat{s}' \) to the \( \hat{s} \) direction. The unit of \( L \) in Eq. (13) is \( \text{W/m}^2 \text{sr} \). The solid-angle integration of the phase function is defined as

\[
\frac{1}{4\pi} \int_{4\pi} p(\hat{s}, \hat{s}') \, d\omega' = \frac{\sigma}{\gamma}
\]

(14)

where the attenuation coefficient \( \gamma \) is the sum of the absorption coefficient (\( \kappa \)) and the scattering coefficient \( \sigma \).

An exact analytical solution of Eq. (13) is impossible for multidimensional geometries. Although the Kubelka-Munk approach [30] is a useful tool for solving Eq. (13), it is limited to 1-D geometries. The diffusion theory [31] provides a fast approach to approximating some physical quantities of light transport in turbid media; however, it is not valid near the photon source or a boundary, where laser intensity is strongly anisotropic. Monte Carlo simulation [28] offers a flexible and accurate approach, but it is quite computationally intensive. The seven-flux model proposed by Yoon et al. [32], on the other hand, is an accurate and fast algorithm for computing laser light distribution in tissues, in which the differential-integral Eq. (13) can be converted to a linear system of equations.

The governing equations for the seven fluxes in a cylindrical coordinate system \((r, z, \phi)\), depicted in Figure 1, are expressed as follows [33]:

\[
\frac{\partial F_c(r, z, \phi)}{\partial z} = -\gamma F_c(r, z, \phi)
\]

(15)

\[
\frac{\partial F_{x_i}(r, z, \phi)}{h \partial x_j} + \gamma F_{x_i} = \gamma [p_{x_i,+r} F_{r} + p_{x_i,-r} F_{-r} + p_{x_i,+\phi} F_{+\phi} + p_{x_i,-\phi} F_{-\phi} + p_{x_i,+\phi} F_{+\phi} + p_{x_i,-\phi} F_{-\phi}]
\]

(16)

where \( F_c \) is the collimated light flux in the \( +z \) direction; \( F_{\pm r}, F_{\pm z}, \) and \( F_{\pm \phi} \) are the six components of the scattered light flux in the \( \pm r, \pm z, \) and \( \pm \phi \) directions, respectively; \( h = 1, 1, r \) for \( x_i = \pm r, \pm z, \pm \phi \), respectively; the phase function \( p(\hat{s}, \hat{s}') \) in Eq. (13) is represented by \( p_{ij} \), denoting the portion of the flux scattered from the \( j \) direction into the \( i \) direction. Because of the axisymmetric nature considered here, the fluxes
in the $\pm \phi$ directions are identical. Thus, the number of the governing Eq. (16)
reduces to 5. Detailed description for determination of $p_{ij}$ can be found in [33].

2.3. Damage Prediction

To quantify the extent of thermal damage in a living tissue, a damage parameter $\Omega$ is defined as [1]

$$\Omega = \ln \left( \frac{C_0}{C_0 - C_d} \right)$$

where $C_0$ is the protein concentration in a nonirradiated tissue and $C_d$ is the concentration of denatured protein. A tissue is assumed to be irreversibly damaged when $\Omega = 1.0$ [1], which corresponds to denaturation of 63% of the protein molecules.

The protein denaturation process can be considered as a chemical reaction. Thus, the damage parameter $\Omega$ is evaluated according to the Arrhenius equation [1],

$$\Omega(r, z, t) = A \int_{t_1}^{t_f} \exp \left( -\frac{E}{RT} \right) dt$$

where $A$ is the frequency factor; $E$ is the energy of activation of denaturation reaction; $R$ is the universal gas constant, 8.314 J/mol · K; $T$ is the absolute temperature of the tissue at the point where $\Omega$ is calculated; $t_1$ is the time at the onset of laser exposure; and $t_f$ is the time when the thermal damage is evaluated. Numerical values for the frequency factor and the activation energy are given as [1] $A = 3.1 \times 10^9$ s$^{-1}$ and $E = 6.28 \times 10^5$ J/mol.

3. NUMERICAL ALGORITHMS

3.1. Numerical Solution of the TWMBHT

In the non-Fourier hyperbolic heat conduction, thermal energy transport is dominated by thermal wave propagation rather than diffusion. Numerous algorithms have been proposed for capturing the sharp thermal wave fronts [e.g., 17, 18, 34–37]. Among them, the oscillation-free TVD method is a highly accurate and stable approach [17, 18, 37]. Therefore, we use it to solve the 2-D axisymmetric TWMBHT in this study.

