Intermediate Macroeconomics  
Economics 4353/7353  
Summer 2011  

Handout 7  
Chapter 4: Consumption, Saving, and Investment

I. Two reasons for macroeconomists to study investment  
A. Investment is the most volatile component of GDP over the business cycle. Although it amounts to only about 14% – 16% of GDP, the decline in investment spending typically constitutes one-half or more of the overall decline in spending in a recession. Hence, to understand business cycles, one must understand the behavior of investment.  
B. Capital accumulation is crucial to economic growth, and it is the flow of investment over time that determines capital accumulation.  
   Note: Recall our brief discussion, and the discussion in the book, regarding the fact that a portion of government spending consists of investment, (at least some of it referred to as “infrastructure”). Here, unless otherwise stated, we ignore investment on the part of government.

II. We proceed in two steps in studying investment  
A. We look at the determination of a firm’s desired capital stock.  
B. Then, given a firm’s desired capital stock, we look at what determines investment.

III. The desired capital stock  
A. Desired Capital Stock: The amount of capital that allows a firm to earn the largest expected profit.  
   B. Determination of the capital stock that maximizes expected profit turns on a comparison of expected costs and expected benefits of using additional capital. Broadly speaking, this comparison and resulting decisions are analogous to those in the context of the determination of the optimal quantity of labor, (see Chapter 3).  
      1. Because current investment usually becomes capital stock with a lag, (e.g. installation lags), the relevant measure of expected benefit is expected future benefit. This is given by the expected future marginal physical product of capital, $MPK_f$. (Ignore the superscript if in a particular case, there is no lag.) Here, $MPK$ and $MPK_f$ are measured holding employment, the state of technology as represented by the parameter $A$, and other factors of production fixed.  
      2. User Cost of Capital: The expected real cost of using an additional unit of capital for a specified period of time. Construction of an appropriate measure of the user cost of capital can be very complex. However, in our approach, we shall focus on just two components, and we shall treat these in a very simple fashion. The two components are: depreciation and interest.  
         (a) Depreciation: The depreciation cost of using capital is the value lost as the capital wears out.  
         (b) Interest: The interest cost of using capital equals the real interest rate times the price of capital. This component of user cost may be either a direct cost, (i.e. the interest cost of funds borrowed to purchase the capital), or, if a firm purchases the capital with internal funds, an opportunity cost, (i.e. the interest income that would have been earned if the funds had instead been used to buy interest-earning assets).
Let \( p_K \) be the real price per unit of capital, let \( d \) be the rate of depreciation per period, and let \( r \) be the real interest rate. Then the user cost of capital is given by:

\[
uc = rp_K + dp_K = (r + d)p_K.
\]

Notes: (i) For simplicity, we often set the price per unit of capital equal to one: \( p_K = 1 \). The price per unit of capital is measured in real terms, that is, the same terms in which \( MPK^f \) is measured. (ii) An important simplification here is that we are ignoring depreciation due to economic obsolescence, something that might be reflected in either the current real price of a particular type of capital or in the depreciation rate deemed relevant. (See footnote 9, p.35.)

3. The desired capital stock is the level of capital stock such that:

Example: Determining the desired capital stock. In this example, we focus on the quantity of forklifts in a very large lumber warehouse (equipped with robots) as the size of the capital stock.

Suppose that the real cost per forklift is 100 units of real output, that the rate of depreciation is 5\%, and that the real interest rate is 4\%. Suppose that the relationship of the size of the capital stock and its (expected) future marginal physical product, (measured somehow, but in terms of real output), are related as in the following table:

<table>
<thead>
<tr>
<th>Number of forklifts</th>
<th>Expected future marginal physical product of capital</th>
<th>User cost Of capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

What is the user cost of capital? What is the optimal size of the capital stock, (optimal number of forklifts)? What property is exhibited by the column of numbers for the expected future marginal physical product of capital? Sketch a graph.

C. Changes in the desired capital stock

1. Factors that shift the \( MPK^f \) curve or that change the user cost of capital change the level of the desired capital stock.

