Money and internal specialization

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Abstract

In this paper, we demonstrate that money is not necessarily benign in terms of trade patterns and specialization within the household. In this paper, we examine model economies in which households consist of vendor-shopper pairs. There is a distance-related transaction cost between traders. We derive the equilibrium range of goods that will be traded in a symmetric equilibrium under two alternative means of payment: (i) goods for goods and (ii) money for goods. Our key finding is that in the money-for-goods economies, we show that the incidence of the transaction cost matters. Money creates incentives for household members to specialize. This intra-household specialization is identified along the extensive margin of consumption. Shoppers specialize in acquiring a greater variety of goods when they bear the transaction cost. In contrast, in the vendor-pays setting, the cost-minimizing allocation is to trade along the intensive margin for a single consumption good.
1 Introduction

At least as far back as Smith [10], researchers have recognized the potential for money to encourage specialization. In Cole and Stockman [3], for instance, differentiated consumption goods are acquired either through self-production or exchange with other agents. With money, the measure of goods that are self-produced declines, expanding the measure of goods acquired through exchange. Because production costs are convex in the measure of goods produced, the set of available consumption bundles expands with money. In Camera, Reed and Waller [2], money encourages production specialization and results in more efficient consumption. Both Cole and Stockman and Camera, et al. contribute to our understanding of the ways in which money induces production specialization and results in higher welfare: in equilibrium in which money is valued, the production possibility set expands relative to the barter equilibrium.

The purpose of this paper is to examine monetary economies in which specialization extends to intra-household actions and how this specialization can affect welfare. In contrast to the literature mentioned above, in the economies we consider, the pattern of external production specialization—which goods are produced by which households—is predetermined: each household is endowed with one type of good which it wishes to sell in order to consume the goods of other households. At the same time, there is a meaningful division of labor within the household, which we refer to as internal specialization. As in many models following Lucas and Stokey [6], households in our model consist of two members, in our case a shopper and a vendor. We consider two alternative means of payment: goods-for-goods (or barter) and money-for-goods. In economies where goods are the means of payment, the distinction between the two household members is mainly semantic. When money is present, however, the shopper-vendor distinction takes on force. In particular, in economies in which money is the means of payment, we demonstrate that internal specialization can interact with transactions costs and limitations on inter-household communication in a way that affects the equilibrium margin on which households trade. Our
analysis identifies a link between internal specialization and money. Interestingly, limited communication combined with money is embodied in the difference between equilibrium that are distinguished along the intensive margin and the extensive margin for consumption goods.

We start with a fairly standard model economy in which a large number of vendor-shopper households are located at points along a circle. In each period, a household’s vendor remains at the home location and is charged with selling units of the good that are specific to the home location. Meanwhile, the shopper travels along the circle purchasing quantities from other locations. Transaction costs are introduced as a fixed fee, measured in units of households’ endowment goods, that must be paid prior to a transaction taking place.\(^1\) We assume that the cost of transacting for a given vendor and shopper is increasing in the distance between the shopper’s home and the vendor’s location. We consider two cases as regards the transaction cost: one specifies that the shopper pays the transaction cost prior to executing trades with a vendor while the other specifies that the vendor bears the transaction cost before executing trades with a visiting shopper.\(^2\) Combining the two possible transaction cost specifications with the two possible means of payment, we have four distinct environments to consider: two barter economies in which either the shopper or vendor bears the transaction costs and two monetary economies in which either the shopper or vendor bears the transaction costs. Throughout our analysis, we focus on competitive equilibrium.

\(^1\)The monetary literature is dominated by studies in which transactions costs are crucial to explaining fiat money is valued. For instance, Baumol \(1\) and Tobin \(8\) established a “demand” for fiat money based on the presence of transactions costs. In both cases, agents must pay a fixed fee to change from interest-bearing bonds to non-interest-bearing fiat money. Saving \(7\) later developed the shopping-time model in which fiat money is valued because it reduces shopping time, freeing more time for enjoying leisure or producing. See also Schrefl \(9\). In our model, transactions costs are not central to the explanation of why fiat money is valued. Rather, we are comparing two different payment systems.

\(^2\)For our purposes, the transaction costs represent quantities of the endowment good that are used up in the trade process. It is equivalent to think of the transactions costs as goods that are being thrown into the ocean.
Our results are easily summarized. An equilibrium consists of a range of locations over which trade will occur (i.e., a range of goods each household will consume), the quantity of goods traded and the prices that clear the markets. We compare the equilibrium outcomes by pairs: barter exchange under the two assignments of transaction costs and monetary exchange under the two assignments of transaction costs. In the barter economies, the two transaction cost assignments result in identical equilibrium outcomes. In contrast, the equilibrium outcomes obtained in the two monetary economies are different. Specifically, the vendor-pays case is characterized by a monetary equilibrium in which the intensive margin is highlighted. In contrast, the shopper-pays monetary equilibrium is characterized by the extensive margin. We interpret the difference in equilibrium outcomes that emerge between the two monetary economies as being a by-product of the internal specialization. In the monetary economy, a household’s vendor specializes in obtaining money while its shopper specializes in acquiring goods for the household to consume. When the shopper pays the distance-dependent transaction costs, he maximizes household utility by incurring costs in order to purchase goods from a range of locations; when the vendor pays the transaction cost, he maximizes household utility by selling the household’s endowment to the household’s closest neighbors (thus minimizing transaction costs). Which party to an exchange bears the transaction cost thus has a significant effect on the equilibrium range of goods that get consumed.

When the vendor pays the transaction cost in the monetary economy, households face a type of prisoners’ dilemma. In this case, each household would ideally like to sell its endowment only to shoppers from the closest neighboring location, while still shopping at a range of locations. But, this “sell near, buy far” strategy cannot be a symmetric equilibrium—in fact, “selling near” is a dominant strategy for every household, and a symmetric equilibrium has each household consuming only goods from its nearest neighbor. This is not the result when shoppers pay the transaction cost. In that case households balance the costs of more distant transactions against the gains from increased variety. We can show that welfare cannot be higher under the vendor-pays case than under the shopper-pays case. Moreover, the prisoners’
dilemma character of the vendor-pays monetary equilibrium suggests a mechanism that eliminates the inefficiency: some form of “pre-play” communication and commitment.

Our results demonstrate two things. First, money creates specializations that heretofore have gone unnoticed. Even when goods production is already highly specialized, there are additional forms of specialization that emerge when money is introduced. Second, in settings in which the monetary equilibrium is inefficient, one can interpret the inefficiency as being the outcome of a limited communication problem. Thus, monetary economies embed internal specialization and limited communication frictions that can result in inefficiencies that are subtle. With limited communication between household members, internal specialization exists in the sense that the two monetary equilibrium result in different allocations, depending on the whom the burden of the transaction cost is assigned. Interestingly, such communication frictions are characterized by consumption-extensive margin.

The remainder of the paper is organized as follows: Section 2 describes the economic environment. We derive the equilibrium outcomes for the pair of economies—the vendor pays and the shopper pays—in Section 3. Similarly, Section 4 derives the equilibrium outcomes for the two monetary economies. We offer a brief summary in Section 5.