Figure 2 depicts a control volume $(i, j)$ with the surrounding control volumes. All the control volumes are equally spaced in the $(r, z)$ computational domain with a size of 1 rad in the direction perpendicular to the $r$–$z$ plane. In the time integration of Eq. (8) from time $t^n$ to $t^{n+1}$, the explicit finite difference over the control volume centered at node $(i, j)$ is in the form

$$Q_{i,j}^{n+1} = Q_{i,j}^n - \frac{\Delta t}{\Delta z} \left( \{E\}_{i+1/2,j}^n - \{E\}_{i-1/2,j}^n \right) - \frac{\Delta t}{\Delta r} \left( \{F\}_{i,j+1/2}^n - \{F\}_{i,j-1/2}^n \right) + \Delta t \cdot \{S\}_{i,j}^n$$

(19)
Using the fractional step (time-splitting) method, Eq. (19) becomes

\[ \{ Q \}_{i,j}^* = \{ Q \}_{i,j}^n - \frac{\Delta t}{\Delta z} (\{ E \}_{i+1/2,j}^n - \{ E \}_{i-1/2,j}^n) + \frac{1}{2} \Delta t \{ S \}_{i,j}^n \]  

(20)

\[ \{ Q \}_{i,j}^{n+1} = \{ Q \}_{i,j}^* - \frac{\Delta t}{\Delta r} (\{ F \}_{i,j+1/2}^* - \{ F \}_{i,j-1/2}^*) + \frac{1}{2} \Delta t \{ S \}_{i,j}^* \]  

(21)

In this manner, the 2-D operator has been split into the product of two 1-D operators. In Eqs. (20) and (21), the superscript * denotes the middle time step.

Now, one can apply the high-resolution, oscillation-free TVD scheme to each of the above two-step Eq. (20) and (21). Defining the characteristic variables

\[ \{ W_z \} = [R_z]^{-1} \{ Q \} \quad \{ S_z \} = [R_z]^{-1} \{ S \} \]  

(22)

and multiplying Eq. (20) by \([R_z]^{-1}\) leads to

\[ \{ W_z \}_{i,j}^* = \{ W_z \}_{i,j}^n - \frac{\Delta t}{\Delta z} ([A_z]_{i+1/2,j} \{ W_z \}_{i+1/2,j}^n - [A_z]_{i-1/2,j} \{ W_z \}_{i-1/2,j}^n) + \frac{1}{2} \Delta t \{ S_z \}_{i,j}^n \]  

(23)
Since \([A_z]\) is a diagonal matrix, the above vector equation actually includes three decoupled equations of the form

\[
(w^k)^n_{ij} = (w^k)^n_{ij} - \frac{\Delta t}{\Delta z} [ (f^k)^n_{i+1/2,j} - (f^k)^n_{i-1/2,j}] + \frac{1}{2} \Delta t (s^k)^n_{ij} \quad k = 1, 2, 3
\]

where the superscript \(k\) represents the \(k\)th wave. From Eqs. (9), (12), and (22), one has

\[
w^1 = -q_r \quad w^2 = \frac{1}{2} (T + q_z) \quad w^3 = \frac{1}{2} (T - q_z)
\]

\[
f^1 = 0 \quad f^2 = w^2 \quad f^3 = -w^3
\]

\[
s^1 = 2q_r \quad s^2 = \frac{1}{4} g' - \frac{q_r}{2r} - q_z \quad s^3 = \frac{1}{4} g' - \frac{q_r}{2r} + q_z
\]

The unknown quantities \(f^k\) at the interfaces of a control volume are evaluated using an interpolation scheme proposed by Yang [17, 18]. The temperature and heat flux at the boundary of the analyzed domain, except for those \(q_z\) and \(q_r\) specified by the boundary conditions, are extrapolated from the Lagrange interpolation with the values at the interior nodes of four adjacent control volumes along the direction normal to that boundary. Since \(s^k\) are evaluated at the interior points of control volumes, no singular term \(q_r/r(r = 0)\) is involved.

Once \(\{W_z\}^n\) is obtained, \(\{Q\}^n\) can be determined by multiplying \(\{W_z\}^n\) by \([R_z]:\)

\[
\{Q\}^n = [R_z] \{W_z\}^n
\]

In the same way, the above schemes can be applied to solve \(\{Q\}_{ij}^{n+1}\) in Eq. (21). This completes the time marching from \(t^n\) to \(t^{n+1}\).

