1. Although one could argue that the major focus in section 3.1 is on the marginal product of capital, (pp.62-64), there is also some focus on the marginal product of labor, (pp.64-66). In any event, everything is strictly analogous for the case of labor. For the graphs, see Handout 4, p.2, for the case of capital, but again, the case of labor is strictly analogous. That said, do note the content of footnote 5, p.65.

2.(a) Given that the exponents on capital and labor add to 1, (0.3 + 0.7 = 1), it is easy to see that total factor productivity did not grow in this case.
   (b) For this case, \( A = 1 \) in 1990, and \( A = Y / (K^{0.3} N^{0.7}) = 300 / [(200)^{0.3} (200)^{0.7}] = 1.5 \) in 2000, so that total factor productivity would have grown by 50%.

3.(a) The labor demand curve is determined by the function: \( ND = (22.5)^2 (w)^{-2} \). Setting labor demand equal to labor supply, with the tax rate set equal to zero, yields an equilibrium wage of \( w = 1.5 \). Equilibrium employment is then given by \( N = (22.5)^2 (1.5)^{-2} = 225 \), and hence full-employment output is \( Y = 675 \). With the tax rate equal to zero, after-tax wage income is given by \( (1.5)(225) = 337.5 \).
   (b) Labor supply is now given by \( NS = 16w^2 \), while labor demand is as in part (a), so that the equilibrium real wage is now \( w = 2.37 \), and equilibrium employment is \( N = 90 \). Hence, full-employment output is given by \( Y = 426.91 \). Finally, after-tax labor income is given by: \( (0.4)(2.37)(90) = 85.32 \). Note that there is a big decline in employment, after-tax wage income, and output, although the (before-tax) real wage = MPN is higher.

4. (a) Consumption in the working, (that is, first), period of life is equal to $1,000,000 – \$S\), where \$S\ represents saving in the first period of life. Consumption in the first period of life is equal to twice that in the retirement, (that is, second), period of life. Given that statement of the problem, consumption in the second period of life is equal to \( (1.2)S\). Hence, one has the equation: \$1,000,000 – S = 2(1.2)S. This yields the result that \$S\ = \$294,118. It follows that consumption in the first period of life is \$705,882, while that in the second period of life is \$352,941. (I have adjusted the numbers slightly to remove the very slight discrepancy that would result from rounding.)
   (b) In this problem, the income effect dominates the substitution effect. That is, when the real interest rate increases, the quantity of saving decreases. One can gain some intuition for this result as follows.
   The general algebraic formula that is relevant is:

\[
\frac{1,000,000 - S}{2} = (1 + r)S.
\]
That is, there is a “target” relationship to be met between consumption in the first period of life and consumption in the second period of life, the vehicle for meeting this target is saving during the first period of life, and, given the parameters of the problem, there is only a limited manner in which that vehicle can be used to meet the target. All of this is summarized in the equation above. The nature of this target is essentially that of an “income target.” It can be met with a lower quantity of saving, leaving more for consumption in both periods of life, if the real interest rate increases. Hence, the income effect dominates.

5. The tax-adjusted user cost is equal to:

\[
\frac{(r + d)p_K}{(1 - \tau)} = \frac{[(.03 + .10)(10,000)]}{(1 - 0.05)} = 1368.42.
\]

6. (a) Since the price of capital (in real terms, that is, in units of output) declines from 60 to 51 in one year, one can take the depreciation rate to be \(d = (9/60) = 0.15\). Hence:

\[
u_c = (0.1 + 0.15)(60) = 15 \text{ (units of output) per year)}.
\]

(b) The general condition is: \(MPK^l = uc\). In the present context, this becomes: \(165 - 2K = 15\), or \(K^* = 75\).

(c) The tax-adjusted user cost of capital is: \(uc/(1 - \tau) = 15 / 0.6 = 25\). Hence, the desired capital stock, which is lower than that in part (b), due to the tax, is given by: \(165 - 2K = 25\), or \(K^* = 70\).

7. Given the values for \(G\) and \(Y\), \(S^d = 500r + 30\), while \(I^d = 120 - 400r\), so that national saving = investment implies that the equilibrium interest rate is \(r = 0.1\). It follows that equilibrium investment and equilibrium national saving are equal to 80.

Given the values of government spending and full-employment output, one then has:

\[
S^d = 80 = Y - C^d - G = 500 - C^d - 100\text{, which implies that } C^d = 320.
\]

Turning to the last part of the question, this represents a good example where the material on pp.36-38 is useful within the context of our theoretical model, as well as in the context of national income accounting.

8. (a) A temporary increase in government spending for military purposes.

First Case: Suppose that the increase in \(G\) is tax-financed.