2 The Environment

The model is a modified version of Townsend’s [11] economy in which infinitely-lived households are spatially separated. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). In our case, there are a finite number \( N > 2 \) of locations equally spaced on a circle of circumference \( N \), and each location is occupied by a large number of identical households. Thus, \( N \) is also the number of household types. The households at each location are endowed with units of a nonstorable, location-specific good, so that there are also a \( N \) types of commodities at each date. The physical environment can be interpreted as a group of households, each living at a specific stretch of beach on an
atoll. Trade takes place as agents from each household move around the atoll to visit the locations of other households. Let \( i \in \{0, 1, 2, ..., N - 1\} = \mathcal{N} \) index locations on the atoll; hence, \( i \) indexes both the locations from which household hails and the goods.

We make the standard assumption of bias against consuming the home-location good—i.e., the households at \( i \) derive no utility from consuming good \( i \). On the other hand, we assume that households at each location do derive utility from the goods at all other locations, and that households have identical preferences defined over the full range of goods (modulo the home good). Each household can be thought of as consisting of a ‘vendor-shopper pair’. At the start of each period, the shopper travels to other locations on the circle to purchase goods, either with cash or units of the home good, while the vendor remains at the home location to transact with the shoppers from other households.\(^3\) Travel by the shopper half of the household is restricted to one direction; after visiting as many locations he or she chooses to visit in this direction, the shopper returns home by the same route. For concreteness, assume the locations are arranged in ascending order clockwise around the circle, with \( i = 0 \) at the top, and that the direction of travel by shoppers is also clockwise.

In order for exchange to take place between households at locations \( i \) and \( j \), a direct resource cost must be borne by one of the parties to the trade. More specifically, the cost is paid in units of the endowment good of whichever party is assumed to bear the cost in a particular environment. The cost is independent of the quantity of goods traded and is increasing in the distance—measured along the circumference of the circle—between locations \( i \) and \( j \). In short, the cost is a fixed transaction cost at each location and is increasing in the distance from the shopper’s home. Throughout our analysis, we assume that the transaction cost assignment is never communicated between the vendor and the shopper.

In order to keep the model simple, so as to focus on the role of the transaction

\(^3\)As in the Lucas-Stokey framework, the key feature is that the pair cannot perfectly coordinate their activities to overcome trading frictions. In the monetary economy of section 4 below, the pair is similar as well to the ‘vending machines’ and shoppers of Cole and Stockman [3].
cost, we assume that the number of households of each type is sufficiently large that each household acts as a price-taker at all locations which its shopper visits and with all shoppers visiting the household’s location.

With this interpretation in mind, we proceed to lay out the model in more precise detail.

The structure of preferences is identical across households, and the preferences of each household treat all goods symmetrically. The momentary utility function for each household type $i \in \mathcal{N}$ is represented by

$$U_t = \left[ \sum_{j \in \mathcal{N} \setminus \{i\}} [c_t(j)]^\alpha \right]^{1/\alpha}$$

(1)

where $0 < \alpha < 1$ and $c_t : \mathcal{N} \setminus \{i\} \to \mathbb{R}_+$ is the consumption ‘bundle’ at date $t$. Each household seeks to maximize the discounted sum

$$\sum_{t=0}^\infty \beta^t U_t,$$

where $0 < \beta < 1$. Identical preferences makes the analysis substantially more tractable. For one thing, given the further assumptions we make below on transactions costs, we can conduct our analysis for a representative household without loss of generality.

Each household $i$ is endowed with $e_t(i) > 0$ units of commodity $i$ at each date $t$. The endowment goods are perishable. We will assume that endowment levels are identical across households and across time; that is $e_t(i) = e$ for all $i$ and $t$.

We have not yet developed a specific role for spatial separation. Here, its force derives from the transaction cost’s dependence on distance. We will consider environments where this cost is borne by either the vendor or the shopper in a given transaction. In the ‘shopper pays’ environment, a shopper who travels from the home location to a location $k$ units away—e.g., from location 0 to location $k$—must pay a cost $a(k)$ before trade can take place. In the ‘vendor pays’ environment, the vendor at any location who wishes to trade with a shopper coming from a location $k$ units away—e.g., from location $N-k$ to 0—must incur the cost $a(k)$ before trade can take
A trading range is defined as the range of locations for which the household member will seek to trade. For the shopper-pays case, this means the shopper is willing to pay the transaction cost to trade with the vendor. For the vendor-pays case, the vendor will bear the transaction cost to trade with the visiting shopper. In either environment, these costs are borne out of the payer’s endowment. Thus, for example, in the ‘shopper pays’ environment, if the shopper from location 0 visits locations 1 through \( k \), the household’s endowment net of transactions costs is

\[
e - \sum_{j=1}^{k} a(j).
\]

Note that this also the net endowment of the household at location \( i = 0 \) in the ‘vendor pays’ environment if the household trades with shoppers coming from locations \( N - k \) through \( N - 1 \). We assume that \( a(k) \) is increasing in \( k \) and that there is a \( \hat{k} \) such that \( 1 < \hat{k} < N - 1 \) with \( e > \sum_{i=1}^{k} a(i) \) for \( k \leq \hat{k} \) and \( e < \sum_{i=1}^{k} a(i) \) for \( k > \hat{k} \).

Given our assumptions on the transaction cost function \( a \), there are \( c > 0 \) and \( k > 1 \) such that

\[
e - \sum_{j=1}^{k} a(j) \geq 2kc. \tag{2}
\]

This condition implies that there are gains from trade: a feasible allocation exists in which each household’s shopper visits the first \( k \) locations in the direction of travel from the home location, each household’s vendor trades with visitors from the \( k \) locations lying in the counterclockwise direction, and each household consumes \( c \) units of every good from these \( 2k \) locations. The utility each household receives from this allocation is

\[
[kc^\alpha + kc^\alpha]^{1/\alpha} = [2kc^\alpha]^{1/\alpha} > 0.
\]

The inequality (2) simply says that this is feasible—in any of our environments, the transaction cost associated with these trades is \( \sum_{i=1}^{k} a(i) \) for every household; the inequality then states that the endowments net of this cost are sufficient for \( 2k \) households to each consume \( c \) units of each good.

In this paper, we now investigate whether it matters if the shopper or the vendor pays the transaction costs. The different assumptions are seemingly innocuous in
terms of affecting equilibrium outcomes. In the next section, we present a case in which the equilibrium is identical regardless of whether the shopper or the vendor pays the transactions fee.

3 Using endowment goods as payment

In this section, we consider trading environments in which the vendor and shopper exchange units of their endowment goods—\textit{i.e.}, barter economies. Given symmetric transaction costs and preferences—and preferences which, moreover, treat all goods identically—it is natural to focus on equilibria which are ‘symmetric’. In particular, we construct equilibria in which: all households trade with households from $k$ adjacent locations lying in both directions from the home location; households’ consumption bundles are identical and constant across locations; and all relative prices are unity. Because of the symmetry of preferences and transaction costs, we can discuss the problem from the perspective of a representative household located at $i = 0$, without loss of generality.