The stability criterion of the preceding numerical method is that the Courant-Friedrichs-Lewy (CFL) numbers in the \(z\) and \(r\) directions must be satisfied by

\[
\text{CFL}_z = \frac{\Delta t}{\Delta z} \leq 1 \quad \text{and} \quad \text{CFL}_r = \frac{\Delta t}{\Delta r} \leq 1
\]

In Eq. (29), \(\Delta t\), \(\Delta r\), and \(\Delta z\) are the dimensionless quantities introduced in Eq. (7). In this study, the Courant-Friedrichs-Lewy number is set at 0.02, based on the numerical tests for ensuring the solutions to be oscillation-free.

### 3.2. Numerical Solution of the Seven-Flux Light Propagation

The governing equations for the seven fluxes, (15) and (16), are solved using the finite-difference method. Once the values of all seven discrete fluxes are obtained, laser intensity \(L(\tilde{r}, \tilde{s})\) can be reconstructed from the relations between the discrete fluxes and laser intensity. The collimated flux \(F_c\) is related to the collimated
intensity $L_c(r, \hat{s})$ \cite{32} by

$$F_c(\vec{r}) = \frac{L_c(\vec{r}, \hat{s})}{\delta(\cos \theta - 1) \delta(\varphi)} \quad (30)$$

where the direction vector $\hat{s}$ is expressed as the directional angles $\theta$ and $\varphi$, and $\delta$ is the delta function. The relations between the scattered flux $F_s$ and the scattered intensity $L_s(r, \hat{s})$ are given by

$$F_s(\vec{r}) = \int_{2\pi \hat{s}} L_s(\vec{r}, \hat{s}') (\hat{s} \cdot \hat{s}') d\omega' \quad (31)$$

where $\hat{s} = \pm r, \pm z, \pm \phi$.

The solution of the collimated intensity $L_c$ in Eq. (30) is straightforward. Although the scattered intensity $L_s$ is implicitly included in Eq. (31), it can be solved by expanding the intensity $L_s$ as the associate Legendre polynomials. After introducing the Legendre polynomial expansion of $L_s$ into Eq. (31), a linear system of equations can be derived in which the unknown coefficients in the Legendre polynomial expansions can be solved.

The rate of heat generation $g$ in Eq. (1) due to absorption of light by the tissue is proportional to the fluence rate and the absorption coefficient:

$$g(\vec{r}) = \kappa \int_{4\pi} L(\vec{r}, \hat{s}) d\omega = \kappa \int_{4\pi} [L_c(\vec{r}, \hat{s}) + L_s(\vec{r}, \hat{s})] d\omega \quad (32)$$

### 3.3. Verification of the Computer Code

The TWMBHT computer code is first validated with a cylindrical tissue heated by a Gaussian heat flux on the upper surface ($z = 0$). For comparison, the 2-D axisymmetric PBHT is also simulated with the code developed in our other study [6]. The blood perfusion rate and the metabolic heat generation are the same for the TWMBHT model and the PBHT model. All the calculations are performed using Roe’s superbee limiter [18]. It is found that the thermal wave characteristics are clearly exhibited for higher values of thermal relaxation time $\tau$, as reported in other studies (e.g., [38]). As $\tau$ decreases, the TWMBHT approaches the PBHT model asymptotically. The above numerical tests verify the credibility of the algorithms and the computer code developed in the present work. For brevity, those results are not presented here.

To verify the subroutines developed for the seven-flux model for laser light propagation, the calculated light distribution is compared with that obtained from a Monte Carlo computer code that has been validated elsewhere [39]. In this test, a Gaussian laser beam of radius 1.0 mm with a peak intensity of 100 W/cm$^2$ is considered. The thickness of the tissue is 2.0 mm. The optical properties of the tissue used are $\sigma = 10.6$ cm$^{-1}$, $\kappa = 0.4$ cm$^{-1}$, and the anisotropy factor $\gamma = 0.882$. Figure 3 compares the calculated laser fluence rate $\Phi$ [defined as $\Phi = \int_{4\pi} L(\vec{r}, \hat{s}) d\omega$] along the beam center line ($r = 0$). It appears that the present result is in good agreement with
that obtained from the Monte Carlo method [39]. The laser fluence rate distribution over the 2-D axisymmetric domain is presented in Figure 4.

