Flow Patterns and Thermal Drag in a One-Dimensional Inviscid Channel with Heating or Cooling

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In this paper investigations on the flow patterns and the thermal drag phenomenon in one-dimensional inviscid channel flow with heating or cooling are described and discussed; expressions of flow rate ratio and thermal drag coefficient for different flow patterns and its physical mechanism are presented.

Keywords: flow patterns and thermal drag, one-dimensional inviscid channel.

INTRODUCTION

Recently, more attention has been paid to the thermal drag and thermal roundabout flow phenomena in heat transfer community. In order to reveal the physical mechanism of thermal drag, a series of investigations have been made on one-dimensional, inviscid channel flow[1–3]. However, all of these investigations were focused on the fluid flow controlled by back pressure, while little research has been done on fluid flow and thermal drag controlled by sonic pressure. In fact, there will be different flow regimes in a channel with the change of pressure ratio. In order to reveal the physical mechanism of thermal drag, investigation on the thermal drag phenomenon for various flow patterns must be performed systematically.

Therefore, an investigation on the flow patterns for one-dimensional inviscid channel flow with heating and cooling was made. Based on this study, expressions of the flow rate ratio $\xi$ and the thermal drag coefficient $C_t$ in various flow regimes are presented and analyzed.

ANALYSIS OF FLOW REGIMES

1. Mathematical Formulations

A constant cross-section channel with heating or cooling is fed by very large vessel filled with gas at $P_0, T_0, \rho_0$ and then the gas flows into the surroundings with back pressure, $P_2$, as shown in Fig.1. Assume the gas viscosity is negligible. In view of the governing equations of gas dynamics[3], we obtain the relations of gas parameters on section 1 and section 2, and the Mach numbers on the two sections can be calculated as follows:

$$\alpha = \frac{P_0}{P_2} = \frac{(1 + \frac{\gamma - 1}{2} M_1^2) \gamma^{-1} (1 + \gamma M_2^2)}{1 + \gamma M_1^2}$$

(1)

$$= \frac{(1 + \frac{\gamma - 1}{2} M_1^2) M_2^2}{(1 + \gamma M_2^2)^2} (1 + He)$$

(2)

**Fig.1 1-D thermal drag in constant area channel**
Then the flow rate ratio $\xi$ and the thermal drag coefficient $C_t$ can be obtained:

$$\xi = \sqrt{\frac{\frac{\gamma - 1}{2} M_1^2 (1 + \frac{\gamma - 1}{2} M_2^2)}{\alpha \frac{\gamma - 1}{\gamma} (\alpha \frac{\gamma - 1}{\gamma} - 1)}} \cdot \frac{1}{\sqrt{1 + He}}$$

(3)

$$C_t = \frac{2 \left[ 1 + He - \frac{(1 + \gamma M_1^2) (1 + \frac{\gamma - 1}{2} M_2^2)}{(1 + \gamma M_2^2) (1 + \frac{\gamma - 1}{2} M_1^2)} \right]}{(1 + He)}$$

(4)

The above relations are valid for both heating and cooling, and the dimensionless heating number $He < 0$ for cooling.

2. Flow regimes

(1) $He > 0$

It is obvious from the above equations that the flow is subsonic when the channel is heated; if the pressure ratio $P_2/P_0$ is greater than the critical pressure ratio, there will be $M < 1$ at all cross-sections in the channel, and the flow is said to be in a back pressure controlled regime. The flow investigated in [1–3] is just this kind of flow pattern. If the pressure ratio $P_2/P_0$ is less than the critical value, $M_1 < M_2 = 1$, the flow is in the sonic controlled regime. Since $P_2/P_0$, there is an expansion wave at the exit.

Under the control of the sonic state, namely $M_2 = 1$, and substituting it into Eq.(2) we have

$$M_1^2 = \frac{\sqrt{1 + He} - \sqrt{He}}{\sqrt{1 + He} + \gamma \sqrt{He}}$$

(5)

Substituting this result into Eq.(1) gives

$$\alpha = \left[ \frac{\gamma + 1}{2} \frac{\sqrt{1 + He} + \sqrt{He}}{\sqrt{1 + He} + \gamma \sqrt{He}} \right]^{\frac{\gamma - 1}{\gamma}}$$

(6)

In fact, this is just the borderline between the sonic controlled regime and the back pressure controlled regime, as shown in Fig.2, where zone I is the sonic controlled regime, and zone II is the back pressure controlled regime.

(2) $He < 0$

Mach number of the subsonic flow decreases but that of the supersonic flow increases along the channel when the channel is cooled, and there is back pressure controlled regime for subsonic flow. However, if $P_2/P_0$ is less than the critical value, $M_1 = 1$, $P_2 = P_b$, the flow is in the sonic controlled regime. At the same time, there is restriction of $He_{\min} = -0.407$ which is different from heating. So there are two modes for $He > He_{\min}$, namely, the back pressure controlled regime and the sonic controlled regime, depending on whether the back pressure is greater or less than the critical pressure. On the other hand, if $He < He_{\min}$, only the flow under the sonic controlled regime exists.