We first consider the case where the vendor pays the transaction cost associated with any exchange. When the vendor is responsible for the transaction fee, each household will choose a set of visiting shoppers with whom its vendor is willing to trade—\textit{i.e.}, a set of locations, lying in the counterclockwise direction from the home location, from which the household will accept goods in exchange for units of the home endowment. Each household will also take as given the set of locations, lying in the clockwise direction from the home location, which are ‘open’ to its own shopper—\textit{i.e.}, those locations around the atoll where other households have incurred the fixed cost to trade the goods which the shopper carries from the home location. For the household located at $i = 0$, call the set of locations visited by the shopper $S_t$ and the set of locations from which the vendor accepts visitors $S'_t$. Because the transaction cost increases with distance, and all goods are treated symmetrically in households’ preferences, we may assume without loss of generality that the sets $S_t$ and $S'_t$ are ‘connected’ in the sense that $S_t$ consists of all locations 1 through $k_t$ for some $k_t$, and
$S_t'$ consists of all locations $N - 1$ through $N - k_t'$ for some $k_t'$. Vendors and shoppers will never skip over a location to trade with one that is more distant; rather, they trade incrementally, choosing a set of adjacent locations and balancing the desire to eat each of the $N - 1$ non-home, differentiated goods against the transactions costs.

Let $A(S'_t)$ denote the fixed of trading goods from locations in $S'_t$—i.e.,

$$A(S'_t) = \sum_{i=1}^{k_t'} a(i).$$

Suppose that all relative prices are unity. We will construct a feasible allocation where each household is maximizing its utility given these relative prices. Since the good is nonstorable, and exchange of goods for goods is the only means of trade, each household’s lifetime utility-maximization problem amounts to a static problem of maximizing momentary utility at each date. Because of the transaction cost, taking as given $S_t$ the household will always choose $S'_t$ such that $S_t \cap S'_t = \emptyset$. Moreover, given the form of preferences (1), at unit relative prices the household’s optimal consumption bundle will obey $c_t(i) = c_t(j)$ for all $i, j \in S_t$ and $c_t(i) = c_t(j)$ for all $i, j \in S'_t$.\footnote{In a symmetric equilibrium, three conditions will be satisfied: (i) households will make mutually beneficial trades that satisfy the first-order conditions; (ii) households will take prices as given; and (iii) markets clear; that is, $e(i) - A(S') = \sum c(j)$. The appendix shows that there is a unique equilibrium with a unit relative equilibrium vector.} Let $c_t$ denote the constant level of consumption on the set $S_t$ and $c'_t$ the constant level of consumption on $S'_t$. The household’s budget constraint is

$$e - A(S'_t) \geq c_t |S_t| + c'_t |S'_t|,$$

where $|S_t|$ and $|S'_t|$ are the numbers of locations in the sets $S_t$ and $S'_t$, respectively.

Using this notation, the household’s momentary utility from consuming $c_t$ on $S_t$ and $c'_t$ on $S'_t$ can be written as

$$U_t = \left[ (c_t)^\alpha |S_t| + (c'_t)^\alpha |S'_t| \right]^{1/\alpha}.$$  

Given $S_t$, the household chooses $c_t$, $c'_t$ and $S'_t$ to maximize utility (4) subject to the budget constraint (3).
Because of the fixed cost, this problem is not convex, but may be approached as follows. Taking both $S'_t$ and $S_t$ as given, we can calculate optimal choices of $c_t$ and $c'_t$. This gives rise to an indirect utility function in terms of $S'_t$ and $S_t$, and we can find the optimal choice of $S'_t$ given $S_t$. Finally, we impose symmetry—$S_t = S'_t$—to arrive at a characterization of a symmetric competitive equilibrium.

It is straightforward from the form of (3) and (4) that, given $S_t$ and for a given choice of $S'_t$, the optimal choices of $c_t$ and $c'_t$ must obey

$$c_t = c'_t = \frac{e - A(S_t)}{|S_t| + |S'_t|}$$

—that is, consumption levels on the two sets are equated, and the budget constraint is satisfied with equality. The household’s momentary utility can then be written in terms of $S_t$ and $S'_t$ as

$$U_t = (e - A(S'_t)) \left[ |S_t| + |S'_t| \right]^{\frac{1}{1-\alpha}}. \quad (5)$$

From (5), a household’s utility is increasing in the cardinality of the set $S'_t$ and decreasing in the transaction cost associated with $S'_t$. It follows that if $S'_t$ is an optimal choice, it must have the smallest transaction cost $A(S)$ over all sets $S$ with cardinality $|S'_t|$, from which it becomes clear that the optimal choice does indeed have the form $\{N - 1, N - 2, ..., N - k'_t\}$. If $S_t$ also has the ‘connected’ form $\{1, 2, ..., k_t\}$, then $|S'_t| = k'_t$, $|S_t| = k_t$, and the optimal choice of the integer $k'_t$ solves

$$\max_{k'_t} \left( e - \sum_{i=1}^{k'_t} a(i) \right) \left[ k_t + k'_t \right]^{\frac{1-\alpha}{\alpha}}.$$

This is an integer-programming problem, the solution of which can be characterized by a set of inequalities. For our purposes in this paper, having very tight characterizations of equilibria is inessential for showing how equilibria either differ or do not differ across different environments. It is sufficient to note that a symmetric competitive equilibrium in the current environment, if one exists, is characterized by

$$k_t = \arg\max_h \left( e - \sum_{i=1}^{h} a(i) \right) \left[ k_t + h \right]^{\frac{1-\alpha}{\alpha}}. \quad (6)$$
Equilibrium consumption by each household from each of the $2k_t$ locations is given by
\[ c_t = \frac{e - \sum_{i=1}^{k_t} a(i)}{2k_t}. \] (7)

Note that the number of goods each household consumes is $2k_t$.\(^5\)

Now suppose that it is the shopper who pays the fixed cost associated with any exchange. In this case, the typical household takes as given a set $S_t'$ of shoppers from other locations who will be visiting the home location, and chooses a set $S_t$ of locations which its shopper will visit. Again assume that all relative prices are unity, from which it follows that the household sets $S_t \cap S_t' = \emptyset$ and chooses constant consumption levels $c_t$ and $c_t'$ on the two sets. The budget constraint again takes the form
\[ e - A(S_t) \geq c_t |S_t| + c_t' |S_t'|, \]
where $A(S_t)$ is the sum of the transactions costs which the household incurs from shopping at the locations in $S_t$. By way of comparison with the previous environment, note that the household’s cost of visiting a set $\{1, 2, ..., k_t\}$ of locations in this environment would be identical to its cost of transacting with shoppers from an interval $\{N - 1, N - 1, ..., N - k_t\}$ in the previous environment.