4. RESULTS AND DISCUSSION

Unless otherwise mentioned, the following properties of a biological tissue are used in the numerical analysis: \( \rho = 1000 \text{ kg/m}^3 \), \( k = 0.628 \text{ W/m} \cdot \text{K} \), \( c = 4.187 \text{ J/kg} \cdot \text{K} \), \( \tau = 16 \text{ s} \) for the thermophysical properties [40]; \( \sigma = 10.6 \text{ cm}^{-1} \), \( \kappa = 0.4 \text{ cm}^{-1} \), \( \gamma = 0.9 \) for the optical properties. The thermal physical properties and temperature of blood are [40] \( \rho_b = 1.06 \times 10^3 \text{ kg/m}^3 \), \( c_b = 3.860 \text{ J/kg} \cdot \text{K} \), \( w_b = 1.87 \times 10^{-3} \text{ m}^3/\text{m}^3 \text{ tissue} \cdot \text{s} \), \( T_b = 37^\circ \text{C} \). The metabolic heat generation is \( g_m = 1.19 \times 10^3 \text{ W/m}^3 \). The radius and thickness of the cylindrical tissue are 5 mm, and its initial temperature is 37°C. Grid refinement studies are first carried out. It is found that a mesh of 210 x 210 points is adequate. The time step of \( \Delta t \) is determined from the Courant-Friedrichs-Lewy number of 0.02.

Figure 5 shows the evolutions of temperature \( T \) and damage parameter \( \Omega \) at the center (0, 0) of the irradiated surface of the tissue. In this calculation, the power,
radius \((r_p)\), and exposure time of the laser beam are 5 W, 2 mm, and 2.3 s, respectively. For comparison, the results computed from the PBHT model are also given in Figure 5. When the bioheat non-Fourier effect is considered, as seen in Figure 5a, temperature increases rapidly with time and reaches a maximum value of about 67°C at the time the laser is turned off. After falling to the minimum value, temperature raises again. Such a temperature transient has also been observed in nonbiological hyperbolic heat conduction (e.g., [38]). On the other hand, the PBHT model computes a smaller value of the maximum temperature, about 57.6°C. The steady temperatures obtained from the two models are identical. The fact that the TWMBHT results in higher temperature response than the PBHT model is similar to that observed from the non-Fourier and Fourier heat conduction models for nonbiological materials (e.g., [41]).

It is worth noting that the temperature fluctuation occurring at about 80–120 s in Figure 5a simulated from the TWMBHT is not a fictitious numerical oscillation caused by the TVD algorithm. Rather, it is caused by the superposition of the thermal waves reflected from the boundaries \((r = r_0)\) and \((z = z_0)\), by examining the evolution of temperature distribution over the computational domain. The thermal wave speed is \(c = \sqrt{\alpha / \tau} \approx 0.097 \text{ mm/s}\) for the tissue considered here. For the thermal wave that propagates in the radial direction, it takes about 80 s for the wave to travel...
from the laser spot radius to the boundary $r = r_0$ and then reflect back to the center point of the laser spot. For the thermal wave that propagates in the thickness direction, on the other hand, the total time for the wave to travel from the irradiated surface ($z = 0$) to the boundary $z = z_0$ and then reflect back to the irradiated surface is about 100 s. These are in agreement with the time period when the temperature fluctuates.

Figures 5b and 5c show the thermal damage transients. Apparently, the time histories of the damage parameter $\Omega$ obtained from the two models are very different. The PBHT model predicts no irreversible thermal damage since the maximum value of $\Omega$ is only about 0.61 (less than 1); see Figure 5c. On the contrary, the saturated damage parameter $\Omega$ computed from the TWMBHT is quite high, about 250 (Figure 5b), indicating that the induced thermal damage is very serious. The above results indicate that the tissue damage predicted with the temperature computed with the PBHT model could be conservative. This suggests that the bioheat non-Fourier effect be taken into account in prediction of laser-induced thermal damage. Otherwise, the damage assessment may not be reliable.

The nonhomogeneous inner structures of a biological tissue are essentially due to the existence of vasculature, pigments, pathological changes, etc., in the tissue. In the TWMBHT, those nonhomogeneous inner structures are collectively characterized by a thermal characteristic time $\tau$. To examine the impact of the thermal relaxation effect, the temperature and damage transients are plotted in Figure 6 for different $\tau$ values. Again, the laser beam of power 5 W, spot radius 2 mm, and exposure time 2.3 s is considered. It is evident from Figure 6a that the longer the thermal relaxation time, the higher the temperature rises. Although the change in temperature is not so pronounced, the impact on the damage parameter is quite different. As seen in Figure 6b, with the laser parameters studied here, irreversible thermal damage will occur when the thermal relaxation time $\tau$ is longer than 0.1 s. This, again, reveals that nonhomogeneous structures of a biological tissue could play an important role in determination of thermal damage in tissues induced by laser irradiation.