**Step 1.** This implies that the current disposable income of households decreases at any given real interest rate.

**Step 2.** Therefore, at any given real interest rate, desired consumption falls, but not by the full amount of the drop in disposable income. The smoothing motive is at work here. The mpc is less than one.

**Step 3.** Therefore, the national saving curve shifts leftward. This can be deduced in either of two ways. The first alternative is to make use of the equation: \(S^d = Y - C^d - G\). Since desired consumption does not fall by as much as government spending increases, desired national saving decreases at any given real interest rate. The second alternative is simply to look at private saving. Private-sector disposable income decreases, but the decrease in desired consumption is not as great as the decrease in disposable income, so that the quantity desired of private saving falls as well, at any given real interest rate.
Given that there is no change in government saving, desired quantity of national saving decreases at any given real interest rate.

**Step 4.** Either way, the national saving curve shifts leftward. Given that the investment curve does not shift, this implies that in the new equilibrium, the real interest rate is higher, and the quantity of investment is lower. Hence, (private-sector) investment is “crowded out.” Note that consumption is also crowded out in this case, (the decrease in current disposable income together with the increase in the equilibrium real interest rate imply this).

Second Case: Suppose that the increase in $G$ is fully deficit-financed.

**Step 1.** Maintaining the assumption that there is no change in households’ expectations regarding future incomes; and since the issue of Treasury securities to finance the increase in government spending has no impact on current private-sector disposable income; it follows that at any given real interest rate, there is no change in desired private saving and no change in desired consumption.

**Step 2.** However, there is a change in public saving, that is, a decrease in $T – G$. Hence, the national saving curve shifts leftward. Given that the desired private-sector investment curve does not shift, this implies that the real interest rate is higher and private-sector investment lower in the new equilibrium. Hence, once again, private-sector investment is crowded out. Given that disposable income is unchanged and the real interest rate higher, consumption is also lower in the new equilibrium.

(b) A temporary increase in government spending for infrastructure.

Note: When going over the results in this part of the problem, keep in mind the three strong maintained assumptions (especially the second and third ones) listed in the statement of the question.

First Case: Suppose that the increase in government spending is tax-financed.

**Step 1.** As stated in the question, the spending on infrastructure increases $MPK^f$. This causes the investment curve to shift rightward.

**Step 2.** However, the temporary increase in government spending implies a leftward shift in the national saving curve. The reasoning is exactly the same as in Steps 1-3 under part (a), First Case, above.

**Step 3.** The result is that there is a higher real interest rate in the new equilibrium. However, absent further analysis, (see Step 4), one cannot say whether the quantity of national saving = quantity of private investment is higher or lower in the new equilibrium (sketch the graph). Also, therefore, one cannot say whether there is crowding out of private investment.

**Step 4.** However, again from Steps 1-3 under part (a), First Case above, one can say that consumption is lower in the new equilibrium, because current disposable income is lower and the real interest rate is higher. Thus, one does have the result that consumption is crowded out.
If the effect of the real interest rate on consumption is sufficiently strong, then it could be that the drop in equilibrium consumption is greater than the drop in disposable income, in which case quantity of national saving = quantity of private-sector investment would be unambiguously higher in the new equilibrium. However, in keeping with the treatment in class, one presumes that the impact of the real interest rate on consumption (and private saving) is not that strong, so that one presumes that the drop in consumption is less than that in disposable income, and hence that private-sector investment is, in fact, crowded out.

Second Case: Suppose that the increase in government spending is deficit-financed.

   **Step 1.** As in part (a), Second Case, Step 1, there is no change in the quantity desired of private-sector saving at any given real interest rate, since there is no change in disposable income. However, as in part (a), Second Case, Step 2, there is a leftward shift in the national saving curve. This, along with the rightward shift in the investment curve, results in the same conclusions as under part (b), First Case, Step 3 just above: the real interest rate is higher in the new equilibrium; but one cannot determine whether quantity of national saving = quantity of private-sector investment are higher or lower in the new equilibrium (see Step 2 just below for further discussion).

   **Step 2.** Consumption spending is crowded out, given no change in disposable income and the higher real interest rate in the new equilibrium. At the same time, and for the same reasons, quantity of private-sector saving is higher in the new equilibrium, offsetting somewhat the fall in the quantity of government saving. Maintaining the presumption that this interest-rate effect on the quantity desired of private-sector saving and the quantity desired of consumption is “not too big”, then the quantity of national saving and (current private-sector) investment are lower in the new equilibrium, so that current private-sector investment is crowded out, despite the fact of the initial shift in the investment curve (work through this result). However, if we treat the size of the interest-rate effect on consumption and private-sector saving as of ambiguous magnitude, then, as concluded in part (b), Second Case, Step 1 just above, one cannot determine whether national saving and investment are higher or lower in the new equilibrium.