The household’s momentary utility is again given by
\[ U_t = \left[ (c_t)^\alpha |S_t| + (c_t')^\alpha |S_t'| \right]^{1/\alpha}. \]

It’s immediate that we again have $c_t = c_t'$ at an optimum. An argument similar to that above shows that $S_t$ takes the form $\{1, 2, ..., k_t\}$, and that the optimal choice of

\(^5\)Heuristically, one can get a feel for the equilibrium by imagining, for a moment, that there are a continuum of locations, in which case the household’s maximization would give the following first-order condition:
\[ a(k_t) [k_t + a(k_t')] = \frac{1 - \alpha}{\alpha} \left( e - \int_0^{k_t} a(i) di \right) \]
In a symmetric equilibrium $k_t = k_t'$, and the common value $k_t$ would be characterized by
\[ a(k_t) k_t = \frac{1 - \alpha}{2\alpha} \left( e - \int_0^{k_t} a(i) di \right). \]
$k_t$, given $S'_t = \{N - 1, N - 2, ..., N - k'_t\}$, is the solution to

$$\max_{k_t} \left( e - \sum_{i=1}^{k_t} a(i) \right) \left[ k_t + k'_t \right]^{\frac{1-\alpha}{\alpha}}.$$

Consequently, a symmetric equilibrium is again characterized by

$$k_t = \arg\max_{h} \left( e - \sum_{i=1}^{h} a(i) \right) [k_t + h]^{\frac{1-\alpha}{\alpha}},$$  \hspace{1cm} (8)

and

$$c_t = \frac{e - \sum_{i=1}^{k_t} a(i)}{2k_t}.$$  \hspace{1cm} (9)

It is important to verify that indeed we have an equilibrium. We define a symmetric competitive equilibrium as one in which the representative households chooses the range of locations with which to trade and the quantity that maximizes utility represented by equation (1), taking prices and ranges of goods that will trade with the representative home location as given. The following proposition verifies that the equilibrium is indeed one that satisfies the conditions to be an equilibrium.

**Proposition 1** In the barter economy, there exists a symmetric competitive equilibrium in which the location-specific goods trade at a relative price equal to one and the range of locations and the quantity are represented by equations (6) and (7).

See Appendix.

Note that the expressions in (6) and (7) are identical to equations 8) and (9). Thus, the analysis shows that the equilibrium outcomes are identical for the two versions of this barter model economy. More specifically, the representative household maximizes utility in equilibrium by choosing the same consumption bundle—that is, the same level of consumption from each location and the same range of locations with which to trade. Hence, the ‘vendor-pays’ economy is equivalent to the ‘shopper-pays’ economy. This is a general feature of economies in which exchange is a trade of endowment goods for endowment goods.\(^6\)

\(^6\)Suppose that the transactions costs are borne according to the following rule: the seller pays
4 Using fiat money as payment

In this section, we consider an environment in which there is a store of value. Suppose there is a good, call it fiat money, which is an intrinsically useless, noncounterfeitable piece of paper. We assume that there is a constant stock of fiat money in the economy. Trade takes place as before, with shoppers from each household moving clockwise around the atoll. In this economy however, all trades take the form of shoppers offering cash to vendors in exchange for goods. Note that in this environment, a household only consumes goods lying in the shopper’s direction of travel from the home location.

As in exchanges in which the endowment goods are used as payment, we assume that there is a fixed cost that is related to the distance between two potential traders. We consider the same two cases: either the shopper or vendor pays a fixed fee to trade with persons that live \( j \) locations away.

In this economy, the separation of the shopper-vendor pair at the start of each period presents a timing issue. The vendor must offer the home-good for cash while the shopper uses the household’s previously accumulated cash balances to finance the pair’s current-period consumption. At the end of the period, the vendor gives the shopper the proceeds from this period’s sales to finance next-period’s consumption. *De facto* a cash-in-advance condition arises.

Analogous to our notation of the last section, let \( S_t \) and \( S'_t \) denote, respectively, the set of locations to which the shopper will carry cash to exchange for goods and the set of locations from which other households’ shoppers will visit bearing cash to exchange for the home endowment good. If the shopper is responsible for the cost of verifying that the goods received satisfy the conditions for trade, the household chooses the set \( S_t \) of locations to visit, and takes as given the set \( S'_t \) of visitors. In this case, for the household at location zero, trading at a set of locations \( S_t \) incurs a cost of \( A(S_t) = \sum_{i \in S_t} a(i) \), where \( a(i) \) is again increasing in \( i \), with \( \sum_{i=1}^{k} a(i) < \epsilon \) for \( k \theta a(i) \) and the shopper pays \((1 - \theta) a(i)\), for \( 0 \leq \theta \leq 1 \). It is fairly straightforward to show that the results, in terms of range of goods consumed \((s)\) and the quantity of each good consumed \((c)\) would be identical for any \( \theta \).
small, and $\sum_{i=1}^{k} a(i) > e$ for $k$ large. Conversely, if the vendor pays the distance-related fixed cost associated with any potential trading partner, the household chooses the set $S'_t$ of shoppers from whom the household’s vendor will accept cash in exchange for the home good and takes as given the set $S_t$ of markets to which the shopper carries money. In this case, the household would incur a cost $A(S'_t) = \sum_{i \in S'_t} a(i)$, which comes out of the pair’s endowment of the home good. (N.B. While we use the same notation for the transactions costs here as in the barter environment, we do not mean this to imply that the cost of transacting is necessarily the same when using money and when using goods. Rather the notation facilitates comparing equilibrium allocations across the two environments. In principle, the transactions cost function can be treated as being the same in both environments. As such, the transaction cost does not account for different equilibrium allocations between the barter economy and the monetary economy.)

In either case, assuming that the de facto cash-in-advance constraint is binding, the household’s money balances at the start of the next period will be the nominal value of the household’s endowment, less transactions costs.

Again, we will assume that all relative prices are unity and construct a symmetric equilibrium with this feature. The household starts the period with a quantity of real cash balances, denoted by $m_t$. Given the set $S_t$ of markets to which the household carries cash, consumption on that set—which will be uniform given unit relative prices—obeys:

$$c_t |S_t| \leq m_t. \tag{10}$$

This is the household’s cash-in-advance constraint: purchases of current consumption by the shopper must be financed with previously accumulated cash balances. Assuming that (10) binds, the household’s real money balances in the subsequent period are given by either

$$m_{t+1} = \frac{p_t}{p_{t+1}} \left[ e - A(S_t) \right], \tag{11}$$

Arguments similar to those given in the appendix for the barter economy can be used here to show that unit relative prices are the only possibility for a symmetric equilibrium.
depending on whether the household incurs the transaction costs through shopping (11) or vending (12). Here, $p_t$ is the price in units of cash of a unit of the home endowment at date $t$. Given unit relative prices between all goods, this is identical with the price level at date $t$. The endowment net of the transaction cost—$e$ less either $A(S_t)$ or $A(S_t')$—is sold by the vendor to shoppers from locations in $S_t'$ in exchange for cash, yielding either $p_t [e - A(S_t)]$ or $p_t [e - A(S_t')]$ units of currency for the household. The real purchasing power of the household’s currency next period is then either $p_t [e - A(S_t)] / p_{t+1}$ or $p_t [e - A(S_t')] / p_{t+1}$.