In the following, the effects of laser exposure time as well as the light absorption and scattering in the tissue are studied. All the laser and material properties used

![Figure 6. Temperature and damage transients for different thermal relaxation times.](image)
in the calculation shown in Figure 5 are employed except for the laser parameter or the optical property whose effect is investigated.

Figures 7a and 7b show the effects of laser exposure time on the temperature and damage at the point (0, 0). As observed from Figure 7b the tissue would be irreversibly damaged ($\Omega \geq 1.0$) when the exposure time is in between 1.5 and 1.6 s. For the PBHT model, the exposure time needed to cause irreversible damage to the tissue is between 2.4 and 2.5 s; see Figure 7c. The predicted laser energy threshold for causing irreversible damage is about 50% higher when the bioheat non-Fourier effect is excluded. A similar conclusion can be drawn for the case in which the laser exposure time is fixed but the power is varied.

Optical properties may vary significantly from one tissue to another [42], thereby resulting in different temperature fields and subsequently causing different thermal damage. Figure 8 shows the effects of light scattering in the tissue on the temperature response and damage parameter at the point (0, 0). As shown in Figure 8, both the temperature and damage parameters increase as the scattering coefficient increases, leading to more severe damage. Figure 8 also shows that the increases in both temperature and damage parameters come more slowly when the scattering coefficient further increases.

Figure 7. Effects of laser exposure time on temperature and damage at point (0, 0).
Distribution of the saturated damage parameter over the analyzed domain is plotted in Figure 9 for two scattering coefficients, $\sigma = 5$ and $10.6 \text{ cm}^{-1}$. Obviously, thermal damage might be more serious in a tissue with a higher scattering coefficient. However, there is no considerable change in the total volume of thermally damaged tissue between the two scattering coefficients.

Figure 8. Effects of scattering coefficient ($\sigma$) on temperature and damage at point (0, 0).

Figure 9. Distribution of damage parameter for (a) $\sigma = 5 \text{ cm}^{-1}$ and (b) $\sigma = 10.6 \text{ cm}^{-1}$.
Figure 10 illustrates the effect of tissue absorption coefficient on temperature and thermal damage at the point (0, 0). It can be seen from Figure 10 that the larger the absorption coefficient, the earlier and steeper is the damage parameter rise and the greater the saturated value. This is attributed to more laser energy being absorbed in the subsurface layer of the tissue due to the smaller optical penetration

Figure 11. Distribution of damage parameter for (a) $\kappa = 0.4 \text{ cm}^{-1}$ and (b) $\kappa = 0.5 \text{ cm}^{-1}$.
depth (the larger absorption coefficient). As a result, the resulting temperature is higher and, in turn, thermal damage becomes more severe in this thin layer. The same trend can also be seen in Figure 11. Unlike the tissue scattering coefficient, the total volume of damaged tissue can be enlarged remarkably, besides the escalation in the degree of tissue damage.

5. CONCLUSIONS

A 2-D axisymmetric thermal wave model of bioheat transfer, together with the seven-flux model for light propagation and the rate process equation for tissue damage, has been formulated for the first time to investigate thermal damage in laser-irradiated biological tissues. The computer code was first verified with comparison of (1) laser light distribution with the Monte Carlo photon transport model and (2) temperature response with the classical parabolic bioheat conduction model. To capture the thermal wave phenomena, the oscillation-free total variation diminishing method, a highly accurate and stable approach, was employed for the numerical analysis. Detailed quantitative comparisons of temperature and damage transients were made between the present results and those obtained from the classical parabolic bioheat conduction model. It was shown that the bioheat non-Fourier effect can be important when the thermal relaxation time of biomaterials is moderately long. Without considering this non-Fourier effect, error and misinterpretation of the predicted temperature and thermal damage may result. A parametric study was also performed for various laser exposure times and tissue optical properties. The predicted laser energy threshold for causing irreversible damage to the tissue studied in this work was about 50\% higher when the bioheat non-Fourier effect was neglected. The degree of thermal damage to tissue increases with increase of either light scattering or absorption coefficient. However, the absorption coefficient has greater impact than the scattering coefficient on the total volume of damaged tissues. Experiments on laser-induced damage in \textit{in vivo} biological tissues are scarce. For further investigations, experiments with \textit{in vivo} tissues are suggested.

REFERENCES