2. Changes in the state of technology change the expected marginal physical product of capital. If there is technological progress such that the expected marginal physical product of capital increases, then the \( MPK^f \) curve would shift up, and the desired level of capital would increase. For instance, in the case in the example, suppose that there is an improvement in the technology of the robots that help to sort the wood on the shelves, in readiness for the forklifts to pick it up. This might increase the expected \( MPK^f \) of an additional forklift, for any given number of forklifts, so that the \( MPK^f \) curve would shift up, increasing the desired number of forklifts to use. Sketch the graph.
3. Changes in the real interest rate, the depreciation rate, or the price of capital change the user cost of capital. For example, if the (real) purchase price of capital should fall, the user cost line would shift down, and the desired level of the capital stock would increase. Sketch the graph.

D. The Tax-adjusted User Cost of Capital: The cost of capital divided by \((1 - \tau)\), where \(\tau\) is the effective tax rate. The effective tax rate is defined to be a single tax rate that provides an adjustment such that the result is equivalent to what emerges under the provisions of the actual tax code, (involving depreciation allowances, etc.). This leads to the revised optimality condition for the desired capital stock:

\[
MPK^f = \frac{uc}{(1 - \tau)} = \frac{(r + d)p_K}{(1 - \tau)}
\]

IV. From the desired capital stock to investment

A. Gross Investment: The actual purchase of new capital goods in a given period.

B. Net Investment: The change in the capital stock over a given period, or equivalently, the difference between gross investment and depreciation in a given period.

C. Let \(I_t\) be gross investment in period \(t\), \(K_t\) the capital stock at the beginning of period \(t\), and \(K_{t+1}\) the capital stock at the beginning of period \((t + 1)\). Then net investment is given by:

\[
K_{t+1} - K_t = I_t - dK_t,
\]

and gross investment satisfies:

\[
I_t = K_{t+1} - K_t + dK_t = K_{t+1} - (1 - d)K_t.
\]

For our purposes, we shall assume that gross investment is always nonnegative. However, net investment can be negative, that is, the quantity of capital that a firm has can decrease. At the aggregate level however, for about the past sixty-five years anyway, for the U.S. and most other of the currently-wealthy countries in the world, the case of a decreasing aggregate capital stock has not been relevant. See Figure 4.6, p.129.

Suppose that in the aggregate, the desired capital stock at the beginning of period \((t + 1)\) is given by \(K^*\). Then in the aggregate, desired gross investment is given by:

\[
I_t^d = K^* - K_t + dK_t.
\]

As stressed in the text, installment lags may be such that a firm may not always be able to attain its desired capital stock in one period. However, what is most important for our purposes is that the determinants of \(K^*\) are also the determinants of current gross investment \(I_t\).

V. Goods market equilibrium

A. The condition for goods market equilibrium can be written in two equivalent forms:
\[ Y = C^d + I^d + G \]
\[ \text{or} \quad S^d = I^d. \]

B. The real interest rate adjusts so as to bring the goods market into equilibrium.

*Example 1.* Suppose that full-employment GDP is given by \( \bar{Y} = 6000 \), government purchases by \( G = 1200 \), desired consumption by \( C^d = 3600 - 2000r + 0.1Y \), and desired investment by \( I^d = 1200 - 4000r \). Making use of the first equilibrium condition above, find the equilibrium real interest rate. What are the equilibrium levels of desired consumption, desired national saving, and desired investment?

Repeat the exercise making use of the second form of the equilibrium condition.

C. Shifts of the (desired) national saving curve and shifts of the (desired) investment curve

1. The saving curve shifts if there is a change in current (full-employment) output, a change in expected future output (that is, expected future income), a change in wealth, a change in current government spending, or a change in current (lump-sum) taxes. (The last does not lead to a shift if Ricardian equivalence holds.)

2. The investment curve shifts if there is a change in the effective tax rate, a change in the depreciation rate, or a shift in the curve representing expected marginal physical product of capital.

*Example 2.* Suppose that full-employment GDP and the levels of government purchases, taxes and transfers are fixed. Suppose that \( MPK^f \) increases at every level of \( K \). That is, the \( MPK^f \) curve shifts upward. What is the impact of this on goods market equilibrium?

*Example 3.* Suppose that full-employment GDP is given by \( Y = \bar{Y} \) and that the level of government purchases, taxes and transfers are fixed. Suppose that there is an increase in household expectations of future income. For simplicity, assume that this has no impact on labor supply, no impact on current \( MPK \), and no impact on \( MPK^f \). What is the impact of this on goods market equilibrium?