Substituting $c_t$ from (10), as an equality, into the household’s momentary utility function (1), gives the following expression for the household’s within-period utility, in terms of $S_t$ and $m_t$:

$$U_t = \left[ m_t^\alpha |S_t|^{1-\alpha} \right]^{1/\alpha}.$$ (13)

We then can cast the household’s lifetime utility-maximization problem as one of the following two dynamic programs, depending on whether we are in the ‘shopper-pays’ or the ‘vendor-pays’ environment:

$$v(m_t; z_t) = \max_{S_t} \left\{ \left[ m_t^\alpha |S_t|^{1-\alpha} \right]^{1/\alpha} + \beta v \left( \frac{p_t}{p_{t+1}} [e - A(S_t)]; z_{t+1} \right) \right\}$$ (14)

or

$$v(m_t; z_t) = \max_{S_t'} \left\{ \left[ m_t^\alpha |S_t|^{1-\alpha} \right]^{1/\alpha} + \beta v \left( \frac{p_t}{p_{t+1}} [e - A(S_t')]; z_{t+1} \right) \right\}$$ (15)

The $z_t$ in these Bellman equations denotes the vector of all exogenous variables which condition the household’s decision at each date, in particular the price level $p_t$. The character of equilibria in the two environments hinges on the very different natures of the solutions to these two problems.

Consider (14) first, which corresponds to the ‘shopper-pays’ environment. Assuming $S_t$ takes the form $S_t = \{1, 2, ..., k_t\}$—i.e., an interval in the direction of travel from the home location to some $k_t$—the Bellman equation becomes

$$v(m_t; z_t) = \max_{k_t} \left[ m_t^\alpha k_t^{1-\alpha} \right]^{1/\alpha} + \beta v \left( \frac{p_t}{p_{t+1}} \left[ e - \sum_{i=1}^{k_t} a(i) \right], z_{t+1} \right).$$
The household’s momentary utility is increasing in the range of locations visited by the shopper, but a greater range of locations comes at the cost of smaller real cash balances for next period. The maximization on the right-hand side of this Bellman equation is an integer-programming problem, as \( k_t \) is restricted to integer values. It would be straightforward to add enough additional structure to fully characterize a solution; however, as in our analysis of the barter economy, having very tight characterizations of equilibria is not important for demonstrating how, in broad terms, equilibria differ across different environments.

Even without explicitly solving this problem, we can draw some conclusions about the character of the solution. The most important feature to note is that there is no explicit dependence of the household’s problem on \( S'_t \), the set of visitors to the home location. This follows from the faceless nature of the household’s monetary transactions—its endowment net of transactions costs is worth \( p_t [e - A (S_t)] \) independent of the identity of the buyers who purchase it. This feature of transactions using money proves to be important for comparing the nature of equilibria in the
'shopper-pays' versus 'vendor-pays' environments.\footnote{As in our analysis of the barter economy, further intuition can be gained by assuming for a moment, that locations are continuous, so that the problem is not integer-constrained. If the value function is differentiable, the first-order condition for the right-hand-side maximization is}

Now, consider (15), the dynamic program which the household faces in the 'vendor-pays' environment. The key differences between the problems described by (14) and (15) are that in the latter, the $|S_t|$ entering the household’s one-period reward—the set of locations which are open to the household’s shopper—is taken as given, while the quantity of real balances the household takes into the subsequent period now depends on the household’s choice of $S_t'$, the set of locations from which the household will accept cash in exchange for the home endowment. That is, the range of goods available to the household’s shopper depends on other households’ decisions as to whether or not to incur the cost of transacting with the shopper, while the household’s vendor makes a similar decision regarding transacting with other households’ shoppers.

If $S_t$ and $S_t'$ are intervals of the form $\{1, \ldots, k_t\}$ and $\{N - k_t', \ldots, N - 1\}$, this

In a steady-state equilibrium, with $m_t$, $k_t$ and $p_t$ constant, these combine to give:

$$a(\ell) = \frac{1 - \alpha}{\alpha \beta} m$$

or

$$a(\ell) = \frac{1 - \alpha}{\alpha \beta} \left[ e - \int_0^\ell a(i) \, di \right],$$

where the last equality follows from the fact that $m = e - \int_0^k a(i) \, di$ when $p_t$, $m_t$ and $k_t$ are all constant. The household’s steady state consumption (per location) is given by $kc = m$, or

$$c = \left( e - \int_0^k a(i) \, di \right) / k.$$
problem can be written as—

\[
v(m_t; z_t) = \max_{k_t} \left\{ \left[ m_t^{\alpha} k_t^{1-\alpha} \right]^{1/\alpha} + \beta v \left( \frac{p_t}{p_{t+1}} \left[ e - \sum_{i=1}^{k_t} a(i) \right]; z_{t+1} \right) \right\}.
\]

This problem has a simple solution—since \( k_t \) is given, and the household’s next-period money balances are decreasing in \( k_t' \), the household chooses the smallest possible set on which to sell its endowment. That is, the household will set \( k_t' = 1 \), or \( S_t' = \{N - 1\} \), offering its endowment in exchange for cash only to shoppers from the nearest adjacent location.

The assumption of price-taking behavior means that the vendor can sell any amount of the home good at \( p_t \) dollars per unit on any \( S_t' \). Given that the verification cost \( A(S_t') \) is increasing in \( S_t' \)—and thus next-period’s real balances are decreasing in \( S_t' \)—the best thing for the household to do is to sell \( e - a(1) \) to the shopper from location \( N - 1 \)—i.e., vend the whole endowment to the shopper from next door.

The preceding problem highlights what it means for fiat money to serve as a generally acceptable medium of exchange. The problem seems to be the combination of having the person who accepts money in exchange for goods being responsible for paying the transaction cost together with the idea of money as generalized purchasing power—i.e., indifference by the household as to the identity (or home goods) of the bearer. In other words, the vendor specializes in acquiring one good—fiat money—that does not directly enter into the household’s utility function. When the shopper pays the transaction cost, utility gained from the variety of goods available outweighs the cost. In contrast, the vendor does not observe any variety of goods in the environment in which the vendor pays the transaction costs. In the absence of acquiring goods that directly enter into the household’s utility function, it is not surprising that the vendor eschews variety, trading with shoppers that minimize the total transaction costs paid by the household.

Note, though, that while specialization arises in the monetary economies, there is no coordination problem within the household. It should be clear from the two cases just considered that vendor and shopper solve a single household maximization problem. In a sense, though, there is a coordination failure across households: each
household would like to consume from a range of locations, while selling to only one location, but this is clearly not possible in equilibrium.

If households take as given the markets in which they can use money—and all relative prices are one, and the household’s preferences treat all these goods identically—then the household takes it real balances at the start of the period and spends them uniformly on this set. Whose money does the household accept? The household accepts money in this economy in order to have it to spend next period. Absent the verification cost $a$—ignoring it for a moment—the household would offer its endowment to anyone with cash. When the cost $a$ is present, if the household can sell $e - \sum_{i \in S'} a(i)$ for $p_t \left[ e - \sum_{i \in S'} a(i) \right]$ on any set $S'_t$, then the household would want to make $S'_t$ a singleton.