Under the sonic controlled regime for cooling, $M_1 = 1$, and substituting this into Eq.(2) we obtain:

$$M_2 = \begin{cases} \frac{1 + \sqrt{-He}}{1 - \gamma \sqrt{-He}} \\ \frac{1 - \sqrt{-He}}{1 + \gamma \sqrt{-He}} \end{cases}$$

(7)
Eq.(7) represents the supersonic solution which exists only in the range $-0.51 < He < 0$ and the subsonic solution which exists in the range $-1 < He < 0$.

We can then calculate the corresponding pressure ratios for two solutions:

$$\alpha = \begin{cases} \frac{(\gamma + 1) \frac{-1}{2} \gamma - 1}{1 - \gamma \sqrt{-He}} & \text{for supersonic solution} \\ \frac{(\gamma + 1) \frac{-1}{2} \gamma + 1}{1 + \gamma \sqrt{-He}} & \text{for subsonic solution} \end{cases}$$  \tag{8}

These are the two boundary line equations for cooling. As shown in Fig.2, there are the back pressure controlled regime in zone III and the sonic controlled regime in zone IV where $M_1 = 1, M_2 < 1$, and it is obvious that there is a shock wave in the tube and an expansion wave at the exit. There is another sonic controlled regime in zone V where $M_1 = 1, M_2 < 1$, and an expansion wave exists at the exit but no shock wave exists in the tube.

From the above analysis we know that one-dimensional, inviscid channel flow with heating or cooling can have five flow patterns. That is, the back pressure controlled regime in zones I, III and the sonic controlled regime in zones II, IV, V. So we can obtain the flow patterns as shown in Fig.2.

**FLOW RATE RATIO AND THERMAL DRAG COEFFICIENT**

1. The Back Pressure Controlled Regime (zone I, III)
   
   Generally, $\xi$ and $C_t$ should be calculated by (3) and (4). These two expressions can be simplified for small Mach numbers. The expressions of the flow rate ratio and the thermal drag coefficient for small Mach numbers are given$^{[9]}$:

$$\xi = \frac{1}{\sqrt{1 + 2He}}$$  \tag{9}

$$C_t = \frac{2He}{1 + He}$$  \tag{10}

2. The Sonic Controlled Regime
   
   The sonic controlled regime includes three zones II, IV, V. They are given as follows:

   Zone II ($M_1 < M_2 = 1, P_2 \neq P_b$)
   
   Substituting $M_2 = 1$ and Eq.(5) into Eq.(3), we can obtain the flow rate ratio

$$\xi = \left(\frac{1 + \gamma}{2}\right)^{\gamma - 1} \cdot \frac{1}{\alpha} \cdot \frac{1}{\sqrt{1 + He}}$$  \tag{11}

Substituting $M_2 = 1$ and Eq.(5) into Eq.(4), we can obtain the thermal drag coefficient

$$C_t = \frac{2\sqrt{He}}{\sqrt{1 + He}}$$  \tag{12}

Zone IV ($1 = M_1 < M_2, P_2 \neq P_b$)

Substituting Eq.(8) and Eq.(9) into Eq.(3), we can obtain the flow rate ratio

$$\xi = \frac{\gamma - 1}{(1 + \gamma \sqrt{-He})^2 \left[(\gamma + 1) - 2(1 + \gamma \sqrt{-He})^{\gamma-1}\gamma\right]}$$  \tag{13}

For $He < -0.407, \alpha = P_0/P_2 < 1$, the upstream flow phenomena may exist only when the channel is cooled. But the flow rate ratio will have no physical significance, and therefore, Eq.(13) is valid only in the range $-0.407 < He < 0$.

Substituting $M_1 = 1$ and Eq.(8) into Eq.(4), we can obtain the thermal drag coefficient

$$C_t = \frac{2(He - \sqrt{-He})}{1 + He}$$  \tag{14}

Eq.(14) is valid in the range $-1 < He < 0$.

Zone V ($1 = M_1 < M_2, P_2 \neq P_b$)

Substituting Eq.(7) and Eq.(9) into Eq.(3), we can obtain the flow rate ratio

$$\xi = \frac{\gamma - 1}{(1 - \gamma \sqrt{-He})^2 \left[(\gamma + 1) - 2(1 - \gamma \sqrt{-He})^{\gamma-1}\gamma\right]}$$  \tag{15}

For $He < -0.51, M_2$ has no supersonic solution, and therefore, Eq.(15) is valid in the range $-0.51 < He < 0$.

Substituting $M_1 = 1$ and Eq.(8) into Eq.(4), we can obtain the thermal drag coefficient

$$C_t = \frac{2(He + \sqrt{-He})}{1 + He}$$  \tag{16}

Eq.(16) is valid in the range $-0.51 < He < 0$.

**CONCLUDING REMARKS**

A one-dimensional, inviscid channel flow with heating or cooling has been investigated systematically in this paper. The principle of identifying flow patterns, the expressions of the flow rate ratio and the thermal drag coefficient for different flow patterns are given. Especially, the numerical investigation of the flow rate
ratio and the thermal drag coefficient at the sonic controlled regime may be seen as a supplement to the previous research work on this subject. These results are helpful to reveal the physical mechanism of thermal drag.

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REFERENCES