By inspection, it is obvious that the equilibrium outcomes for the vendor-pays case are not identical to those in the shopper-pays case when fiat money is present. In short, it matters who pays the fixed costs. In the monetary version of this economy, we have the extreme result that an equilibrium exists when one party to a transaction bears the cost and is different when the other party bears the cost. Moreover, the following proposition compares the welfare outcomes associated with the two monetary economies.

**Proposition 2** In the two monetary economies, $v^s(m_t; z_t) \geq v^v(m_t; z_t)$; that is, welfare under the vendor-pays case cannot be less than welfare under the shopper-pays case.

Let $v^i(m_t; z_t)$ where $i = v, s$ denote the value function computed for the vendor-pays and shopper-pays case, respectively. Proposition 2 simply states that $v^s(m_t; z_t) \geq v^v(m_t; z_t)$. Note that shopper could always choose to consume the good of just the next door neighbor. So, if the shopper-pays case chooses a range of goods such that $k > 1$, it follows that welfare is strictly greater under the shopper-pays case than under the vendor-pays case.

The intuition is straightforward. The cost to the shopper from going to an additional location, call it the $k^{th}$, is twofold. First, there is the marginal utility foregone
from consuming less at each of the \( k - 1 \) locations so that the shopper can acquire some goods at the \( k^{th} \) location. Second, there is the marginal utility foregone because some goods are used up by transaction costs at the \( k^{th} \) location. To offset these two marginal costs, there is the marginal utility associated with quantities from the new location. As long as the marginal benefit exceeds the sum of the two marginal costs, welfare is higher. Hence, if the shopper chooses multiple locations, it follows that total welfare is greater by buying at these locations than if the shopper were to stop after trading with the first location.

What Proposition 2 indicates is that an internal specialization acts in a way similar to a constraint. Here, the most obvious constraint is the lack of a mechanism that will overcome the communication friction associated with internal specialization. In the vendor-pays case, the household’s behavior differs from when the shopper bears the burden of the transaction cost. If there was a mechanism that operated across locations so that any household that accepts a shopper from \( j \) locations would ensure that the home shopper could trade at a location \( j \) distance away, then the welfare achieved under the shopper-pays case could be achieved under the vendor-pays case.

The location-0 vendor takes what other locations do as given. As such, the location-0 vendor’s dominant strategy is to only accept shopper’s from the \( N - 1 \) locations will be willing to trade. It is in the sense that internal specialization is analogous to the prisoner’s dilemma: no location-0 vendor will unilaterally agree to accept shoppers from \( N - j \), for \( j > 1 \), unless the location-0 shopper is guaranteed rights at locations of further from the home location. Money, by itself, can raise welfare compared to the barter economies, but at a cost of introducing internal specialization. Here, the internal specialization adds a communication friction that is embedded in the monetary economy. There is simply no feature of money that can eliminate the communication friction.

To illustrate this point, it is straightforward to show how that there exists a mechanism—symmetric trade rights—that eliminates the inefficiency of the vendor-pays setting with money. The following proposition formalizes this point.
Proposition 3  With a costless intermediary to enforce symmetric trade rights, \( v^V (m_t; z_t) = v^S (m_t; z_t) \).

To prove this point, we begin by describing an environment in which an intermediary can costlessly enforce a welfare improving symmetric equilibrium. Indeed, our characterization will show that the difference between the monetary economies represented by the shopper-pays and vendor-pays cases will amount to showing which of the two yields the highest welfare.

We assume that there is a central island in the middle of the atoll. On that central island there is a large number of agents that can costlessly ensure symmetric trade rights; more concretely, if a location-0 shopper wishes to trade with a vendor that is \( j \) locations away, then the location-0 vendor will agree to trade with the shopper that is \( j \) locations away, where \( j \leq N \). In all of the symmetric equilibria we study, symmetric trade rights are an equilibrium outcome. Here, an intermediary is a mechanism that costlessly enforces symmetric trading rights. Consequently, a representative, say location-0, household takes this mechanism into account when it solves their maximization problem. We assume the intermediaries are Nash competitors. In short, the intermediary will maximize the welfare of the representative household.

The intermediary, therefore, is to examine the two environments—the shopper-pays and vendor-pays cases—while controlling for symmetry.

We consider both cases separately, demonstrating that the two problem are identical under the intermediary framework. To illustrate this point, note that when the shopper pays, the maximization problem for the intermediary agent is

\[
v^S (m_t; z_t) = \max_{k_t} \left[ m_t^{oa} k_t^{1-a} \right]^{1/a} + \beta v \left( \frac{p_t}{pt+1} \left[ e - \sum_{i=1}^{k_t} a (i) \right] ; z_{t+1} \right)
\]

s.t. \( k_t = k'_t \)

As the reader can see, the sole difference between the problem with and without the intermediary is clear; there is an additional constraint which guarantees symmetry in trading ranges. With \( k_t = k'_t \), the intermediary solves the problem under the
constraint that the range of location at which the representative household’s shopper wishes to purchase goods is identical to the range of locations at which the representative household’s vendor will trade with shopper from locations up to \( k \) locations away.

Note that \( k' \) does not appear in the Bellman equation for the shopper-pays case. The implication is that the constraint is costlessly satisfied in the monetary economy in which the shopper-pays the transaction cost. In short, the constraint is irrelevant. In words, it is costless for the household to sell to the same length defined by \( \sum_{0}^{k} \{ \epsilon_{i} - k_{0} \} \leq \sum_{i=1}^{k} a(i) \) where \( k < k' \). Because the shopper bears the burden, the vendor’s action in accumulating fiat money is costless to the representative household. Since the households are otherwise identical across locations, no shopper will trade with the location-0 agent from farther away than \( k \) locations just as no location-0 shopper will trade with a vendor farther than \( k \) locations away. Thus, in equilibria, \( k = k' \) and the choice is the same as it was for the shopper-pays setting without an intermediary.

Next, we examine the intermediary’s problem in the vendor-pays economy. We write down the problem for the vendor-pays case with the same additional constraint that we used in the shopper-pays setting; formally,

\[
v^{v}(m_t; z_t) = \max_{k'_{t}} \left[ m_{t, k}^{\alpha} \left( \frac{p_{t}}{p_{t+1}} \right) \right]^{1/\alpha} + \beta v \left( e - \sum_{i=1}^{k'_{t}} a(i) ; z_{t+1} \right)
\]

\[
s.t. \ k'_{t} = k_{t}
\]

With the equality constraint, it is straightforward to substitute into the Bellman equation. Thus, the problem can either be written as \( v^{v}(m_t; z_t) = \max_{k'_{t}} \left[ m_{t, k}^{\alpha} \left( \frac{p_{t}}{p_{t+1}} \right) \right]^{1/\alpha} + \beta v \left( e - \sum_{i=1}^{k'_{t}} a(i) ; z_{t+1} \right) \) or equivalently as \( v^{v}(m_t; z_t) = \max_{k_{t}} \left[ m_{t, k}^{\alpha} \right]^{1/\alpha} + \beta v \left( e - \sum_{i=1}^{k_{t}} a(i) ; z_{t+1} \right) \). Clearly, the choice will yield the same value of locations that the representative location household will accept or visit. Indeed, as one can see from the latter representation of the unconstrained Bellman equation, the
vendor-pays case will yield the same outcome as the shopper-pays case. Thus, the existence of the intermediary is sufficient to eliminate the difference between the two cases.

4.1 Remarks

The result says that the monetary equilibrium allocation is not invariant to the transaction-cost mechanism. In contrast, the equilibrium allocation is invariant to the transaction-cost mechanism in the barter economy. We refer to the invariance present in the two monetary equilibrium allocations as identifying internal specialization. Internal specialization manifests itself by the consumption allocation along the intensive margin in the vendor-pays case and the consumption along the extensive margin in the shopper-pays case. Clearly, internal specialization differs from the notion of external production specialization evident in the literature dealing with money and specialization. To be clear, internal specialization is associated with the margin upon which the household chooses to consume whereas external specialization is associated with the range of goods that the household will produce.\footnote{Our results share a common feature with Cole and Stockman's results. Consider only the shopper-pays case, and compare the equilibrium allocations in our monetary economy with our barter economy. In both our setting and the Cole-Stockman model, the equilibrium allocation is different in the monetary economy from that in the barter economy. Hence, our model economy exhibits a kind of specialization akin to that in Cole and Stockman, though the definition of the two types of specialization are different.} In our setting, specialization is akin to a form of individualism; that is, there are fundamental differences in the nature of the household’s problem in the monetary economy that are suppressed in the barter economy. In the monetary economy, the vendor-shopper distinction matters because each member contributes a specialized activity to maximizing the household’s welfare function; the shopper specializes in buying goods while the vendor would specialize in acquiring the money balances. Thus, specialization captures the distinct roles that could emerge in the presence of money. In the barter economy, no such distinctive roles exist, both parties contribute to household welfare by acquiring goods. In short, there is no individualism in the barter economy, but...
is permitted in the monetary economy.

One might argue that our result pertains to the failure of a particular type of equilibrium—namely, one in which distance-specific prices are ruled out. While we did exclude distance-specific prices from our analysis, we do not think that it limits the force of our argument. In particular, while money prices which increase with distance might certainly salvage equilibrium in the ‘vendor-pays’ case, such prices could not come about in the ‘shopper-pays’ case. To see why, note that in the ‘shopper-pays’ economy, a household’s vendor takes as given the set of visitors who will be arriving with currency in exchange for the home good, and the vendor has a fixed amount of the home good—namely the household’s endowment less the costs which the household’s shopper incurs—from which to make offers of goods for currency. Unless all visitors offer the same money price for the household’s good, the vendor—whose goal is to maximize next-period’s money balances—will only sell the home good to the visitor offering the highest price. Thus, even if one allows for distance-specific prices, it is clear that the character of the equilibria which result must differ dramatically across our two specifications of who pays the transaction cost.

5 Summary and conclusion

In this paper, we specify a simple general equilibrium model with differentiated goods in which traders face a fixed fee to acquire goods. Transactions costs are distinguished by whether they are paid by the vendor or the shopper. We then ask whether the party responsible for the transaction cost matters in the sense of differences in equilibrium exchange. Our two main results are:

(i) In barter economies—i.e., ones in which goods are exchanged for goods—it does

\footnote{An example of distance-specific pricing would be a simple two-part scheme. One part is the marginal cost of the endowment good while the other part is the marginal cost of the verification fee. The marginal cost of the endowment good is constant, while the marginal cost of the verification fee is increasing in the distance. Hence, each location would be charged a different price, one that is increasing in the distance from the home location.}
not matter who pays the transaction costs in the sense that the equilibrium outcomes are invariant to who bears the burden;

(ii) In monetary economies, it does matter in the sense that households will consume a wider range of the differentiated products when the shopper pays the transaction costs than when the vendor pays the transaction fee.

(iii) A mechanism, in the form of an intermediary that enforces a symmetric trading range, can resolve the differences between the two monetary equilibrium; that is, \( v^s (m_t; z_t) = v^v (m_t; z_t) \).

The differences that emerge in the two monetary economies owe chiefly to two factors. One is the presence of internal specialization. Within the household communication is severely limited so that money induces vendors to specialize in acquiring money while shoppers specialize in acquiring consumption goods. These two forms of internal specialization account for why the equilibrium allocations are different when the vendor bears the transaction cost compared with when the shopper bears the transaction costs. If the vendor-shopper pair were allowed to communicate, the household would choose the allocation that maximizes welfare. Note that internal specialization is muted in the barter economies because vendors and shoppers are acquiring consumption goods. In other words, only the extensive margin is operational in the barter economies whereas both intensive and extensive margins are operational in the monetary economies. Lastly, in all four economies, external specialization if already assumed insofar as households only produce one, location-specific good.

Second, welfare is generally not the same in the two monetary economies. In particular, welfare under the shopper-pays case is an upper bound for welfare under the vendor-pays case. We show that any inefficiency associated with the vendor-pays economy can be eliminated by introducing an intermediary to eliminate the limited communication problem that drives a wedge between the shopper-pays and vendor-pays cases. For instance, consider a household identified at some arbitrary location, call it location 0. If there exists an intermediary who ensures that if a vendor accepted a shopper from \( j > 1 \) locations upstream, then a vendor \( j > 1 \) locations
downstream would accept the location-0 shopper. This intermediary would avoid the costs of internal specialization that adversely affect the household in the monetary economy when the vendor pays the fixed transaction cost. Hence, the intermediary mechanism overcomes the limited communication feature embedded in vendor-pays monetary economy.
References


6 Appendix I

Consider the barter economy in which the shopper pays the transaction cost. For the other cases, the analysis is similar. Let \( p(l, h) \) denote the price of good \( h \) (in units of good \( l \)) paid by a shopper from location \( l \), and \( q(l, h) \) denote the price of good \( h \) (also in units of good \( l \)) paid by a vendor at \( l \). Suppose the household at \( l \) visits locations \( l + 1 \) through \( l + k \) and is visited by shoppers from locations \( l - 1 \) through \( l - k' \). The household at \( l \) then faces the budget constraint

\[
e - \sum_{i=1}^{k} a(i) \geq \sum_{i=1}^{k} p(l, l + i) c(l + i) + \sum_{i=1}^{k'} q(l, l - i) c(l - i) \quad (A.1)
\]

The agent maximizes utility subject to this constraint (for given \( k \) and \( k' \)), which yields the following demand functions

\[
c(l + i) = \left( \frac{1}{\lambda p(l, l + i)} \right)^{\frac{1}{\tau - \alpha}} \quad (A.2)
\]

for all \( i \in \{1, 2, \ldots, k\} \) and

\[
c(l - i) = \left( \frac{1}{\lambda q(l, l - i)} \right)^{\frac{1}{\tau - \alpha}} \quad (A.3)
\]

for all \( i \in \{1, 2, \ldots, k'\} \), where \( \lambda \) is the Lagrange multiplier on the household’s budget constraint. Substituting (A.2) and (A.3) into (A.1), as an equality, gives

\[
e - \sum_{j=1}^{k} a(j) = \left( \frac{1}{\lambda} \right)^{\frac{1}{\tau - \alpha}} \left( \sum_{j=1}^{k} p(l, l + j)^{-\frac{\alpha}{\tau - \alpha}} + \sum_{j=1}^{k'} q(l, l - j)^{-\frac{\alpha}{\tau - \alpha}} \right)
\]

or

\[
\left( \frac{1}{\lambda} \right)^{\frac{1}{\tau - \alpha}} = \frac{e - \sum_{j=1}^{k} a(j)}{\sum_{i=1}^{k} p(l, l + i)^{-\frac{\alpha}{\tau - \alpha}} + \sum_{i=1}^{k'} q(l, l - i)^{-\frac{\alpha}{\tau - \alpha}}}
\]

Thus,

\[
c(l + i) = \frac{p(l, l + i)^{-\frac{1}{\tau - \alpha}} \left( e - \sum_{j=1}^{k} a(j) \right)}{\sum_{j=1}^{k} p(l, l + j)^{-\frac{\alpha}{\tau - \alpha}} + \sum_{j=1}^{k'} q(l, l - j)^{-\frac{\alpha}{\tau - \alpha}}}
\]

and

\[
c(l - i) = \frac{q(l, l - i)^{-\frac{1}{\tau - \alpha}} \left( e - \sum_{j=1}^{k} a(j) \right)}{\sum_{j=1}^{k} p(l, l + j)^{-\frac{\alpha}{\tau - \alpha}} + \sum_{j=1}^{k'} q(l, l - j)^{-\frac{\alpha}{\tau - \alpha}}}
\]
In a symmetric equilibrium, all households choose the same number of locations to visit (so \( k = k' \)), and \( p(l, l + i) \) and \( q(l, l - i) \) depend only on \( i \). Consequently, we suppress the dependence on \( l \) without loss of generality, letting \( p(i) \) and \( q(i) \) denote these relative prices.

Since the endowment at each location (net of the transaction cost) is divided between the \( k \) visitors and the \( k \) locations visited, material balance requires:

\[
e - \sum_{i=1}^{k} a(i) = \frac{\sum_{j=1}^{k} p(j)^{\alpha} q(i)^{1-\alpha} (e - \sum_{j=1}^{k} a(j))}{\sum_{j=1}^{k} p(j)^{\alpha} + \sum_{j=1}^{k} q(j)^{1-\alpha}} + \frac{\sum_{j=1}^{k} p(j)^{\alpha} q(i)^{1-\alpha} (e - \sum_{j=1}^{k} a(j))}{\sum_{j=1}^{k} p(j)^{\alpha} + \sum_{j=1}^{k} q(j)^{1-\alpha}}
\]

or

\[
\sum_{i=1}^{k} p(i)^{\alpha} + \sum_{i=1}^{k} q(i)^{1-\alpha} = \sum_{i=1}^{k} p(i)^{1-\alpha} + \sum_{i=1}^{k} q(i)^{\alpha}.
\]

Next, note that relative prices are related by

\[
q(i) = \frac{1}{p(i)}
\]

—that is, in a symmetric equilibrium, the relative price paid by the vendor for a good brought from \( i \) locations away is the inverse of the relative price paid by a shopper for a good purchased \( i \) locations away. Relative prices for a symmetric equilibrium must then obey

\[
\sum_{i=1}^{k} p(i)^{\alpha} + \sum_{i=1}^{k} p(i)^{1-\alpha} = \sum_{i=1}^{k} p(i)^{1-\alpha} + \sum_{i=1}^{k} p(i)^{\alpha}
\]

which simplifies further to

\[
\sum_{i=1}^{k} \left[ (p(i)^{1-\alpha} + p(i)^{-1}) - (p(i)^{1-\alpha} + p(i)^{\alpha}) \right] = 0
\]

It is then straightforward to show that when \( \alpha \in (0, 1) \), each term \( i \) in this sum is nonnegative for all \( p(i) > 0 \) and strictly positive for \( p(i) \neq 1 \). If, however, \( \alpha > 1 \), then each term \( i \) is nonpositive for \( p(i) > 0 \), and strictly negative for \( p(i) \neq 1 \). In either case, only \( p(i) = 1 \) for all \( i \) satisfies the last equation.

To verify this claim, note that each term in the sum has the form

\[
z + z^{-1} - (z^\alpha + z^{-\alpha}),
\]

30
where $z = p(i)^{\frac{1}{1-\alpha}}$. Since $f(z) = z + z^{-1}$ is convex, we have, for all positive $z$ and $x$,

$$f(z) \geq f(x) + f'(x)(z - x),$$

or

$$z + z^{-1} \geq x + x^{-1} + (1 - x^{-2})(z - x). \quad (A.4)$$

In the case of $\alpha \in (0,1)$, letting $x = z^\alpha$ in (A.4) yields

$$z + z^{-1} \geq (z^\alpha + z^{-\alpha}) + (1 - z^{-2\alpha})(z - z^\alpha). \quad (A.5)$$

If $z > 1$, then $1 - z^{-2\alpha}$ and $z - z^\alpha$ are both positive. Alternatively, if $z < 1$, $1 - z^{-2\alpha}$ and $z - z^\alpha$ are both negative. Hence, the product on the right-hand side of (A.5) obeys $(1 - z^{-2\alpha})(z - z^\alpha) > 0$ for $z \neq 1$. Thus, inequality (A.5) implies that $z + z^{-1} > z^\alpha + z^{-\alpha}$ for $z \neq 1$.

Next, consider the case in which $\alpha > 1$. Substituting $z = x^\alpha$ into inequality (A.4) yields

$$x^\alpha + x^{-\alpha} \geq x + x^{-1} + (1 - x^{-2})(x^\alpha - x)$$

or

$$-(1 - x^{-2})(x^\alpha - x) \geq x + x^{-1} - (x^\alpha + x^{-\alpha}). \quad (A.6)$$

With $\alpha > 1$, $x^\alpha - x$ and $1 - x^{-2}$ are both either strictly positive (if $x > 1$) or strictly negative (if $x < 1$). Thus, the left-hand side of inequality (A.6) is less than zero for any $x \neq 1$, implying that $x + x^{-1} < (x^\alpha + x^{-\alpha})$ for all $x \neq 1$.

Finally, for any $\alpha > 0$, it is easy to show that inequalities (A.5) and (A.6) hold as equalities when $z = x = 1$. Thus, the material balance condition for a symmetric equilibrium is satisfied if and only if the list of relative prices has $p(i) = 1$ for all $i$. 